

Nonstabilizerness in open XXZ spin chains: Universal scaling and dynamics

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(Received 28 April 2025; revised 19 June 2025; accepted 26 September 2025; published 3 November 2025)

Magic, or nonstabilizerness, is a crucial quantum resource, yet its dynamics in open quantum systems remain largely unexplored. We investigate magic in the open XXZ spin chain under either boundary gain and loss or bulk dephasing using the stabilizer Rényi entropy M_2 . To enable scalable simulations of large systems, we develop a unique, highly efficient algorithm for computing M_2 within the matrix product state formalism while maintaining constant bond dimension—an advancement over existing methods. For boundary driving, we uncover universal scaling laws, $M_2(t) \sim t^{1/z}$, linked to the dynamical exponent z for several distinct universality classes. We also disentangle classical and quantum contributions to magic by introducing a mean-field approximation for magic, thus emphasizing the prominent role of quantum-critical fluctuations in nonstabilizerness. For bulk dephasing, dissipation can transiently enhance magic before suppressing it and drive it to a nontrivial steady-state value. These findings position magic as a powerful diagnostic tool for probing universality and dynamics in open quantum systems.

DOI: 10.1103/96bk-xf8p

I. INTRODUCTION

Nonstabilizerness quantifies the deviation of a quantum state from the set of stabilizer states, serving as a key resource for quantum advantage [1–4] in tasks such as quantum computing and simulation [5–8]. In closed systems, unitary evolution generates and propagates magic through coherent interference, with chaotic dynamics typically leading to faster magic growth compared to localized phases [9–19]. However, in open quantum systems, the interplay between coherent dynamics and decoherence fundamentally alters the behavior of magic. While dissipation and dephasing generally suppress magic by destroying quantum coherences, driven-dissipative systems can exhibit nonequilibrium steady states (NESS) that preserve or even generate magic, particularly in the presence of interactions [20–22]. Understanding the dynamics of magic in open systems is, therefore, crucial for harnessing quantum resources in realistic, noisy environments [23–25].

In this work, we investigate the fate of magic in open quantum systems, focusing on the XXZ spin chain under boundary driving and bulk dephasing [26–31]. Using the

stabilizer Rényi entropy M_2 as a measure of magic [32], we explore how decoherence and dissipation reshape the Pauli spectrum and influence the growth and saturation of magic.

First, we develop a highly efficient algorithm for computing the stabilizer Rényi entropy (magic) in open quantum systems. While previous approaches scale as M^4 or M^2 in the bond dimension M [33–35], our method keeps the bond dimension constant, resulting in linear memory scaling with M and significantly reduced computational cost. This ensures that large bond dimensions, up to $M \sim 1000$, can be handled efficiently in practice.

With this algorithm in hand, we next investigate the open XXZ spin chain. For boundary gain and loss, we identify a universal scaling law for magic in the XXZ chain, showing $M_2(t) \sim t^{1/z}$ with z being the dynamical critical exponent. It reflects various transport regimes, including ballistic ($z = 1$), diffusive ($z = 2$), and even the Kardar-Parisi-Zhang (KPZ) dynamics [36–38] at the isotropic point with $z = 3/2$. Having established the boundary-driven case, we then move to bulk dephasing, which reveals qualitatively different physics and additional insights. To disentangle classical and quantum contributions to nonstabilizerness, we introduce a mean-field approximation for magic that eliminates quantum correlations while preserving local magnetization contributions. This approach provides a classical baseline for magic, allowing us to quantify the role of quantum correlations and identify their impact on transport dynamics and magic generation.

In the presence of bulk dephasing when all sites are coupled to environment through coupling γ_z , we find purely

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dissipative dynamics with no coherent evolution at the spin rotational invariant limit. This occurs when starting from a fully polarized state in any direction. In general, we find that the time evolution is governed by two distinct timescales: the dephasing timescale $\tau_d \sim 1/\gamma_z$, which drives the exponential suppression of magic, and the coherence timescale τ_c . The latter is the inverse of the $SU(2)$ symmetry-breaking interaction and controls transient oscillations in magic. Remarkably, dephasing can transiently enhance magic before driving it to zero. However, in magnetization-conserving systems, magic does not fully decay but stabilizes at a nontrivial steady-state value. Furthermore, when the dynamics is restricted to the zero magnetization sector, starting from certain initial states such as the Néel configuration, magic exhibits a robust power-law decay over time, reflecting the underlying gapless nature of the Lindbladian [39,40].

