

Diffusion with local resetting and exclusion

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Stochastic resetting models diverse phenomena across numerous scientific disciplines. Current understanding stems from the renewal framework, which relates systems subject to *global* resetting to their nonresetting counterparts. Yet, in interacting many-body systems, even the simplest scenarios involving resetting give rise to the notion of *local* resetting, whose analysis falls outside the scope of the renewal approach. A prime example is that of diffusing particles with excluded volume interactions that *independently* attempt to reset their position to the origin of a one-dimensional lattice. With renewal rendered ineffective, we instead employ a mean-field approach whose validity is corroborated via extensive numerical simulations. The emerging picture sheds first light on the nontrivial interplay between interactions and resetting in many-body systems.

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I. INTRODUCTION

As allegorized in the popular nursery rhyme on a small spider's Sisyphean attempt to ascend a waterspout during questionable weather, surmounting a long and volatile endeavor is often quite challenging and full of setbacks. Similar setbacks also arise in many natural phenomena that are prone to “resetting”: the abrupt cessation of a dynamical process, which consecutively starts anew. Examples for resetting phenomena are abundant and come from a diverse range of research fields including search, first-passage, and animal foraging [1–10]; algorithmics [11–13]; optimization [14–18]; and reaction kinetics [19–25]. The renewal framework constitutes the current state-of-the-art in the analysis of resetting phenomena [6,7,26]. This powerful approach utilizes the understanding that a resetting event forces the system back to its initial state to formulate stochastic renewal equations, relating the properties of resetting systems to those of their nonresetting counterparts. Such relations have proven salient in uncovering steady-state, transport, relaxation, and first-passage properties in a long list of model systems [27–46]. Moreover, they have been instrumental in the discovery of universal phenomena that emerge as a result of resetting [6,7,9,15,47–51].

In spite of its centrality to the study of resetting phenomena, the renewal framework's applicability is limited. Specifically, in many-body systems, it only applies when

resetting acts to bring the *entire* system back to its initial configuration, a process which we hereby term “global resetting” [52–54]. Yet, generically, resetting will have a more local nature that can, in turn, be affected by interactions. As a concrete example, consider diffusive particles, subject to repulsive short-ranged interactions that move along a one-dimensional (1D) track immersed in a fluid. In this case, resetting can be used to model the unbinding of a particle from the track and its subsequent rebinding at a uniquely favorable location. However, due to interactions, rebinding cannot occur if this location is already occupied by a different particle. This, in fact, is precisely the picture that arises in biopolymerization, where RNA polymerases and ribosomes play the role of resetting particles. Since a resetting event leaves the system's configuration mostly intact, aside from the new position of the resetting particle, the renewal framework does not apply and alternative approaches must be sought. The absence of a unifying theoretical framework renders the study of “local” resetting in interacting many-body systems extremely challenging.

In this Letter, we take a first step toward establishing an understanding of local resetting within an analytically tractable model. In particular, we study a 1D ring lattice of L sites, occupied by diffusive particles which interact via volume exclusion and *independently* attempt to reset their position to the origin site, if it is vacant (see Fig. 1). These dynamics give rise to a nontrivial steady-state density profile, whose analysis lies well beyond the scope of renewal theory. Instead, taking a mean-field (MF) approach, we are able to derive a closed-form solution for the stationary density profile and analyze its scaling properties in the limit of large L . The MF description is corroborated against extensive numerical simulations to remarkable accuracy. The results established herein, which are expected to directly generalize to higher spatial dimensions, pave the way to the extension of current

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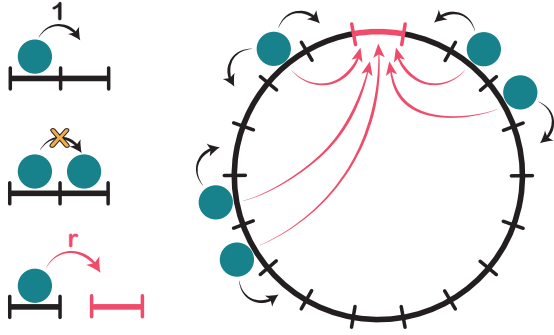


FIG. 1. An illustration of diffusion with exclusion and local resetting on a ring lattice. Each particle *attempts* to hop to its left and right neighboring sites with rate 1 and to reset its position to the origin with rate r . In both cases, an attempt is successful only if the target site is vacant.

experimental studies of single-particle resetting [55,56], to interacting many-body systems.

