

Symmetry-induced decoherence-free subspaces

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Preservation of coherence is a fundamental, yet subtle, phenomenon in open systems. We uncover its relation to symmetries respected by the system Hamiltonian and its coupling to the environment. We discriminate between local and global classes of decoherence-free subspaces for many-body systems through the introduction of “ghost variables”. The latter are orthogonal to the symmetry and the coupling to the environment depends solely on them. Constructing them is facilitated in classical phase space and can be transferred to quantum mechanics through the equivalent role that Poisson and Lie algebras play for symmetries in classical and quantum mechanics, respectively. Examples are given for an interacting spin system.

DOI: [10.1103/PhysRevResearch.5.L012003](https://doi.org/10.1103/PhysRevResearch.5.L012003)

Introduction. A physical system interacting with an environment [1–4] relaxes to an equilibrium or a nonequilibrium steady state on time scales much longer than the relaxation time [5]. In the steady state the system no longer evolves in time due to its coupling to the environment and *all* associated physical observables reach a constant value. This characteristic is inherent to classical and quantum open systems.

Quantum mechanically, relaxation is accompanied by decoherence, which remains a major obstacle in putting physical devices to work for quantum computation [6–8]. Therefore, decoherence-free subspaces [9–12] (DFS), which are protected against decoherence effects [13], play a crucial role in realizing quantum computing [6,14,15]. Understanding and formulating the general conditions underlying DFS is the prerequisite to tame decoherence and to establish open systems, which exhibit nonstationary long-time dynamics (NLD), that is, remain out of equilibrium despite their interaction with an environment [12,16,17].

In closed systems, it is well known that symmetries, i.e., constants of motion, are crucial to characterize their dynamics. Here, we will show that this is also the case for open systems: Symmetries can be used to formulate simple conditions for DFS in terms of the dependence of the system and the coupling to the environment on these symmetries. Moreover, these conditions can be identified and formulated in an intuitive way via classical dynamics since exact symmetries hold quantum mechanically as well as classically through their equivalent formulation in terms of commutators and Poisson brackets, respectively. The classical phase-space perspective will also allow us to introduce naturally new classes of DFS where the coupling to the environment depends only on so

called “ghost variables”, orthogonal to the symmetries. First, we will present the quantized conditions for the DFS in terms of eigenstates of the symmetry operator.

Our starting point is the Lindblad master equation [4] $\dot{\rho} = -i\mathcal{L}(\mathbb{H}, \mathbb{L})\rho$ for the systems’s density matrix ρ with Hamiltonian $\mathbb{H}$ coupled to the environment in Markov approximation through the Lindblad operators $\mathbb{L}_\alpha$. In the framework of the master equation DFS exist if and only if the Lindbladian $\mathcal{L}$ has nonzero real eigenvalues [10,16,18,19]. Here, we will work with the adjoint Lindbladian $\mathcal{L}^\dagger(\mathbb{H}, \mathbb{L})$, which has identical spectral properties, since we are interested in operators, most prominently a symmetry $\mathbb{J}$. As a constant of motion for the open system its dynamics is governed in the Heisenberg picture by $\mathcal{L}^\dagger$ through

$$0 = \frac{d\mathbb{J}}{dt} = i\mathcal{L}^\dagger \mathbb{J}, \quad (1)$$

with $\mathcal{L}^\dagger = \mathcal{L}_\mathbb{H}^\dagger + \mathcal{L}_\mathbb{L}^\dagger$, where

$$\mathcal{L}_\mathbb{H}^\dagger \mathbb{J} = \frac{1}{\hbar} [\mathbb{H}, \mathbb{J}], \quad (1a)$$

$$\mathcal{L}_\mathbb{L}^\dagger \mathbb{J} = \frac{i}{\hbar} \sum_\alpha (\mathbb{L}_\alpha^\dagger [\mathbb{J}, \mathbb{L}_\alpha] + [\mathbb{L}_\alpha^\dagger, \mathbb{J}] \mathbb{L}_\alpha). \quad (1b)$$

