Letter

Cooling and state preparation in an optical lattice via Markovian feedback control

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We propose and investigate a scheme based on Markovian feedback control that allows for the preparation of single targeted eigenstates of a system of bosonic atoms in a one-dimensional optical lattice with high fidelity. It can be used for *in situ* cooling the interacting system without particle loss, for both weak and strong interactions, and for experimentally preparing and probing individual excited eigenstates. For that purpose the system is assumed to be probed weakly via homodyne detection of photons that are scattered off-resonantly by the atoms from a structured probe beam into a cavity mode. By applying an inertial force to the system that is proportional to the measured signal, the system is then guided into a pure target state. The scheme is found to be robust against reduced measurement efficiencies.

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Introduction. Atomic quantum gases in optical lattices constitute a unique experimental platform for studying quantum many-body systems under extremely clean and flexible conditions [1]. An important property of these systems is their extremely high degree of isolation from the environment, which is provided by the optical trapping of atoms inside ultrahigh vacuum. It offers the rather unique opportunity to experimentally study coherent nonequilibrium dynamics of many-body systems [2] over hundreds of tunneling times, before eventually residual heating processes make themselves felt. However, suitable dissipation can also be beneficial and the lack of it a limitation. For instance, whenever excitations are created, e.g., by time-dependent parameter variations into a parameter regime of interest, the induced energy remains in the system and causes heating. Here the ability to cool the system after a parameter ramp *in situ* (without particle loss) would be highly desirable. Also the controlled preparation of states beyond the strict constraints of equilibrium in driven-dissipative systems offers intriguing perspectives, like the investigation of transport [3,4], the engineering of non-Hermitian Hamiltonians [5], Bose condensation in nonequilibrium steady states [6,7], etc. Therefore, various routes for engineering dissipation in quantum-gas experiments have been pursued already, such as the engineering of reservoirs [8] and heat baths [9], the implementation of dephasing noise via inelastic scattering of lattice photons [10], or engineered local particle loss/detection via ionization [11].

Here we propose and investigate a scheme for controlled dissipative state preparation and cooling in bosonic optical-lattice systems by means of measurement-based feedback control [12,13]. It is based on the continuous measurement of photons that are off-resonantly scattered by the atoms from a structured probe beam into a cavity mode [14,15] in combination with a simple lattice measurement-conditioned acceleration and can be used to prepare individual eigenstates of the system with high fidelity. Controlling the probe beam as well as the feedback strength allows for engineering an artificial environment with tailored properties. It permits preparing the ground state (i.e., cooling) of the system with high fidelity not only for non- or weakly interacting bosons, but, remarkably, at integer filling also for arbitrarily strong repulsion. Moreover, the approach is capable also of preparing individual excited eigenstates of the system, which would open the possibility to study their statistical properties as they are predicted, e.g., for ergodic systems by the eigenstate thermalization hypothesis (ETH) [16–18].

Measurement-based feedback control has been proposed and used already successfully in other systems for quantum state preparation [19–31], cooling [32–41], simulating nonlinear dynamics [42–44], as well as controlling dynamics [45–50] and phase transitions [51–55], just to name a few. In this paper, we consider Markovian feedback control as proposed by Wiseman [56], where a feedback Hamiltonian proportional to the measurement signal is continuously added to the system. It has been employed already for the stabilization of arbitrary one-qubit quantum states [57,58], the control of two-qubit entanglement [59–62], as well as optical and spin squeezing [55,63,64].

Model. We consider a system of N interacting bosonic atoms in a one-dimensional optical lattice described by the Bose-Hubbard model,

$$H = -J\sum_{l=1}^{M-1}(a_l^\dagger a_{l+1} + a_{l+1}^\dagger a_l) + \frac{U}{2}\sum_{l=1}^{M} n_l(n_l-1), \quad (1)$$

with annihilation and number operators a_l and $n_l = a_l^\dagger a_l$ for bosons on site l. Here J quantifies nearest-neighbor tunneling and U on-site interactions.

