

Observation of supersensitivity of non-Hermitian skin effect

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Non-Hermitian systems exhibit an extraordinary sensitivity to boundary conditions, with a notable example being the non-Hermitian skin effect (NHSE), where a significant number of eigenstates accumulate at their open boundaries. In this work, we reveal a supersensitivity feature associated with the NHSE. In an acoustic system with dissipative couplings that preserve the anomalous time-reversal symmetry, we observe a k -dependent skin effect, in which acoustic waves localize at either the left or right open boundaries depending on the perpendicular wave vector. Remarkably, this supersensitivity is so extreme that adding a single unit to one boundary transforms the skin effect from bidirectional to unidirectional. More intriguingly, introducing another single unit to the opposite boundary can restore the original bidirectional skin effect. We directly observe this supersensitivity by detecting acoustic field distributions, consistent with theoretical predictions. Our findings offer a powerful approach to non-Hermitian manipulation and pave the way for high-sensitivity measurement using non-Hermitian systems.

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Recent advancements in non-Hermitian physics have significantly expanded the scope of traditional Hermitian systems [1–4]. This progress is driven by the intrinsic instability of non-Hermitian spectra and wave functions, which give rise to unique and unconventional phenomena. One notable example occurs in the vicinity of exceptional points (degeneracies with coalescence of eigenstates), leading to enhanced sensitivity through square root responses [5–11]. Furthermore, the emergence of non-Hermitian skin effect (NHSE) renders states exhibiting an anomalous accumulation at the boundaries [12–20], which disrupts extended Bloch waves and challenges the conventional bulk-edge correspondence [21–25]. The NHSE is highly sensitive to boundary conditions; in one-dimensional (1D) systems, even small modifications to boundaries can drastically alter the spectra and the localization behavior of skin modes [26–31]. In higher dimensions, the NHSE becomes more intricate, with its behavior depending on the dimensionality and global geometry [32–40].

In recent years, advancements in non-Hermitian physics have stimulated extensive experimental research in open systems, especially across diverse classical wave platforms [31,41,42]. A notable advantage of classical wave systems, especially those on a macroscopic scale, lies in the precise control of their boundaries, enabling exact exploration

of the system's sensitivity to these modifications. Recently, the NHSE for sound waves has been demonstrated within phononic crystals, where non-Hermiticity arises from acoustic dissipative couplings [37,43], asymmetric couplings [44–49], and on-site gain/loss of sound energy [50,51]. However, existing studies have predominantly focused on the experimental realization of the NHSE, with limited attention devoted to leveraging the highly sensitive properties of non-Hermitian systems. In fact, unlocking the full potential of high sensitivity of the NHSE could provide crucial insights and valuable inspiration for advancing non-Hermitian effects toward practical device applications.

In this work, we unveil the supersensitivity of the NHSE, where the skin effect can switch between bidirectional and unidirectional in response to subtle boundary modifications. In contrast to the NHSE driven by global geometry or parameter variations [34–38], the NHSE behavior in two-dimensional (2D) systems can be tuned by one single unit cell, associated with the redistribution of the overall wave functions across the system. Specifically, we design a rolled 2D phononic crystal (PC) with a pair of open boundaries, which exhibits a k -dependent NHSE. The states at $\pm k$, related by time-reversal symmetry, correspond to skin effects with opposite directions, resulting in a bidirectional skin effect. Strikingly, this bidirectional skin effect collapses to unidirectional upon the addition of a single unit cell to one of the open boundaries. Furthermore, the bidirectional skin effect is restored by adding an additional unit cell to the opposite open boundary. This supersensitivity of the NHSE in our acoustic system highlights its potential for developing high-precision sensors. All the simulation results are performed by COMSOL Multiphysics.

