

Non-Hermitian topology in monitored quantum circuits

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We demonstrate that genuinely non-Hermitian topological phases and corresponding topological phase transitions can be naturally realized in monitored quantum circuits, exemplified by the paradigmatic non-Hermitian Su-Schrieffer-Heeger model. We emulate this model by a 1D chain of spinless electrons evolving under unitary dynamics and subject to periodic measurements that are stochastically invoked. The non-Hermitian topology is visible in topological invariants adapted to the context of monitored circuits. For instance, the topological phase diagram of the monitored realization of the non-Hermitian Su-Schrieffer-Heeger model is obtained from the biorthogonal polarization computed from an effective Hamiltonian of the monitored system. Importantly, our monitored circuit realization allows direct access to steady-state biorthogonal expectation values of generic observables, and hence, to measure physical properties of a genuine non-Hermitian model. We expect our results to be applicable more generally to a wide range of models that host non-Hermitian topological phases.

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Introduction. The interplay between nonintegrable unitary quantum dynamics, which evolve generic states into volume law entangled states, and measurements, which effectively reduce quantum entanglement, have recently been shown to facilitate novel “measurement-induced phase transitions” [1–30]. A unitary quantum circuit interspersed with local measurements, each invoked with a classical probability p , undergoes a measurement-induced phase transition visible in the entanglement dynamics of typical product states evolved by the quantum circuit. Various aspects, such as emergent conformal symmetry [4] and effective statistical mechanics descriptions [7,11,31], of the transition as a function of p have already been understood. In this paper, we show that in addition to entanglement transitions, monitored quantum circuits (MQCs) can realize topological phase transitions, in particular between non-Hermitian topological phases. Such topological transitions in MQCs are visible in topological diagnostics such as those related to the topological bulk-boundary correspondence.

Non-Hermitian topological systems have received considerable attention [32–37] in the recent past. In an important development, the landmark Altland-Zirnbauer classification [38–41], which provides an exhaustive topological classification of free fermion and Bogoliubov-de-Gennes systems with combinations of particle-hole, time reversal, and chiral symmetry, has been extended to non-Hermitian systems [34]. Additionally, the bulk-boundary correspondence and bulk topology for candidate models of several topological

non-Hermitian phases have been explored [35–37,42–47]. These models exhibit various phenomena, such as localization transitions, non-Hermitian skin effect, and the emergence of exceptional points, unique to non-Hermitian systems. Among these models, the non-Hermitian Su-Schrieffer-Heeger (nH-SSH) model remains a paradigmatic example that illustrates various novel features of non-Hermitian topology [43–45,47–52].

Alongside theoretical progress there have been several developments in realizing non-Hermitian physics in experimental platforms. These include photonic [53–61], phononic (mechanical) [62–65], and topoelectric circuit [66] setups, where the evolution of the electric, acoustic, and current modes determined by Maxwell’s, Newton’s, and the generalized Ohm’s equation, respectively, are formally equivalent to the Schrödinger equation and can furthermore be made non-Hermitian owing to the ubiquity of dissipation, loss, and gain in such systems. Despite these experimental milestones, examples of truly quantum physical realizations of non-Hermitian phases remain scarce. Additionally a quantum-mechanical protocol to simulate the biorthogonal expectation value of generic operators—standard in the description of non-Hermitian systems [67]—is lacking.

Two main avenues expected to generate effective non-Hermiticity in the single-particle evolution of a quantum particle are (i) dissipative processes involving interactions with an external bath [50,68–74] and (ii) strong interactions that lead to finite lifetimes of quasiparticles appearing as a non-Hermitian correction to the noninteracting Hamiltonian via the self energy [33,75–82]. In the context of dissipative dynamics, it has been shown [73,83] that postselected quantum trajectories can be modelled as nonunitary dynamics governed by an effective non-Hermitian Hamiltonian. In such a setup the quantum trajectory without any quantum jumps is postselected.

