Letter

Compiling arbitrary single-qubit gates via the phase shifts of microwave pulses

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We give an arbitrary single-qubit gate compilation scheme on superconducting processors that takes advantage of tuning the phase shift of microwave pulses to obtain a continuous gate set. This scheme is compatible with any two-qubit gate, and we only need to calibrate the X_π and $X_{\pi/2}$ pulses. We report an experimental implementation on a fluxonium device in [Phys. Rev. Lett. **130**, 070601 (2023)] and obtain state-of-the-art fidelities. We give two other schemes: the first requires one X_π pulse and one pulse with a variable rotation angle, and the second requires four $X_{\pi/2}$ pulses. We also find that if we can do virtual Z gates, then we can also do virtual gates around any axis. Our results apply to any physical platform that natively supports virtual Z.

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I. INTRODUCTION

The conventional way to realize universal quantum computing is to compile a quantum circuit using arbitrary single-qubit gates, that is, all of $SU(2)$, and a fixed two-qubit gate, such as CNOT. Hence, arbitrary single-qubit gates are ultimately necessary to run quantum algorithms. Furthermore, arbitrary single-qubit gates can help alleviate two-qubit gate errors [1] and are necessary for certain benchmarking schemes [2,3]. However, it is a nontrivial task to realize arbitrary single-qubit gates on an experimental device. One way to realize arbitrary single-qubit gates is via approximate compilation using a finite universal single-qubit gate set, such as $\{H, S, T\}$ [4]. However, the gate fidelities on current experimental devices are limited, with decoherence still being a significant issue. This approach would therefore lead to unacceptably low fidelities.

Another way to realize arbitrary single-qubit gates is via exact compilation using continuous gate sets, such as the Euler angle decomposition:

$$Z_\theta X_\sigma Z_\psi. \tag{1}$$

The problem with this, however, is that we need to physically implement infinitely many single-qubit gates. This is difficult because the physical control parameters on an experimental device can have complicated relationships with the parameters of the abstract unitary gate being implemented. For example, for a superconducting qubit, which is the physical platform we will focus on, single-qubit gates are usually implemented via a microwave pulse sent on a drive line capacitively coupled to the qubit. For illustration, the circuit diagram for a Cooper pair box qubit is shown in Fig. 1. This pulse implements an X_σ gate, where the rotation angle σ is determined by the amplitude and duration of the pulse. However, there are complicating factors: To implement the desired rotation angle to high precision we would also need to take into account the pulse shape, as well as pulse distortion at short time scales, and even complex processes such as leakage and phase errors induced by the AC-Stark shift [5]. In experiments, it is therefore necessary to calibrate the gate parameters, such as duration, amplitude, and detuning, for every rotation angle we want to implement (see for instance [6]).

There is a known way to accurately implement a continuous gate set without an infinite calibration overhead for superconducting qubits. It is wellknown that in the rotating frame, by adding a phase shift ϕ to the microwave pulse implementing an X_σ gate, in the frame rotating with the qubit we instead obtain the gate [7]

$$X_\sigma(\phi) \equiv Z_{-\phi} X_\sigma Z_\phi. \tag{2}$$

We call σ the rotation angle of $X_\sigma(\phi)$, and below we will often fix σ and take ϕ to be a free variable. This is very exciting observation; by calibrating an X_σ gate, we get a continuous gate set $\{X_\sigma(\phi)\}_\phi$ by tuning ϕ. Furthermore, the correspondence between the phase shift ϕ and the conjugation angle ϕ in $Z_{-\phi} X_\sigma Z_\phi$ is not complicated by other factors. That is, to high accuracy, the phase shift ϕ is equal to the conjugation angle ϕ [8]. Furthermore, the phase shift ϕ can made be very precise experimentally [8] using a global frequency reference such as an atomic clock [9]. We will refer to this trick as phase-shifted microwave (PMW) pulses.