II. THE MODEL

Consider a 1D periodic lattice of L sites labeled $\ell = 0, \dots, L-1$, occupied by N particles of average density $\bar{\rho} \equiv N/L$. The particles are subject to hard-core, exclusion interactions by which each site may hold one particle at most [57–60]. The system then evolves in continuous time via the dynamical rules illustrated in Fig. 1: Each particle *attempts* to hop to its left and right neighboring sites with rate 1 and to reset its position to site $\ell = 0$ with rate r . In both cases, an attempt is successful only if the target site is vacant.

These dynamics can be formulated in terms of a Markov chain. Let $\tau_\ell(t)$ denote the occupation of site ℓ at time t , taking the value 0 if the site is vacant and 1 otherwise. The model's dynamics imply the following evolution of $\tau_\ell(t)$,

$$\tau_\ell(t + dt) - \tau_\ell(t) = \Gamma_\ell(t), \quad (1)$$

where $\Gamma_\ell(t)$ is given by

$$\Gamma_\ell(t) = \begin{cases} \tau_{\ell\pm 1} & \text{w.p. } (1 - \tau_\ell)dt \\ -\tau_\ell & \text{w.p. } (1 - \tau_{\ell\pm 1})dt \\ \sigma_\ell & \text{w.p. } (1 - \tau_0)R_\ell dt \end{cases} \quad (2)$$

and w.p. denotes “with probability.” Clearly, a distinction must be made between site $\ell = 0$, which experiences an influx of resetting particles and the remaining sites $\ell \neq 0$. Equation (2) for $\Gamma_\ell(t)$ indeed serves as short-hand notation for

$$\begin{aligned} \sigma_\ell &= -1 \text{ and } R_\ell = r\tau_\ell & \text{for } \ell \neq 0 \\ \sigma_\ell &= +1 \text{ and } R_\ell = r \sum_{m=1}^{L-1} \tau_m & \text{for } \ell = 0. \end{aligned} \quad (3)$$

Note that the expression for R_0 is not arbitrary, originating from the model's particle conservation, i.e., $\sum_{\ell=0}^{L-1} [\tau_\ell(t + dt) - \tau_\ell(t)] = 0$.

III. MAIN RESULTS

Using an analytical MF approach and extensive numerical simulations, we show that the stationary density profile of

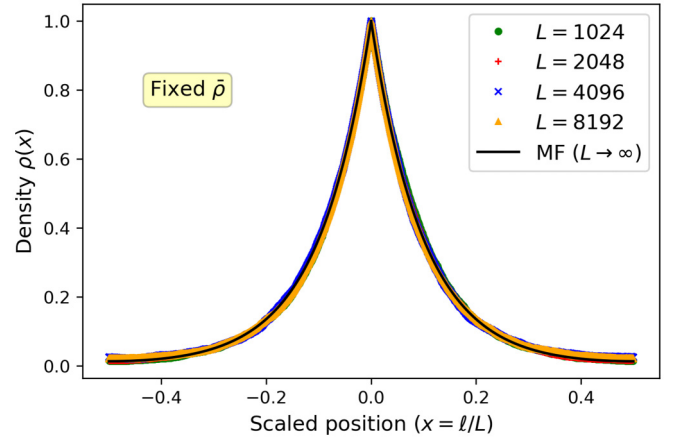


FIG. 2. Data collapse of the density profile for $r = 1$ and $\bar{\rho} = 0.2$ versus the scaling variable $x = \ell/L$. The different markers denote various values of the system size L while the solid black curve denotes the theoretical MF prediction in Eq. (12) for $L \rightarrow \infty$. Note that the site indices ℓ have been shifted to $\ell = -L/2 + 1, \dots, 0, \dots, L/2$ for convenience of presentation. Correspondingly, $x \in (-0.5, 0.5]$.

resetting and interacting random walkers behaves very differently from that found in the absence of interactions. As in the noninteracting case, resetting acts to concentrate particles at the origin site $\ell = 0$. However, here exclusion prevents the origin from being occupied by more than a single particle. Moreover, resetting cannot occur when the origin is occupied, giving rise to a nontrivial interplay among exclusion, diffusion, and resetting.