For a constant of motion $\mathbb{J}$, with $[\mathbb{H}, \mathbb{J}] = [\mathbb{L}_\alpha, \mathbb{J}] = 0$, we find that DFS exist if there exist two eigenspaces $\{|n\rangle\}$ and $\{|m\rangle\}$ of $\mathbb{J}$ with eigenvalues J_n and J_m , respectively, such that

$$\Delta \mathbb{H}_{nm} \neq 0 \quad \text{and} \quad \Delta \mathbb{L}_{\alpha nm} = 0, \quad (2)$$

for all α , with the explicit forms of $\Delta \mathbb{H}_{nm}$ and $\Delta \mathbb{L}_{\alpha nm}$ given below in Eq. (4).

While the condition on $\mathbb{H}$ is necessary for (oscillatory) nonstationary dynamics in the first place, the condition on $\mathbb{L}_\alpha$ makes sure that this dynamics is preserved for long times and therefore establishes NLD. Interestingly, since $\mathbb{L}_\alpha$ is in principle a function of all quantum operators $\mathbb{J}$ and $\{\mathbb{Z}_i\}$, the *ghost operators*, being orthogonal to $\mathbb{J}$, the condition $\Delta \mathbb{L}_{\alpha nm} = 0$ can be achieved in two qualitatively different ways:

(i) the $\mathbb{L}_\alpha$ depend explicitly on $\mathbb{J}$ (local realization) and

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(ii) the $\mathbb{L}_\alpha$ do not depend on $\mathbb{J}$ but can depend on all the operators $\{\mathbb{Z}_i\}$ (global realization).

We have a set of commuting operators: $\mathbb{J}$, its conjugate, and the ghost operators $\{\mathbb{Z}_i\}$. Together, they span the Hilbert space for the open-system dynamics. The Hamilton operator $\mathbb{H}$ (and similarly all Lindblad operators $\mathbb{L}_\alpha$) with the symmetry $\mathbb{J}$ can be written as a direct sum of irreducible (square) matrices

$$\mathbb{H} = \bigoplus_n \mathbb{H}_n, \quad \mathbb{L}_\alpha = \bigoplus_n \mathbb{L}_{\alpha n}, \quad (3)$$

where $\mathbb{X}_n$ is associated with the eigenspace of J_n , an eigenvalue of $\mathbb{J}$. They are defined as $\mathbb{X}_n = \text{tr}_n \mathbb{P}_n \mathbb{X} \mathbb{P}_n$ where $\mathbb{P}_n = \prod_{j(J_j \neq J_n)} (\mathbb{J} - J_j \mathbb{1}) / (J_n - J_j)$ are projectors [20] rendering all subspaces to null operators but the one associated with the eigenvalue J_n of $\mathbb{J}$. tr_n , finally, takes the partial trace over all complementary subspaces, reducing the dimension to the irreducible one of J_n . Corresponding to the multiplicity η_n of the eigenvalue J_n , the $\mathbb{H}_n$ and $\mathbb{L}_{\alpha n}$ are $\eta_n \times \eta_n$ matrices. Due to the block-diagonal form (3), the superoperator $\mathcal{L}^\dagger$ can be written as blocks associated with a pair of eigenvalues (J_n, J_m) such that $\mathcal{L}^\dagger = \bigoplus_{n,m} \mathcal{L}_{nm}^\dagger$. Each block of the superoperator

$$\begin{aligned} \mathcal{L}_{nm}^\dagger &= \frac{1}{\hbar} \Delta \mathbb{H}_{nm} + \frac{i}{\hbar} \sum_\alpha ((\mathbb{1}_n \otimes \mathbb{L}_{\alpha m}^\dagger) \Delta \mathbb{L}_{\alpha nm} \\ &\quad - (\mathbb{L}_{\alpha n}^\dagger \otimes \mathbb{1}_m) \Delta \mathbb{L}_{\alpha nm}) \end{aligned} \quad (4)$$

is of size $(\eta_n \times \eta_m) \times (\eta_n \times \eta_m)$ with [21]

$$\Delta \mathbb{H}_{nm} \equiv \mathbb{H}_n \otimes \mathbb{1}_m - \mathbb{1}_n \otimes \mathbb{H}_m^\dagger, \quad (4a)$$

$$\Delta \mathbb{L}_{\alpha nm} \equiv \mathbb{L}_{\alpha n} \otimes \mathbb{1}_m - \mathbb{1}_n \otimes \mathbb{L}_{\alpha m}^\dagger. \quad (4b)$$

For a graphical representation of the Lindbladian structure discussed here, see the Supplemental Material [22].