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In order to control the system via quantum feedback control, we consider a homodyne measurement of an operator c. The dynamical evolution of the system is then described by the stochastic master equation (SME) [12] ($\hbar = 1$ hereafter), $d\rho_c = -i[H, \rho_c]dt + \mathcal{D}[c]\rho_c dt + \mathcal{H}[c]\rho_c dW$, with $\mathcal{H}[c]\rho := c\rho + \rho c^\dagger - \mathrm{Tr}[(c + c^\dagger)\rho]\rho$ and $\mathcal{D}[c]\rho := c\rho c^\dagger - \frac{1}{2}(c^\dagger c\rho + \rho c^\dagger c)$. Here ρ_c denotes the quantum state conditioned on the measurement result, $I_{\mathrm{hom}} = \mathrm{Tr}[(c + c^\dagger)\rho] + \xi(t)$, with $\xi(t) = dW/dt$ and dW being the standard Wiener increment with mean zero and variance dt. The quantum backaction of a weak measurement can be used for tailoring the system's dynamics and state. For instance, a quantum nondemolition measurement of light allows for the preparation of different types of atom-number squeezed and macroscopic superposition states [15,65]. Even more control is obtained by introducing measurement-dependent feedback.

Here we consider Markovian feedback control, where a feedback term $I_{\mathrm{hom}}F$ proportional to the instantaneous signal is added to the Hamiltonian, giving rise to an evolution governed by the modified SME [56] $d\rho_c = -i[H + H_{\mathrm{fb}}, \rho_c]dt + \mathcal{D}[A]\rho_c dt + \mathcal{H}[A]\rho_c dW$, with operators

$$A = c - iF, \quad H_{\mathrm{fb}} = \tfrac{1}{2}(c^\dagger F + Fc). \tag{2}$$

Taking the ensemble average over the stochastic measurement outcomes gives the feedback-modified ME [56]

$$\dot{\rho} = -i[H + H_{\mathrm{fb}}, \rho] + \mathcal{D}[A]\rho. \tag{3}$$

The effect induced by the feedback loop is seen to replace the collapse operator c by A and to add an extra term H_{fb} to the Hamiltonian. Our goal is to find the proper measurement operator c and feedback operator F so that the steady state of the ME (3) is (as close as possible to) an eigenstate $|E\rangle$ of the Hamiltonian H (1). Reference [66] points out that the ME (3) has a pure steady state if and only if the effective Hamiltonian $H_{\mathrm{eff}} = H + H_{\mathrm{fb}} - iA^\dagger A/2$ and the collapse operator A have a common eigenstate, which is then the steady state. Since the additional feedback-induced term in the Hamiltonian H_{fb} is proportional to the measurement strength, for weak measurement, we can safely neglect H_{fb} relative to H and construct a collapse operator A so that $A|E\rangle = 0$. The dissipator cannot be neglected since it is the only dissipative term. This strategy is confirmed in our simulations below, where H_{fb} is fully taken into account. In the following we will first discuss the preparation of the ground state $|G\rangle$, before considering also excited eigenstates.

Feedback scheme for noninteracting case. In order to get an insight of how to choose the measurement and feedback operator, let us first take a look at the double-well system ($M = 2$). It can be mapped to a spin-$N/2$ system [67] with the collective spin operators defined as $J_x = \frac{1}{2}(a_1^\dagger a_2 + a_2^\dagger a_1)$, $J_y = -\frac{i}{2}(a_1^\dagger a_2 - a_2^\dagger a_1)$, and $J_z = \frac{1}{2}(a_1^\dagger a_1 - a_2^\dagger a_2)$, which are angular momentum operators satisfying the commutation relations $[J_\mu, J_\nu] = i\epsilon_{\mu\nu\sigma}J_\sigma$ for $\mu, \nu, \sigma = x, y, z$ and the Levi-Civita symbol $\epsilon_{\mu\nu\sigma}$. The ground state of the noninteracting Hamiltonian $H_2 = -J(a_1^\dagger a_2 + a_2^\dagger a_1) = -2JJ_x$ is the eigenstate of J_x, $|j = N/2, m = N/2\rangle$, with the largest eigenvalue $m = N/2$. From the theory of angular momentum, we know that this state is also an eigenstate of operator $J_- = J_z - iJ_y$ with eigenvalue 0. Hence, by choosing the measurement operator c as $\sqrt{\gamma}J_z$ and feedback operator F as $\sqrt{\gamma}J_y$, so that $A = \sqrt{\gamma}(J_z - iJ_y)$, we will reach a steady state very close to the ground state of the Hamiltonian H_2 for weak measurement with strength $\gamma \ll J$.