For a fixed mean density $\bar{\rho}$, the model's exclusion interactions and diffusive dynamics yield a density profile that is a scaling function of ℓ/L , asserting that the density substantially deviates from $\bar{\rho}$ throughout the *entire* system. This stands in stark contrast to the noninteracting picture, where the width of the profile is $\sim \sqrt{D/r}$, with D denoting the diffusion coefficient [2]. Here we find that, for large systems, the density profile is entirely independent of the resetting rate r . In fact, the only external parameter which enters the profile is the mean density $\bar{\rho}$, as seen in Eqs. (12)–(14). A different picture emerges when the number of particles $N = \bar{\rho}L$ is instead kept *fixed*, in which case the density profile is shown to be a function of ℓ alone, as obtained in Eq. (9). These MF predictions are verified, to remarkable precision, in data collapses obtained from extensive numerical simulations. Figure 2 demonstrates the density profile's scaling form for a fixed mean density while Fig. 3 shows its behavior for fixed particle number N (additional data for different parameters and a comparison to the case of a single resetting particle are provided in Ref. [61]).

IV. MF ANALYSIS

To evoke the MF approximation, we first average over the Markov chain in Eqs. (1) and (2), replacing the mean occupation $\langle \tau_\ell \rangle$ by the corresponding density field $\rho_\ell \in [0, 1]$. The MF approximation is then manifested in the factorization of products of the form $\langle \tau_\ell \tau_k \rangle \approx \langle \tau_\ell \rangle \langle \tau_k \rangle \rightarrow \rho_\ell \rho_k$. The rates $R_{\ell=0}$ and $R_{\ell \neq 0}$ in Eq. (3), respectively, become

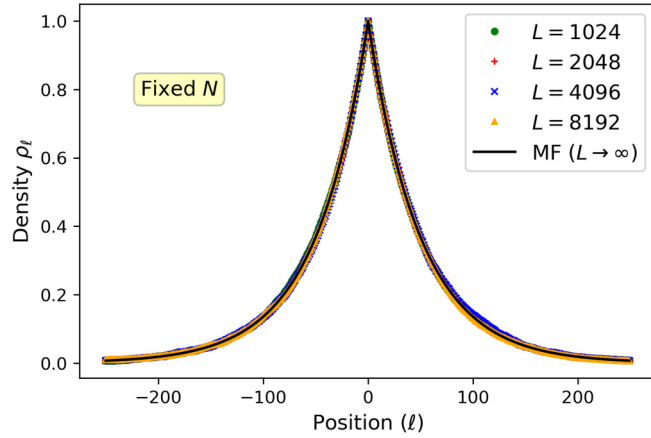


FIG. 3. Data collapse of the density profile for $r = 1$ and $\bar{\rho}L = N = 100$ plotted versus the lattice index ℓ for 250 sites to the left and right of the origin. The different markers denote various values of the system size L while the solid black curve denotes the theoretical MF prediction in Eq. (9) for fixed N . Note that the site indices ℓ have been shifted to $\ell = -L/2 + 1, \dots, 0, \dots, L/2$ for convenience of presentation.

$\langle R_{\ell=0} \rangle = r \sum_{m=1}^{L-1} \rho_m$ and $\langle R_{\ell \neq 0} \rangle = r \rho_\ell$. While it is generally quite difficult to rigorously justify the MF approximation, this approach has proven to be remarkably successful for analyzing numerous lattice models with exclusion interactions [62–68]. As we demonstrate below, MF also provides a remarkably accurate description of the present model.

In the limit $dt \rightarrow 0$, the MF approximation yields the following equation for $\rho_\ell(t)$,

$$\partial_t \rho_\ell = \rho_{\ell+1} - 2\rho_\ell + \rho_{\ell-1} + \sigma_\ell(1 - \rho_0)\langle R_\ell \rangle. \quad (4)$$

Our interest lies in the stationary behavior of the density profile. To this end, we set $\partial_t \rho_\ell = 0$ and *separately* analyze Eq. (4) at sites $\ell \neq 0$, termed the “bulk” equation, and at site $\ell = 0$, which we call the “boundary” equation. Solving the stationary bulk equation gives

$$\rho_\ell = c_1 A_-^\ell + c_2 A_+^\ell, \quad (5)$$

where $c_{1,2}$ are constants and we have defined

$$A_\pm = 1 + \frac{a}{2} \left(1 \pm \sqrt{1 + \frac{4}{a}} \right), \quad (6)$$

and

$$a = r(1 - \rho_0). \quad (7)$$

Using the dynamic’s symmetry around site $\ell = 0$, i.e., $\rho_\ell = \rho_{L-\ell}$, gives $c_2 = c_1 A_+^{L-\ell}$. The particle conservation condition

$$N = \bar{\rho}L = \rho_0 + \sum_{\ell=1}^{L-1} \rho_\ell \quad (8)$$

is then used to determine c_1 , such that ρ_ℓ becomes

$$\rho_\ell = \frac{(1 - A_-)(L\bar{\rho} + a/r - 1)}{2(A_- - A_-^L)} (A_-^\ell + A_+^{\ell-L}). \quad (9)$$