II. STABILIZER RÉNYI ENTROPY IN OPEN SYSTEMS

In open quantum systems, magic can be extended to mixed states through the stabilizer Rényi entropy, defined using the density matrix [32]. For a general state ρ , the stabilizer Rényi entropy M_2 is given by

$$M_2 = -\log_2 \frac{\sum_P |c_P|^4}{\sum_P |c_P|^2}, \quad (1)$$

where P represents elements of the N -qubit Pauli group [41–43] and the Pauli coefficients c_P are $c_P = \text{Tr}(\rho P)$. In open systems, the Pauli spectrum exhibits both delocalizations due to coherent dynamics and decay from nonunitary processes. Even for closed systems, calculating the full Pauli spectrum is computationally expensive for large N due to the exponential growth of the Pauli group. Efficient methods like Monte Carlo sampling [44] and tensor networks [33,45,46] allow for practical evaluation of M_2 in closed systems.

We introduce a unique approach to compute magic in open systems, using a vectorized density-matrix structure while maintaining a constant bond dimension. Details of the algorithm are presented in the Appendix.

This method, extending the framework of Ref. [46], avoids the bond dimension increase required by closed-system algorithms, offering a scalable and computationally efficient tool for quantifying magic in open quantum systems.

We represent the vectorized density matrix $|\rho\rangle\rangle$ as a matrix product state (MPS) [47–49], with explicit tracking of the physical index pairs (s_i, s'_i) :

$$|\rho\rangle\rangle = \sum_{s_i, s'_i} A_1^{s_1, s'_1} A_2^{s_2, s'_2} \cdots A_L^{s_L, s'_L} |s_1 s'_1 \cdots s_L s'_L\rangle. \quad (2)$$

This form allows for efficient manipulation while preserving the system's structure and normalization properties. To compute the stabilizer Rényi entropy, we construct a matrix product operator (MPO) W that encodes the squared action of local Pauli operators according to Eq. (A5). The entropy is then given by

$$M_2 = -\log_2 \frac{\langle P\rho | W^2 | P\rho \rangle}{\langle P\rho | P\rho \rangle}. \quad (3)$$

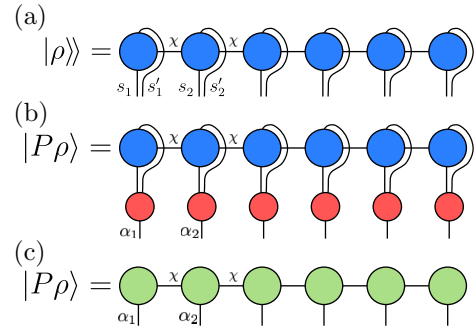


FIG. 1. (a) Illustration of the MPS structure for the vectorized density matrix, with explicit site labels shown. (b) Depiction of the Pauli vector construction and its corresponding MPS structure. Red dots represent the Pauli tensors. (c) After contracting the physical indices, the Pauli vector is expressed as an MPS with the same bond dimension χ as the original vectorized density matrix.

Here, $|P\rho\rangle$ is a Pauli-transformed version of $|\rho\rangle\rangle$, constructed such that the bond dimension remains unchanged. The structures of $|\rho\rangle\rangle$ and $|P\rho\rangle$ are illustrated in Fig. 1.

