

Stability of algebraic spin liquids coupled to quantum phonons

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Algebraic spin liquids are quantum disordered phases of insulating magnets that exhibit fractionalized gapless excitations and power-law correlations. Quantum spin liquids in this category include the experimentally established one-dimensional (1D) Luttinger liquid, as well as the U(1) Dirac spin liquid (DSL) that has been a focus of recent candidate materials searches. Most notably, several exchange-frustrated Heisenberg materials on the triangular lattice have shown evidence of the U(1) DSL. In this work, we measure the algebraic correlations of spin-singlet excitations in the J_1 - J_2 antiferromagnetic Heisenberg model on the triangular lattice, prompting a detailed investigation of this model's stability under spin-lattice coupling with dynamical quantum phonons using variational Monte Carlo. As seen before in 1D spin chains, we observe a low-temperature transition from a U(1) DSL to valence bond order and predict the parameter regime where the model realizes a stable DSL ground state. To achieve this, we employ a series of finite-size scaling Ansätze inspired by the low-energy DSL's conformal description in terms of quantum electrodynamics and show that emergent monopole operators drive the instability. We compare the physics of this transition to the 1D Luttinger liquid throughout our analysis. We derive the regime of stability against spin-Peierls ordering and argue that the DSL ground state might still be achievable in candidate materials, despite its tendency to valence bond solid ordering.

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Introduction. Quantum spin liquids (QSLs) are exotic phases of matter characterized by fractionalized excitations and a long-ranged entanglement structure, possibly realized in magnetic systems where the spins avoid magnetic ordering down to zero temperature due to frustration effects [1,2]. QSLs and their dynamics evade a description in terms of a Landau framework of order parameters for symmetry-broken states [3]. Instead, they can be portrayed as deconfined phases of emergent gauge theories, which make the fractionalization of spins into *spinon* quasiparticles, coupled to emergent dynamical gauge fields [4], manifest. Assessing the intrinsic stability of these states as well as their robustness against external perturbations is a challenging task, but remains a critical question on the road to their realization in materials.

The focus of this letter is the U(1) Dirac spin-liquid (DSL) state, where this challenge is complicated further by

the strongly interacting nature of the appropriate emergent gauge theory, quantum electrodynamics in $2 + 1$ dimensions (QED₃) [3,5]: At low energies, this theory of gapless Dirac fermions (spinons) interacting with a dynamical compact U(1) gauge field is believed to flow to an interacting fixed point with conformal symmetry [6]. Recently, a wealth of numerical evidence has accumulated that the U(1) DSL could be realized in the frustrated J_1 - J_2 Heisenberg model on the triangular lattice, as an effect of the competition of first- and second-neighbor antiferromagnetic exchange. Different numerical methods reported a stable, extended spin-liquid regime around the highly frustrated point $J_2/J_1 = 1/8$ [7–11]. Most importantly, spectral functions within the QSL phase show indication of Dirac spinon excitations [12–14] and energy-level spectroscopy yields evidence for low-lying monopole states [15,16], consistent with QED₃.

The emergent conformal symmetry implies that correlation functions must have a power-law decay $C_i(\mathbf{r}) \propto \|\mathbf{r}\|^{-2\Delta_i}$ at large distances. The emergent SU(4) symmetry of the low-energy theory requires that this exponent Δ_Φ must be the same for both spin-spin and dimer-dimer correlations (at appropriate wave vectors) and is given by the scaling dimension of the monopole Φ . This power-law behavior motivated the name “algebraic” QSL [5,17–19], suggesting an analogy with the gapless spin-liquid ground state of the spin-half Heisenberg chain [6,20], which is predicted to have power-

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law correlations controlled by conformal data and a similar emergent $SO(4)$ symmetry [21]. Such critical correlations of the one-dimensional (1D) spin liquid can render it unstable to perturbations; dimer-dimer correlations couple to lattice distortions to precipitate a spin-Peierls instability toward a valence bond solid (VBS) dimerized state, and spin-spin correlations lead to Néel ordering under weak interchain coupling, as may appear in a bulk 3D sample [22]. In 1D, the prediction of correlation functions in gapless spin liquids and the properties of the corresponding instabilities are facilitated by both powerful analytical frameworks (such as bosonization and the Bethe Ansatz) and accurate numerical tools, like the density matrix renormalization group (DMRG) [23].