We start by building a tight-binding model on a square lattice, as illustrated in Fig. 1(a). The unit cell, outlined in a light blue frame, consists of two inequivalent sites with

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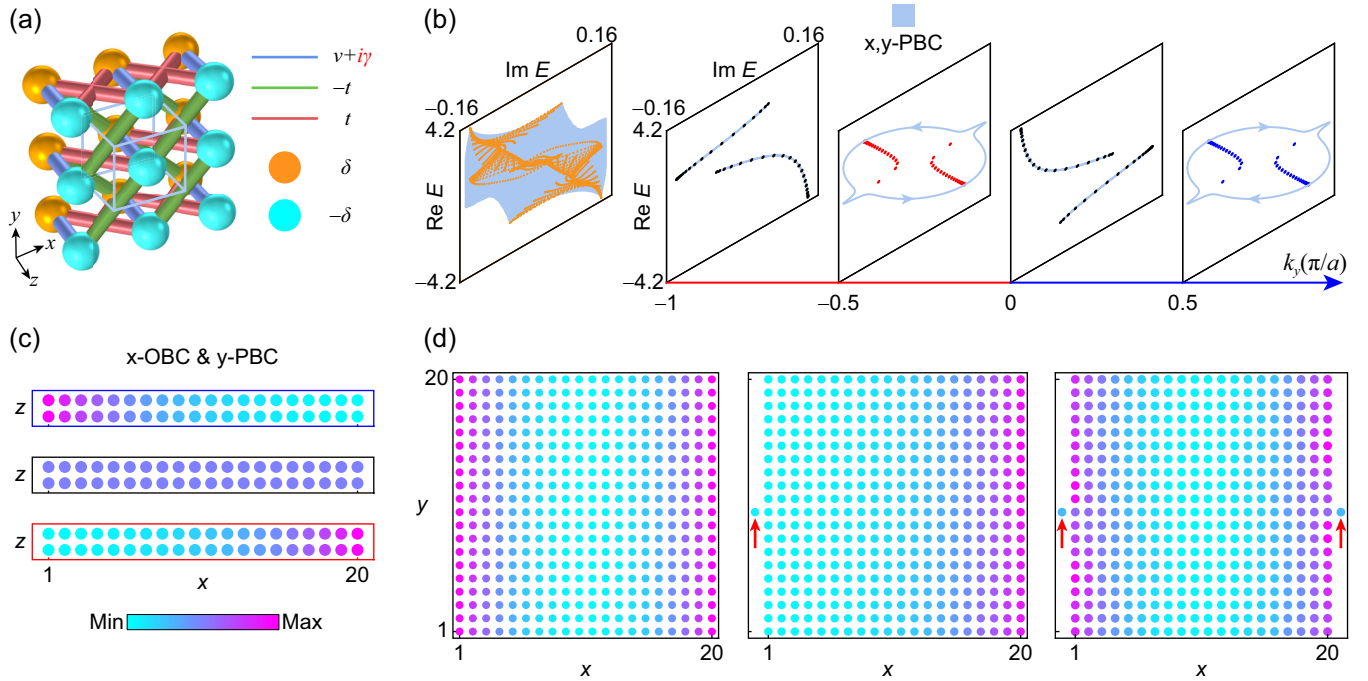


FIG. 1. Non-Hermitian k_y -dependent skin effect and supersensitivity to boundary modifications. (a) Illustration of the unit cell in the non-Hermitian lattice model. (b) Left panel: The full complex spectrum of H as a 2D model under y -PBC. The full spectrum with mixed boundaries x -OBC and y -PBC (orange dots) lies within the spectral area obtained under x , y -PBCs (blue region). Other panels: Complex spectra for four representative fixed k_y . (c) Spatial field distribution of the eigenstates under x -OBC and y -PBC, the fixed k_y corresponding to cases of panel (b). (d) Spatial field distribution of the eigenstates under x -OBC and y -PBC with different boundary modifications. The parameters used are $\delta = 1$, $v = -1$, $t = -0.7$, and $\gamma = 0.15$.

on-site potentials $\pm\delta$. The intercell hoppings are of strengths $\pm t$, while the non-Hermiticity enters as intracell hoppings of strength $v + i\gamma$. The corresponding Bloch Hamiltonian takes the form

$$H = \begin{pmatrix} \delta - 2t \cos(k_x - k_y) & v + i\gamma + 2t \cos k_x \\ v + i\gamma + 2t \cos k_x & -\delta - 2t \cos(k_x + k_y) \end{pmatrix}. \quad (1)$$

