

Hilbert space geometry and quantum chaos

Rustem Sharipov^{1,*}, Anastasiia Tiutiakina,² Alexander Gorsky³, Vladimir Gritsev⁴, and Anatoli Polkovnikov⁵¹Physics Department, Faculty of Mathematics and Physics, *University of Ljubljana*, Ljubljana, Slovenia²Laboratoire de Physique Theorique et Modelisation, CNRS UMR 8089, *CY Cergy Paris Universite*, 95302 Cergy-Pontoise Cedex, France³Laboratory of Complex Networks, Center for Neurophysics and Neuromorphic Technologies, Moscow, Russia⁴Institute for Theoretical Physics, *University of Amsterdam*, Science Park 904, Postbus 94485, 1090 GL Amsterdam, The Netherlands⁵Department of Physics, *Boston University*, 590 Commonwealth Avenue, Boston, Massachusetts 02215, USA

(Received 4 February 2025; accepted 4 March 2026; published 9 April 2026)

The quantum geometric tensor (QGT) characterizes the Hilbert space geometry of the eigenstates of a parameter-dependent Hamiltonian. In recent years, the QGT and related quantities have found extensive theoretical and experimental utility, in particular for quantifying quantum phase transitions both at and out of equilibrium. Here, we consider the symmetric part (quantum Riemannian metric) of the QGT for various multiparametric random matrix Hamiltonians and discuss the possible indication of ergodic or integrable behavior. We found for a two-dimensional parameter space that, while the ergodic phase corresponds to the smooth manifold, the integrable limit marks itself as a singular geometry with a conical defect. Our study thus provides more support for the idea that the landscape of the parameter space yields information on the ergodic-nonergodic transition in complex quantum systems, including the intermediate phase.

DOI: [10.1103/92z8-bqx9](https://doi.org/10.1103/92z8-bqx9)

Introduction. Over the past few decades, there has been significant research into the statistical properties of quantum systems. This has led to the crucial distinction between two types of phases: quantum chaotic and integrable. One method of identifying whether a system is in a chaotic or integrable regime involves the use of a quantum geometric tensor (QGT). The QGT is a tool that quantifies the responses of a system to changes in its parameters, making it a valuable approach to analyze these distinct phases [1–3].

The real part of the QGT defines a Riemann metric in the parameter space known as the quantum metric tensor or quantum information metric, while the imaginary part corresponds to the Berry curvature. This metric measures the distances in the space of the eigenfunctions bundled over the parameter space. The singular behavior of the Ricci scalar or a change in the topology of the parameter space is associated with quantum phase transitions in the system [4]. The symmetries of this metric can also be used for the classification of quantum phases [5]. Some recent examples of the identification of critical points via singularities in the quantum metric can be found in Refs. [6,7].

More recently, the quantum metric tensor has been used as a probe of quantum chaos [8]. In particular, it was found that at integrable points for the integrability breaking direction of the perturbation, the metric tensor generically scales

exponentially with the system size, similarly to its scaling in ergodic systems that satisfy the eigenstate thermalization hypothesis. However, there are possible exceptions related to weak integrability breaking perturbations [9–12]. In contrast, for integrability preserving perturbations, the scaling of the QGT was found to be polynomial, reflecting the vanishing of spectral weight at low frequencies [13,14]. In general, integrable and ergodic/mixing regimes are separated by a Kolmogorov–Arnold–Moser-like chaotic but nonthermalizing region [8,13]. A similar intermediate nonergodic extended (NEE) phase has also been identified in the context of disordered systems. This phase is characterized by a multifractal structure of eigenstates, first observed in the Rosenzweig-Porter (RP) model [15], and later confirmed through the replica approach [16] and the Krylov basis viewpoint [17]; see also Refs. [18–22]. In Refs. [14,23], it was further argued that the geometric tensor becomes highly anisotropic near integrable points and that such integrable points serve as attractors of geodesic flows determined by the metric.

At least three other measures of the “chaos-intermediate-integrability” patterns have to be mentioned. Out-of-time-order correlators [24] related to the operator growth work in some situations, while the approach based on the Krylov complexity was suggested in Ref. [25]. Finally, the dependence of the inverse participation ratio $\text{IPR}_q(L)$ on the system size L distinguishes the localized and delocalized states and corresponds to the integrable and nonintegrable regimes, respectively [26]. There are some links between the approaches: The measure similar to the fidelity susceptibility in context of localization was proposed in Ref. [27]. The QGT can be related to the late-time behavior of autocorrelator tensor of the operators $\partial_{\lambda_j} H$, where λ_j denotes the coordinates in the parameter space [28]. For this reason, there must be some

*Contact author: rustem.sharipov@fmf.uni-lj.si

connection between the operator spreading in Krylov space and the QGT. At the same time, there are clear examples that restrict the validity of the OTOC and operator growth as chaos measures [28–31].