III. BOUNDARY-DRIVEN XXZ SPIN CHAIN

The open XXZ spin chain is a paradigmatic model for studying transport and nonequilibrium steady states in open quantum systems [22,27,50]. Its Hamiltonian is given by

$$H = \frac{J}{4} \sum_{j=1}^{L-1} (X_j X_{j+1} + Y_j Y_{j+1} + \Delta Z_j Z_{j+1}), \quad (4)$$

where X_j, Y_j , and Z_j are Pauli operators at site j , and Δ is the anisotropy parameter. We investigate the generation and depletion of magic in the XXZ chain coupled to external reservoirs. Specifically, we focus on boundary-driven dynamics, where interactions with reservoirs at the chain's edges drive the system into a nonequilibrium steady state [26,50], as displayed in Fig. 3(a). The dissipation at the boundaries is described by the Lindblad operators

$$F_{1,1} = \sqrt{\gamma} \sqrt{1 - \mu} S_1^-, \quad F_{1,2} = \sqrt{\gamma} \sqrt{1 + \mu} S_1^+, \\ F_{L,1} = \sqrt{\gamma} \sqrt{1 + \mu} S_L^-, \quad F_{L,2} = \sqrt{\gamma} \sqrt{1 - \mu} S_L^+, \quad (5)$$

which act at the first and last sites of the chain. The system is initialized in an infinite-temperature density matrix, $|\rho_0\rangle\rangle = |\mathbb{1}_\infty\rangle\rangle$, a stabilizer state with zero magic. At $t = 0$, the couplings to the reservoirs are switched on, and we track the time evolution of the magic measure $M_2(t)$, along with the local magnetization profile $\langle S_z(x, t) \rangle$. In our numerical simulations, the driving parameter is kept small, $0.01 \leq \mu \leq 0.05$.

Following the quench, the system evolves toward an NESS characterized by a steady magnetization current [50]. During this evolution, two distinct light-cone structures emerge from the system's boundaries and propagate into the bulk. These expanding fronts are clearly visible in the magnetization profiles shown in the insets of Figs. 2(a)–2(c). Their spatial extent Δx exhibits a universal scaling $\Delta x \sim t^{1/z}$, reflecting the underlying transport regime: ballistic ($z = 1$) in the easy-plane phase, KPZ-type scaling ($z = 3/2$) at the isotropic point,

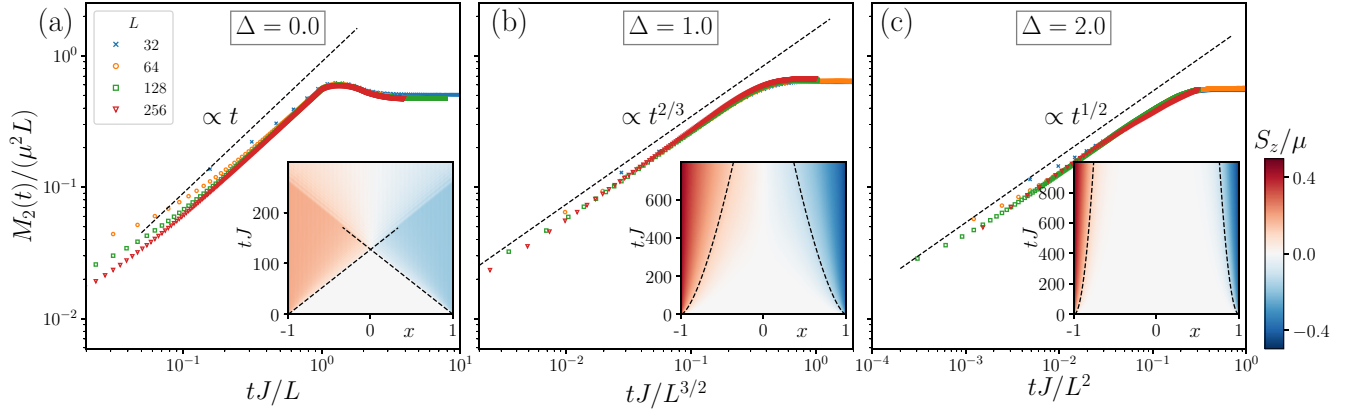


FIG. 2. (a), (b), (c) Time evolution of magic for different anisotropy parameters Δ and for various system sizes. The insets in each panel present the light-cone formation in magnetization for the respective Δ as a function of position $x = 2j/L - 1$, with j numbering the sites. The dashed lines in the insets mark the boundaries of the emerging light cones, defined as the points where the magnetization reaches 1/4 of its maximum value. The initial density matrix corresponds to the infinite-temperature state, $|\rho_0\rangle\rangle = |\mathbb{1}_\infty\rangle\rangle$. For $\Delta = 0$, the magic and magnetization light cone exhibit a linear growth, characteristic for the ballistic transport regime. At $\Delta = 1$, the growth follows a $t^{2/3}$ scaling, indicative of the KPZ universality class, while for $\Delta = 2$, it becomes diffusive with a dynamical exponent $z = 2$. The legend and color bar are shared for all panels.