We can take advantage of this continuous gate set to compile arbitrary single-qubit gates. This was already done in [8];

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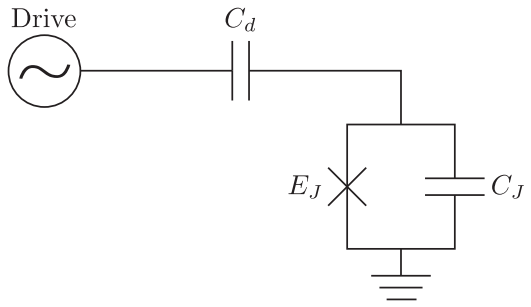


FIG. 1. A microwave drive line capacitively coupled to a Cooper pair box.

arbitrary single-qubit gates are compiled using

$$Z_\theta X_{\pi/2} Z_\psi X_{\pi/2} Z_\omega. \tag{3}$$

This can be compiled via two $X_\sigma(\phi)$ rotations, save for an extraneous Z rotation $Z_{-(\theta+\psi+\omega)}$. We will refer to this as the *virtual Z scheme*, as the rightmost Z_ϕ in Eq. (2) is effectively used as a Z rotation. The extraneous Z rotation can be taken care of with classical post-processing that tracks all the phases in a procedure known as *phase carrying*, which usually assumes that the two-qubit gate used has a special matrix structure such that the extraneous phases can be carried across it. We call such a gate a *phase carrier*. For example, for an iSWAP gate, the two extraneous phases can be carried across the gate, but the phases will be exchanged between the two qubits. Another common phase carrier is the CZ gate. For a detailed explanation of phase carrying as well as a mathematical characterization of these special two-qubit gates and their local equivalence classes, see Sec. I of the Supplemental Material [10].

The assumption that the two-qubit gate being used is a phase carrier does not always hold. Although microwave-activated gates can get around this, the usual approach for realizing gates such as SQiSW $\equiv \sqrt{\text{iSWAP}}$ or other general iSWAP family gates via flux tuning is explicitly incompatible with phase carrying. Note that this incompatibility is also with the use of the virtual Z gate to realize arbitrary Z rotations as corrections to two-qubit gates [7,8,11,12]. Other examples of incompatible two-qubit gate schemes include the dynamical decoupled two-qubit gate (we refer to this as DDCZ) in [13] and gates realized through tunable couplers, such as general fSim family [14] and bSWAP family [15] gates. Furthermore, new gate schemes are still being being developed, and compatibility with the virtual Z scheme can be a quite restrictive requirement.

In this Letter we propose compilation schemes that still take advantage of using PMW pulses to exactly compile arbitrary single-qubit gates but without incurring extraneous Z rotations [16]. The resulting compilation schemes are compatible with any two-qubit gate. In particular, one of the schemes requires just three $X_\sigma(\phi)$ gates with fixed rotation angles. This is the minimum necessary since $SU(2)$ has three real dimensions. It requires only the calibration of X_π and $X_{\pi/2}$ pulses, which are already necessary for Clifford-based randomized benchmarking and the measurement of T_1 and T_2 coherence times. This scheme was experimentally realized in a separate work where it was used as a single-qubit compilation scheme compatible with the SQiSW gate [2]. Benchmarking results indicate that state-of-the-art single-qubit gate fidelities can be achieved. We additionally provide other variants of this scheme, one that requires two $X_\sigma(\phi)$ gates, but one of which with a variable rotation angle. The other scheme requires four $X_\sigma(\phi)$ gates but with the benefit that only the $X_{\pi/2}$ pulse needs to be calibrated. Lastly, we prove that the ability to do virtual Z implies we can do single-qubit virtual rotations *around any axis*, not just Z. We give compilation schemes based on this fact that are not only compatible with phase carriers but also with gates locally equivalent to one, such as CNOT, the cross-resonance (CR) gate [17], or even the Mølmer-Sørensen (MS) gate [18] on trapped-ion systems, all of which are locally equivalent to CZ. Note that since we treat everything at an abstract level, our compilation schemes can be used on any quantum computing platform which natively supports $X_\sigma(\phi)$ gates. This includes trapped-ion systems [19,20], quantum dots [21], and nitrogen-vacancy centers [22].