Coherent oscillations of a beating operator $\mathbb{A}_{nm}$, which is an eigenoperator of $\mathcal{L}_{nm}^\dagger$ with nonzero real eigenvalues λ , occur for those $\Delta \mathbb{H}_{nm}$ and $\Delta \mathbb{L}_{\alpha nm}$ that fulfill condition (2). The eigenoperators $\mathbb{A}_{nm}$ are matrices with an $\eta_n \times \eta_m$ nonvanishing part such that $\mathbb{A}_{nm} = \mathbb{P}_n \mathbb{A}_{nm} \mathbb{P}_m$ and can have multiple beating frequencies. Furthermore, the $\mathbb{A}_{nm}$ are also eigenoperators of the Lindbladian since under condition (2), with (4b), $[\mathbb{L}_{\alpha n}, \mathbb{L}_{\alpha m}^\dagger] = [\mathbb{L}_{\alpha m}, \mathbb{L}_{\alpha n}^\dagger] = 0$. Since symmetries are respected equivalently by a quantum system and its classical counterpart (here, via Poisson brackets), we can give an intuitive account of conditions (2) in classical phase space [23,24]. There, local versus global realization of these conditions become clearer.

Phase-space perspective of symmetry-induced DFS. For a constant of motion J with $\{H, J\} = \{L_\alpha, J\} = 0$, we find that semiclassically DFS exist if there is at least one layer $J = J_0$ on which

$$\omega(J_0) \equiv \partial H / \partial J|_{J_0} \neq 0, \quad \ell_\alpha(J_0) \equiv \partial L_\alpha / \partial J|_{J_0} = 0, \quad (5)$$

where $\omega(J)$ is the Hamiltonian frequency and $\ell_\alpha(J)$ can be interpreted as the decay rate induced by the environment. Equation (5) is the classical analog of the quantum condition (2) and can be obtained by taking the limit $\hbar$ very small or derived semiclassically from dynamical conditions as is shown below. Note that we denote quantum operators with $\mathbb{X}$ and analogous (classical) functions with X .

We consider phase-space variables $(J, \theta, \mathbf{Z})$ with J conserved, $\{J, H\} = \{J, L_\alpha\} = 0$ for all α . The angle θ is canonically conjugate to the action-like J such that $\{\theta, J\} = 1$. Then, the phase space is foliated by manifolds on which J is conserved. The $\mathbf{Z}$ are a set of conjugate *ghost variables* such that $\{\mathbf{Z}, J\} = \{\mathbf{Z}, \theta\} = 0$, whose relevance for DFS will become clear below.

From the action-angle variables we can construct “beating variables”

$$A = f(J) \exp(-i\theta) \quad \text{and} \quad A^* = f(J) \exp(i\theta), \quad (6)$$

with $A^*A = f^2(J)$ [22,25]. The beating variables describe dynamics along a path on which J is conserved. In the semiclassical limit of the adjoint Lindbladian [22–24] the dynamics of A on the manifold of constant J is given by $A = i\mathcal{L}^*A$ with

$$\mathcal{L}^*A = -\left(\partial_J H + \sum_\alpha (2\text{Im}(L_\alpha \partial_J L_\alpha^*) - i\hbar |\partial_J L_\alpha|^2)\right)A, \quad (7)$$

which follows from the Poisson brackets $\{A, H\} = -i(\partial_J H)A$ and $\{A, L_\alpha\} = -i(\partial_J L_\alpha)A$, where $\partial_J H$ is real and $\partial_J L_\alpha$ can be complex (since L_α can be complex). Hence, A evolves in time with an oscillatory and a decaying part, regardless of the dynamics of the ghost variables $\mathbf{Z}$. The oscillatory part comes from a combination of the Hamiltonian and the dissipative components, while the decay part is a consequence of dissipation only. Note, that the decaying part is of order $\hbar$ and is a consequence of diffusion or quantum noise.