In this Letter, we investigate the geometry of the perturbed Hilbert space for matrix Hamiltonians. To explore the metric in the neighborhood of the chaotic point, we start with a matrix Hamiltonian for a finite-dimensional system and add random matrix perturbations. This analysis is a generalization of [32–34], where only the susceptibility to fidelity at a single point of the parameter space was considered, which is not enough to capture the geometry of the quantum manifold. To examine the metric near an integrable point, we consider diagonal matrices with independent Gaussian distributions of eigenvalues, also known as the random energy model [35]. In both cases, we perturb the initial system by a pair of random matrices multiplied by parameters, which play the role of the coordinates. We then evaluate the quantum metric in a two-dimensional (2D) parameter space in both cases and find a clear-cut indication of the difference in geometries between the integrable and chaotic cases: There is a universal conical singularity emerging at the integrable point, which is related to a long time tail of the autocorrelation function of these random matrices. Our analysis also clearly identifies the presence of an intermediate regime in the behavior of the metric, which is interpreted most naturally as the NEE phase.

Quantum geometric tensor. Let us begin by briefly recalling the definition of the QGT and introducing the relevant notations. Consider the eigenspace of some parameter-dependent Hamiltonian $H(\vec{\lambda})$. With energy eigenvalues $E_n(\vec{\lambda})$ and eigenvectors $|n(\vec{\lambda})\rangle$ depending on the parameters,

$$H(\vec{\lambda})|n(\vec{\lambda})\rangle = E_n(\vec{\lambda})|n(\vec{\lambda})\rangle, \quad (1)$$

where the parameter $\vec{\lambda} = \{\lambda_1, \lambda_2, \dots\}$ is generally multicomponent. In the space of eigenvectors, one can define the distance between nearby states with infinitesimally different parameters in the following way [1]:

$$ds^2 \equiv 1 - |\langle n(\vec{\lambda}) | n(\vec{\lambda} + d\vec{\lambda}) \rangle|^2. \quad (2)$$

Then, the quantum geometric tensor for the n th eigenstate is defined as the leading contribution in $d\vec{\lambda}$ expansion of the distance defined above:

$$\begin{aligned} ds^2 &\equiv g_{\alpha\beta}^{(n)} d\lambda_\alpha d\lambda_\beta + \mathcal{O}(|d\vec{\lambda}|^3) \\ &= \langle \partial_\alpha n | \partial_\beta n \rangle - \langle \partial_\alpha n | n \rangle \langle n | \partial_\beta n \rangle + \mathcal{O}(|d\vec{\lambda}|^3), \end{aligned} \quad (3)$$

where $\partial_\alpha \equiv \partial_{\lambda_\alpha}$. Notice that the quantum geometric tensor is invariant under arbitrary phase transformations of the eigenfunctions $|n(\lambda)\rangle \rightarrow e^{i\phi_n(\lambda)} |n(\lambda)\rangle$, which is expected from the gauge invariance of the QGT. In the absence of degeneracies, it is straightforward to rewrite Eq. (3) in the following form (see Supplemental Material (SM) [36]):

$$g_{\alpha\beta}^{(n)} = \sum_{m \neq n} \frac{\langle n | \partial_\alpha H | m \rangle \langle m | \partial_\beta H | n \rangle}{(E_n - E_m)^2}, \quad (4)$$

which we shall use in the next sections.

The real part of quantum geometric tensor is the quantum metric tensor, while the imaginary part is the Berry curvature.

In this work, we will focus on the *averaged geometric tensor* (or the Hilbert-Schmidt norm of the adiabatic gauge potential [8]):

$$G_{\alpha\beta} = \frac{1}{N} \sum_{n=1}^N g_{\alpha\beta}^{(n)}, \quad (5)$$

where N is Hilbert-space dimension, e.g., $N = 2^L$ for qubit spin chains. It is straightforward to verify that the imaginary, antisymmetric part of this averaged geometric tensor is zero when the parameter-dependent Hamiltonian is Hermitian. Additionally, we note that the Hilbert space average is equivalent to the average with respect to the infinite temperature density matrix.