While quasi-1D spin-liquid behavior is well established experimentally [24,25], candidate materials in 2D are less well characterized. Recently, however, several candidates have also been identified in 2D that show signs of a gapless ground state and are potentially predominantly described by the triangular-lattice J_1 - J_2 model, such as $\text{YbZn}_2\text{GaO}_5$ [26] and Ytterbium-delafoosite compounds AYbSe_2 , where A is an Alkali metal [27–29]. A theoretical challenge is understanding the stability of the 2D $U(1)$ DSL in more realistic models, which account for the experimental realities such as disorder or spin-lattice coupling. This is particularly pressing, given the algebraic nature of the $U(1)$ DSL and recent field-theory-based predictions of a weak-coupling instability in bilayer systems [30] or when coupled to lattice distortions [31]. Some of us recently argued that such static deformations, or *classical* phonons, render the field theory unstable to VBS formation. While DMRG simulations indicated [31] that certain lattice distortions are relevant perturbations to *microscopic models*, a direct observation of such instability and its fate at *finite temperature* and in the presence of dynamical lattice-scale phonons remains an open question of direct experimental relevance.

To this end, we study in this Letter the stability of the DSL in a triangular-lattice J_1 - J_2 model when spins are coupled to dynamical phonons: First, we use a series of variational Monte Carlo (VMC) calculations to study the model's dimer-dimer correlations and argue to find good evidence that the response of the microscopic model exhibits algebraic behavior, which, remarkably, is consistent with correlations of monopole operators in the low-energy field theory. Second, we assess the behavior of the spin-Peierls transition, considering both the classical limit of static distortions and a full dynamical treatment of quantum phonons. We *directly* observe a $U(1)$ DSL-VBS transition that is well described by conformal field theory (CFT) predictions: the critical spin-lattice coupling goes to zero in the limit of adiabatic phonons, but remains finite when the phonon energy scale ω is large. A series of scaling collapses inspired by the conformal data of the QED_3 continuum theory show a good agreement with monopole scaling dimension $\Delta_\Phi \approx 1.25$ predicted from analyses of pure QED_3 with $N_f = 4$ fermions [33–35]. The induced VBS order has a 12-site unit cell and the scaling of the measured critical spin-lattice coupling is described by a power law $\lambda_c \sim \omega^{3-2\Delta_\Phi}$. We plot our predictions of the coupled spin-lattice phase diagram in Fig. 1 for a moderate coupling strength $\lambda = 0.3$ (on the order seen in 1D systems [36–38]). Together, this confirms that a finite-temperature spin-Peierls phase transition

