

Time reflection and refraction in synthetic frequency dimension

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The duality of space and time in Maxwell's equations has prompted interest in time boundaries and the accompanying temporal analog of spatial reflection and refraction. However, achieving observable time-boundary effects at optical frequencies in real materials is challenging. In this Letter, we demonstrate that time reflection and refraction can be observed in a two-band model centered around a nonzero reference energy. Our model can be physically implemented in the synthetic frequency dimension as a system of two coupled dynamically modulated ring resonators. We find that modulation at microwave frequencies is sufficient to observe time-boundary effects for optical waves in synthetic frequency dimension. Our Letter shows that implementing multiband models in synthetic dimensions opens a new avenue for further exploration of time boundaries.

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The space-time duality inherent in Maxwell's equations suggests that there are temporal analogs to many of the more familiar spatial phenomena [1,2]. Such parallels between space and time include the laws of reflection and refraction. Similar to a spatial boundary, a temporal boundary induces scattering of incident electromagnetic waves. Previous theoretical works have demonstrated that at a temporal boundary created by changing the refractive index, time-refracted and time-reflected waves are generated [3–6]. Such time boundaries have been further explored to form photonic time crystals [7–10] and to achieve temporal aiming [11], inverse prisms [12], and higher-order transfer functions [13].

However, to observe time-boundary effects experimentally for optical waves, the change in refractive index must be comparable to that of the material and occur on the time scale of the optical wave period (few femtoseconds) [10,14,15]. These requirements are difficult to satisfy in real materials. To date, time refraction has been experimentally achieved at optical frequencies by using epsilon-near-zero media [15]. However, time reflection has only been observed in water waves [16].

In this Letter, we propose using a two-band model to achieve time reflection and refraction. In this model, the time boundary is implemented by changing the coupling constants in time. We show that the required timescale for the modulation is no longer controlled by the frequency of the waves themselves but rather by the band gap in the two-band model. Consequently, time-reflection and refraction effects can be observed in optical waves by applying a time boundary oc-

curing on the microwave timescale, in contrast to previous works. Specifically, we find that the time boundary can occur as slow as 0.72 MHz, about nine orders of magnitude slower than those of previous works (terahertz).

Our system can be implemented naturally in synthetic dimensions. The concept of synthetic dimension uses the coupling of a system's internal degrees of freedom to create extra dimensions [17,18]. In photonics, this concept allows one to transcend limits of the spatial dimension and to explore higher-dimensional systems [17–31]. In this Letter, we propose to implement the two-band model in a system of coupled ring resonators to observe time reflection and refraction in synthetic frequency space.

We start by reviewing the basic working principles of a time boundary in a medium with dispersion $E(k)$. A time boundary can be implemented by a temporally sudden, but spatially uniform change in the medium's permittivity. For a wave with momentum k and energy $E(k)$, such a time boundary will excite both the wave at $E(k)$ and its time-reversed counterpart at $-\tilde{E}(k)$, where $\tilde{E}(k)$ is the energy after the time boundary [4,32]. Importantly, to achieve such excitation, the temporal change in the permittivity must be fast enough to be comparable with the energy difference $2\tilde{E}(k)$. For $\tilde{E}(k)$ in the optical frequencies, it becomes very difficult to achieve such energy transitions as the required switching speed is too fast.

In contrast to previous works, our approach is based on a periodic system with two energy bands that satisfy $E_+(k) + E_-(k) = 2E_0$, where $E_+(k)$ and $E_-(k)$ denote the upper and lower bands at momentum k , respectively, and E_0 is a constant reference energy. We will show that time reflection and refraction can be achieved by imposing a change of the energy band structure, which only needs to occur on the timescale of the band separation. While E_0 can be at optical frequencies, it is possible to have $E_+(k) - E_-(k) \ll E_0$. The simultaneous excitation of the states in the two bands after the change