Geometry of a random matrix model. Following Refs. [32,34], we consider a two-parameter family of $N \times N$ random Hamiltonians:

$$H = H_0 + xH_x + yH_y. \quad (6)$$

We focus on the case where random matrices H_0 , H_x , and H_y are drawn independently from the Gaussian unitary ensemble (GUE) [37] with the distribution

$$\rho(H_a) = e^{-\frac{N}{2} \text{Tr}(H_a^2)}, \quad a = 0, x, y. \quad (7)$$

In this section, we calculate the QGT, for this purely random model. We treat the QGT as an induced metric on a two-dimensional surface embedded in the three-dimensional Euclidean space. First, notice that it is convenient to move to the polar coordinates, since our system has no preferred direction on average. That is, we redefine

$$H = H_0 + r \cos \phi H_x + r \sin \phi H_y. \quad (8)$$

It is clear that the metric tensor averaged over the random matrices H_0 , H_x , H_y must be independent of the angle ϕ . In the course of straightforward calculations (details in Ref. [36]), the components of the averaged QGT are given by

$$\bar{G}_{rr}(r) = \frac{N-1}{2(r^2+1)^2}, \quad \bar{G}_{\phi\phi}(r) = r^2 \frac{N-1}{2(r^2+1)}, \quad (9)$$

and $\bar{G}_{r\phi} = 0$.

Topology by embedding. Finally, the two-dimensional first fundamental form can be written as

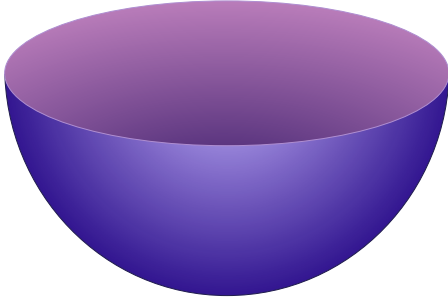
$$ds^2 = \bar{G}_{rr}(r) dr^2 + \bar{G}_{\phi\phi}(r) d\phi^2. \quad (10)$$

We analyze the topological characteristics of the corresponding manifold by constructing an isometric surface and plotting its shape. In order to do this, we consider a three-dimensional Euclidean space and a two-dimensional submanifold $\mathcal{M}_{\text{ch}}$ embedded within it, with the metric set by the standard relation:

$$dZ^2 + dR^2 + R^2 d\phi^2 = \bar{G}_{rr}(r) dr^2 + \bar{G}_{\phi\phi}(r) d\phi^2. \quad (11)$$

Since the metric components are independent of ϕ , the submanifold $\mathcal{M}_{\text{ch}}$ can be parametrized as follows:

$$\mathcal{M}_{\text{ch}} : Z = Z(r), \quad R = R(r).$$

FIG. 1. Isometric manifold $\mathcal{M}_{\text{ch}}$.

Then, by using Eq. (11) we obtain the corresponding system of equations, which reads as

$$\left(\frac{dZ}{dr}\right)^2 + \left(\frac{dR}{dr}\right)^2 = \bar{G}_{\pi}(r), \quad (12a)$$

$$R(r)^2 = \bar{G}_{\phi\phi}(r). \quad (12b)$$

This system of equations can be easily solved for the metric tensor we derived:

$$R(r) = R_0 \frac{r}{\sqrt{r^2 + 1}}, \quad Z(r) = R_0 \left(1 - \frac{1}{\sqrt{r^2 + 1}}\right), \quad (13)$$

where $R_0 = \sqrt{\frac{N-1}{2}}$. We fixed the integration constant by assuming $Z(0) = 0$. This system defines a two-dimensional surface parametrized by r . By excluding r from these equations, we find that Z and R satisfy the following constraint relation:

$$(Z - R_0)^2 + R^2 = R_0^2, \quad (14)$$

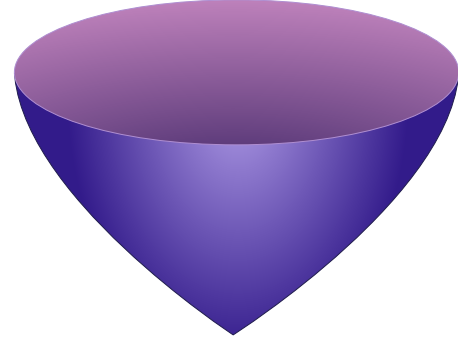
where $0 \leq R \leq R_0$ and $0 \leq Z \leq R_0$. This represents the equation of a lower hemisphere centered at $Z = R_0$ with radius R_0 . In Fig. 1, we illustrate the shape of this isometric manifold.