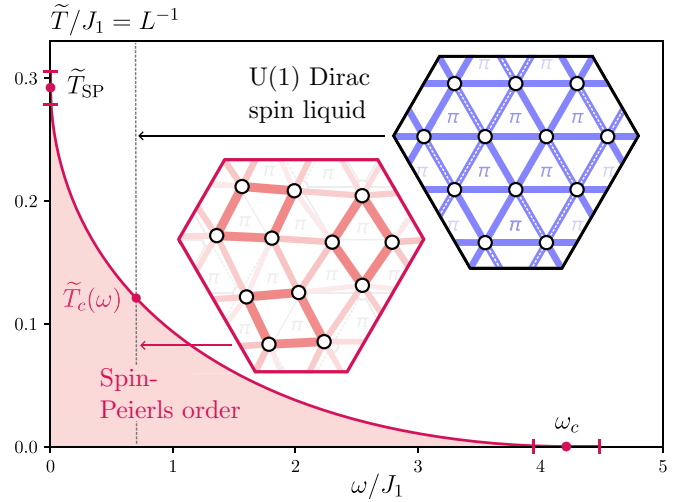


FIG. 1. Phase diagram of the frustrated triangular-lattice Heisenberg model coupled to dynamical phonons with energy ω , for fixed spin-lattice coupling $\lambda = 0.3$. Below ω_c , we measure a finite-temperature phase transition $\tilde{T}_c(\omega)$ from the $U(1)$ Dirac spin liquid to spin-Peierls order (i.e., simultaneous valence bond solid order and lattice distortion). We use a variational method, where the spin wave function in the liquid regime is the fermionic state with alternating 0 and π fluxes (see upper-right inset for our gauge choice: on dotted bonds, the fermion hoppings have a negative sign) [32]. The strengths of the $\langle S_i^z S_j^z \rangle$ correlations of nearest neighbors are shown in the insets through the thickness/shading of the bonds. Finite-temperature responses are inferred from the finite-size behavior of the system. We find that a conformal scaling Ansatz well describes the scaling of critical temperature, using which we measure the scaling dimension of monopoles $\Delta_\Phi = 1.23(2)$.

away from a liquid state may be expected in 2D as a direct result of the emergent monopole operators in the low-energy theory.

Model and correlations. The starting point of our study is the $S = 1/2$, J_1 - J_2 Heisenberg model on the triangular lattice:

$$\mathcal{H}_{J_1 J_2} = J_1 \sum_{\langle i, j \rangle} \vec{S}_i \cdot \vec{S}_j + J_2 \sum_{\langle\langle i, j \rangle\rangle} \vec{S}_i \cdot \vec{S}_j, \quad (1)$$

with $J_1, J_2 > 0$. According to different numerical approaches, the phase diagram of the model features an extended QSL region for $0.07(1) \lesssim J_2/J_1 \lesssim 0.16(1)$ [7–11]. A variational description of the QSL can be formulated within the Abrikosov representation, in which spin operators are expressed in terms of fermionic partons, $\vec{S}_i = \frac{1}{2} f_{i,\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i,\beta}$, subjected to the one-fermion-per-site constraint [3]. Following the projective symmetry group classification of QSL phases on the triangular lattice [32], the VMC study of Ref. [10] showed that the lowest-energy variational Ansatz is a $U(1)$ DSL, defined by the fermionic Hamiltonian

$$\mathcal{H}_0 = \sum_{\langle i, j \rangle} \sum_{\alpha=\uparrow, \downarrow} t_{i,j} f_{i,\alpha}^\dagger f_{j,\alpha} + \text{H.c.} \quad (2)$$

The nearest-neighbor hoppings $t_{i,j} = t e^{i a_{i,j}}$ produce alternating 0 and π fluxes through adjacent triangular plaquettes (see Fig. 1), and the $U(1)$ gauge redundancy is given by $f_{i,\alpha}^\dagger \rightarrow e^{i \theta_i} f_{i,\alpha}^\dagger$ and $a_{i,j} \rightarrow a_{i,j} - \theta_i + \theta_j$. The *bona fide* DSL wave