TABLE I. List of the compilation schemes in this Letter. In the "Pulses" column, the different letters stand for fixed (F) rotation angle and variable (V) rotation angle. The "Example gates" column lists native two-qubit gates for which the scheme would be suitable for.

Scheme	Pulses	2Q compatibility	Example gates
vZ-1	1V	Phase carrier	CPHASE(θ)
vZ-2	2F		iSWAP, bSWAP
PMW-3	3F	All-compatible	SQiSW, iSWAP(θ)
PMW-4	4F		bSWAP(θ)
PMW-2	1V+1F		fSim(θ, ϕ)
vR-1	1V	Leaky gate	CNOT, CR
vR-2	2F		DDCZ, MS

We summarize all single-qubit gate compilation schemes that use PMW pulses in Table I, along with the pulses needed for each and their two-qubit gate compatibilities. We also list examples of native two-qubit gates for which these compilation schemes would be suitable for. Note this only takes into account the matrix form and not the physical implementation. Other than the vZ schemes [8], all compilation schemes are elaborated on in Sec. II. We conclude in Sec. III with a discussion.

II. PMW COMPILATION SCHEMES

In this section we give the different all-compatible compilation schemes based on PMW pulses as well as our generalization of the virtual-Z scheme to arbitrary single-qubit gate axes. For completeness, we first provide the standard exposition on single-qubit gates via microwave pulses starting with the circuit Hamiltonian [7]. In Fig. 1 we have a standard example of a microwave drive line coupled to a superconducting qubit circuit, this one being the Cooper pair box. Note that a resonant pulse on the flux line would lead to the same qubit Hamiltonian, and so our PMW compilation schemes would apply in that case as well.

The Hamiltonian of such a system projected onto the computational subspace (spanned by ground state $|0\rangle$ and first excited state $|1\rangle$) is given by

$$H(t) = -\frac{\omega_{01}}{2}Z + \Omega(t)\cos(\omega_D t - \phi)X,$$

where $\hbar\omega_{01} \equiv E_1 - E_0$ is the energy difference between $|1\rangle$ and $|0\rangle$, $\Omega(t)$ is the envelope of the microwave pulse, ω_D its frequency, and ϕ its initial phase. Then, in the frame rotating with the qubit, when the drive is resonant the Hamiltonian becomes

$$H_{\rm rf}(t) = \frac{\Omega(t)}{2}(\cos\phi X + \sin\phi Y).$$

Note that here we applied the rotating wave approximation. When implemented for a time T, the gate effected is

$$\begin{aligned} U(T) &= \exp\left[-\frac{i}{2}(\cos\phi X + \sin\phi Y)\int_0^T \Omega(t)dt\right] \\ &= X_\sigma(\phi), \end{aligned}$$

where $\sigma \equiv \int_0^T \Omega(t)dt$. We obtain the desired $X_\sigma(\phi)$ rotation. Hence, by varying the initial phase ϕ of the microwave pulse, we have a direct handle on the parameter of the unitary we want to implement.

A. Three fixed angle rotations (PMW-3)

The first scheme we provide can compile an arbitrary single-qubit gate using three $X_\sigma(\phi)$ rotations with fixed rotation angles. An arbitrary single-qubit gate $U \in SU(2)$ can be parametrized by three real parameters:

$$U(\alpha,\beta,\gamma) = \begin{bmatrix} e^{i\alpha}\cos\gamma & -e^{-i\beta}\sin\gamma \\ e^{i\beta}\sin\gamma & e^{-i\alpha}\cos\gamma \end{bmatrix}. \tag{4}$$