If $\partial_J L_\alpha = 0$ the decaying part in (7) vanishes and the beating variable A exhibits NLD, provided that $\partial_J H \neq 0$, which leads to the conditions (5) specifying the existence of DFS. If the Lindblad function depends explicitly on the conserved quantity J and there exists a manifold $J = J_0$ on which $\ell_\alpha(J_0, \mathbf{Z}) = 0$, the environment couples to the degree of freedom associated with J , but not on the subspace $J = J_0$. Although J is conserved, NLD is restricted to the manifold $J = J_0$, and therefore we call this realization (i) of DFS *local*. One also gets $\ell_\alpha = 0$, if L_α does not depend explicitly on J at all. In this case, the environment does not couple to the degree of freedom associated with J . These conditions apply to all manifolds labeled by J , and therefore this constitutes a *global* realization (ii) of DFS.

In both cases, the environment does not affect the oscillatory dynamics of the beating variable A characterized by the (nonzero) real eigenvalues $\omega(J, \mathbf{Z})$ of the Lindbladian. If the Hamiltonian frequency ω depends explicitly on the ghost variables $\mathbf{Z}$, this oscillatory dynamics can be very complicated. If $\omega = \omega(J)$ only, one can directly solve $\dot{A} = i\mathcal{L}^*A$ to obtain $A(t) = \exp(-i\omega(J)t)A(0)$. In case (i) the environment extinguishes for long times all oscillatory motion linked to J but one with frequency $\omega(J_0)$ on the manifold J_0 .

From the semiclassical perspective as developed above we can draw further conclusions. The conditions (5) do not require integrability (or near integrability) of the system. Therefore, also classically chaotic systems can have DFS, which we demonstrate here explicitly with the Heisenberg spin model. Furthermore, if DFS exist due to a symmetry, small perturbations of the system will not destroy them. This follows from the KAM theorem [26], which ensures the

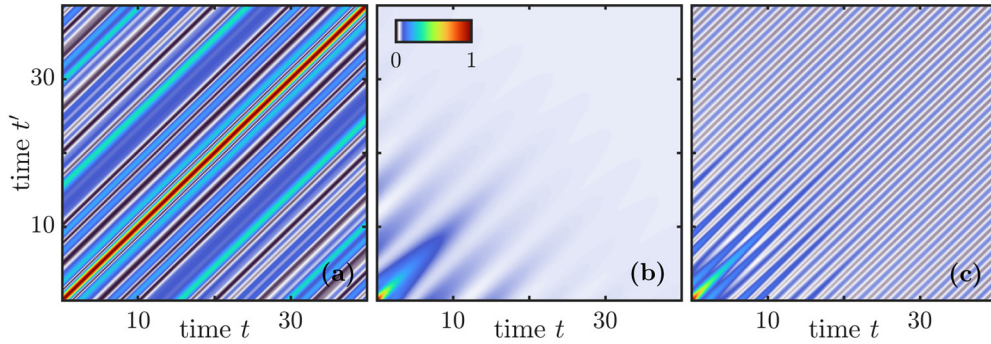


FIG. 1. Autocorrelation function $|\text{tr} \rho(t) \rho(t')|$ for Hamiltonian (8) with $N=6$. Parameters ω_i , δ_{ij} , and Δ_{ij} are given within the Supplemental Material [22]. (a) No dissipation, $\mathcal{L}_{\mathbb{L}} = 0$. (b) Dissipation with Lindblad operators $\mathbb{L}_{\alpha} = \gamma \tilde{S}_{\alpha}^z$, $\alpha = 1 \dots N$. (c) Dissipation with ghost operators such that $\mathbb{L}_{\alpha} = \gamma \tilde{S}_{\alpha}^z$, $\alpha = 1 \dots N-1$. In both cases $\gamma=1/3$.

existence of modified action-angle variables for open systems subjected to perturbations. Therefore, we construct now explicitly a coupling to the environment according to (ii) for a chaotic Hamiltonian.