For a fixed k_y , this model can be treated as a 1D system. Specifically, when $k_y = \pm 0.5\pi$, the system, after applying the unitary transformation of $U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$, becomes equivalent to a non-Hermitian Su-Schrieffer-Heeger (SSH) with asymmetric left/right intracell couplings of strengths $v + i(\gamma \pm \delta)$ [44,52]. The energy spectra under periodic boundary conditions in the x direction (x -PBC; light blue curves and region) and open boundary conditions in the x direction (x -OBC; colored dots) are displayed in Fig. 1(b). More spectra with different k_y values are provided in the Supplemental Material [53]. The PBC spectrum forms a closed loop in the complex plane, with its nonzero winding number indicating the emergence of NHSE [13,14,54]. For a certain base energy E_b , the winding number is given by $w = \frac{1}{2\pi i} \int_{-\pi}^{\pi} \partial_{k_x} \ln \det[H(k_x) - E_b] dk_x$. For instance, at $k_y = -0.5\pi$, the spectrum exhibits an anticlockwise circulation in the complex plane, corresponding to $w = -1$. This results in the right localization of the eigenmodes, as shown in Fig. 1(c). Conversely, at $k_y = 0.5\pi$, the spectrum circulates clockwise, corresponding to $w = 1$, which leads to the left localization of the eigenmodes. Note that the NHSE occurs for $k_y \neq 0$ or π , as represented by the blue and red segments in Fig. 1(b).

Moreover, the Hamiltonians $H(k_y)$ and $H(-k_y)$ form a time-reversal pair characterized by opposite winding numbers w , causing opposite directional skin effects and collectively giving rise to the $\mathbb{Z}_2$ skin effect [55].

The k_y -dependent NHSE induces a bidirectional skin effect in a 2D system with PBC along the y direction. As shown in the left panel of Fig. 1(b), the full spectrum under x -OBC is represented by orange dots. The blue-shaded region corresponds to the spectrum area under both x -PBC and y -PBC. The significant discrepancy between these two spectra implies the emergence of the skin effect along the x direction. Such skin effect is bidirectional, with the eigenstates spatially localized at both the left and right boundaries, as illustrated in the left panel of Fig. 1(d). The bidirectional NHSE arises from the fact that the 2D finite system with y -PBC can be viewed as a superposition of multiple 1D $\mathbb{Z}_2$ systems, each formed by paired states with $(k_y, -k_y)$.

Minor boundary modifications can significantly affect the distribution of eigenmodes due to the extreme sensitivity of the NHSE. For example, adding a single unit cell to the left boundary of a finite system dramatically alters the spatial field distribution compared to the unmodified system, as shown in the middle panel of Fig. 1(d). This minor boundary modification induces a unidirectional rightward skin effect—a phenomenon previously unobserved in time-reversal symmetric systems. The added unit cell creates hoppings that connect all chains with different k_y at the left end, mimicking the effects of removing the left boundary. Furthermore, when unit cells are added to both open boundaries of the system, the bidirectional NHSE reemerges and shows similar localization

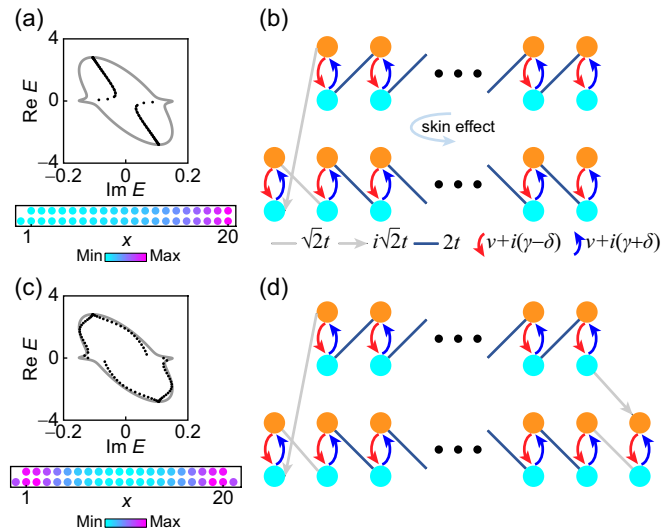


FIG. 2. Supersensitivity of the non-Hermitian $\mathbb{Z}_2$ skin effect in a minor system. (a) Top panel: Complex spectra with PBC (curve) and OBC (dots) with a left boundary modification. Bottom panel: Spatial field distribution of the eigenstates under x -OBC. (b) Schematic of a finite lattice with a left boundary modification after applying the unitary transformation. (c), (d) The same as panels (a) and (b) but for the case with both left and right boundary modifications.