A direct calculation yields that multiplying three $X_\sigma(\phi)$ rotations with rotation angles $\pi/2, \pi,$ and $\pi/2$ gives

$$\begin{aligned} &X_{\pi/2}(\theta)X_\pi(\phi)X_{\pi/2}(\omega) \\ &= \begin{bmatrix} -e^{i(\theta-\omega)/2}\cos\frac{\theta-2\phi+\omega}{2} & -e^{i(\theta+\omega)/2}\sin\frac{\theta-2\phi+\omega}{2} \\ e^{-i(\theta+\omega)/2}\sin\frac{\theta-2\phi+\omega}{2} & -e^{-i(\theta-\omega)/2}\cos\frac{\theta-2\phi+\omega}{2} \end{bmatrix}. \end{aligned} \tag{5}$$

We can therefore identify Eq. (5) with Eq. (4) by setting

$$(\theta - 2\phi + \omega)/2 = \pi - \gamma(\theta-\omega)/2 \quad = \alpha(\theta+\omega)/2 = -\beta,$$

which is solved by

$$\theta = \alpha - \beta, \quad \omega = -\alpha - \beta, \quad \phi = -\beta + \gamma - \pi. \tag{6}$$

We will refer to this as the *PMW-3 scheme*.

The variable parameters α, β, γ are directly related to the phase shifts of the microwave pulses θ, ϕ, ω according to Eq. (6). However, unlike the virtual Z scheme, this scheme does not incur an extraneous Z rotation. Furthermore, only the pulses for $X_{\pi/2}$ and X_π need to be calibrated, which are already necessary for realizing Clifford-based randomized benchmarking and T_1, T_2 measurements [7]. Now, although we need three $X_\sigma(\phi)$ rotations to compile an arbitrary single-qubit gate, certain single-qubit gates can be compiled using fewer rotations, as we show in Sec. II of the Supplemental Material [10]. We also show in Sec. III of the Supplemental Material [10] that for excitation number conserving two-qubit gates, we can reduce the number of pulses needed for a pair of single-qubit gates adjacent to the two-qubit gate from six pulses to five. Lastly, it is not a coincidence that we come upon this PMW-3 scheme; in Sec. IV of the Supplemental Material [10] we surprisingly find that any compilation scheme for arbitrary single-qubit gates using the minimial number of three $X_\sigma(\phi)$ rotations is essentially the same as our scheme.

In [24], the authors study the two-qubit gate SQiSW. To benchmark this gate, they use fully randomized benchmarking (FRB), which requires Haar random gates. Each Haar random gate is compiled using SQiSW and arbitrary single-qubit gates. The single-qubit gates are compiled using PMW-3. Furthermore, through various calibration techniques, they can measure the single-qubit gate errors on their implementation of SQiSW, and they correct for this at compilation time. For the arbitrary single-qubit gates compiled using PMW-3, fidelities per pulse of more than 99.9% are achieved, which is comparable with the state-of-the-art.

B. Four fixed angle rotations (PMW-4)

Since any single-qubit gate has three real parameters, we need at least three phase shifts to cover all the degrees of freedom. Interestingly, by using one more pulse we can further alleviate the burden of calibration but at the cost of incurring a greater overall error. More specifically, the PMW-3 scheme only requires calibrating two fixed X rotation angles: π and $\pi/2$. We can reduce this to a single angle $\pi/2$ by splitting the π rotation into two $\pi/2$ rotations. To be specific, we propose a *PMW-4 scheme*:

$$U(\alpha,\beta,\gamma) = X_{\pi/2}(\theta)X_{\pi/2}(\phi_1)X_{\pi/2}(\phi_2)X_{\pi/2}(\omega). \tag{7}$$

Since $X_{\pi/2}X_{\pi/2} = X_\pi$, we can verify from Eqs. (6) that a solution to Eq. (7) is

$$\theta = \alpha - \beta, \quad \omega = -\alpha - \beta, \quad \phi_1 = \phi_2 = -\beta + \gamma - \pi.$$