Application to the Heisenberg XXZ spin model. Since systems, which we classify as local realizations (i) have been discussed [16,17], we focus on global realizations (ii), which require the Lindblad operators to be independent of the symmetry $\mathbb{J}$. Therefore, only many-body systems render this case nontrivial.

We consider the Heisenberg XXZ spin model [27–30]

$$\mathbb{H} = \sum_i \omega_i S_i^z + \sum_{ij} [\sigma_{ij} (S_i^+ S_j^- + S_i^- S_j^+) + \Delta_{ij} S_i^z S_j^z], \quad (8)$$

where the $S_i = (S_i^x, S_i^y, S_i^z)$ are spin-1/2 operators on site i with Pauli matrices, and $S_i^{\pm} = S_i^x \pm i S_i^y$ being ladder operators, with $i = 1 \dots N$ and N the number of spins. Combinations $S_i^{\pm} S_j^{\mp}$ evoke spin flips between site i and j . The total spin $\mathbb{J} = S^z = \sum_{k=1}^N S_k^z / N$ is conserved in time as $\mathcal{L}^{\dagger} \mathbb{J} = 0$. Thus one can construct Lindblad operators that generate NLD, as shown in Fig. 1. For $\mathbb{H}$ of Eq. (8) with the specific parameters given within the Supplemental Material [22], we compare pure Hamiltonian dynamics, i.e., $\mathcal{L}_{\mathbb{L}} = 0$ [Fig. 1(a)], with an “arbitrary” dissipative part [Fig. 1(b)]

$$\mathbb{L}_{\alpha} = \gamma S_{\alpha}^z \quad \alpha = 1 \dots N, \quad (9a)$$

and with a dissipative part constructed from ghost operators only [Fig. 1(c)]

$$\mathbb{L}_{\alpha} = \gamma \tilde{S}_{\alpha}^z \quad \alpha = 1 \dots N-1. \quad (9b)$$

While the autocorrelation function decays for (9a), it leads to pertinent oscillations for (9b), since the $\mathbb{L}_{\alpha}$ commute with $\mathbb{J} = S^z$. In order to construct the ghost operators $\tilde{S}_{\alpha}^z$, see Eq. (11) below for their explicit form, we resort to classical phase space.

To this end, we consider the classical description of the Heisenberg XXZ spin model [22,27–30] for spin variables $S_i = (S_i^x, S_i^y, S_i^z)$ where $i = 1 \dots N$. The conserved quantity is the total spin $J = S^z = \sum_i S_i^z / N$. We derive the associated classical ghost variables using polar coordinates $S_i = [S_i^2 - (S_i^z)^2]^{1/2} (\mathbf{e}_x \cos \theta_i + \mathbf{e}_y \sin \theta_i) + \mathbf{e}_z S_i^z$ for the spin on site i , where the azimuthal angles θ_i are canonically conjugate to the S_i^z and $S_i^2 = |\mathbf{S}_i|^2$. Akin to relative center-of-mass motion

for massive particles we take advantage of this fact by introducing relative angles $\Delta \theta_k = \theta_{k+1} - \theta_k$ for $k = 1 \dots N-1$, and the total angle $\theta = \sum_i \theta_i$. They are related to the Jacobi coordinates for celestial many-body systems [31].

The canonical transformation from phase-space variables (θ_i, S_i^z) to phase-space variables $(J, \theta, \mathbf{Z})$, with ghost variables $\mathbf{Z} = (\Delta \theta_k, \tilde{S}_k^z)$ for $k = 1 \dots N-1$, and its inverse are given within the Supplemental Material [22]. The transformation is obtained by using the canonical rules [32] with a suitable generating function associated with the center of mass and the relative angles. Besides $\Delta \theta_k$, we obtain as ghost variables (see Supplemental Material [22])

$$\tilde{S}_k^z = \frac{k}{N} \sum_{i=1}^N S_i^z - \sum_{i=1}^k S_i^z, \quad (10)$$