This result can be anticipated from heuristic reasoning: By adding a random matrix deformation $xH_x + yH_y$ to an existing random matrix H_0 , we generate another random matrix equivalent to H_0 in addition to the rescaling of the variance σ^2 . The geometry of the manifold cannot depend on the overall scale; therefore, locally the shape should be both r and ϕ independent. Hence, $\mathcal{M}_{\text{ch}}$ is locally equivalent to the surface of a sphere, where all points are also equivalent.

Geometry of integrability breaking. Now, let us consider a different Hamiltonian, where the unperturbed Hamiltonian H_0 is replaced by a diagonal matrix Λ_0 with independent random entries while the perturbation is still a random matrix. Such a diagonal matrix exhibits Poisson level statistics and can therefore be regarded as a representation of an integrable model. The Hamiltonian H now takes the form

$$H = \Lambda_0 + xH_x + yH_y, \quad (15)$$

where $\Lambda_0 = \text{diag}(\lambda_i)$ with $\rho(\lambda_i) = e^{-\frac{1}{2}\lambda_i^2}$, and H_x and H_y are drawn from GUE as before. This setup is comparable to the RP model [15] with the addition that we also allow deformations along the ϕ direction, which obviously keeps the spectrum invariant but has a nontrivial metric associated with eigenstate deformations. For $N = 2$, the model is similar to the one considered in Ref. [38], where the angular parameter

FIG. 2. Isometric manifold $\mathcal{M}_{\text{int}}$.

is also present. The QGT for this model at integrable point $r = 0$ has been studied in Ref. [34]. It is intuitively clear that the radial and angular directions should no longer be equivalent near the integrable point, and thus, the spherical geometry found in the previous section must be distorted near $r = 0$.

Let us first consider $N = 2$, where the entire analysis can be carried out analytically. We find that the quantum metric tensor in polar coordinates is given by

$$\bar{G}_{\phi\phi} = r \frac{1}{2\sqrt{2}} \arctan\left(\frac{\sqrt{2}}{r}\right), \quad (16a)$$

$$\bar{G}_{\pi\pi} = \frac{1}{4} \left(\frac{\arccot\left(\frac{r}{\sqrt{2}}\right)}{\sqrt{2}r} - \frac{1}{2+r^2} \right), \quad (16b)$$

and $\bar{G}_{r\phi} = 0$. The fact that the metric tensor is ϕ independent and that its mixed component is zero again allows us to perform the embedding in pseudo-Euclidean space (see the SM [36]).

Near the origin, i.e., at $r \ll 1$, we can Taylor expand the metric and obtain the following expression for the first fundamental form:

$$ds^2 \approx \frac{\pi}{8\sqrt{2}} \left(\frac{dr^2}{r} + 2r d\phi^2 \right) = \frac{\pi}{2\sqrt{2}} \left(d\rho^2 + \frac{1}{2} \rho^2 d\phi^2 \right), \quad (17)$$

where $\rho = \sqrt{r}$. Embedding this metric, as before, into Euclidean space, we find

$$R(\rho) \approx Z(\rho) \approx \sqrt{\frac{\pi}{4\sqrt{2}}} \rho. \quad (18)$$

This surface represents a cone with angle $\pi/2$ (see Ref. [36]), we illustrate the shape of the isometric manifold in Fig. 2. In the limit of $r \gg 1$, the components of the metric tensor [Eqs. (16a) and (16b)] approach those of the random model [Eq. (9)], and the shape crossovers to the hemisphere. The presence of the conical singularity indicates a change in the geometric properties of the quantum manifold. In particular, the quantum eigenstates become parametrically more sensitive to small perturbations in the ϕ direction than to the radial (random) direction close to the integrable point. Having identified the conical geometry at the origin of $N = 2$, let us turn to numerical simulations, increasing the size of the system. The corresponding behavior of the metric components near the