behavior as in the case without boundary modification, as illustrated in the right panel of Fig. 1(d). In this case, additional hoppings are established at both open boundaries, with hopping strengths weaker than those in the bulk, causing states to accumulate at the boundaries. This extraordinary behavior underscores the supersensitivity of the NHSE.

A minimal model that exhibits a $\mathbb{Z}_2$ skin effect is presented to explicitly explain the supersensitivity. The tight-binding model and its corresponding Hamiltonian are provided in Appendix A, which formally is two copies of the Hamiltonian in Eq. (1) with $k_y = 0.5\pi$ and $k_y = -0.5\pi$, as illustrated in Fig. 5(b). Applying the same unitary transformation U , this model gives rise to a pair of non-Hermitian SSH chains with intercell couplings of $2t$ and intracell couplings of $v + i(\gamma \pm \delta)$, exhibiting unidirectional leftward and rightward skin effects, respectively. The $\mathbb{Z}_2$ skin effect here stems from the block diagonalization of the entire Hamiltonian, which is forbidden when the chain boundaries are modified. As in the 2D case, attaching a unit cell at the left end of this finite chain, the complex spectrum with OBC (dots) remains almost the same compared to the unmodified case, while the spatial field distribution significantly differs, exhibiting a rightward unidirectional skin effect [Fig. 2(a)]. This boundary modification under U is explicitly reflected in the last two columns and rows of Eq. (B1) in Appendix B, corresponding to the two leftmost sites in Fig. 2(b). These two sites are connected to the chains via gray hoppings of strength $\sqrt{2}t$, serving as additional channels that couple the two originally independent chains. This coupling enables the skin modes in the upper chain to subsequently experience the rightward skin effect through the lower chain. With an additional modification at the opposite end, similar channels are formed, creating a closed loop of coupled chains [27,28,53]. As shown

in Fig. 2(d), the hoppings with these boundary channels are weaker than those in the bulk, resulting in the OBC spectrum closely approaching the PBC spectrum. Despite this, the OBC spectrum remains enclosed by the PBC spectrum, and the bidirectional skin effect emerges, as shown in Fig. 2(c). A detailed Hamiltonian-based analysis is provided in Appendix B.

To experimentally observe the supersensitivity of the NHSE, we construct a PC based on the proposed tight-binding model. The schematic of the unit cell and a photograph of the entire PC are presented in Figs. 3(a) and 3(b), respectively. The PC sample, fabricated by 3D printing technology, consists of $20 \times 20 \times 2$ acoustic cavities. The entire sample is rolled into a cylindrical shape by joining its two end boundaries to ensure PBC along the y direction. This PBC is crucial for observing supersensitivity [53]. Additionally, the non-Hermitian intracell hopping is implemented by introducing carefully designed losses, achieved through holes in the acoustic waveguides that are sealed with sound-absorbing sponges [48,49]. In this configuration, the acoustic cavities represent the lattice sites, while the coupling waveguides correspond to the hopping terms in the tight-binding model [37,43]. Detailed structural parameters are provided in the Supplemental Material [53].

The simulated spectrum of the PC is shown in Fig. 3(c). The orange dots represent the spectral area of the sample under x -OBC and y -PBC, while the light blue dots correspond to the spectrum under x -PBC and y -PBC. These two spectra do not overlap, implying the existence of the skin effect [34]. As depicted in Fig. 3(d), the spatial field distribution of the eigenstates exhibits localization at the left and right boundaries under x -OBC. Furthermore, the projected dispersion is shown in Fig. 3(e), where the color indicates the expectation value of the position operator in real space, effectively capturing the degree of eigenstate localization at the left and right boundaries. It is evident that the eigenstates with $0 < k_y < \pi$ are localized at the left boundary, while those with $-\pi < k_y < 0$ are localized at the right boundary, both forming skin modes.