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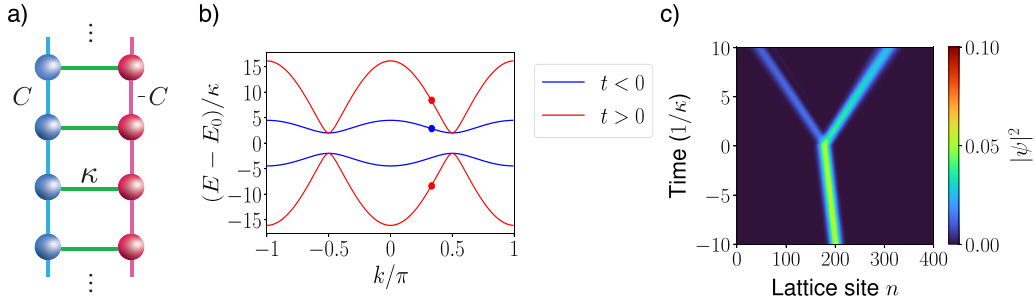


FIG. 1. (a) Schematic of the two-leg ladder lattice model. Blue and red spheres represent a and b lattice sites, respectively. C and $-C$ are the inter-rung coupling constants for the a and b sublattices, respectively. κ is the interleg coupling constant. (b) Band structure of Hamiltonian given by Eq. (1) where blue indicates before the time-boundary ($t < 0$) with coupling constant $C/\kappa = 1$, whereas red indicates after the time-boundary ($t > 0$) with $C/\kappa = -4$. Dots denote the states on the band structures with central wave-vector $k_0 = \pi/3$. (c) Temporal evolution of the system initialized in the state described by Eq. (4). The system consists of $N = 400$ total lattice sites, composed of $M = 200$ unit cells of a and b sites. The even and odd lattice sites are a and b sites, respectively. The coupling constant C changes instantaneously from $C/\kappa = 1$ to $C/\kappa = -4$ at the time-boundary $t = 0$. Simulation employs the matrix exponential method.

yields the time-reflected and time-refracted waves. Thus, the required modulation speed is significantly lower than that of the previous approach.

We illustrate our approach by using a two-legged ladder model [Fig. 1(a)] as described by

$$\begin{aligned}
 H = & \sum_m E_0 [\hat{a}_m^\dagger \hat{a}_m + \hat{b}_m^\dagger \hat{b}_m] + \sum_m [C \hat{a}_m^\dagger \hat{a}_{m+1} + \text{H.c.}] \\
 & + \sum_m [-C \hat{b}_m^\dagger \hat{b}_{m+1} + \text{H.c.}] \\
 & + \sum_m [\kappa \hat{a}_m^\dagger \hat{b}_m + \text{H.c.}]
 \end{aligned} \quad (1)$$

In Eq. (1), E_0 is the on-site energy of each lattice site and H.c. stands for the Hermitian conjugate. $\hat{a}_m^\dagger$, $\hat{b}_m^\dagger$ ($\hat{a}_m$, $\hat{b}_m$) are the creation (annihilation) operators for a and b lattice sites in the m th unit cell, respectively. C is the coupling constant between nearest-neighbor a sites. The coupling constant between the nearest-neighbor b sites are fixed at $-C$. The a and b sublattices, thus, each form a leg of the ladder model. κ is the interleg coupling constant between a and b sites within a unit cell [Fig. 1(a)]. In our system, the parameters C and κ are real.

The dispersion relation for Eq. (1) is as follows:

$$E_{\pm}(k) = E_0 \pm \sqrt{(2C \cos k)^2 + \kappa^2}, \quad (2)$$

where the positive and negative signs in Eq. (2) correspond to the upper and the lower bands, respectively. With our choice of the coupling constants in the a and b sublattices, the two bands are symmetric around E_0 . The corresponding eigenstates are as follows:

$$|\psi_{\pm}(k)\rangle = \frac{1}{\sqrt{N_{\pm}(k)}} \left(2C \cos k \pm \sqrt{(2C \cos k)^2 + \kappa^2} \right), \quad (3)$$

where $|\psi_+(k)\rangle$, $|\psi_-(k)\rangle$ correspond to the upper band $E_+(k)$ and the lower band $E_-(k)$, respectively. $N_{\pm}(k)$ is the corresponding normalization constant.