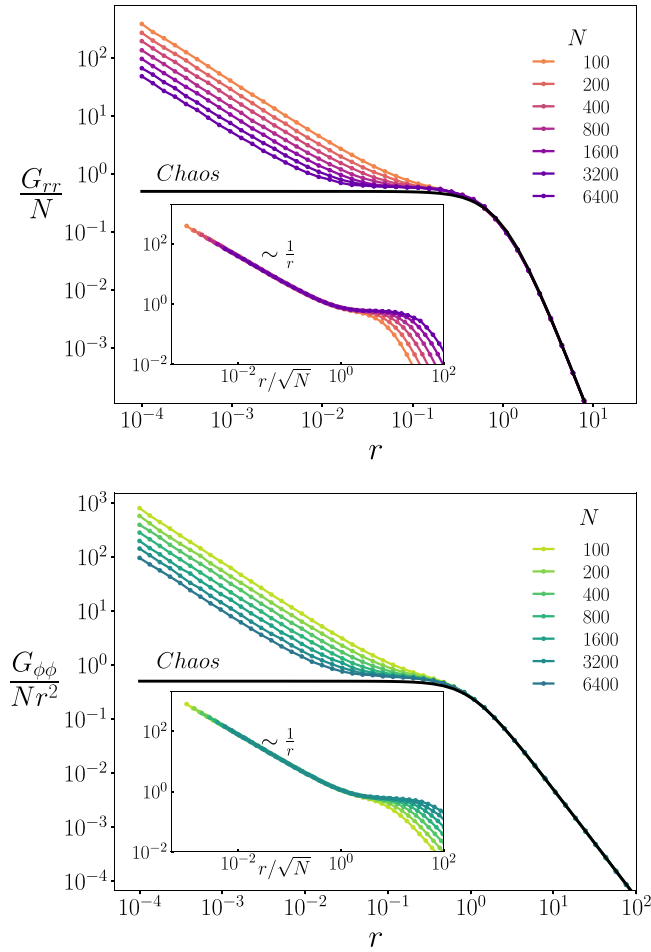


FIG. 3. G_{rr} and $G_{\phi\phi}$ components of the QGT as a function of radial parameter r . The inset on the plots represents metric components as function of rescaled parameter $r/\sqrt{N}$. The black line corresponds to the fully chaotic case results given by Eq. (9).

origin is presented in Fig. 3. For $r \ll r^* = 1/\sqrt{N}$, there is an evident $1/r$ behavior of the metric, similar to the $N = 2$ case. Here,

$$ds^2 \sim \sqrt{N} \left(\frac{dr^2}{r} + 2rd\phi^2 \right), \quad (19)$$

which, in addition to the general prefactor, agrees with Eq. (17). This regime corresponds to the conical geometry. We note that r^* is the localization transition point (see details in Ref. [36]), so physically, the conical geometry describes the localized phase. For large $r \gg 1$, which corresponds to the ergodic phase, G_{rr} and $G_{\phi\phi}$ approach the corresponding values of the RMT model [Eq. (9)]. This regime obviously represents spherical geometry.

Finally, we analyze the behavior of the QGT in an intermediate regime at $r^* \ll r \ll 1$. We consider the scaling of the parameter $r = N^{(1-\gamma)/2}$, for which the localized, ergodic, and fractal phases of the integrability-breaking model correspond to $\gamma > 2$, $\gamma < 1$, and $1 < \gamma < 2$, respectively. To confirm this phase identification, we evaluated the fractal dimension D_2 (see Ref. [36] for details and its relation to the RP model). In Fig. 4, we show G_{rr} and $G_{\phi\phi}/r^2$ as a function of γ across the different phases. The values of QGT in extended regime