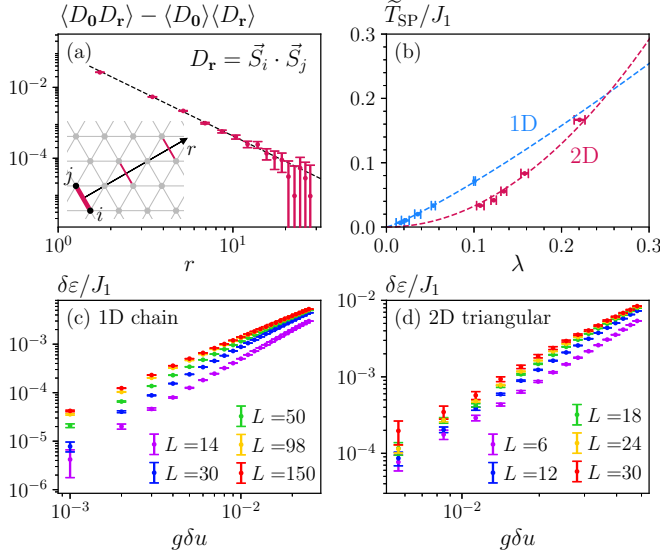


FIG. 2. Static limit of lattice distortions and weak-coupling physics. (a) Dimer-dimer correlations of the triangular-lattice DSL (on the $L = 42$ cluster), fitted with a power law $r^{-2\Delta_\Phi}$, $\Delta_\Phi = 1.20(3)$. (b) Spin-Peierls temperature $\tilde{T}_{SP}$ as a function of the dimensionless spin-lattice coupling λ , for the 1D antiferromagnetic Heisenberg chain and the DSL ansatz for the triangular-lattice J_1 - J_2 model (2D). See the Supplemental Material [47] for details of the fitted curves, obtained from the data collapse of Fig. 3. (c), (d) The energy gain per site $\delta\epsilon$, as a function of static distortions, for various linear sizes L in 1D and 2D (where $N = L \times L$ sites).

function is constructed by applying the Gutzwiller projector $\mathcal{P}_G = \prod_i (n_{i,\uparrow} - n_{i,\downarrow})^2$ (with $n_{i,\alpha} = f_{i,\alpha}^\dagger f_{i,\alpha}$) to the ground state of $\mathcal{H}_0$. This state, which has no free variational parameters, turns out to be remarkably stable to symmetry breaking and lowering of the U(1) gauge structure to $\mathbb{Z}_2$ [10]. In the low-energy (infra-red, IR) limit, this is a theory of $N_f = 4$ Dirac cones coupled to a U(1) gauge field, or QED₃, which is believed to flow to a conformal fixed point [34]. Importantly, there are gapless excitations of the (compact) gauge field as monopoles Φ , i.e., tunneling events of the emergent magnetic flux. These are highly relevant operators about the conformal fixed point, estimated to have a scaling dimension Δ_Φ within the range 1.18–1.34 [35].

Theoretical works predict that a subset of the monopole excitations of the DSL have zero spin and transform as VBS order parameters with momentum $\mathbf{X} = \mathbf{K}/2$, where $\mathbf{K}$ is the corner of the Brillouin zone [6,39], as seemingly confirmed by numerical results of the J_1 - J_2 model on small clusters [15]. Motivated by these findings, we look at the dimer-dimer correlations $\mathcal{C}(r)$ of the triangular-lattice DSL, shown in Fig. 2(a) as a function of the spatial separation r . We find a power-law decay $\mathcal{C}(r) \propto r^{-2\Delta_\Phi}$, where $\Delta_\Phi = 1.20(3)$, and a spatial pattern consistent with the wave vector $\mathbf{X}$. This is compatible with a scenario where the dimer-dimer correlations are dominated by spin-singlet monopoles [6,15,39], and we hence interpret Δ_Φ as the monopole scaling dimension.

Static distortions. In this section, we evaluate the response of the DSL to static, longitudinal, spatially uniform lattice distortions with momentum $\mathbf{X}$ commensurate with the

monopoles. A consequence of their power-law correlations is the potential divergence of the system's susceptibility and a corresponding instability [31].

The distortions are coupled to the spins by assuming, without loss of generality, that the first-neighbor exchange interactions depend linearly on the relative sites displacements as follows:

$$\mathcal{H}[\mathbf{u}_i] = \mathcal{H}_{J_1 J_2} - g J_1 \sum_{\langle i,j \rangle} \hat{\mathbf{r}}_{i,j} \cdot (\mathbf{u}_i - \mathbf{u}_j) \vec{S}_i \cdot \vec{S}_j, \quad (3)$$