To implement the time boundary in our system, we impose a change on some of the coupling constants in Eq. (1). As

a numerical example, we consider a change in C from $C = \kappa$ for $t < 0$ to $C = -4\kappa$ for $t > 0$, creating a time boundary at $t = 0$. The band structures corresponding to $C = \kappa$ and $C = -4\kappa$ are shown in Fig. 1(b) as blue and red lines, respectively. For each band structure at a given k , there are two allowed eigenstates with different energies that propagate with opposite-signed group velocities of the same magnitude. Suppose at $t = -10/\kappa$, we excite one of the states with wave-vector k [blue dot in Fig. 1(b)]. For $t > 0$, after the time boundary, since the change is spatially uniform, the wave-vector is conserved. We thus expect the system to be in the two states indicated by the red dots in Fig. 1(b). These two states are the time-refracted wave with the same signed group velocity as the initial state and the time-reflected wave with the opposite sign.

The effect of such a time boundary is demonstrated numerically in Fig. 1(c) where we simulate $N = 400$ total lattice sites, consisting of $M = 200$ unit cells of a and b sites. At $t = -10/\kappa$, we initialize the system to be in a state that is in the upper band,

$$\Psi(m) = \begin{pmatrix} a_m \\ b_m \end{pmatrix} = \frac{1}{\sqrt{\sigma} \sqrt{\pi}} e^{-(m-m_0)^2/2\sigma^2} e^{ik_0 m} |\psi_+(k_0)\rangle, \quad (4)$$

where a_m and b_m are the amplitudes of the a and b sites in the m th unit cell, respectively. The state is normalized such that $\sum_m \langle \Psi(m) | \Psi(m) \rangle = 1$. In Fig. 1(c), we choose $k_0 = \pi/3$, $m_0 = M/2$, and $\sigma = 5$ so that this state centers around the blue dot in Fig. 1(b) and, hence, has a nonzero group velocity. As time evolves, the pulse initially propagates to the left in the $-n$ direction, consistent with the negative group velocity $v_g = dE/dk$ at the wave-vector k_0 [blue dot in Fig. 1(b)]. At the time boundary at $t = 0$, the pulse splits into two states which propagate with opposite-signed group velocities, in consistency with our discussion on the time-refracted and time-reflected waves.

We now assume that both coupling constants C and κ can be varied at the time boundary. In the case of an instantaneous change in the coupling constants, the time-refraction and time-reflection probabilities can be computed analytically. For an incident wave $|\psi_+(k)\rangle$, which corresponds to the state in the upper band at momentum k for the system before the time

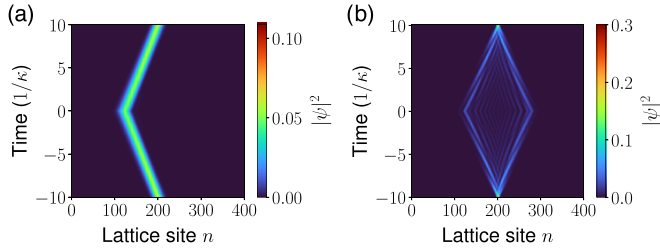


FIG. 2. Numerical demonstration of complete time reflection. Shown here are the temporal evolutions of the state described by (a) Eq. (4) and (b) Eq. (8). In both panels, the Hamiltonian parameters are $C = 5$ and $\kappa = 2$ before the time boundary. At the boundary $t = 0$, the Hamiltonian instantaneously changes according to Eq. (7).

boundary, the probability of time refraction is as follows:

$$T = |\langle \tilde{\psi}_+(k) | \psi_+(k) \rangle|^2 = \frac{1}{N_+(k) \tilde{N}_+(k)} \{ [\mu(k) + \sqrt{\mu(k)^2 + \kappa^2}] \times [\tilde{\mu}(k) + \sqrt{\tilde{\mu}(k)^2 + \tilde{\kappa}^2}] + \kappa \tilde{\kappa} \}^2, \quad (5)$$

where tildes indicate parameters after the time boundary and $\mu(k) \equiv 2C \cos k$. $|\tilde{\psi}_+(k)\rangle$ corresponds to the eigenstate of the upper band $\tilde{E}_+(k)$ at momentum k after the time boundary. Equation (5) measures the probability of photons remaining in the upper band after the time boundary. It is a time-refraction coefficient since the sign of the group velocity remains the same after the time boundary. The probability of time reflection is $R = 1 - T$. The analogous probabilities for an incident state $|\psi_-(k)\rangle$ can be similarly computed.