We experimentally observe the k_y -dependent NHSE. In the experiment, an acoustic source [denoted by stars in Fig. 3(b)] is placed near the left or right boundary to selectively excite the skin modes, and the response of the acoustic pressure field is measured at the corresponding boundary. By performing a Fourier transformation of the measured field, the resulting band dispersion reflects the localization of the bulk modes at the left boundary, as shown in the left panel of Fig. 3(f), with the color indicating the degree of localization. The excited modes are primarily observed in $0 < k_y < \pi$, confirming that these bulk states correspond to skin modes localized at the left boundary. Similarly, the right panel of Fig. 3(f) verifies that the measured bulk states in $-\pi < k_y < 0$ are localized at the right boundary. Furthermore, Fig. 3(g) presents the spatially resolved intensity of the localized acoustic response stimulated by acoustic chiral sources, consisting of two point sources with phase differences [53]. In the experiments, the microphone, as well as the chiral source, scans across the entire sample to capture the acoustic signals, and the measurement frequency ranges from 3.3 to 4.7 kHz. These measured results manifest boundary accumulation of the acoustic pressure fields, providing direct evidence of the skin effect.

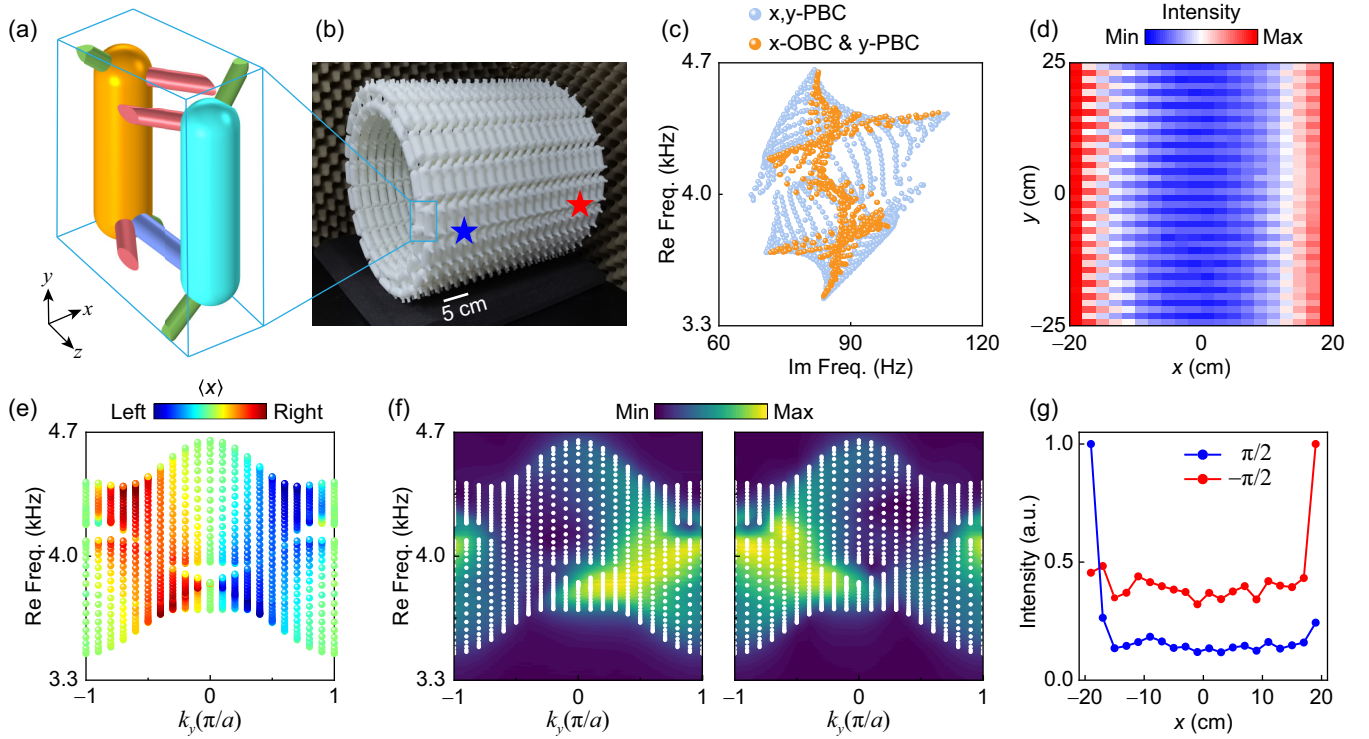


FIG. 3. Acoustic realization of k_y -dependent skin effect. (a) Schematic of the unit cell. Acoustic loss is introduced in the narrow blue tube. (b) Photograph of the PC sample. (c) Complex spectra for the PC. Light blue dots correspond to the spectrum under x -PBC and y -PBC, while orange dots are obtained under x -OBC and y -PBC. (d) Spatial field distribution of all the eigenstates. (e) Simulated projected dispersion for a ribbon with colors indicating to expectation value of position operator. (f) Measured (color map) and simulated (points) projected dispersions obtained by the pressure field at the left and right boundaries. (g) Distribution of acoustic responses corresponding to different chiral sources.

Based on the measured k_y -dependent NHSE, we show its supersensitivity to boundary modifications. Since the skin modes are localized at the left and right boundaries under x -OBC, we measure the localized acoustic response along the x direction to detect the NHSE. The result for the unmodified case is shown in the bottom panel of Fig. 4(a), showing a clear