and diffusive spreading ($z = 2$) in the easy-axis regime. The exponent z dictates the transport behavior, distinguishing different universality classes. A crucial aspect of this evolution is the initial state choice. The system is initialized in the infinite-temperature density matrix, $|\rho_0\rangle\rangle = |\mathbb{1}_\infty\rangle\rangle$, which ensures an uncorrelated and homogeneous starting point. This setup allows the observed transport properties and the associated dynamical scaling to emerge purely from the interplay between unitary evolution and boundary driving, rather than from any pre-existing correlations. In particular, it guarantees that the KPZ universality class at $\Delta = 1$ arises intrinsically from the system's dynamics, reinforcing the robustness of the observed scaling behavior.

For $\Delta < 1$, the system exhibits ballistic transport [51], with a linear light cone ($z = 1$). At the critical point $\Delta = 1$, the dynamics enters the Kardar-Parisi-Zhang universality class [38,52,53], where transport becomes superdiffusive with a characteristic light-cone scaling $z = 3/2$. This anomalous behavior reflects the emergence of strong correlations and fluctuations. In contrast, for $\Delta > 1$, transport is diffusive ($z = 2$), resulting in a broader and slower-growing light cone.

Remarkably, the time evolution of magic follows the same universal scaling as the magnetization profile, with $M_2(t) \sim t^{1/z}$. Rescaling by μ^2 and the system size L reveals a universal collapse of the magic dynamics across different transport regimes, underscoring its role as a robust measure of the system's universality class. In the KPZ regime ($\Delta = 1$), magic exhibits a characteristic $t^{2/3}$ growth, in precise agreement with the expected scaling. This highlights magic as a sensitive probe of quantum transport, capturing the emergence of quantum correlations that define the KPZ universality class, beyond what is accessible through conventional observables like magnetization currents. Similar to the magnetization profile, magic does not develop uniformly but instead emerges within the two light cones propagating from the system's edges. These expanding regions serve as the primary sources of magic generation before it eventually saturates in the NESS. As shown in Fig. 2 (insets), the transition to the steady state coincides with the collision of the two light cones at the center, marking the completion of the transport-driven magic buildup.

One of the key findings of this work is the emergence of a universal scaling law for magic, as demonstrated in Figs. 2(a)–2(c). Our results establish that the magic follows the scaling form

$$M_2(L, t) \sim L f\left(\frac{t}{L^z}\right), \quad (6)$$

where the scaling function $f(x)$ governs the dynamical evolution across different regimes. In the early-time regime ($t \ll L^z$), magic exhibits a universal growth $M_2(t) \sim t^{1/z}$,

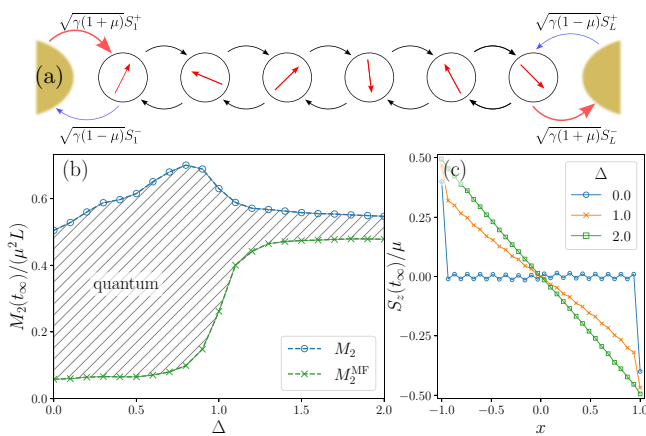


FIG. 3. (a) Sketch of the boundary driven XXZ spin chain in the infinite temperature limit. (b) The steady-state values of the total magic, $M_2(t_\infty)$, and the mean-field magic, $M_2^{\text{MF}}(t_\infty)$, as functions of Δ . The hashed region represents the contribution to magic arising from quantum correlations. (c) Steady-state magnetization profile $S_z(t_\infty)$ for three different values of Δ . In both panels, the system size is set to $L = 32$.