Achieving complete reflection ($T = 0$) requires that $\langle \tilde{\psi}_+ | \psi_+ \rangle = 0$. Using Eq. (5), we obtain the condition for total time reflection,

$$-\kappa \tilde{\kappa} = (\mu + \sqrt{\mu^2 + \kappa^2})(\tilde{\mu} + \sqrt{\tilde{\mu}^2 + \tilde{\kappa}^2}). \quad (6)$$

This condition is satisfied when

$$H(t > 0) - E_0 = -[H(t < 0) - E_0]. \quad (7)$$

Since the band structure of H in Eq. (7) is symmetric with respect to E_0 , the band structure remains the same before and after the time boundary. However, the eigenstate corresponding to the upper band before the boundary becomes that of the lower band after the boundary. In other words, $|\tilde{\psi}_+(k)\rangle = |\psi_-(k)\rangle$ and $|\tilde{\psi}_-(k)\rangle = |\psi_+(k)\rangle$.

We confirm this effect of total time reflection numerically in Fig. 2(a) where we initialize the system in the state described by Eq. (4). We use the same initialization parameters as in Fig. 1(c). Before the time boundary ($t < 0$), the Hamiltonian has coupling constants $C = 5$ and $\kappa = 2$. After the time boundary ($t > 0$), $C = -5$ and $\kappa = -2$. We note the complete absence of the time-refracted wave after the time boundary.

For the case when Eq. (7) is satisfied, the time boundary performs a time reversal of the pulse envelope. To illustrate this, in Fig. 2(b), we use the same time-boundary setup as in Fig. 2(a), but we initialize the system at $t = -10/\kappa$ with a state described by

$$\Psi(m) = \begin{pmatrix} a_m \\ b_m \end{pmatrix} = \frac{1}{\sqrt{2\sigma^4\pi}} e^{-(m-m_0)^2/2\sigma^2} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (8)$$

where $m_0 = M/2$ and $\sigma = 1$. In contrast to the initial state for the case of Fig. 2(a), here both bands are excited. Consequently, at $-10/\kappa < t < 0$, we see both forward and backward propagation in space. At the time-boundary ($t = 0$), complete time reflection is achieved for every k component. As a result, after evolving the state for the same length of time after the time-boundary ($t = 10/\kappa$), the system returns to the initial state.

The effect that we report here in Fig. 2(b) is reminiscent of Pendry's perfect lens [33]. However, in our system, we employ one time and one spatial dimension. A similar refocusing phenomenon at a source point has also been observed