bidirectional NHSE. Each data point represents the average of five measurements, with error bars indicating the associated standard deviation. Figure 4(b) shows a PC sample with one modified boundary, where a unit cell is added to the left boundary (highlighted in red). This boundary modification disrupts the original skin effect, establishing a new equilibrium pattern and leading to a unidirectional NHSE, where the skin modes shift to the right boundary. This phenomenon is experimentally validated through the measured response field, as shown in the bottom panel of Fig. 4(b). In the absence of boundary modifications, the rolled 2D PC exhibits a lattice rotational symmetry along the y direction, allowing it to be decomposed into a set of independent chains and each supports a k_y -dependent NHSE. However, the introduced boundary modification breaks the rotational symmetry, establishing new channels between these chains [53]. Furthermore, when the boundary modification of adding one unit cell is applied to both the left and right boundaries, as illustrated in Fig. 4(c), the bidirectional NHSE is restored due to the establishment of new channels at both boundaries. The measured response field confirming this restoration is presented in the bottom panel of Fig. 4(c). These results highlight the supersensitivity of the NHSE to subtle changes in boundary conditions. Notably, due to the intrinsic sensitivity of the non-Hermitian system, these modifications can add curves of new eigenstates in the complex energy spectrum [39,40,53], corresponding to spatial channels linking the two boundary modifications.

In conclusion, we have theoretically calculated and experimentally demonstrated the supersensitivity of the NHSE.

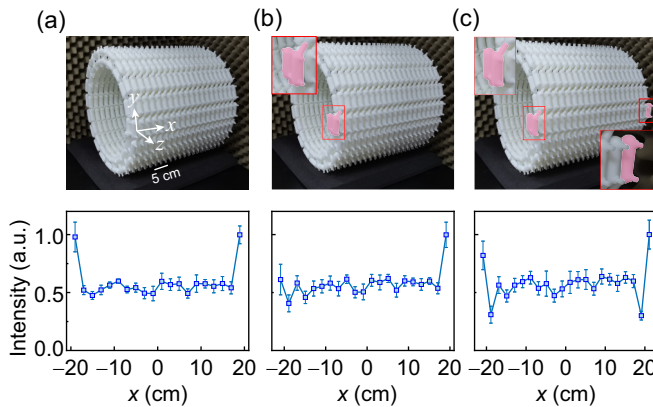


FIG. 4. Observation of the supersensitivity to boundary modifications. (a)–(c) Top panels: Photographs of the PC samples with varying boundary configurations. The cavities marked in red indicate the unit cells added to boundaries. Bottom panels: Measured distribution of acoustic responses with the uncertainty in the frequency range of 4.0–4.2 kHz, showing the impact of boundary modifications on acoustic wave localization.