C. One variable angle and one fixed angle rotation (PMW-2)

We also give a compilation scheme involving one variable rotation angle and one fixed rotation angle π. In particular, we can do a direct calculation to conclude

$$X_\sigma(\theta)X_\pi(\omega) = U(\alpha,\beta,\gamma)$$

by setting

$$\sigma = 2\gamma - \pi, \quad \theta = 3\pi/2 + \alpha - \beta, \quad \omega = 3\pi/2 - \beta.$$

Although this involves a $X_\sigma(\phi)$ rotation with a variable rotation angle and therefore would have calibration issues, we only need two rotations instead of three as in the PMW-3 scheme. We will refer to this as the *PMW-2 scheme*. This scheme is to the ZXZ scheme via Eq. (1) using a $X_\sigma(\phi)$ gate with an extraneous phase as the PMW-3 scheme is to the virtual Z scheme. Since it involves a $X_\pi(\phi)$ rotation, additional Z rotations can be incorporated in a similar manner to that of the PMW-3 scheme.

D. Compatibility with leaky gates (virtual *R*)

Here we generalize the virtual Z scheme to make it compatible with any leaky gate. A two-qubit gate V is called *leaky*

[25] on the first wire, if there exists an arbitrary axis single-qubit rotation R_ϕ and single-qubit gate families $C(\phi), D(\phi)$ such that

$$V(R_\phi \otimes I) = (C(\phi) \otimes D(\phi))V.$$

Leaky gates are a direct generalization of phase carriers, and there are many experimentally relevant gates that are leaky but are not phase carriers, such as the DDCZ, the CNOT gate and the or the Mølmer-Sørensen gates. In these cases the virtual Z compilation scheme is not directly applicable. Instead, we propose virtual R compilation schemes that decomposes a single-qubit gate into a pulse and an extraneous rotation along a given axis. The freedom to choose this axis implies we can perform virtual rotations around any axis. In particular we prove the following: given an arbitrary rotation axis $R = aX + bY + cZ$, for arbitrary $U \in SU(2)$, there exists angles ϕ, ω, θ such that

$$U = R_\theta X_{\pi/2}(\phi) X_{\pi/2}(\omega).$$

The resulting compilation scheme uses two phase-shifted $X_{\pi/2}$ pulses, just like the virtual Z scheme in Eq. (3). We can also obtain a generalization of the compilation scheme via Eq. (1) using one $X_\sigma(\phi)$ gate with an extraneous phase. To be explicit, we call these schemes *vR-2* and *vR-1,* respectively, and to parallel our vR scheme names, we call the corresponding schemes *vZ-2* and *vZ-1*. The number of pulses is the same as the virtual Z schemes, saving one pulse per gate compared to the PMW compilation schemes. The proof that we can perform virtual R compilation and the corresponding generalization of phase carrying can be found in Sec. V and Sec. VI of the Supplemental Material [10], respectively.

III. DISCUSSION

Using our PMW-3 scheme we can compile single-qubit gates where the gate's three real parameters are directly related to the phase shifts ϕ of the microwave pulses that implements the gate $X_\sigma(\phi) \equiv Z_{-\phi} X_\sigma Z_\phi$. Experimentally these phase shifts can be made extremely precise, unlike the angle of rotation σ, which *a priori* needs to be calibrated for each possible value [6]. The PMW-3 scheme in comparison only requires calibrating the X_π and $X_{\pi/2}$ rotations. The PMW-4 scheme only requires calibrating $X_{\pi/2}$, albeit at the cost of adding another pulse. The PMW-2 scheme only requires two pulses, but requires calibrating an arbitrary X_σ rotation. We also gave virtual R schemes that extended the virtual Z schemes to two-qubit leaky gates.

Altogether, our schemes help make it possible to add general two-qubit gates to the quantum instruction set, which underlies any computational task. Note this implies our results are relevant not only in a noisy intermediate-scale quantum (NISQ) setting, but also in fault-tolerant settings. For example, to realize stabilizer codes, we usually need to compile CNOT. This may require multiple applications of the hardware level two-qubit gate with single-qubit gates in between. The phase errors on the two-qubit gates are in general dependent on the time elapsed during the computation (see for instance Appendix H in [23]), and so a continuous set of single-qubit gates is necessary to correct them in between two-qubit gate applications. Our schemes can compile such a set of gates.