directly reflecting the system's transport properties, with $f(x \ll 1) \sim x^{1/z}$. At late times ($t \gg L^z$), magic saturates as $M_2 \sim L$, signaling the approach to the nonequilibrium steady state, which gives $f(x \gg 1) \sim \text{const}$ for the scaling function. The saturation time t_0 , at which M_2 reaches a plateau, is closely connected to the growth of correlations in the system; it marks the point where the correlation length, constrained by the system size, approaches $\xi \sim L$. This universal scaling structure provides a unifying framework for understanding its growth across different dynamical universality classes. To disentangle classical from quantum contributions, we further introduce a mean-field approximation, which provides a natural baseline for comparison.

IV. MEAN-FIELD APPROXIMATION FOR MAGIC

To quantify the quantum nature of the system, we introduce a mean-field approximation that isolates classical contributions to magic. The mean-field density matrix, $\rho_{\text{MF}}(t)$, is a product state with bond dimension one, constructed to reproduce the local magnetization while discarding all correlations: $\rho_{\text{MF}} \approx \bigotimes_{i=1}^L \rho_i$, where each local density matrix ρ_i is given by $\rho_i = \frac{1}{2}(\mathbb{1}_2 + \sum_{\alpha} m_i^{\alpha} \sigma_i^{\alpha})$. Here, $\mathbb{1}_2$ is the identity matrix, σ_i^{α} ($\alpha = x, y, z$) are the Pauli matrices at site i , and $m_i^{\alpha} = \langle \sigma_i^{\alpha} \rangle$ are the local magnetization components. This formulation allows us to compute the mean-field magic $M_2^{\text{MF}}(t)$ via Eq. (1) and compare it directly with the full magic $M_2(t)$, thereby quantifying the contribution of quantum correlations.

Figure 3(b) highlights the fundamental role of transport in determining the steady-state magic. In the ballistic regime ($\Delta \ll 1$), quantum correlations are dominant, maximizing the gap between $M_2(t_{\infty})$ and $M_2^{\text{MF}}(t_{\infty})$.

For small values of Δ (specifically $\Delta \lesssim 0.6$), the magnetization profile remains largely unchanged (as presented in Ref. [40], Fig. 6(b); see also Ref. [54]), resulting in an almost constant value of the mean-field magic. In this regime, the local magnetization is negligible throughout the bulk and only remains significant near the boundaries. Consequently, the mean-field magic $M_2^{\text{MF}}(t_{\infty})$, which depends solely on local expectation values, is primarily supported by contributions from the edge sites, leading to its overall smaller magnitude, in contrast to the total M_2 . Near $\Delta \approx 1$, both measures of magic rise sharply, signaling enhanced entanglement and a fundamental shift in the magnetization profile. In the diffusive regime, the gap between full and mean-field magic narrows, reflecting the suppression of quantum correlations. In the large- Δ limit, the steady-state magnetization profile converges to $\langle Z(x) \rangle \sim \arcsin(x)$, with vanishing transverse components [54]. While the mean-field magic is computed only from local observables, the full quantum magic retains contributions from nonlocal correlations such as $\langle \cdots Z_i \cdots Z_j \cdots \rangle$, resulting in a finite and persistent gap between the full and mean-field values even as $\Delta \rightarrow \infty$, as confirmed by our simulations for large Δ .