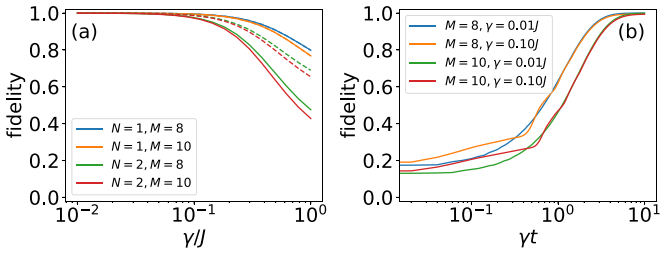


FIG. 1. (a) Fidelities f (solid) and $\tilde{f}$ (dashed) between the steady state of Eq. (3) and the ground state of Eq. (1) vs measurement strength γ. (b) Fidelity between the instantaneous state and the ground state of Eq. (1), starting from the state with one particle on the leftmost site.

Generalizing this scenario to the multisite case, we consider the measurement and feedback operator

$$c = \sqrt{\gamma}\sum_{l=1}^{M} z_l n_l, \quad F = \sqrt{\gamma}\sum_{l=l}^{M-1}(ia_{l+1}^\dagger a_l - ia_l^\dagger a_{l+1}). \tag{4}$$

Here c is a weighted sum of site occupations and can be implemented via homodyne detection of the off-resonant scattering of structured probe light from the atoms [14,15,68]. The detection efficiency can be enhanced by placing the system inside an optical cavity [14,50,69–71], so that via the Purcell effect [72] the photons are scattered predominantly into one cavity mode. The feedback F is given by an imaginary component of the tunneling parameter and can be realized by accelerating the lattice, as discussed below. For the noninteracting case ($U = 0$), the ground state of (1) reads $|G\rangle = \frac{1}{\sqrt{N!}}(\sum_l g_l a_l^\dagger)^N|0\rangle$ with $g_l = \sqrt{\frac{2}{M+1}}\sin(\frac{\pi l}{M+1})$. The condition $A|G\rangle = (c - iF)|G\rangle = 0$ then fixes the coefficients in the measurement operator to $z_l = (g_{l+1} - g_{l-1})/g_l$.

In order to check the proposed scheme, we calculate the fidelity f between the steady state ρ_{ss} of Eq. (3) and the ground state $|G\rangle$ of Eq. (1), i.e., $f = \sqrt{\langle G|\rho_{\mathrm{ss}}|G\rangle}$, which takes values between 0 and 1. To compare results for different particle numbers, we also introduce the fidelity "per particle" $\tilde{f} \equiv f^{1/N}$. Figure 1(a) shows the fidelity as a function of the measurement strength γ. As expected, for small enough γ, H_{fb} has no detrimental impact and a close-to-one fidelity is approached [73]. This is confirmed when considering the excitation energy $\Delta E = \mathrm{Tr}[\rho_{\mathrm{ss}}H] - \langle G|H|G\rangle$ as a probe of deviations from the target state [74]. In the presence of an external trapping potential, the scheme can directly be adapted, simply by replacing the coefficients g_l by the discrete single-particle ground-state wave function of trapped system and even without being adapted it is robust against weak potential modulations (see [74] for more details).

The coupling γ also influences the time required to reach the steady state. For one particle in a double-well system ($N = 1$, $M = 2$), it is easy to verify that $f^2 = 1 - \frac{1}{2}e^{-4\gamma t}$ [74]. Hence, a close-to-one fidelity can be reached at $\gamma t \gtrsim 1$. In Fig. 1(b) we show the time-dependent fidelity

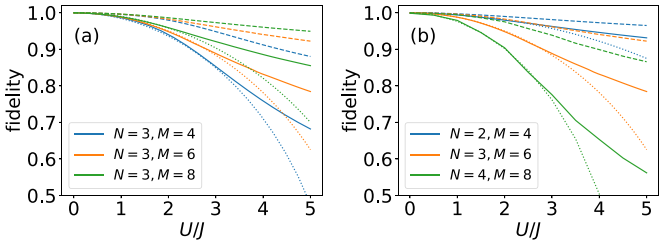


FIG. 2. Steady-state fidelities f (solid) and $\tilde{f}$ (dashed) vs interaction strength U for different site numbers M at (a) $N = 3$ and (b) half filling $N = M/2$, with $\gamma = 0.01J$. The dotted lines denote the fitting curves $f^2 = 1 - \chi[(nU)/(2\Delta E)]^2$ with χ being the fitting parameter [74].