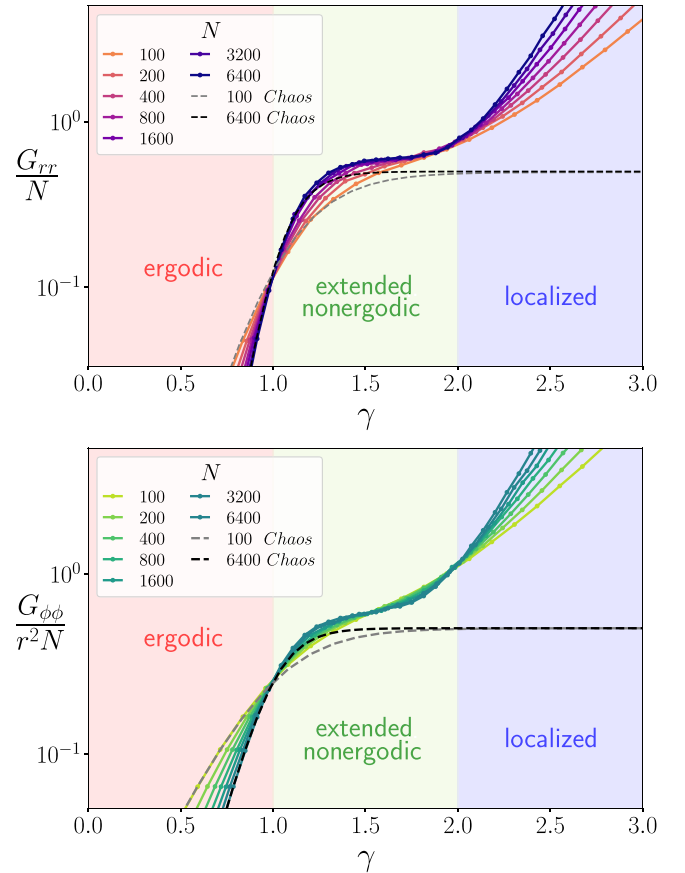


FIG. 4. Components of QGT as a function of scaling parameter. The dashed lines correspond to chaotic model given by Eq. (6).

do not diverge like in localized phase, but saturate to a value slightly different from the one computed for fully chaotic model [Eq. (6)]. Notably, $G_{\phi\phi}$ identifies both transitions separating the localized, fractal, and ergodic regimes, whereas G_{rr} provides a less sharp diagnostic of these crossovers.

It is worth noting the apparent analogy between the conical singularity near the integrable point and that (with the same conical angle) found previously in Ref. [4] for the ground state metric close to the isotropic quantum critical point in the ferromagnetic regime of the anisotropic XY spin chain. Our study shows an apparent similarity between the integrable and quantum critical points, with both characterized by the critical slowing down of dynamics and hence by divergent metric, as well as by a strong anisotropy along the radial and azimuthal directions for perturbations. In both situations, we observe condensation of the level crossings, near the origin (i.e., near the critical or integrable point), which are equivalent to magnetically charged defects producing vortex Berry connections in two-dimensional parameter space or the monopole Berry connection in three-dimensional parameter space.

For random matrix perturbations considered in this work, the matrix elements entering Eq. (4) for $x = y = 0$ are random, and hence the scaling of the geometric tensor is entirely determined by the spectral properties of the Hamiltonian. In this case, the diagonal components of the geometric tensor become equivalent to the spectral complexity $C(t)$, defined in Sec. 7 of Ref. [39]. The inverse of the time

t used to regularize the spectral complexity is equivalent to the energy cutoff μ introduced in Refs. [8,23]. Physically, the regularized geometric tensor determines the distance between the energy shells of width μ . Then, t or $1/\mu$ represents the minimal time required to measure this regularized metric. From these considerations, we conclude that the divergence $1/r$ of the geometric tensor leading to the conical singularity (see Fig. 4) originates from the loss of the level repulsion and therefore corresponds to μ of the order of the level spacing: $\mu \sim 1/N$ (see Ref. [36] for details).

Conclusions and outlook. In this Letter, we discussed the geometry of the two-dimensional parameter spaces for matrix Hamiltonians. Combining the analytical and numerical tools, we found that the associated geometry is smooth and locally equivalent to the surface of the sphere when the perturbation is large and the model is effectively in the ergodic regime. At the same time, near the integrable point, the metric develops a conical defect both for the Hilbert space dimension $N = 2$ and large N matrices. We also observed three different regimes of the metric behavior, corresponding to the localized, ergodic, and intermediate phases. The observed conical singularity is reminiscent of the one observed for the ground state near a continuous quantum phase transition. This similarity highlights the close analogy between the integrable and quantum critical points, both characterized by the emergence of divergent timescales and singular geometry.

It would be interesting to analyze in more detail the geometry by adding “integrable” perturbations commuting with

the unperturbed Hamiltonian. There, the metric is expected to diverge even more strongly due to an additional enhancement of the matrix elements of the perturbation near the integrable point [13,40]. We leave the corresponding analysis of the associated geometry for future work. Another interesting direction concerns the investigation of the metric for the perturbed matrix Russian Doll Model with time-reversal symmetry breaking, which also hosts the NEE phase [38]. Finally, we note that the adiabatic gauge potential and hence the metric can be directly computed from the Lanczos coefficients defining the Krylov basis [41,42]. For this reason, it can also be very interesting to study geometry for the matrix Hamiltonians enjoying the exact results [43,44]. The large N matrix Hamiltonians are also studied in the context of low-dimensional holography as Hamiltonians in the boundary one-dimensional model; see, for instance, Ref. [45]. Hence, it would be interesting to recognize the meaning of our findings in dual 2D gravity.