To evaluate our PMW compilation schemes, we need to compare them to the existing alternatives. For phase carriers (or locally equivalent), the compilation scheme of choice is the virtual $Z(R)$ scheme as it only involves two pulses. For two-qubit gates that are not locally equivalent to phase carriers, an alternative to our PMW-3 scheme is the virtual Z scheme plus an arbitrary Z_θ rotation [11]. In general, Z_θ rotations can be performed via fine flux tuning of the qubit frequency. However, the calibration problem of X_σ applies here as well. Our scheme is the only known arbitrary single-qubit gate compilation scheme that does not carry a calibration overhead and is compatible with any two-qubit gate. Even if we fix the amount of calibration we are allowed to perform, the question of how the precision of the flux pulse compares to that of the microwave phase shift, and whether the additional X_π rotation in the PMW-3 scheme under-compensates or over-compensates for the precision difference, is something that would have to be empirically evaluated for individual devices. The comparison could depend on the specific superconducting circuit parameters and the quality of the control hardware. Note that fixed frequency qubits by definition cannot be tuned and so cannot implement Z rotations via flux pulses.

It can be argued that the necessity for additional gates, either Z_θ or $X_\pi(\phi)$ rotations, may suggest a preference for phase carriers in experimental settings. However, phase carriers and even their local equivalence classes are an extremely restricted class of gates, as proved in Sec. I of the Supplemental Material [10]. Although most standard two-qubit gates such as iSWAP and CZ are phase carriers, there is growing interest in gates such as SQiSW [11], and even continuously parametrized two-qubit gates [14,26] for NISQ applications.

Another question that needs to be addressed is the issue of calibrating arbitrary X rotations. One of the advantages of our PMW-3 scheme is that we only need to calibrate the X_π and $X_{\pi/2}$ pulses (only $X_{\pi/2}$ for the PMW-4 scheme). However, it is not completely clear if a general X_σ rotation really needs individual calibration. In [6], a good number of rotation angles were calibrated using the rapid technique in [27], and it was claimed that these calibrated parameters provide an interpolation table for intermediate rotation angles. Although intuitive, we are not aware of such an interpolation method that can achieve fidelities comparable to that of the calibrated angles. The notion of geometric gates [28] addresses this somewhat by choosing compilation schemes which are innately robust to control parameter errors. However, how much this robustness affects the final gate fidelities of arbitrary single-qubit gates is still relatively unstudied.

We conclude with a few open questions. Although we found that we can compile all of $SU(2)$ using three $X_\sigma(\phi)$ gates of fixed angles, we do not give a comprehensive theory of the relationship between the fixed angles available and what subsets of $SU(2)$ can be compiled. In particular this could answer what minimal fixed set of calibrated angles would be optimal for a specific algorithm, or for Haar random gates. Another interesting avenue is to find optimal compilation

schemes for specific two-qubit gates. As shown in Sec. III of the Supplemental Material [10], even gates that are not locally equivalent to phase carriers do not have to cost six pulses for a pair of single-qubit gates. As another example, the cross-resonance gate is not a phase carrier but is still compatible with virtual Z because it is microwave-activated. This allows us to perform procedures analogous to adding a phase shift to microwave pulses for single-qubit gates [8]. The full implications of this for compilation, along with the fact that it is locally equivalent to a phase carrier and is therefore compatible with virtual X on the second qubit, is worth further study.

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[10] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevResearch.5.L022031 for a mathematical characterization of phase carriers and their locally equivalent gates; special single-qubit gates and how to compile them using fewer PMW pulses; a compilation scheme for excitation number conserving gates using fewer PMW pulses; a proof of the uniqueness of the PMW-3 scheme; a proof of the existence of virtual R schemes; and a compilation scheme using virtual R for leaky gates.

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