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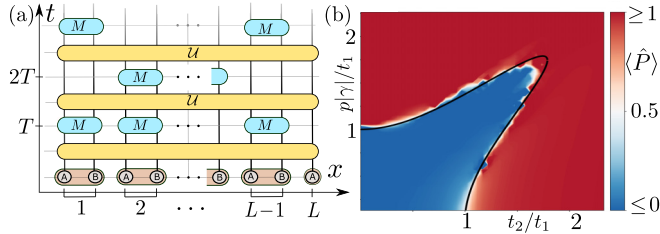


FIG. 1. (a) A schematic of the applied sequence of unitary time evolution ($\mathcal{U}$) and measurement (M) for a chain with a single A atom on the right end, and (b) topological phase diagram of $\mathcal{H}_{\text{eff}}$ [defined in Eq. (6)] associated with a circuit of depth $N = 200$ cycles; we display the biorthogonal polarization $\hat{P}$, defined in Eq. (2), of the midgap state as a function of $p|\gamma|/t_1$ and t_2/t_1 , where p is the classical probability of measurement. The solid-black line indicates the theoretically expected non-Hermitian topological phase transition at $t_1^4 = (t_2^2 - |p\gamma|^2)^2 + (2p\gamma t_2)^2$. We attribute the small deviations to statistical fluctuations $\sim \mathcal{O}(T\gamma/\sqrt{N})$ emerging from the probabilistic circuit. Further parameters are $L = 26$, $Tt_1 = 0.001$, and $\gamma/t_1 = 2.5 + 0.4i$.

In this paper, we propose an alternate route towards realizing and detecting quantum non-Hermitian topological phases: Instead of designing a non-Hermitian topological Hamiltonian in the first place, we construct the generator of non-Hermitian time evolution for a given non-Hermitian Hamiltonian using MQCs with postselected trajectories of generalized measurements [1–4, 6–19, 23–25]. The Hermitian part of the corresponding non-Hermitian topological Hamiltonian is incorporated in the unitary component of the MQC while the non-Hermiticity is modelled using generalized measurements. These are implemented locally with some classical probability p and punctuate the otherwise unitary dynamics [cf. Fig. 1(a)]. Using the nH-SSH model as a case study, we show that distinct topological features, diagnosed using the biorthogonal bulk-boundary correspondence, can be attributed to the MQC. Interestingly, signatures of the topology are directly produced by the circuit itself: At half filling the system evolves into a steady state that carries profound information on the topology. To access this information we sandwich the observable of interest by MQCs [cf. Fig. 3(a)] and investigate the autocorrelator of the concatenated circuit, which then corresponds to a biorthogonal expectation value of the steady state.

Non-Hermitian Su-Schrieffer-Heeger model. The nH-SSH model describes noninteracting spinless electrons hopping on a 1D lattice. Each unit cell contains two orbitals denoted A and B . The Hamiltonian $\mathcal{H}(t_1, t_2, \gamma) = \mathcal{H}_1(t_1) + \mathcal{H}_2(t_2, \gamma)$ includes intra- and intercell hopping terms

$$\begin{aligned}\mathcal{H}_1(t_1) &= \sum_j t_1 [c_{j,B}^\dagger c_{j+1,A} + c_{j+1,A}^\dagger c_{j,B}], \\ \mathcal{H}_2(t_2, \gamma) &= \sum_j [(t_2 + \gamma) c_{j,A}^\dagger c_{j,B} + (t_2 - \gamma^*) c_{j,B}^\dagger c_{j,A}],\end{aligned}\quad (1)$$

where $c_{j,A}^\dagger$ ($c_{j,B}$) are fermionic creation (annihilation) operators on the A (B) atom of site j . $\gamma = \gamma_R + i\gamma_I$ and the parameters $t_{1,2}$ and $\gamma_{I,R}$ are real-valued such that $\mathcal{H}_1$ is a

Hermitian operator while $\mathcal{H}_2$ has both Hermitian and anti-Hermitian terms proportional to t_2 and γ , respectively.