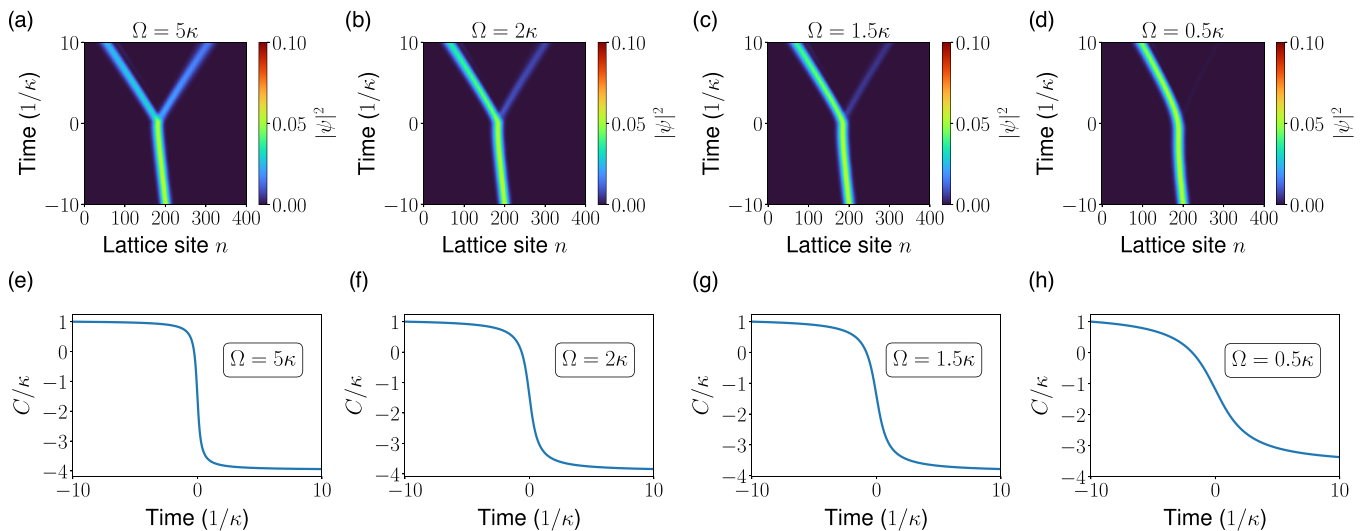


FIG. 3. Numerical demonstration of smoothly varying time boundary. (a)–(d) Temporal evolution of the wave function initialized in the state described by Eq. (4), using the same parameters as Fig. 1(c). In the Hamiltonian, C is varied smoothly from $C = \kappa$ to $C = -4\kappa$, where $C(t)$ is given by Eq. (9). The band structures for $C = \kappa$ and $C = -4\kappa$ are shown in Fig. 1(b). Simulations were performed using the fourth-order Runge-Kutta method. (e)–(h) Change in coupling constant C as a function of time, depicted below its corresponding dynamical simulation.

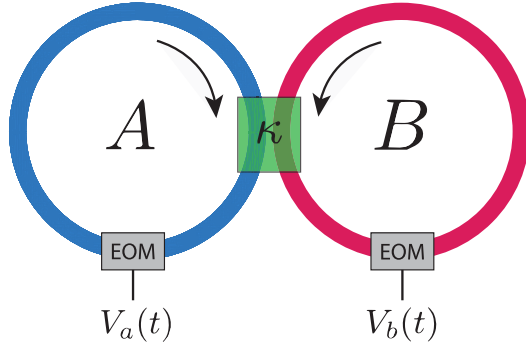


FIG. 4. Synthetic frequency dimension implementation using a dynamically modulated coupled two-ring resonator system. $V_a(t)$ and $V_b(t)$ are the applied voltages, EOM is the electro-optical modulator, and κ represents the directional coupler between the rings. Arrows indicate the counterpropagating modes needed to implement the two-band model.

experimentally in water waves [16]. Our demonstration here differs in that we show complete time reflection for all wave vectors, whereas Ref. [16] shows both time reflection and time refraction. Time reversal of the wave envelope has also been considered in Refs. [34–36]. None of these works, however, employ a sharp time boundary relative to the timescale of the system as we present here.

We now investigate the sharpness of the time boundary needed to observe time refraction and reflection. To facilitate the comparison, we use the same parameters as in Fig. 1(c), but we vary $C(t)$ smoothly from $C = \kappa$ at $t = -10/\kappa$ to $C = -4\kappa$ at $t = +\infty$ using the functional form

$$C(t) = -\frac{\delta}{\pi} \arctan(\Omega t) - \alpha(\Omega), \quad (9)$$

where $\delta = 5\kappa$ and $\alpha(\Omega)$ is a constant offset to ensure that $C(-10/\kappa) = \kappa$. As we can see in Figs. 3(e)–3(h), most of the variation of C occurs around $t = 0$ within a time duration that is inversely related to Ω . With Eq. (9), the sharpness of the time boundary is controlled by the parameter Ω . Using different values of Ω , we compare the temporal evolution of an initial pulse described by Eq. (4) [Figs. 3(a)–3(d)]. The corresponding change in the value of $C(t)$ is depicted in Figs. 3(e)–3(h).