Acknowledgments. We thank C. Jonay, T. Prosen, S. Grozdanov, L. Vidmar, P. Łydzba, and D. Kurlov for useful discussions. R.S. acknowledges support from ERC Advanced Grant No. 101096208—QUEST, and Research Programme P1-0402 of Slovenian Research and Innovation Agency (ARIS). A.T. was funded by the ERC Starting Grant No. 101042293 (HEPIQ).

Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

-
- [1] J. P. Provost and G. Vallee, Riemannian structure on manifolds of quantum states, *Commun. Math. Phys.* **76**, 289 (1980).
 - [2] P. Zanardi and N. Paunković, Ground state overlap and quantum phase transitions, *Phys. Rev. E* **74**, 031123 (2006).
 - [3] P. Zanardi, P. Giorda, and M. Cozzini, Information-theoretic differential geometry of quantum phase transitions, *Phys. Rev. Lett.* **99**, 100603 (2007).
 - [4] M. Kolodrubetz, V. Gritsev, and A. Polkovnikov, Classifying and measuring geometry of a quantum ground state manifold, *Phys. Rev. B* **88**, 064304 (2013).
 - [5] D. Liska and V. Gritsev, Hidden symmetries, the Bianchi classification and geodesics of the quantum geometric ground-state manifolds, *SciPost Phys.* **10**, 020 (2021).
 - [6] S. Peotta, K.-E. Huhtinen, and P. Törmä, Quantum geometry in superfluidity and superconductivity, in *Quantum Mixtures with Ultra-cold Atoms*, Proceedings of the International School of Physics “Enrico Fermi”, Vol. 211 (IOS Press, Amsterdam, 2025), pp. 373–404.
 - [7] A. Alexandrov and A. Gorsky, Information geometry and synchronization phase transition in the Kuramoto model, *Phys. Rev. E* **107**, 044211 (2023).
 - [8] M. Pandey, P. W. Claeys, D. K. Campbell, A. Polkovnikov, and D. Sels, Adiabatic eigenstate deformations as a sensitive probe for quantum chaos, *Phys. Rev. X* **10**, 041017 (2020).
 - [9] F. M. Surace and O. Motrunich, Weak integrability breaking perturbations of integrable models, *Phys. Rev. Res.* **5**, 043019 (2023).
 - [10] S. Vanovac, F. M. Surace, and O. I. Motrunich, Finite-size generators for weak integrability breaking perturbations in the Heisenberg chain, *Phys. Rev. B* **110**, 144309 (2024).
 - [11] P. Orlov, A. Tiutiakina, R. Sharipov, E. Petrova, V. Gritsev, and D. V. Kurlov, Adiabatic eigenstate deformations and weak integrability breaking of Heisenberg chain, *Phys. Rev. B* **107**, 184312 (2023).
 - [12] B. Pozsgay, R. Sharipov, A. Tiutiakina, and I. Vona, Adiabatic gauge potential and integrability breaking with free fermions, *SciPost Phys.* **17**, 075 (2024).
 - [13] D. Sels and A. Polkovnikov, Dynamical obstruction to localization in a disordered spin chain, *Phys. Rev. E* **104**, 054105 (2021).
 - [14] H. Kim and A. Polkovnikov, Integrability as an attractor of adiabatic flows, *Phys. Rev. B* **109**, 195162 (2024).
 - [15] V. E. Kravtsov, I. M. Khaymovich, E. Cuevas, and M. Amini, A random matrix model with localization and ergodic transitions, *New J. Phys.* **17**, 122002 (2015).
 - [16] D. Venturelli, L. F. Cugliandolo, G. Schehr, and M. Tarzia, Replica approach to the generalized Rosenzweig-Porter model, *SciPost Phys.* **14**, 110 (2023).
 - [17] B. Bhattacharjee and P. Nandy, Krylov fractality and complexity in generic random matrix ensembles, *Phys. Rev. B* **111**, L060202 (2025).
 - [18] E. Bogomolny and M. Sieber, Eigenfunction distribution for the Rosenzweig-Porter model, *Phys. Rev. E* **98**, 032139 (2018).