Although we have obtained a formal solution for the MF density profile ρ_ℓ , our work is not yet done since ρ_ℓ still

depends on the density at site $\ell = 0$ through a in Eq. (7). To determine ρ_0 , we revisit Eq. (4) for the density profile and consider its behavior at $\ell = 0$. In the stationary limit, this equation can be written as

$$\rho_1 = \rho_0 - \frac{r}{2}(1 - \rho_0)(N - \rho_0), \quad (10)$$

where $N - \rho_0 = \sum_{m=1}^{L-1} \rho_m$ and we have again used the $\ell \rightarrow L - \ell$ symmetry to replace $\rho_1 + \rho_{L-1} \rightarrow 2\rho_1$. To make progress, we separately consider two distinctly different physical scenarios: the case of a constant particle density $\bar{\rho}$ and the case of a constant particle number N .

V. FIXED DENSITY $\bar{\rho}$

In this case the number of particles in the system N grows linearly with system-size L . For large L , and a correspondingly large number of particles, there is a small time gap between the instance site $\ell = 0$ is vacated by a particle hopping to a vacant neighboring site and the time it is reoccupied due to a resetting event. Reoccupation due to hopping from neighboring sites is negligible, since it is attempted with rate 1 while resetting events are attempted with rate $L\bar{\rho}r \equiv Nr \gg 1$. In the limit of $L \rightarrow \infty$, where the system contains infinitely many particles, the total rate of resetting attempts becomes infinite and reoccupation is *immediate*. We thus expect that the density near the origin be unity, up to small finite- L corrections. We thus consider the ansatz

$$\begin{aligned} \rho_0 &\cong 1 - (\alpha/L)^\mu \\ \rho_1 &\cong 1 - (\beta/L)^\nu, \end{aligned} \quad (11)$$

generally allowing for a different scaling with L at sites $\ell = 0$ and $\ell \neq 0$. Substituting this ansatz into Eq. (10) yields the relation $\mu = 1 + \nu$. We can then determine the value of ν by substituting Eq. (11) into Eq. (9) for ρ_ℓ , evaluated at site $\ell = 1$. This self-consistency requirement sets $\nu = 1$ and, correspondingly, $\mu = 2$ (see Ref. [61] for additional details and a complementary approach for determining the scaling of ρ_0). With this, we obtain the density profile at sites $\ell \neq 0$ as

$$\rho_\ell \cong 2^{-1} \alpha \bar{\rho} (e^\alpha - 1)^{-1} [e^{\alpha(L-\ell)/L} + e^{\alpha\ell/L}]. \quad (12)$$

The parameter α is determined by demanding that the ansatz for ρ_0 in Eq. (11) be consistent with the density profile ρ_ℓ in Eq. (12) at site $\ell = 0$. This gives

$$\bar{\rho} \alpha \coth[\alpha/2] = 2, \quad (13)$$

providing a transcendental equation for α that we must numerically solve, given the value of $\bar{\rho}$. With this, the density profile assumes the simpler form

$$\rho_\ell \cong \cosh \left[\frac{\alpha}{2} \left(\frac{L - 2\ell}{L} \right) \right] / \cosh \left[\frac{\alpha}{2} \right]. \quad (14)$$

Although the relation between α and $\bar{\rho}$ cannot be inverted analytically, we can still obtain important insight regarding the dependence of α on $\bar{\rho}$ in the asymptotic limits of $\alpha \rightarrow 0$ and $\alpha \rightarrow \infty$. A straightforward analysis yields

$$\bar{\rho} \cong \begin{cases} 1 - \alpha^2/12 & \alpha \ll 1 \\ 2/\alpha & \alpha \gg 1 \end{cases}, \quad (15)$$