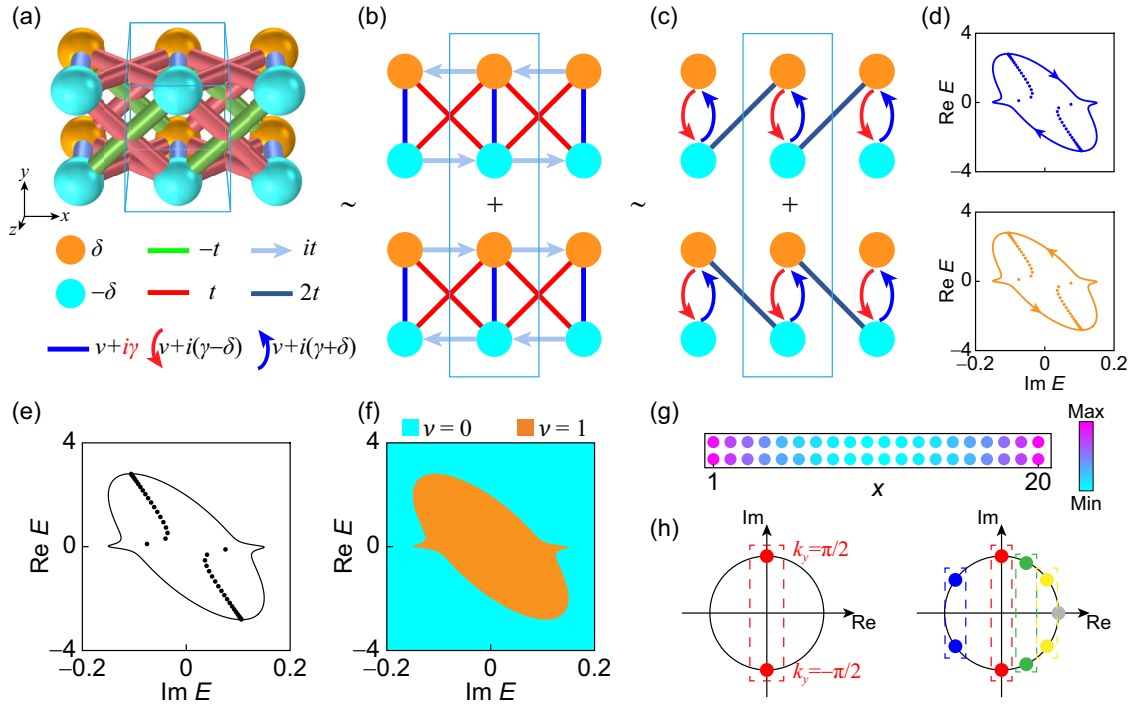


FIG. 5. Non-Hermitian $\mathbb{Z}_2$ skin effect. (a)–(c) Lattice model that can be reduced into two subsystems. Blue boxes denote the unit cells. (d) Complex spectra of the two subsystems under PBC (curve) and OBC (dots). (e) Complex spectra of the entire system under PBC (curve) and OBC (dots). (f) $\mathbb{Z}_2$ invariant v of H . (g) Spatial field distribution of the eigenstates of $H_{\mathbb{Z}_2}$ under x OBC. (h) Left panel: k_y values corresponding to two subsystems of $H_{\mathbb{Z}_2}$. Right panel: Multiple k_y values corresponding to two subsystems of H in the main text. The parameters are $\delta = 1$, $v = -1$, $t = -0.7$, and $\gamma = 0.15$.

A k -dependent non-Hermitian skin effect is realized through dissipative couplings where the non-Hermiticity corresponds to $i\sigma_x$. Acoustic waves are localized at the left or right open boundaries, depending on the wave vector. Furthermore, the transition between bidirectional and unidirectional skin effects is achieved by boundary modifications, such as adding or removing a unit cell or introducing impurity-type perturbations [53]. In our experiments, we observe the k -dependent NHSE and its extreme sensitivity to boundary modifications. Our findings can be extended to other non-Hermitian systems such as photonic crystals [38,53,56–61], cold atoms [62–65], elastic media [66,67], and electric circuits [68–72]. The supersensitivity of the NHSE offers an efficient mechanism for mode manipulation based on non-Hermiticity, enabling its application in complex non-Hermitian networks hosting multiple accumulations simultaneously.