with g denoting the spin-lattice coupling constant and $\mathbf{u}_i$ the local displacement operators. The unit vectors $\hat{\mathbf{r}}_{i,j}$ are defined as $\hat{\mathbf{r}}_{i,j} = \frac{\mathbf{r}_i - \mathbf{r}_j}{\|\mathbf{r}_i - \mathbf{r}_j\|}$, with $\{\mathbf{r}_i\}$ indicating the site coordinates in the undistorted triangular lattice. This form of the spin-phonon interaction is a strong-coupling analog of the Su-Schrieffer-Heeger model for electron-phonon systems [40]. We only modulate the dominant nearest-neighbor bonds, but we confirm that the results agree with a model of bond-dependent J_2 interactions. In this static limit, the lattice dynamics is frozen and the displacement operators $\mathbf{u}_i$ become classical variables with energy cost $\frac{1}{2} K \mathbf{u}_i^2$ per site. The distortion we impose is a 12-site pattern of strength δu , commensurate with the monopole momenta [31]. In parallel, we repeat the same analysis in the 1D Heisenberg chain, where the lattice distortion is a dimerization $u_i = \delta u (-1)^i$. We normalize both cases such that the energy cost (per site) of lattice displacements is $\frac{1}{2} K \delta u^2$ and hence the dimensionless spin-lattice coupling is $\lambda = J_1 g^2 / K$.

We compute how the variational ground-state energy of the J_1 - J_2 model in Eq. (3) changes when the exchange couplings are modulated by lattice distortions, on a set of $L \times L$ geometries. The energy gain per site $\delta\epsilon$ (with respect to the undistorted case $\delta u = 0$) is shown in Fig. 2(d), as a function of the distortion strength and for various linear system sizes L . The same calculation of the 1D Heisenberg chain's energetic response to dimerization as a function of length L is shown in Fig. 2(c). For small distortions, we observe a quadratic gain in energy $\delta\epsilon = A(L) g^2 \delta u^2$ that is strongly L dependent. This energy gain competes with the quadratic energy cost of static distortions to give the effective potential $\epsilon(\delta u) = (K/2)(\delta u)^2 - \delta\epsilon$. Thus, the lattice will spontaneously distort whenever the sign of this quadratic response changes to negative, i.e., when $2A(L_c) = K/g^2$, yielding a critical system size L_c above which the lattice will spontaneously distort. Because algebraic QSLs are relativistic theories with dynamical scaling exponent $z = 1$, L_c can be understood to define an effective spin-Peierls temperature scale $\tilde{T}_{SP}/J_1 = L_c^{-1}$, below which an infinitely large system will distort. This is motivated by the observation that CFTs at finite temperature are described by a finite size ($\beta = T^{-1}$) in the Euclidean time direction [41]. We estimate the spin-Peierls transition line by measuring the response $A(\tilde{T})$ for various effective temperatures; then, the critical (dimensionless) spin-lattice coupling strength is $\lambda_c(\tilde{T}) = J_1 g_c^2 / K = J_1 / [2A(\tilde{T})]$. For a given coupling strength, this equivalently defines an effective transition temperature $\tilde{T}_{SP}$ that is plotted as points in Fig. 2(b) for 1D and 2D.

Scaling collapse. We will now discuss in detail the possibility that this spin-Peierls transition is precipitated by