- [19] W. Tang and I. M. Khaymovich, Non-ergodic delocalized phase with Poisson level statistics, *Quantum* **6**, 733 (2022).
- [20] I. M. Khaymovich and V. E. Kravtsov, Dynamical phases in a “multifractal” Rosenzweig–Porter model, *SciPost Phys.* **11**, 045 (2021).
- [21] P. von Soosten and S. Warzel, Non-ergodic delocalization in the Rosenzweig–Porter model, *Lett. Math. Phys.* **109**, 905 (2018).
- [22] B. Altshuler and V. Kravtsov, Random Cantor sets and minibands in local spectrum of quantum systems, *Ann. Phys.* **456**, 169300 (2023).
- [23] S. Sugiura, P. W. Claeys, A. Dymarsky, and A. Polkovnikov, Adiabatic landscape and optimal paths in ergodic systems, *Phys. Rev. Res.* **3**, 013102 (2021).
- [24] A. I. Larkin and Y. N. Ovchinnikov, Quasiclassical method in the theory of superconductivity, *Sov. Phys. JETP* **28**, 1200 (1969).
- [25] D. E. Parker, X. Cao, A. Avdoshkin, T. Scaffidi, and E. Altman, A universal operator growth hypothesis, *Phys. Rev. X* **9**, 041017 (2019).
- [26] F. Evers and A. D. Mirlin, Anderson transitions, *Rev. Mod. Phys.* **80**, 1355 (2008).
- [27] M. Serbyn, Z. Papić, and D. A. Abanin, Criterion for many-body localization-delocalization phase transition, *Phys. Rev. X* **5**, 041047 (2015).
- [28] C. Lim, K. Matirko, A. Polkovnikov, and M. O. Flynn, Defining classical and quantum chaos through adiabatic transformations, *J. Phys. A: Math. Theor.* **59**, 075702 (2026).
- [29] B. V. Fine, T. A. Elsayed, C. M. Kropf, and A. S. de Wijn, Absence of exponential sensitivity to small perturbations in nonintegrable systems of spins $1/2$, *Phys. Rev. E* **89**, 012923 (2014).
- [30] I. Kukuljan, S. Grozdanov, and T. Prosen, Weak quantum chaos, *Phys. Rev. B* **96**, 060301(R) (2017).
- [31] S. Pappalardi, A. Polkovnikov, and A. Silva, Quantum echo dynamics in the Sherrington-Kirkpatrick model, *SciPost Phys.* **9**, 021 (2020).
- [32] M. Berry and P. Shukla, Quantum metric statistics for random-matrix families, *J. Phys. A: Math. Theor.* **53**, 275202 (2020).
- [33] P. Sierant, A. Maksymov, M. Kuś, and J. Zakrzewski, Fidelity susceptibility in Gaussian random ensembles, *Phys. Rev. E* **99**, 050102(R) (2019).
- [34] A.-G. Penner, F. von Oppen, G. Zaránd, and M. R. Zirnbauer, Hilbert space geometry of random matrix eigenstates, *Phys. Rev. Lett.* **126**, 200604 (2021).
- [35] B. Derrida, Random-energy model: Limit of a family of disordered models, *Phys. Rev. Lett.* **45**, 79 (1980).
- [36] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/92z8-bqx9> for additional details.
- [37] M. L. Mehta, *Random Matrices* (Elsevier, Amsterdam, 2004).
- [38] V. R. Motamarri, A. S. Gorsky, and I. M. Khaymovich, Localization and fractality in disordered Russian Doll model, *SciPost Phys.* **13**, 117 (2022).
- [39] M. A. Skvortsov, M. Amini, and V. E. Kravtsov, Sensitivity of (multi)fractal eigenstates to a perturbation of the Hamiltonian, *Phys. Rev. B* **106**, 054208 (2022).
- [40] L. V. Iliesiu, M. Mezei, and G. Sárosi, The volume of the black hole interior at late times, *J. High Energy Phys.* **07** (2022) 073.
- [41] B. Bhattacharjee, A Lanczos approach to the adiabatic gauge potential, *arXiv:2302.07228*.
- [42] K. Takahashi and A. del Campo, Shortcuts to adiabaticity in Krylov space, *Phys. Rev. X* **14**, 011032 (2024).
- [43] I. Dumitriu and A. Edelman, Matrix models for beta ensembles, *J. Math. Phys.* **43**, 5830 (2002).
- [44] V. Balasubramanian, J. M. Magan, and Q. Wu, Tridiagonalizing random matrices, *Phys. Rev. D* **107**, 126001 (2023).
- [45] P. Saad, S. H. Shenker, and D. Stanford, JT gravity as a matrix integral, *arXiv:1903.11115*.