When the time boundary is sharp ($\Omega = 5\kappa$), the incident pulse splits into time-reflected and time-refracted states [Fig. 3(a)]. The resulting temporal evolution looks similar to that of the instantaneous time boundary [Fig. 1(c)]. As the frequency Ω decreases, we see that the time reflection effect becomes less prominent [Figs. 3(b)–3(d)]. At $\Omega = 0.5\kappa$, time reflection is almost completely absent. Figure 3 indicates that the required switching rate for the time boundary in our system is on the order of κ in order to achieve a significant time reflection effect. The required switching rate, thus, can be far lower than the frequency of the optical wave. We reach a similar conclusion about the switching rate required to achieve

complete time-reflection (Fig. 2). Detailed analyses can be found in the Supplemental Material [37].

The Hamiltonian in Eq. (1) can be straightforwardly implemented using the synthetic frequency dimension. Our proposed system consists of two coupled ring resonators, each modulated by an electro-optic modulator (EOM) driven by an applied voltage (Fig. 4). Each ring supports resonant frequency modes that differ by multiples of the free spectral range $\Omega_R = 2\pi v_g/L$, where v_g is the group velocity and L is the length of each ring. The frequency modes of ring A correspond to the a lattice sites, whereas the modes of ring B are the b sites in the two-leg ladder Hamiltonian [Fig. 1(a)].

We assume that rings A and B are modulated with voltages $V_a(t) = V_0 \cos(\Omega_R t)$ and $V_b(t) = -V_0 \cos(\Omega_R t)$, respectively. Modulating each ring at the frequency of the free spectral range Ω_R couples the nearest-neighbor frequency modes in each resonator [18,38,39]. The choice of the modulation waveforms, where the voltages are of the same modulation magnitude but with opposite modulation phases, ensures that the coupling constants in each leg of the implemented Hamiltonian have equal magnitude and opposite signs. Detailed derivations and simulations of the experiment can be found in the Supplemental Material [37].

In this system, the time boundary is implemented by changing the modulation strength V_0 . Here, we note that the switching rate [Ω in Eq. (9)] needed to observe time reflection is on the order of the coupling constant κ , which can be in the microwave frequency regime if the rings are implemented using optical fibers. Experimental values of $\Omega_R = 2\pi 6$ MHz and $\kappa \approx 0.08 \Omega_R = 2\pi 0.48$ MHz have been realized for such a system [29,40]. Given these values, a switching rate of 0.72 MHz is sufficient to observe time reflection in optical waves [Fig. 3(c)]. Numerical simulations corroborate these numbers [37]. Finally, the concept developed in this Letter, which allows one to create time-reflected and time-refracted waves with relatively slow time variations, can be implemented in any system having multiple bands with opposite-signed group velocities separated by a small band gap, such as the photonic crystal systems as considered in Ref. [34].

To summarize, we have shown that by utilizing a two-band model, time refraction and reflection of optical waves can be observed by imposing a temporal boundary on a timescale that is significantly slower than that of the carrier frequency. By using a system of coupled ring resonators, our model can be naturally implemented in synthetic frequency space.

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[1] B. Kolner, Space-time duality and the theory of temporal imaging, *IEEE J. Quantum Electron.* **30**, 1951 (1994).

[2] S. Akhmanov, A. Sukhorukov, and A. Chirkin, Nonstationary phenomena and space-time analogy in nonlinear optics, *Sov. Phys. JETP* **28**, 748 (1969).