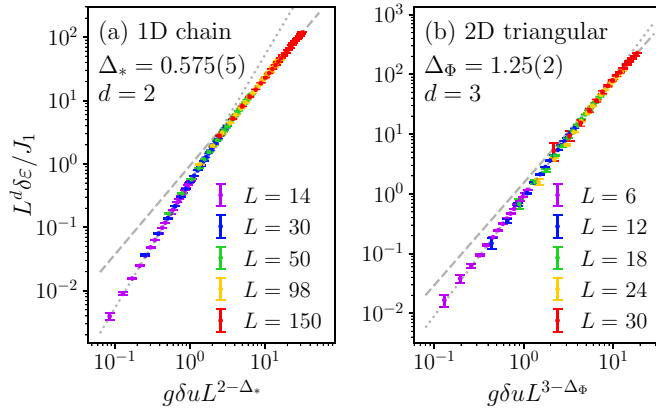


FIG. 3. Data collapse of the energy gain per site $\delta\epsilon$ under static lattice distortion $g\delta u$. Results for (a) the Heisenberg chain and (b) the triangular-lattice J_1 - J_2 model are shown. We plot $L^d \delta\epsilon / J_1$ against $g\delta u L^{d-\Delta}$ for dimension $d = D + 1$ and optimize the scaling dimensions to get the best collapse, shown in inset. Dotted and dashed lines represent asymptotic fitting functions for the regimes of weak and strong coupling, i.e., at small and large values of $g\delta u$.

lattice couplings to emergent gapless monopoles and argue for the presence of a weak-coupling instability. To do this, we highlight that our numerical results on larger systems show significant deviation from a quadratic (weak-coupling) response: the energy gain becomes significantly less L dependent and is best described by a lower exponent $(g\delta u)^\chi$, $\chi < 2$. In the thermodynamic limit, we predict that the static distortion will open a gap by inducing a relevant instanton; the respective field-theoretic coupling has scaling dimensions $d - \Delta$, where $d = D + 1$ and we define $\Delta = \Delta_*$, Δ_Φ in 1D and 2D, respectively. In our calculations, the system size also acts as a relevant operator with the coupling $1/L$ (scaling dimension 1) that opens a finite-size gap [42]. To this end, we make the following scaling Ansatz for the energy density gain:

$$\delta\epsilon(g\delta u, 1/L)/J_1 = L^{-d} \tilde{\mathcal{F}}(L^{d-\Delta} g\delta u), \quad (4)$$

in terms of a universal scaling function $\tilde{\mathcal{F}}(x)$. In the weak-coupling (small distortions, small system) limit, i.e., $x \rightarrow 0$, the function must be quadratic $\tilde{\mathcal{F}}(x) \sim x^2$; conversely, in the large-parameter, *strong-coupling*, limit $x \rightarrow \infty$, the function must follow $\tilde{\mathcal{F}}(x) \sim x^{d/(d-\Delta)}$ in order to be L independent, thereby reproducing nonanalytical scaling laws determined by the CFT fixed point theory.

We apply this analysis to the $d = 2 + 1$ U(1) DSL in Fig. 3(b) and extract the exponent $\Delta_\Phi = 1.25(2)$. This agrees remarkably well with the fit of dimer-dimer correlations and is compatible with independent predictions of the QED₃ monopole scaling dimension [35]. A power-law fit to the weak- and strong-coupling regimes is shown as dotted and dashed lines in Fig. 3(b). The weak-coupling fit allows us to obtain the functional form of the critical temperature as a function of coupling strength by using the same procedure as for individual data points above. We find in 2D $\tilde{T}_{\text{SP}}/J_1 \approx 3.24\lambda^{2.0}$, as plotted with the dashed line in Fig. 2(b). This static calculation lies on the $\omega = 0$ axis of the phase diagram (Fig. 1).

We repeat the analysis for the $d = 1 + 1$ Heisenberg chain data in Fig. 3(a). The extracted exponent is $\Delta_* = 0.575$ and we measure the spin-Peierls temperature $\tilde{T}_{\text{SP}}/J_1 \approx 1.05\lambda^{1.18}$; the exponent is shifted from the bare scaling dimension 0.5 and the transition temperature is suppressed compared to the approximate field theoretic prediction due to logarithmic corrections [43]. Taken together, we find similar behavior of static 1D and 2D spin-Peierls transitions, suggesting that for comparable spin-lattice couplings, the spin-Peierls transition in two dimensions could be an equally important instability as in 1D.