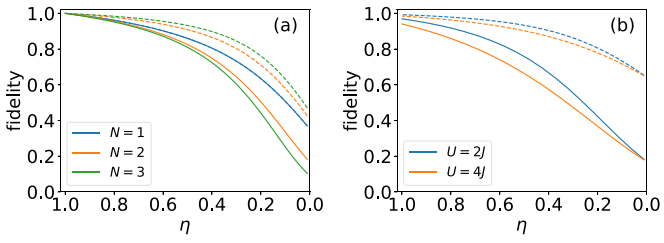


FIG. 4. Steady-state fidelities f (solid) and $\tilde{f}$ (dashed) as a function of the detection efficiency η for (a) noninteracting case with different particle number N at $M = 8$ and for (b) interacting case with $M = N = 4$ and $\gamma = 0.01J$.

$f = \sqrt{\langle G|\rho(t)|G\rangle}$ as a function of the scaled time γt for $N = 1$. For $\gamma t \gtrsim 1$ the curves for different γ collapse and quickly approach the steady state. From Fig. 1 we can infer that $\gamma \lesssim 0.1J$ is sufficient to achieve a high fidelity, corresponding to a preparation in a few tens of tunneling times. For a larger system (with larger M or N) smaller γ and thus also longer times are required [74].

Without any modification the feedback scheme still performs well in the presence of weak interactions $U \lesssim J$, as can be seen from Fig. 2 showing the steady-state fidelity versus U. While for fixed total particle number N the scheme works better for larger system sizes (M) [Fig. 2(a) for $N = 3$], the opposite trend is observed when instead fixing the filling $n = N/M$ [see Fig. 2(b) for $n = 1/2$].

Feedback scheme for interacting particles. To better capture the impact of interactions, we adapt the feedback scheme by adding a prefactor λ to the feedback operator F, so that the collapse operator becomes

$$A = c - i\lambda F. \tag{5}$$

The choice of λ maximizing the fidelity is shown in Fig. 3(a) as a function of U. The corresponding fidelity is shown in Fig. 3(b). We can observe that in the case of integer filling (orange line), this simple rescaling of the feedback strength allows to achieve fidelities close to unity for all interaction strengths. The fact that it is possible to prepare the ground state of an interacting system for every interaction strength, including the strongly correlated regime of intermediate interactions close to the Mott transition [75], is remarkable, especially since we are considering only single-particle (i.e., quadratic) measurement and feedback operators c and F and since it is sufficient to vary a single parameter λ to cover the whole range of interactions.

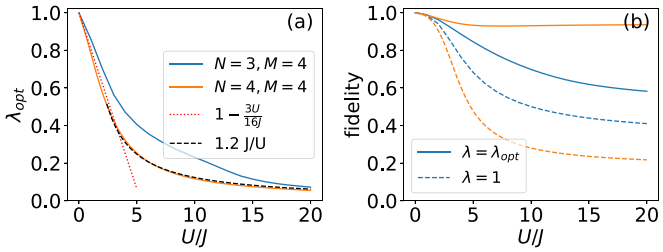


FIG. 3. (a) The optimal parameter λ which gives the highest fidelity between the steady state of Eq. (3) and the ground state of Eq. (1) as a function of the interaction strength U. The corresponding fidelity f is shown in (b) for $\gamma = 0.01J$.

This choice (5) can again be motivated by inspecting the double-well case. For weak interactions, the ground state is given by $|G\rangle \simeq |\psi_0^{(0)}\rangle - \alpha|\psi_2^{(0)}\rangle$ where $|\psi_n^{(0)}\rangle$ denotes the nth eigenstate of the noninteracting Hamiltonian and $\alpha = \sqrt{2N(N-1)}(U/J)/16$. Since $A|G\rangle \propto [\sqrt{N}(1-\lambda) - \alpha\sqrt{2(N-1)}(1+\lambda)]|\psi_1^{(0)}\rangle$, the cooling condition can be satisfied by setting $\lambda = 1 - (U/J)(N-1)/(4M)$, which is a linear function of U/J. For integer filling, the ansatz (5) is also suitable for strong interactions. The ground state is then approximated by $|G\rangle \simeq |n, n\rangle + (nJ/U)(|n+1, n-1\rangle + |n-1, n+1\rangle)$ with integer $n = N/2$. This gives $A|G\rangle = \sqrt{\gamma}[n_1 - n_2 - \lambda(a_1^\dagger a_2 - a_2^\dagger a_1)]|G\rangle \simeq \sqrt{\gamma}n(2J/U - \lambda)(|n+1, n-1\rangle - |n-1, n+1\rangle)$. Therefore, by setting $\lambda = 2J/U$, we have $A|G\rangle \simeq 0$, which leads to a steady state close to the ground state of the system. For half-integer filling (odd N), the condition $A|G\rangle \simeq 0$ does not hold for any λ [74]. Generalizing this reasoning to M sites, we again find $\lambda \propto U/J$ for weak interactions and for integer filling also $\lambda \propto J/U$ for strong interactions [dashed and dotted lines in Fig. 3(a)].