- [3] Y. Xiao, D. N. Maywar, and G. P. Agrawal, Reflection and transmission of electromagnetic waves at a temporal boundary, *Opt. Lett.* **39**, 574 (2014).
- [4] J. T. Mendonça and P. K. Shukla, Time refraction and time reflection: Two basic concepts, *Phys. Scr.* **65**, 160 (2002).
- [5] F. Morgenthaler, Velocity modulation of electromagnetic waves, *IEEE Trans. Microwave Theory Tech.* **6**, 167 (1958).
- [6] B. W. Plansinis, W. R. Donaldson, and G. P. Agrawal, What is the Temporal Analog of Reflection and Refraction of Optical Beams? *Phys. Rev. Lett.* **115**, 183901 (2015).
- [7] E. Lustig, Y. Sharabi, and M. Segev, Topological aspects of photonic time crystals, *Optica* **5**, 1390 (2018).
- [8] J. R. Zurita-Sánchez, P. Halevi, and J. C. Cervantes-González, Reflection and transmission of a wave incident on a slab with a time-periodic dielectric function $\epsilon(t)$, *Phys. Rev. A* **79**, 053821 (2009).
- [9] A. Dikopoltsev, Y. Sharabi, M. Lyubarov, Y. Lumer, S. Tsesses, E. Lustig, I. Kaminer, and M. Segev, Light emission by free electrons in photonic time-crystals, *Proc. Natl. Acad. Sci. USA* **119**, e2119705119 (2022).
- [10] M. Lyubarov, Y. Lumer, A. Dikopoltsev, E. Lustig, Y. Sharabi, and M. Segev, Amplified emission and lasing in photonic time crystals, *Science* **377**, 425 (2022).
- [11] V. Pacheco-Peña and N. Engheta, Temporal aiming, *Light Sci. Appl.* **9**, 129 (2020).
- [12] A. Akbarzadeh, N. Chamanara, and C. Caloz, Inverse prism based on temporal discontinuity and spatial dispersion, *Opt. Lett.* **43**, 3297 (2018).
- [13] D. Ramaccia, A. Alù, A. Toscano, and F. Bilotti, Temporal multilayer structures for designing higher-order transfer functions using time-varying metamaterials, *Appl. Phys. Lett.* **118**, 101901 (2021).
- [14] E. Lustig, S. Saha, E. Bordo, C. DeVault, S. N. Chowdhury, Y. Sharabi, A. Boltasseva, O. Cohen, V. M. Shalaev, and M. Segev, Towards photonic time-crystals: observation of a femtosecond time-boundary in the refractive index, 2021 Conference on Lasers and Electro-Optics (CLEO), San Jose, CA, USA (2021), pp. 1–2 (unpublished).
- [15] Y. Zhou, M. Z. Alam, M. Karimi, J. Upham, O. Reshef, C. Liu, A. E. Willner, and R. W. Boyd, Broadband frequency translation through time refraction in an epsilon-near-zero material, *Nat. Commun.* **11**, 2180 (2020).
- [16] V. Bacot, M. Labousse, A. Eddi, M. Fink, and E. Fort, Time reversal and holography with spacetime transformations, *Nat. Phys.* **12**, 972 (2016).
- [17] T. Ozawa and H. M. Price, Topological quantum matter in synthetic dimensions, *Nat. Rev. Phys.* **1**, 349 (2019).
- [18] L. Yuan, Q. Lin, M. Xiao, and S. Fan, Synthetic dimension in photonics, *Optica* **5**, 1396 (2018).
- [19] A. Dutt, M. Minkov, I. A. D. Williamson, and S. Fan, Higher-order topological insulators in synthetic dimensions, *Light Sci. Appl.* **9**, 131 (2020).
- [20] A. Dutt, Q. Lin, L. Yuan, M. Minkov, M. Xiao, and S. Fan, A single photonic cavity with two independent physical synthetic dimensions, *Science* **367**, 59 (2020).
- [21] Y. Hu, C. Reimer, A. Shams-Ansari, M. Zhang, and M. Loncar, Realization of high-dimensional frequency crystals in electro-optic microcombs, *Optica* **7**, 1189 (2020).