which are linear functions of the spin variables. Note that using canonical transformations ensures that the new set of variables spans the entire phase space. If the interaction with the environment is any analytic function of these variables or operators (i.e., if it does not depend explicitly on θ and J) DFS exist and the open system exhibits NLD. The ghost variables or operators do not affect the oscillatory dynamics associated with the conserved quantity J . Therefore, coherent oscillations persist on long-timescales and the spectrum of the Lindbladian contains nonzero real eigenvalues λ , cf. Fig. 2. The beating variables associated with the conserved quantity, as given in Eq. (6), are obtained in terms of spin variables by performing a noncanonical change of coordinates [22] leading to $A \propto \prod_{i=1}^N S_i^-$ and $A^* \propto \prod_{i=1}^N S_i^+$. The beating variables oscillate at a frequency $\Omega = \sum_{i=1}^N \omega_i$, corresponding to the absolute value of the nonzero real eigenvalues and are not affected by dissipation.

Now, the *ghost operators* as the quantum analogs of the *ghost variables* follow simply from replacing the Cartesian spin variables in the ghost variable expressions by the corresponding Cartesian spin operators. Therefore, the ghost operators are functions of the spin operators (Pauli matrices) and read

$$\tilde{S}_k^z = \frac{k}{N} \sum_{i=1}^N S_i^z - \sum_{i=1}^k S_i^z, \quad (11a)$$

$$\tilde{S}_k^{\pm} = \mathbb{D}_k S_{k+1}^{\pm} S_k^{\mp}, \quad (11b)$$

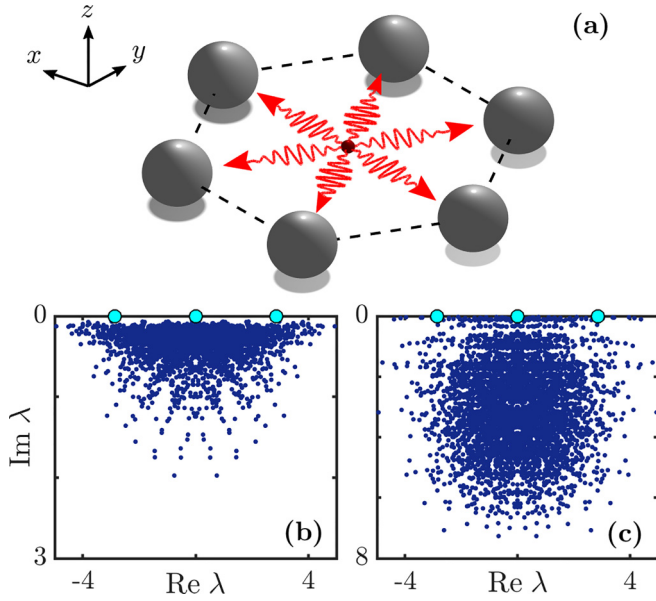


FIG. 2. (a) Schematic of the spin system for $N = 6$ governed by Hamiltonian (8) as used before in Fig. 1. The red lines illustrate the dissipations induced by the ghost operators. [(b), (c)] Spectrum of the adjoint of the Lindbladian $\mathcal{L}^\dagger$ for Lindblad operators as a function of the ghost operators (11) such that $\mathbb{L}_\alpha = \gamma \tilde{S}_\alpha^z$ and $\mathbb{L}_\alpha = \gamma \tilde{S}_\alpha^+$, respectively. In both cases $\alpha = 1 \dots N-1$ and $\gamma = 1/3$. The cyan circles are the purely real eigenvalues.

with $k = 1 \dots N-1$ and a diagonal matrix $\mathbb{D}_k$ [22]. By construction, all combinations of ghost operators are also ghost operators. Any Lindblad operator, which is an analytic function of these ghost operators leads to DFS.