Complex γ leaves the system with only chiral symmetry and reduces the number of topologically distinct phases from three (at $\gamma \in \mathbb{R}$) to two, bounded by $t_1^4 = (t_2^2 - |\gamma|^2)^2 + (2\gamma t_2)^2$ [cf. black-solid line in Fig. 1(b)]. The spectrum of the Hamiltonian depends sensitively on the choice of boundary conditions [35, 36, 43]. The open chain has a bulk line gap and potentially a single midgap zero-mode state depending on the boundary termination, while the periodic chain is either gapped or has exceptional points. For concreteness, we focus on an open 1D chain with L unit cells, which terminates on an A orbital at both ends. For this setup, one always obtains a single zero mode localized on either end of the chain defining two distinct topological phases, distinguished by the biorthogonal polarization [35] $\mathcal{P} = \langle \Psi_L | \hat{P} | \Psi_R \rangle$ where $|\Psi_{L(R)}\rangle$ are the left (right) biorthogonal zero-mode eigenstates and

$$\hat{P} = \frac{1}{L} \sum_{j=1}^L j \Pi_j, \quad (2)$$

where $\Pi_j = c_{j,A}^\dagger |0\rangle \langle 0| c_{j,A} + c_{j,B}^\dagger |0\rangle \langle 0| c_{j,B}$ is the projection operator on the j th unit cell. A topological phase transition is characterized by a jump in $\mathcal{P}$ [35] and is equivalent to a change in the winding number defined in a generalized Brillouin zone [36, 42].

Monitored circuit and measurement protocol. In what follows, we realize the nH-SSH model as a quantum circuit constructed from two ingredients: (i) a unitary evolution operator that corresponds to the Hermitian part of the nH-SSH model and (ii) measurement operators executing generalized measurements that act locally on the unit cells of the nH-SSH chain and emulate the non-Hermiticity of the model, cf. Fig. 1(a). The unitary time-evolution operator is obtained by simply setting $\gamma = 0$ in the nH-SSH evolution operator $\mathcal{U}(T) = \exp\{-iT\mathcal{H}(t_1, t_2, 0)\}$ (we set $\hbar = 1$). A measurement on the j th unit cell of the 1D chain is defined by an operator $M_{j,+}$, which is a member of a set of Hermitian Kraus operators $\{M_{j,\alpha}\}$ with $\alpha \in \{0, \pm\}$. The Kraus operators define a completely-positive trace-preserving map that maps ρ , the density matrix of the fermionic chain, to $\sum_\alpha M_{j,\alpha} \rho M_{j,\alpha}^\dagger$. Explicitly, these take the form

$$\begin{aligned}M_{j,\pm} &= \sqrt{\frac{1}{2(1 + |\gamma T|^2)}} [1 \pm iT(\gamma^* c_{j,B}^\dagger c_{j,A} - \gamma c_{j,A}^\dagger c_{j,B})], \\ M_{j,0} &= \sqrt{\frac{|\gamma T|^2}{1 + |\gamma T|^2}} (1 - c_{j,A}^\dagger c_{j,A} - c_{j,B}^\dagger c_{j,B}),\end{aligned}\quad (3)$$

and satisfy the trace-preserving condition of a quantum map $\sum_\alpha M_{j,\alpha}^\dagger M_{j,\alpha} = 1$. Note that for $|\gamma T| = 1$, $M_{j,\pm}$ implement projective measurements in the single-particle sector. We consider a measurement protocol that postselects the “+” outcome, i.e., the measurement maps the state $|\Psi\rangle$ to $M_{j,+}|\Psi\rangle / \|M_{j,+}|\Psi\rangle\|$. Such a measurement outcome occurs with probability $\|M_{j,+}|\Psi\rangle\|^2$ and in the limit $|\gamma T| \rightarrow 0$ (the regime we work in) only the measurement outcomes corresponding to $M_{j,\pm}$ survive. Postselecting the $M_{j,+}$ outcome over n measurements therefore involves performing 2^n successive experiments. Such a generalized measurement protocol