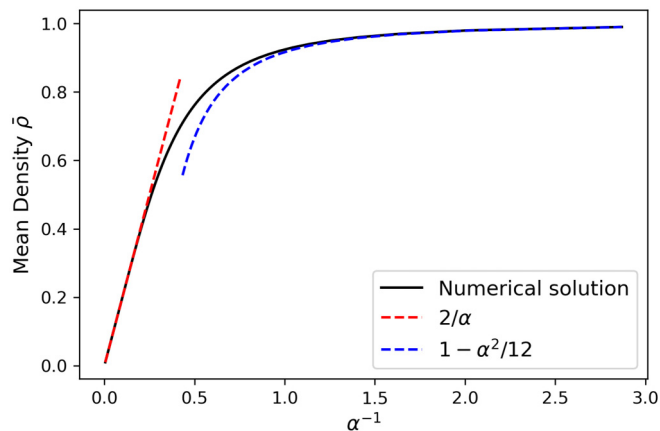


FIG. 4. Plot of $\bar{\rho}$ versus α , as obtained from Eq. (13). The solid black curve is the numerical solution of the equation, the dashed red curve is the large α approximation and the dashed blue curve describes the small α limit.

as demonstrated in Fig. 4. We conclude that for a fixed mean density $\bar{\rho}$, ρ_ℓ becomes a scaling function of ℓ/L , at large L . This implies that the density profile spans the entire system, as is clearly observed in Fig. 2. This scaling behavior also appears for $r \propto L$, as shown in Ref. [61].

VI. FIXED PARTICLE NUMBER N

We next consider the case where the particle number N is fixed, finite, and independent of L . The time gap between the instance site $\ell = 0$ is vacated and the instance it is reoccupied now remains *finite*, even as L is increased. Correspondingly, both ρ_0 and ρ_1 are asymptotically independent of L . With this insight, we return to Eq. (9) in order to relate ρ_ℓ at $\ell = 0$ to ρ_0 for finite N , in the limit of large L . Substituting $\bar{\rho} = N/L$ and $a = r(1 - \rho_0)$ into Eq. (9), setting $\ell = 0$ and equating to ρ_0 asymptotically yields the polynomial equation

$$\frac{r(1 - \rho_0)(N - \rho_0)}{\sqrt{r(1 - \rho_0)[r(1 - \rho_0) + 4]} - r(1 - \rho_0)} \cong \rho_0, \quad (16)$$

where we have used the fact that A_+^{-L} and A_-^L both vanish as $L \rightarrow \infty$, for any L -independent a [see Eq. (6)]. This equation for ρ_0 has three roots, of which only one satisfies $\rho_0 \in [0, 1]$. While its precise analytical form is rather involved and is thus deferred to [61], in the limit $1 \ll N \ll L$, where the system contains many particles but the average density is low, it reduces to $\rho_0 = 1 - 4r^{-1}N^{-2} + \mathcal{O}(N^{-4})$. By Eq. (7), this translates into $a \cong 4/N^2$ which, when substituted into Eq. (9), reveals that the density profile has a width of $\sim N/2$ around site $\ell = 0$. Since N is fixed, the density profile remains a function

of ℓ alone, implying that it only extends over a finite $\sim \mathcal{O}(N)$ region near the origin, as demonstrated in Fig. 3.

VII. CONCLUSIONS

In this Letter, we have gone beyond the popular renewal framework to study the effect of *local* resetting on a many-body interacting system. Employing a MF approach, we derived the stationary density profile of N particles diffusing on a 1D ring lattice of L sites while being subject to exclusion interactions and local resetting with rate r . In the thermodynamic limit with a fixed density, when the system is occupied by infinitely many particles, we find that the profile's width becomes entirely independent of r , being solely a function of the position and average density $\bar{\rho}$. This intriguing behavior directly follows from the delicate interplay between local resetting and exclusion interactions, which prohibit resetting if the origin is already occupied. In contrast, for noninteracting particles the density profile is known to adopt a width of $\sim r^{-1/2}$ [2], irrespective of $\bar{\rho}$ or N . Additional comparison which elucidates the relation between the model studied herein and the case of a *single* resetting particle is provided in Ref. [61].

Two recent experimental studies explored resetting in the context of diffusive single-particle systems, clearly marking interacting many-body systems as the next frontier [55,56]. A coupling between local resetting and interactions is one complication that will surely arise in such systems. Since volume exclusion is the prototypical model for short-range interactions, the results established herein are especially well posed to serve as a benchmark for comparison. Moreover, the approach we presented can be extended further, as required, to capture more realistic scenarios and higher spatial dimensions. The same approach can also be applied to additional exclusion-type models, such as models with asymmetric hopping rates and additional types of interactions. More realistic resetting protocols are also interesting to consider in this regard.

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