The explicit decomposition of magic into classical and quantum components is a key achievement of this work, as it provides a direct measure of nonstabilizerness due to quantum correlations. Notably, despite being entirely classical, $M_2^{\text{MF}}(t)$ exhibits the same scaling behavior as the full magic [40],

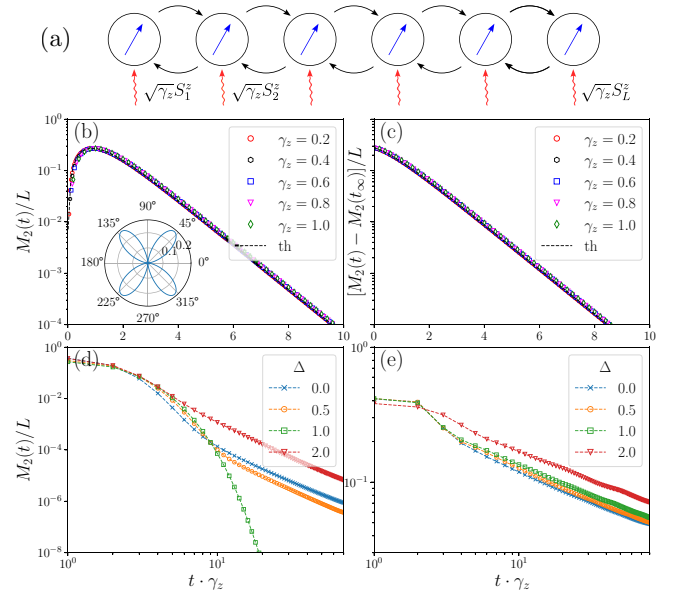


FIG. 4. (a) Sketch of the bulk dephasing modeled with sites coupled to an external reservoir via the jump operator S^z . Time evolution under dephasing affecting all qubits at the isotropic point $\Delta = 1$. (b) Starting from an initial density matrix with zero magic, $\rho(0) = (|+\rangle\langle+|)^{\otimes L}$, the magic initially increases before undergoing an exponential decay to zero at late times. The inset displays the magic density in the NESS state $M_2(t_{\infty}, \theta)$ as a function of the initial polarization angle θ . (c) When the initial density matrix is $\rho(0) = (|T\rangle\langle T|)^{\otimes L}$, the system possesses a high degree of magic. Due to conservation of the total S_z spin component there is a nonzero magic in the NESS $M_2(t_{\infty}) = \log_2(6/5)$, subtracted in panel (c) from $M_2(t)$. The dashed line denotes the analytical result from Eq. (7). Power-law decay of magic M_2 for various initial states within the $\langle S_z \rangle = 0$ sector: (d) a product state $\rho(0) = (|+\rangle\langle+|)^{\otimes L}$ and (e) a Néel state, both evaluated for $\gamma_z = 1$ and $L = 96$. At the isotropic point $\Delta = 1$, the data in panel (d) recover the exponential decay already presented in panel (b).

confirming that universal transport signatures persist even in the absence of quantum correlations.

V. QUANTUM DEPHASING

We investigate bulk dephasing using local Lindblad operators $F_j = \sqrt{\gamma_z} S_j^z$, which act independently at each site [see Fig. 4(a)], suppressing quantum coherences in the S^z basis and leading to information loss. To understand its effects, we first analyze the single-qubit case. The density matrix evolves as $\rho_1(\theta, \varphi, t) = \frac{1}{2}[1 + Z \cos \theta + \sin \theta(X \cos \varphi + Y \sin \varphi)e^{-\gamma_z t/2}]$, where dephasing exponentially suppresses X and Y while preserving Z . The corresponding magic follows

$$M_2(t) = \log_2 \frac{1 + \cos^2 \theta + \sin^2 \theta e^{-\gamma_z t}}{1 + \cos^4 \theta + \sin^4 \theta (\sin^4 \varphi + \cos^4 \varphi) e^{-2\gamma_z t}}. \quad (7)$$

This highlights how dephasing erases quantum coherences while retaining classical information along Z . A similar analysis extends to two interacting qubits, leading to a general expression for magic (see Ref. [40] for details).