can be implemented by coupling the j th unit cell in the 1D chain to a “macroscopic” ancilla qubit $|\sigma\rangle_j$ as follows: First, the total system is prepared in a tensor product state $|\Psi\rangle \otimes |0\rangle_j$, where $|0\rangle_j$ is a reference qubit state and $|\Psi\rangle$ is the initial state of the 1D chain. Then, one acts with the unitary $\mathcal{U}_{M_j} : |\Psi\rangle \otimes |0\rangle_j \mapsto \sum_{\alpha=\pm} M_{j,\alpha} |\Psi\rangle \otimes |\alpha\rangle_j$ [84], where $|\pm\rangle_j$ denote the σ_z eigenstates of the qubit. Finally, a projective measurement for the ancilla qubit onto $|+\rangle_j$ is carried out. The normalized postmeasurement state thus obtained is precisely $M_{j,+} |\Psi\rangle / \|M_{j,+} |\Psi\rangle\|$. Unitary evolution for a single time step T followed by a simultaneous postselective measurement on all the unit cells is described by the operator $F = (\prod_j M_{j,+}) \cdot \mathcal{U}(T)$. Instead of carrying out the measurement on every unit cell, we consider a protocol wherein the measurement operation is invoked on each unit cell with a classical probability $p \in [0, 1]$. Thus one obtains a probabilistic quantum circuit operator that acts on a state $|\Psi\rangle$ as

$$F(p, s) |\Psi\rangle = \frac{(\prod_j M_{j,+}^{s_j}) \cdot \mathcal{U}(T) |\Psi\rangle}{\|(\prod_j M_{j,+}^{s_j}) \cdot \mathcal{U}(T) |\Psi\rangle\|}, \quad (4)$$

with $s_j = 0$ or 1 with probability $1 - p$ and p , respectively. Correspondingly, a depth N MQC with measurement probability equal to p is defined as

$$F_N(p, \{s\}) := F(p, s_N) \cdot F(p, s_{N-1}) \cdots F(p, s_1). \quad (5)$$

Effective Hamiltonian and non-Hermitian topology. Next, we extract the nH-SSH topological phase diagram directly from the MQC. For the deterministic case of $p = 1$, the circuit is periodic with period T and we can apply a Floquet-Magnus expansion to obtain a Floquet Hamiltonian $\mathcal{H}_F$, which governs the dynamics over a unit-time step T $F(p = 1, s)$ [85]. In the high-frequency limit, where $T\nu \ll 1$ ($\nu \in \{t_1, t_2, \gamma\}$), the measurement operator $\prod_j M_{j,+} \propto \exp\{-iT\mathcal{H}_2(0, \gamma)\}$ and consequently the Floquet Hamiltonian (to leading order in inverse frequency [86]) equals the single-particle nH-SSH Hamiltonian, i.e., $\mathcal{H}_F = \mathcal{H}(t_1, t_2, \gamma) + \mathcal{O}(T\nu)$. Moreover, $\mathcal{H}_F$ captures the dynamics of the system for (at least) exponentially long times $\sim \exp[1/(vT)]$ [87–89]. For $p \neq 1$ periodicity is lost; however, an N -cycle effective Hamiltonian can be defined as

$$\mathcal{H}_{\text{eff}}^{(N)}(p, \{s\}) = \frac{i}{NT} \sum_{k=1}^N \log(F(p, s_k)). \quad (6)$$

For increasing N , this effective Hamiltonian $\mathcal{H}_{\text{eff}}^{(N)}$ is self-averaging, such that deviations fall off as $1/\sqrt{N}$ according to the central limit theorem, cf. Fig. 2(a). In this limit, the many-cycle effective Hamiltonian recovers the nH-SSH Hamiltonian (in leading order Magnus expansion) up to corrections $\mathcal{O}(1/\sqrt{N})$

$$\mathcal{H}_{\text{eff}}^{(N)}(p, \{s\}) = \mathcal{H}(t_1, t_2, p\gamma) + \mathcal{O}(1/\sqrt{N}) + \mathcal{O}(T\nu). \quad (7)$$

Consequently, the effective Hamiltonian for a probabilistic circuit $F_N(p, \{s\})$ at large N only depends on γp . This allows to replace a probabilistic circuit with given γ and p with an effective deterministic circuit with $\gamma_{\text{eff}} = p\gamma$ [cf. Fig. 2(a)]. Hence, the dynamics of the probabilistic circuit is equally well captured by $\mathcal{H}(t_1, t_2, p\gamma)$ up to exponentially long