- [22] X.-W. Luo, X. Zhou, C.-F. Li, J.-S. Xu, G.-C. Guo, and Z.-W. Zhou, Quantum simulation of 2d topological physics in a 1d array of optical cavities, *Nat. Commun.* **6**, 7704 (2015).
- [23] H. Chalabi, S. Barik, S. Mittal, T. E. Murphy, M. Hafezi, and E. Waks, Synthetic Gauge Field for Two-Dimensional Time-Multiplexed Quantum Random Walks, *Phys. Rev. Lett.* **123**, 150503 (2019).
- [24] I. Martin, G. Refael, and B. Halperin, Topological Frequency Conversion in Strongly Driven Quantum Systems, *Phys. Rev. X* **7**, 041008 (2017).
- [25] Y. Baum and G. Refael, Setting Boundaries with Memory: Generation of Topological Boundary States in Floquet-Induced Synthetic Crystals, *Phys. Rev. Lett.* **120**, 106402 (2018).
- [26] E. Lustig, S. Weimann, Y. Plotnik, Y. Lumer, M. A. Bandres, A. Szameit, and M. Segev, Photonic topological insulator in synthetic dimensions, *Nature (London)* **567**, 356 (2019).
- [27] L. Yuan, Y. Shi, and S. Fan, Photonic gauge potential in a system with a synthetic frequency dimension, *Opt. Lett.* **41**, 741 (2016).
- [28] L. Yuan, M. Xiao, Q. Lin, and S. Fan, Synthetic space with arbitrary dimensions in a few rings undergoing dynamic modulation, *Phys. Rev. B* **97**, 104105 (2018).
- [29] A. Dutt, L. Yuan, K. Y. Yang, K. Wang, S. Buddhiraju, J. Vučković, and S. Fan, Creating boundaries along a synthetic frequency dimension, *Nat. Commun.* **13**, 3377 (2022).
- [30] B. Bartlett, A. Dutt, and S. Fan, Deterministic photonic quantum computation in a synthetic time dimension, *Optica* **8**, 1515 (2021).
- [31] S. Buddhiraju, A. Dutt, M. Minkov, I. A. D. Williamson, and S. Fan, Arbitrary linear transformations for photons in the frequency synthetic dimension, *Nat. Commun.* **12**, 2401 (2021).
- [32] E. Galiffi, R. Tirele, S. Yin, H. Li, S. Vezzoli, P. A. Huidobro, M. G. Silveirinha, R. Sapienza, A. Alù, and J. B. Pendry, Photonics of time-varying media, *Adv. Photonics* **4**, 014002 (2022).
- [33] J. B. Pendry, Negative Refraction Makes a Perfect Lens, *Phys. Rev. Lett.* **85**, 3966 (2000).
- [34] M. F. Yanik and S. Fan, Time Reversal of Light with Linear Optics and Modulators, *Phys. Rev. Lett.* **93**, 173903 (2004).
- [35] S. Sandhu, M. L. Povinelli, and S. Fan, Stopping and time reversing a light pulse using dynamic loss tuning of coupled-resonator delay lines, *Opt. Lett.* **32**, 3333 (2007).
- [36] A. V. Chumak, V. S. Tiberkevich, A. D. Karenowska, A. A. Serga, J. F. Gregg, A. N. Slavin, and B. Hillebrands, All-linear time reversal by a dynamic artificial crystal, *Nat. Commun.* **1**, 141 (2010).
- [37] See Supplemental Material at <https://link.aps.org/supplemental/10.1103/PhysRevResearch.5.L012046> for the derivation of coupled mode equations for a coupled ring resonator system and details of proposed experimental setup.
- [38] L. Yuan, A. Dutt, and S. Fan, Synthetic frequency dimensions in dynamically modulated ring resonators, *APL Photonics* **6**, 071102 (2021).
- [39] A. Dutt, M. Minkov, Q. Lin, L. Yuan, D. A. B. Miller, and S. Fan, Experimental band structure spectroscopy along a synthetic dimension, *Nat. Commun.* **10**, 3122 (2019).
- [40] K. Wang, A. Dutt, C. C. Wojcik, and S. Fan, Topological complex-energy braiding of non-hermitian bands, *Nature (London)* **598**, 59 (2021).