In both models, the strong-coupling regimes are also well fitted by the proposed power laws [with fixed exponent $d/(d - \Delta) < 2$]. This implies the spin energy gain will always outcompete the lattice stiffness in the thermodynamic limit, such that the *static* model has a gapped ground state for any nonzero spin-lattice coupling. The magnitude of the strong-coupling response controls the size of the equilibrium lattice distortion and spin gap, which may be potentially measured in experiments.

Dynamical phonons. We will now study a full model in which lattice distortions (i.e., phonons) are treated quantum mechanically, allowing us to calculate the phase diagram of the spin-Peierls transition as a function of phonon frequency ω and assess how the static instability is dampened or resolved. We focus here on the U(1) DSL on the triangular lattice, by considering the Hamiltonian

$$\mathcal{H} = \mathcal{H}[\mathbf{u}_i] + \sum_i \left(\frac{\mathbf{p}_i^2}{2m} + \frac{1}{2} m \omega^2 \mathbf{u}_i^2 \right), \quad (5)$$

where $\mathbf{u}_i$ and $\mathbf{p}_i$ are displacement and momentum operators of uncoupled harmonic oscillators (i.e., Einstein phonons); these are coupled to the spins by Eq. (3). We pursue a full quantum treatment of the Hamiltonian (5), by means of a recently developed VMC approach. The variational Ansätze take the form $|\Psi_{\text{sp}}\rangle = \mathcal{J}_{\text{sp}}|\Psi_{\text{p}}\rangle \otimes |\Psi_{\text{s}}\rangle$, where $|\Psi_{\text{s}}\rangle$ is a Gutzwiller-projected fermionic state, $|\Psi_{\text{p}}\rangle$ is a phonon coherent state describing lattice distortions, and $\mathcal{J}_{\text{sp}}$ is a spin-phonon Jastrow factor [44–46]. At variance with the case of static distortions, we do not impose a specific phonon mode, but we instead perform unbiased optimizations of the variational state to find the lowest-energy distortion. Details about the VMC method are provided in the Supplemental Material [47]. At large enough spin-phonon coupling ($g > g_c$), our variational search finds that the system forms a 12-site VBS state with the distortion pattern shown in the inset of Fig. 1, compatible with field theory predictions. We then estimate the critical coupling $g_c(\omega)$ at which phonons condense and distort the lattice, by comparing the energies of the VBS and the (undistorted) DSL Ansätze.

The results are shown for varying system sizes L in Fig. 4(a), where the critical coupling g_c is plotted (in units of $\sqrt{J_1 m}$). First, we see that as we take the adiabatic limit $\omega \rightarrow 0$, the critical coupling approaches zero linearly (with an L -dependent coefficient), in agreement with our static results that find an instability, i.e., $g_c(\omega = 0) = 0$. We observe a deviation from the linear trend for larger ω ; in order to capture the system-size dependence in this strong-coupling regime, we employ another scaling Ansatz. In the field the-

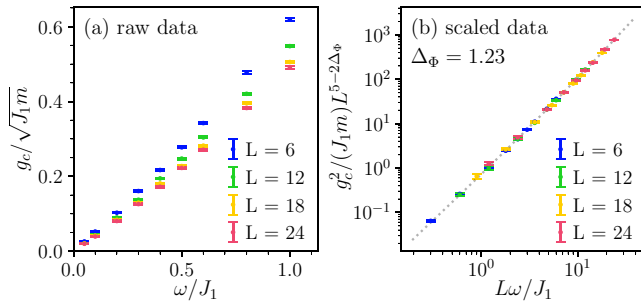


FIG. 4. (a) Measured critical phonon coupling strength g_c of the U(1) DSL as a function of phonon frequency ω (dynamical phonons) and linear system size L . We produce a data collapse in panel (b) by plotting $g_c^2 / (J_1 m) L^{5-2\Delta_\Phi}$ against $L\omega / J_1$.