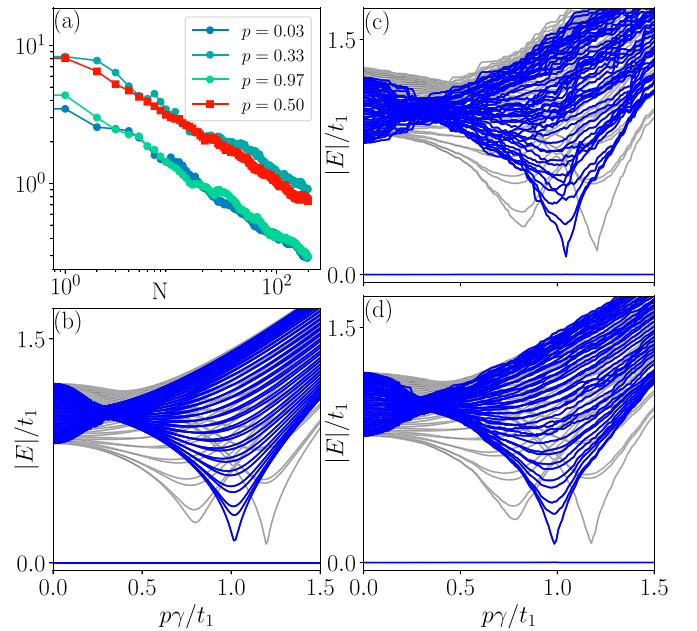


FIG. 2. (a) Matrix norm difference $\|\mathcal{H}_{\text{eff}}^{(N+1)}(p, \{s\}) - \mathcal{H}_{\text{eff}}^{(N)}(p, \{s\})\|/t_1$ with two unrelated realizations $\{s\}$ and $\{\bar{s}\}$ computed according to Eq. (6) for different values of p (green, turquoise, and blue circles) and matrix norm difference $\|\mathcal{H}_{\text{eff}}^{(N)}(p, \{s\})|_{\gamma} - \mathcal{H}_{\text{eff}}^{(N)}(1, \{s\})|_{p\gamma}\|/t_1$ (red squares). (b) Spectrum of the nH-SSH model $\mathcal{H}(t_1, t_2, p\gamma)$ for open (blue) and periodic (grey) boundary conditions. We use a system with complete unit cells in the periodic boundary case to restore translational symmetry. [(c),(d)] Spectrum of the effective Hamiltonian associated with a probabilistic MQC at $N = 20$ (c) and $N = 100$ (d) with open (blue) and periodic (grey) boundary conditions. In (c) and (d) we tune the classical probability p and compute the effective Hamiltonian, while in (b) we tune $\gamma_{\text{eff}} = p\gamma$ correspondingly. In all plots we have $t_2/t_1 = 0.2$, $L = 50$, $\gamma/t_1 = 2 + 0.2i$, and $Tt_1 = 0.001$.

times $\sim \exp[1/(vT)]$, given $1 \ll N < \exp[1/(vT)]$. Interestingly, the (low-energy) topological properties of $\mathcal{H}_{\text{eff}}^{(N)}(p, \{s\})$ match well those of $\mathcal{H}(t_1, t_2, p\gamma)$ even for fairly small values of N [Figs. 2(b) and 2(c)], while larger values of N increase the coincidence also at higher energies [Figs. 2(b) and 2(d)]. Subsequently, we work in the large N regime and use $\mathcal{H}_{\text{eff}}^{(N)}(p, \{s\}) = \mathcal{H}_{\text{eff}}(p)$, $F_N(p, \{s\}) = F_N(p)$ for convenience. Since the circuit $F_N(p)$ conserves particle number microscopically with each circuit component individually commuting with the particle number operator, it is convenient to block-diagonalize the circuit and work in the single-particle Hilbert space. The topological phase diagram of the circuit can then be obtained from the spectrum of $\mathcal{H}_{\text{eff}}(p)$ using the biorthogonal polarization $\mathcal{P}$ as a topological diagnostic [see Fig. 1(b)].