VI. HEISENBERG CHAIN

In the Heisenberg limit ($\Delta = 1$), full $SU(2)$ spin symmetry ensures that any fully polarized product state remains an eigenstate, leading to purely dephasing dynamics with no unitary evolution. As a result, each qubit evolves independently, and the magic density follows the single-qubit result. However, away from this special point, two competing timescales emerge: the dephasing time $\tau_d \sim 1/\gamma_z$, governing the exponential suppression of magic, and the coherence time $\tau_c \sim 1/J|\Delta - 1|$, associated with coherent oscillations in magic (see Ref. [40] for details). At $\Delta = 1$, the absence of coherence-driven dynamics eliminates τ_c , leading to a universal decay dictated solely by τ_d . Figure 4 shows the time evolution for two initial states. For $\rho_0 = (|+\rangle\langle+|)^{\otimes L}$ [Fig. 4(a)], magic starts at zero, peaks at $t \approx 1/\gamma_z$, and then decays exponentially. This transient magic enhancement reflects the interplay between decoherence timescales. When starting from a high-magic state $\rho_0 = (|T\rangle\langle T|)^{\otimes L}$ [Fig. 4(b)], the conservation of $\langle S_z \rangle$ prevents full decay, stabilizing a nontrivial steady-state magic,

$$\lim_{t \rightarrow \infty} \frac{M_2(t)}{L} = \log_2 \frac{1 + \cos^2 \theta}{1 + \cos^4 \theta}. \quad (8)$$

When rescaled by system size, magic dynamics collapse onto a universal curve, matching the single-qubit behavior. The inset of Fig. 4(a) shows a polar plot of the steady-state magic. Taken together, these complementary perspectives build a coherent picture of how magic behaves in open quantum systems.

VII. CONCLUSIONS

In this work, we have uncovered the previously unexplored dynamics of magic in open quantum systems. By developing a unique and efficient algorithm for computing the stabilizer Rényi entropy M_2 in open systems, we achieve scalable simulations of large systems under nonunitary dynamics.

For boundary driving, we reveal a universal scaling law for magic, $M_2(t) \sim t^{1/z}$, directly tied to the dynamical exponent z of the underlying transport regime and demonstrate that magic serves as a sensitive probe of quantum transport, including the Kardar-Parisi-Zhang universality class.

We introduce a mean-field approach for magic, which isolates the classical contribution to magic through local expectation values. The difference between the full and mean-field magic serves as a direct probe of nonstabilizer generated by the many-body correlations built into the system.

For bulk dephasing, we demonstrate that dissipation can transiently enhance magic before ultimately driving it toward its steady-state value. Magic exhibits an initial growth, peaks around a characteristic dephasing timescale $t \approx 1/\gamma_z$, and then decays exponentially. In systems with conserved magnetization, however, magic is not fully erased and stabilizes at a nontrivial steady-state value. In the Heisenberg limit, we derive an analytical expression capturing this time dependence. Away from the $SU(2)$ point, the dynamics are governed by two key timescales: the dephasing time $\tau_d \sim 1/\gamma_z$, which controls the exponential decay, and the coherence time

$\tau_c \sim 1/J|\Delta - 1|$, which accounts for transient oscillations away from the $SU(2)$ point.

Interestingly, when the system is initialized in configurations such as the Néel state or $(|+\rangle\langle+|)^{\otimes L}$, both belonging to the zero-magnetization sector, the magic exhibits a power-law decay in time. This behavior reflects the gapless character of the Lindbladian dynamics in this regime [see Figs. 4(d) and 4(e) and Ref. [40] Sec. IIC]. A more detailed investigation, including the magic dependence on the initial state and system parameters, is the subject of another work.

These results establish magic as a powerful and versatile measure for characterizing quantum dynamics in open systems and highlight the complex interplay between unitary and nonunitary dynamics as well as symmetry in open quantum systems.