Imperfect detection efficiency. So far, we have assumed perfect detection, $\eta = 1$. For an efficiency $\eta < 1$, the ME becomes [56] $\dot{\rho} = -i[H + H_{\rm fb}, \rho] + \mathcal{D}[A]\rho + \frac{1-\eta}{\eta}\mathcal{D}[F]\rho$. In Fig. 4 we investigate the robustness against a reduction of η. For both the noninteracting [Fig. 4(a)] and the interacting [Fig. 4(b)] cases, the fidelity is found to decrease slowly with η. Roughly speaking, large fidelities require efficiencies larger than 50%.

Stabilizing excited eigenstates. It is also possible to prepare excited eigenstates of the system. For the single-particle problem, the nth eigenstate reads $|\psi_n\rangle = \sum_l g_l^{(n)}|l\rangle$, with $g_l^{(n)} = \sqrt{\frac{2}{M+1}}\sin\left(\frac{n\pi l}{M+1}\right)$, with $n = 1, 2, \ldots, M$. We can stabilize each of these eigenstates as steady state by using the feedback and measurement operators (4) with $z_l = [g_{l+1}^{(n)} - g_{l-1}^{(n)}]/g_l^{(n)}$. This scheme can be generalized to N noninteracting particles to prepare the nth "coherent" eigenstate $|n, N\rangle = \frac{1}{\sqrt{N!}}\left(\sum_l g_l^{(n)} a_l^\dagger\right)^N|0\rangle$, where all particles occupy the nth single-particle eigenstate. Figure 5(a) shows the fidelities between the steady state and $|n, N\rangle$ versus γ. By symmetry states $|n, N\rangle$ and $|M - n + 1, N\rangle$ show the same behavior. As expected, the fidelity increases and approaches 1 as γ decreases.

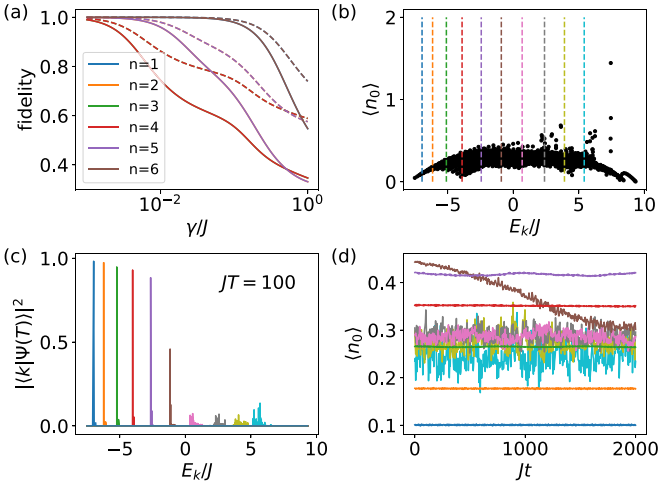


FIG. 5. (a) Fidelities f (solid) and $\tilde{f}$ (dashed) between the steady state of Eq. (3) and the nth "coherent" eigenstate $|n, N\rangle$ vs γ for $N = 2$, $M = 6$, $U = 0$. (b) The expectation value of n_0 in the eigenstates at $U = J$. The dashed lines mark the states corresponding to $|n, N\rangle$. (c) The distribution of the final state from adiabatic preparation (with the interaction strength U linearly ramped up from 0 to J within $JT = 100$ starting from the corresponding "coherent" eigenstate at $U = 0$) on the eigenbasis at $U = J$. (d) Evolution of $\langle n_0\rangle$ under the Hamiltonian (1) with $U = J$ starting from the final state from adiabatic preparation. $N = 4$ and $M = 15$ in (b)–(d).

The preparation of states closer to the center of the spectrum requires smaller γ.