Monitored state evolution. Thus far, we have obtained the topological phase diagram from the effective Hamiltonian defined using the MQC. A natural question is whether the topology can also be inferred directly from the evolution of states under the MQC? Since the MQC emulates time evolution with a genuine non-Hermitian Hamiltonian, all initial states eventually terminate in a steady state corresponding to the eigenstate with the largest imaginary energy eigenvalue

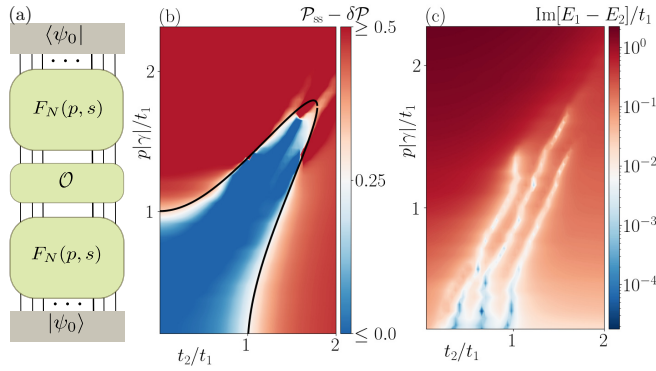


FIG. 3. (a) A schematic of a circuit realizing the biorthogonal steady-state expectation value of the operator $\mathcal{O}$. (b) Biorthogonal polarization $\mathcal{P}_{ss} - \delta\mathcal{P}$ of the steady state and (c) corresponding imaginary gap $\text{Im}(E_1 - E_2)/t_1$, obtained from exact diagonalization [90,91] of a MQC $F_N(p)$ with $N = 150$ cycles. The solid-black line in (b) indicates the analytical result for the non-Hermitian phase transition. Further parameters of the plot are $Tt_1 = 0.001$, $L = 7$, $\gamma/t_1 = 2.5 + 0.4i$.

and a nonzero overlap with the initial state. This can be directly deduced from an eigenstate expansion of the effective non-Hermitian evolution operator

$$F_N(p) : |\psi_0\rangle \mapsto \sum_m \frac{e^{-iNT E_m} |\Psi_{R,m}\rangle \langle \Psi_{L,m} | \psi_0\rangle}{||F_N(p)|\psi_0||}, \quad (8)$$

where we have used the biorthogonal decomposition of $\mathcal{H}_{\text{eff}}(p)$ with complex eigenenergies $\{E_m\}$ ordered according to decreasing imaginary component such that $\text{Im}(E_m) \geq \text{Im}(E_{m+1})$. Clearly, in the limit that $NT\text{Im}(E_1 - E_2) \gg 1$ and assuming $\langle \Psi_{L,1} | \psi_0 \rangle \neq 0$, the initial state $|\psi_0\rangle$ evolves into the steady state $|\Psi_{R,1}\rangle$ under the MQC action, $F_N(p)|\psi_0\rangle \approx |\Psi_{R,1}\rangle \langle \Psi_{L,1} | \psi_0 \rangle$, provided the evolution time NvT is less than the lifetime $\exp[1/(vT)]$ of the effective Hamiltonian.

To reveal the topological properties of the MQC, we want to probe the midgap state, which in the single-particle sector (except for highly fine-tuned initial states) does not correspond to the steady state. Owing to an emergent chiral symmetry of $\mathcal{H}_{\text{eff}}(p)$, this problem can be circumvented at half filling $n = L$, where n is the number of electrons in the chain. The steady state at half filling is a Slater determinant of the eigenstates of $\mathcal{H}_{\text{eff}}(p)$ with imaginary part of the eigenvalue larger or equal to that of the midgap state, and is therefore expected to contain information about the topology of the model. Yet, to extract this inherent information, ordinary expectation values may not suffice: The time evolution associated with the MQC is subject to the non-Hermitian skin effect, rendering ordinary expectation values ineffective as probability currents are not preserved. Instead, we require *biorthogonal* expectation values. A way to design such a biorthogonal steady-state expectation value is through autocorrelators of the MQC. In particular, let us consider the following autocorrelator:

$$\mathcal{A}(\hat{P}, N) = \frac{\langle \psi_0 | F_N(p) \hat{P} F_N(p) | \psi_0 \rangle}{\langle \psi_0 | F_{2N}(p) | \psi_0 \rangle}, \quad (9)$$

corresponding to the circuit of Fig. 3(a) with depth $2N$, consisting of $2NpL$ independent measurements. Notice

that this is twice the number of measurements required for ordinary expectation values.