ACKNOWLEDGMENTS

This work received financial support from CNCS/CCCDI-UEFISCDI, under Projects No. PN-IV-P1-PCE-2023-0159, No. PN-IV-P1PCE-2023-0987, and No. PN-IV-P8-8.3-PM-RO-FR-2024-0059, and by the “Nucleu” Program within the PNCDI 2022–2027, carried out with the support of MEC, Project No. 27N/03.01.2023, component project code PN 23 24 01 04. This research was also supported by the National Research, Development and Innovation Office—NKFIH within the Quantum Technology National Excellence Program (Project No. 2017-1.2.1-NKP-2017-00001), K134437, K142179 by the BME-Nanotechnology FIKP grant (BME FIKP-NAT), the QuantERA ‘QuSiED’ Grant No. 10101773. We acknowledge the Digital Government Development and Project Management Ltd. for awarding us access to the Komondor HPC facility based in Hungary.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: MAGIC CALCULATION WITHIN THE VECTORIZED BASIS

One of the main results of this work is the development of a highly efficient algorithm for computing stabilizer Rényi entropy magic in quantum systems, which maintains a constant bond dimension throughout the calculation, ensuring scalability and computational efficiency even for large systems. We represent the vectorized density matrix $|\rho\rangle\rangle$ in an MPS form [47–49]

$$|\rho\rangle\rangle = \sum_{s_i, s'_i} A_1^{s_1, s'_1} A_2^{s_2, s'_2} \cdots A_L^{s_L, s'_L} |s_1 s'_1 \cdots s_L s'_L\rangle, \quad (A1)$$

where we explicitly track the pairs of physical indices (s_i, s'_i) at each site. The tensors $A_j^{s_j, s'_j}$ are of dimension $\chi \times \chi$ for $1 < j < L$, while the boundary tensors $A_1^{s_1, s'_1}$ and $A_L^{s_L, s'_L}$ are a $1 \times \chi$

row vector and a $\chi \times 1$ column vector, respectively. This structure is illustrated in Fig. 1. The normalization condition $\text{Tr}(\rho) = 1$ translates to $\langle \mathbb{1}_\infty | \rho \rangle = 1$, where $|\mathbb{1}_\infty\rangle$ represents the MPS form of the infinite-temperature density matrix. To compute the stabilizer Rényi entropy, we define the Pauli vector $|P\rho\rangle$ by applying a tensor product of Pauli operators:

$$P_\alpha = P_{\alpha_1} \otimes P_{\alpha_2} \otimes \cdots \otimes P_{\alpha_L}, \quad (\text{A2})$$

and the corresponding vectorized form is

$$|P\rho\rangle = \sum_{\alpha_i} B_1^{\alpha_1} B_2^{\alpha_2} \cdots B_L^{\alpha_L} |\alpha_1 \dots \alpha_L\rangle. \quad (\text{A3})$$

The tensors $B_j^{\alpha_j}$ are defined as

$$B_j^{\alpha_j} = \sum_{s_j, s'_j} \frac{\langle s_j | P_{\alpha_j} | s'_j \rangle}{\sqrt{2}} A_j^{s_j, s'_j}. \quad (\text{A4})$$

Crucially, the bond dimensions of $|P\rho\rangle$ are identical to those of $|\rho\rangle$, ensuring that the computational cost remains

manageable. The Pauli vector is normalized such that $\langle P\rho | P\rho \rangle = \sum_\alpha \text{Tr}(P_\alpha \rho)^2$. Next, we introduce an MPO for efficient computation:

$$W = \sum_{\alpha_i, \alpha'_i} C_1^{\alpha_1, \alpha'_1} C_2^{\alpha_2, \alpha'_2} \cdots C_L^{\alpha_L, \alpha'_L} |\alpha_1 \dots \alpha_L\rangle \langle \alpha'_1 \dots \alpha'_L|. \quad (\text{A5})$$

The MPO tensors are given by $C_j^{\alpha_j, \alpha'_j} = B_j^{\alpha_j} \delta_{\alpha_j, \alpha'_j}$, and this construction allows for the computation of the stabilizer Rényi entropy (magic) as

$$M_2 = -\log_2 \frac{\langle P\rho | W^2 | P\rho \rangle}{\langle P\rho | P\rho \rangle}. \quad (\text{A6})$$

In practice, we apply W to $|P\rho\rangle$ and compute its inner product with itself to obtain the numerator of Eq. (3). The constant bond dimension throughout this process is a central feature of the method, making it a powerful and scalable tool for analyzing quantum systems, particularly in the context of stabilizer entropy.

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