oretical description of the model, dynamical phonons can be integrated out exactly to leave an effective retarded interaction between monopoles, with an effective coupling constant $\eta = g^2 / (J_1 m)$ with field-theoretic scaling dimension $d + 2 - 2\Delta_\Phi$. Our Ansatz for the critical coupling η_c as a function of ω / J_1 and $1/L$ reads

$$\eta_c(\omega / J_1, 1/L) = g_c^2 / (J_1 m) = L^{2\Delta_\Phi - d - 2} \tilde{G}(L\omega / J_1) \quad (6)$$

and the collapse is achieved by plotting $\eta_c L^{d+2-2\Delta_\Phi}$ against $L\omega / J_1$. In the 2D case considered here, we find the best collapse for $\Delta_\Phi = 1.23(3)$, as shown in Fig. 4(b). The scaling function must be quadratic in the small-parameter regime; by fitting the collapsed data to a quadratic function $\tilde{G}(x) = a_1 x^2$, we extract the critical (effective) temperature as a function of the spin-lattice coupling as $\omega \rightarrow 0$. Explicitly, this is done by using the scaling $\eta_c = a_1 (\tilde{T} / J_1)^{d-2\Delta_\Phi} (\omega / J_1)^2$ and then evaluating the dimensionless (critical) coupling $\lambda_c = J_1 g^2 / (J_1 m) = a_1 (\tilde{T} / J_1)^{d-2\Delta_\Phi}$. Here, we used $K = m\omega^2$ and took $\omega \rightarrow 0$ with K fixed. The resulting function $\tilde{T}_{\text{SP}} / J_1 \approx 2.72 \lambda^{1.85}$ agrees numerically with the independent static calculation from Fig. 2(b) (within error bars for small λ). In the large-parameter regime, we similarly extract the critical frequency $\omega_c / J_1 \approx 39 \lambda^{1.85}$ below which the $\tilde{T} = 0$ ground state is spin-Peierls ordered (equivalently the thermodynamic limit in our calculations). In Fig. 1, these critical points $\tilde{T}_{\text{SP}}$ and ω_c are plotted for $\lambda = 0.3$. The phase boundary $\tilde{T}_c(\omega)$ for $0 < \omega < \omega_c$ is plotted using a fit to the whole scaling function that interpolates the two regimes, shown as a dotted line in Fig. 4 and discussed in the Supplemental Material [47].

Outlook. Using large-scale VMC simulations for the J_1 - J_2 triangular-lattice Heisenberg model, we report a power-law decay of the dimer-dimer correlation function. This is consistent with predictions of the conformal continuum theory of the U(1) DSL, serving as evidence that predictions from this

low-energy field theory can be tested and, to good approximation, confirmed in microscopic lattice models. By studying this model coupled to phonons, both in the static limit and in the full quantum regime, we confirm that the spin-Peierls instability is relevant in 2D, but with transition temperatures that are suppressed in the regime of large phonon frequencies, as is potentially significant for current experiments with J_1 - J_2 spin-liquid candidate materials [28,29]. Recently, a very low-energy gap has been observed in NaYbSe₂ that could be related to such a weak spin-Peierls transition. Thus, it is important to search for signs of lattice symmetry breaking compatible with the VBS pattern of our spin-Peierls scenario, an observation of which would serve as a distinctive signature of an underlying U(1) DSL state.

There exist other exotic deconfined phases of emergent U(1) gauge theories in two dimensions such as deconfined quantum critical points, as potentially realized in J - Q -type models [48–50]. It is an important question to understand the stability of these transitions to spin-lattice coupling or alternatively the possibility that they might be driven by varying the phonon energy scale [51,52]. Comparing our quantitative results for microscopic models, our work demonstrates that there exists a strong similarity of spin-Peierls instabilities of algebraic spin liquids in 1D and 2D, both in the static case and upon coupling to quantum phonons. It is an interesting direction for future studies to investigate to what extent such analogies are also present for other deformations of algebraic spin liquids in 1D and 2D, e.g., interchain/interlayer couplings and disorder, which may be controlled by monopoles or potentially other exotic quasiparticles in the spectrum.

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Data availability. The data that support the findings of this article are openly available [53].

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