In Fig. 2(b), we have used $N-1$ Lindblad operators as a linear combination of the ghost operators (11a) such that $\mathbb{L}_\alpha = \gamma \tilde{S}_\alpha^z$ with $\gamma = 1/3$ for $\alpha = 1 \dots N-1$. For Fig. 2(c), we have used $N-1$ Lindblad operators of the ghost operators (11b) such that $\mathbb{L}_\alpha = \gamma \tilde{S}_\alpha^+$ with $\gamma = 1/3$ and $\alpha = 1 \dots N-1$. In both cases, the coherent oscillations associated with the symmetry $\mathbb{J}$ are not affected, neither are the beating operators $\mathbb{A} = \prod_{i=1}^N S_i^-$ and $\mathbb{A}^\dagger = \prod_{i=1}^N S_i^+$. Note also that the (real) eigenvalues (cyan circles) are the same for the cases of Figs. 2(b) and 2(c), since they come from the Hamiltonian part of the Lindbladian $\mathcal{L}_H^\dagger$, which we have not changed. We find $\mathcal{L}_H^\dagger \mathbb{A} = \Omega \mathbb{A}$ and $\mathcal{L}_L^\dagger \mathbb{A} = 0$ corresponding to the cyan circles for positive real eigenvalues (the negative one corresponds to the one associated with $\mathbb{A}^\dagger$) in Figs. 2(b) and 2(c).

Summary. This completes our classical, semiclassical, and quantum treatment of symmetry-induced DFS. It clearly reveals that the notion of nonstationary (oscillating) behavior and its equivalence to real eigenvalues of the Lindbladian applies equally to classical, semiclassical, and quantum systems, which establishes DFS for classical and semiclassical systems. For the latter, coherent oscillations in classical systems can still occur in the presence of dissipation (through the second term in (7), see [33–35] for examples), but diffusion can inhibit long-time oscillatory nonstationary motion (through the third term in (7) being of order $\hbar$). The eigenoperator of the adjoint Lindbladian corresponding to the real nonzero eigenvalues can be constructed classically, see Eq. (6), or quantum mechanically ($\mathbb{A}_{nm}$) from a symmetry. Playing the role of a beating operator A or $\mathbb{A}_{mn}$ connect different elements of the constant of motion J or $\mathbb{J}$ in form of its layers in phase space or its irreducible representations in Hilbert space, respectively. Elements $\mathbb{J}_n$ of the symmetry itself can be expressed in terms of the beating operator as $\mathbb{J}_n = \mathbb{A}_{mn}^\dagger \mathbb{A}_{mn}$. This irreducible representation also plays a crucial role in the context of breaking the ergodicity thermalization hypothesis in Hamiltonian systems [36], which is related to NLD in the absence of dissipation. For DFS, the Lindblad operators must be degenerate with respect to at least two different irreducible subspaces $n \neq m$ of the symmetry, while the hamiltonian must not be degenerate, see (2). This gives a clear picture how DFS emerge.

Guided by this insight, we have identified a new class of couplings to the environment leading to DFS, namely when the Lindblad operators do only depend on ghost operators, which span the Hilbert space but commute with the symmetry $\mathbb{J}$, as explicitly demonstrated with the Heisenberg XXZ spin model. This global realization of DFS requires Lindblad operators only linearly dependent on the spin operators, which can be achieved in cavity QED experiments much easier than what has been proposed so far theoretically based on local realizations [16,17].

In terms of the eigenoperator, DFS exist if $[\mathbb{L}_\alpha, \mathbb{A}_{nm}] = [\mathbb{L}_\alpha^\dagger, \mathbb{A}_{nm}] = 0$ [16]. Note, however, that the latter condition is not necessary since for the adjoint Lindbladian to have real eigenvalues, it follows from (1b) that only the sum $\mathbb{L}_\alpha^\dagger [\mathbb{A}, \mathbb{L}_\alpha] + [\mathbb{L}_\alpha^\dagger, \mathbb{A}] \mathbb{L}_\alpha = 0$ must vanish. How this extends the classes of possible interactions with the environment leading to DFS even further beyond those depending only on ghost operators will be a subject of further research [37].

Acknowledgments. J.D. acknowledges Andreas Buchleitner, Charlie Duclut, Christian Johansen, and Francesco Piazza for helpful discussions.

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