The denominator ensures biorthogonal normalization. In the limit $NT\text{Im}(E_1 - E_2) \gg 1$ we readily obtain

$$\mathcal{A}(\hat{P}, N) = \langle \psi_0 | \Psi_{R,1} \rangle \langle \Psi_{L,1} | \psi_0 \rangle \mathcal{P}_{ss}, \quad (10)$$

where $\mathcal{P}_{ss} = \langle \Psi_{L,1} | \hat{P} | \Psi_{R,1} \rangle / \langle \Psi_{L,1} | \Psi_{R,1} \rangle$ is the biorthogonal polarization of the steady state. Since the many-body ground state at half filling is a Slater determinant of a topological zero-mode state and remaining bulk single-particle eigenstates, the quantization of $\hat{P}$ gets modified. Subtracting off a constant background $\delta\mathcal{P} = (n-1)/2$ the polarization jumps by 0.5 across a topological phase transition. In Fig. 3(b) we display $\mathcal{P}_{ss} - \delta\mathcal{P}$ for a circuit of $N = 150$ driving cycles at half filling. Importantly, $\mathcal{P}_{ss}$ is able to trace the topology of the system. As a guide to the eye we display the same phase boundary lines as shown in Fig. 1(b), corresponding to the analytically expected topological phase transition.

Compared to the single-particle sector, the many-particle sectors are more prone to instabilities emerging from higher-order corrections to $\mathcal{H}_{\text{eff}}(p)$: Such terms of order $\mathcal{O}(vT)$ or higher involve at least two particles and, typically, they do not respect chiral symmetry. Yet, their impact on the zero-mode is restricted to regions where $\text{Im}(E_1 - E_2) \sim vT$. To avoid such situations for most of the parameter regime [cf. Fig. 3(c)] we have chosen $\gamma \in \mathbb{C}$. For real γ , $\text{Im}(E_1) \neq \text{Im}(E_2)$ can be ensured only with $\gamma > t_2$ at odd length L . The reason for this constraint is readily obtained from perturbation theory in terms of the intracell hopping terms t_2 and γ ($\gamma \in \mathbb{R}$). At $\gamma, t_2 = 0$, the Hamiltonian is Hermitian with the real eigenenergies $\{-t_1, 0, +t_1\}$, where the $\pm t_1$ levels are $(L-1)$ -fold degenerate. Introducing finite γ and t_2 intercell hopping terms lifts this degeneracy where imaginary energies are introduced for $\gamma > t_2$. However, if the $(L-1)$ -fold degenerate subspace is odd, one level does not acquire any shift in first-order perturbation theory so that the imaginary part of its energy remains degenerate with the boundary mode. Thus, at half filling we have $\text{Im}(E_1) - \text{Im}(E_2) = 0$ and the steady-state approach is rendered ineffective. Conceptually, real valued γ is particularly interesting as it leads to a different topological phase diagram as compared to complex valued γ : For real γ one finds three distinct topological phases of which one ($\gamma^2 > t_2^2 + t_1^2$) is genuinely non-Hermitian, i.e., it is *not* adiabatically connected to the Hermitian case and has no Hermitian counterpart. Importantly, this transition necessarily happens with $\gamma > t_2$ and is thus tractable with the steady-state approach using the autocorrelator $\mathcal{A}(\hat{P}, N)$ of the MQC.

Conclusions. In this Letter we have proposed a MQC of free fermions built from interspersed generalized measurements and unitary time evolution that effectively implements a non-Hermitian SSH model. This minimal model embraces all genuine non-Hermitian ingredients including the non-Hermitian skin effect, which makes it challenging to obtain physically meaningful observables able to trace the topology of the system. Yet, the MQC is able to provide such a topological signature since the autocorrelator of the form of Eq. (9) resembles a steady state biorthogonal expectation value.

Note added. A recent paper [92] also studies effective non-Hermitian topological phases arising in monitored quantum

systems. While our physical setup is similar to [92], we study observables of a genuine non-Hermitian topological phase exhibiting the skin effect.

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