The preparation of excited eigenstates of the interacting system offers the intriguing perspective to study their individual properties experimentally. This would allow one, for instance, to directly probe ETH [16–18] stating that each individual eigenstate of a generic ergodic system is characterized by expectation values that follow the prediction of the microcanonical ensemble. In Fig. 5(b) we plot the mean occupations $\langle n_0\rangle$ of the leftmost site for all eigenstates of a system with $N = 4$, $M = 15$, and $U = J$, and one can see that the data points mostly form a rather thin stripe, confirming that (up to finite-size effects) the expectation values depend mainly on the energy. Since it turns out that the stabilization of excited eigenstates via feedback control is difficult for the interacting system, we propose to use feedback control for the preparation of noninteracting eigenstates $|n, N\rangle$ and to then linearly switch on the interactions via a Feshbach resonance in a second step. In Fig. 5(c) we plot the overlap of the time evolved states for a ramp time of $T = 100/J$ with the interacting many-body eigenstates $|k\rangle$. The energy expected for perfect adiabatic behavior is indicated by the vertical dashed lines of the same color in Fig. 5(b). While for initial coherent eigenstates with smaller energy, we find a very narrow distribution with almost only a single eigenstate occupied, for larger energies broader distributions indicate nonadiabatic behavior. We attribute the latter to the dynamic instability caused by the combination of repulsive interactions with negative effective mass found for states with large kinetic energy [76]. Even broader distributions are found [74] when preparing the state via a quench from a state with every other site occupied like in a recent experiment [77]. After the interaction ramp, we let the prepared state evolve with the interacting Hamiltonian and plot the mean-occupation versus time [Fig. 5(d), colors like in Fig. 5(c)]. The fact that we hardly see any evolution of that expectation value for the states with the lower energies shows how close they are to an eigenstate. In turn, for the prepared states in the upper half in the spectrum we find strong oscillations indicating the superposition of a few eigenstates.

Experimental implementation. By including the feedback control term $I_{\rm hom}F$ to the Hamiltonian (1), the parameter for tunneling (rightwards) is modified to $J' = J + i\sqrt{\gamma}I_{\rm hom} = \sqrt{J^2 + \gamma I_{\rm hom}^2}e^{i\theta(t)}$, with $\tan\theta(t) = \sqrt{\gamma}I_{\rm hom}/J$. This can be achieved by accelerating the lattice potential $V_L(x)$ according to $V(x,t) = V_L[x - d_L\xi(t)]$, with d_L being the lattice constant. In the reference frame comoving with the lattice, the corresponding inertial force is described by a time-dependent vector potential represented by the time-dependent Peierls phase $\theta(t) = md_L^2\dot{\xi}(t)$ attached to the tunneling matrix element [78]. Now choose $\dot{\xi}(t) = \arctan(\sqrt{\gamma}I_{\rm hom}/J)/(md_L^2) = \frac{2E_R}{\pi^2}\arctan(\sqrt{\gamma}I_{\rm hom}/J)$ with $E_R = \pi^2/(2md_L^2)$. The drift of lattice $\xi(t)$ is dependent on the homodyne current $I_{\rm hom}$. Simulations of the modified SME show that at the time the steady state is reached it is on the order of a lattice constant and can be reduced further (without loosing fidelity) by postselection [74].

Conclusion. We have proposed and investigated a simple scheme for measurement-based feedback control of a system of ultracold bosonic atoms in an optical lattice. Remarkably, at integer filling a simple modification of the feedback strength allows for cooling the system at arbitrary repulsive interaction strengths. Moreover, we showed that it is possible to prepare individual excited eigenstates, which would allow one to experimentally study their statistical properties as they are predicted by the ETH. As an outlook, it will be interesting to generalize the scheme predicted here to scenarios, where *in situ* cooling would be crucial, in order to counteract heating caused by Floquet engineering or imperfect adiabatic state preparation at a quantum phase transition, e.g., for the preparation of topological quantum states. Moreover, also the engineering of artificial thermal baths via feedback control, mimicking finite-temperature reservoirs, and the combination of two of them to study heat-current-carrying steady states in a highly controllable quantum gas is a fascinating perspective. Generally, it will be interesting to employ feedback control to realize driven-dissipative systems and to control the properties of their nonequilibrium steady states (or phases) beyond the constraints of thermal equilibrium.

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[73] For odd site number M, the steady state is not unique due to

degeneracies in the spectrum. However, by adding tiny interactions a unique steady state is recovered.

[74] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevResearch.4.L022045 which includes Ref. [79,80], for the calculation of the fidelity for one particle in a double well, discussion of the nonunique steady state for odd site number, energy difference between the steady state and the ground state, feedback scheme in the presence of an external potential, the robustness of the free particle scheme against disorder, the system size dependence of the measurement strength, perturbation theory for the double-well system under weak interactions, discussion about the double-well system with odd particle numbers and multisite case (general M) at integer filling, state distribution in the eigenbasis, and drift of lattice.

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