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Data availability. The data are not publicly available. The data are available from the authors upon reasonable request.

Appendix A: The $\mathbb{Z}_2$ skin effect in 1D system. The tight-binding model is illustrated in Fig. 5(a), with the corresponding Hamiltonian given by $H_{\mathbb{Z}_2} = \delta\sigma_z + (v + i\gamma + 2t\cos k_x)\sigma_x - 2t\sin k_x\tau_y\sigma_z$, where τ and σ are Pauli matrices representing the interlayer and intralayer degrees of freedom, respectively. The model can be block-diagonalized using an interlayer unitary transformation $U \otimes I_2$, where $U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ and I_2 is the 2×2 identity matrix. This transformation results in two decoupled subsystems, schematically illustrated in Fig. 5(b). Moreover, both subsystems can be further transformed into non-Hermitian SSH models [Fig. 5(c)] via an intralayer unitary transformation $I_2 \otimes U$. These subsystems exhibit left and right skin effects, respectively, with their PBC and OBC complex energy spectra shown in Fig. 5(d). Notably, the PBC spectra possess opposite winding numbers for these two subsystems. Consequently, the complex energy spectrum of the original Hamiltonian $H_{\mathbb{Z}_2}$ is a superposition of the two subsystem spectra [Fig. 5(e)], exhibiting a point gap but with zero winding number. The Hamiltonian $H_{\mathbb{Z}_2}$ has anomalous time-reversal symmetry described by $C_+ H_{\mathbb{Z}_2}^T(k_x) C_+^{-1} = H_{\mathbb{Z}_2}(-k_x)$, where $C_+ = \sigma_y \otimes I_2$ satisfies $C_+ C_+^* = -1$. The corresponding $\mathbb{Z}_2$ invariant $v \in \{0, 1\}$ is thus given as

$$(-1)^{v(E)} = \text{sgn} \left\{ \frac{\text{Pf}[(H_{\mathbb{Z}_2}(\pi) - E)C_+]}{\text{Pf}[(H_{\mathbb{Z}_2}(0) - E)C_+]} \exp \left[-\frac{1}{2} \int_{k_x=0}^{k_x=\pi} d \log \det[(H_{\mathbb{Z}_2}(k) - E)C_+] \right] \right\}.$$

The calculated $\mathbb{Z}_2$ invariants, as shown in Fig. 5(f), reveal a nontrivial topological number enclosed by the loop, indicating the presence of the $\mathbb{Z}_2$ skin effect. To further illustrate the skin effect, we calculate the spatial distribution of all the eigenstates, defined as $\sum_n |\psi_n|^2$, where ψ_n represents the n th normalized eigenstate. As shown in Fig. 5(g), the resulting profile exhibits localization at both ends, which can be viewed as a superposition of the leftward and rightward skin effects of the two subsystems. In fact, the subsystems of $H_{\mathbb{Z}_2}$ correspond to two special cases of the Hamiltonian H in the main text with $k_y = \pm 0.5\pi$. The Hamiltonian H incorporates multiple pairs of chains with opposite k_y [Fig. 5(h)], resulting in a bidirectional skin effect.

Appendix B: Explicit form of the Hamiltonian for the 1D system with OBC. For a system with n unit cells, the Hamiltonian under OBC is represented as a $4n \times 4n$ matrix with form

$$H_{\mathbb{Z}_2}^{\text{OBC}} = \begin{pmatrix} h_0 & h_t^T \\ h_t & h_0 \end{pmatrix}.$$

Here, h_0 denotes the intralayer Hamiltonian and h_t represents the interlayer coupling. Both h_0 and h_t are $2n \times 2n$ matrices, where

$$h_0 = \begin{pmatrix} \delta & v + i\gamma & 0 & t & \cdots & 0 & 0 \\ v + i\gamma & -\delta & t & 0 & \cdots & 0 & 0 \\ 0 & t & \delta & v + i\gamma & \cdots & 0 & 0 \\ t & 0 & v + i\gamma & -\delta & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \delta & v + i\gamma \\ 0 & 0 & 0 & 0 & \cdots & v + i\gamma & -\delta \end{pmatrix}$$

and

$$h_t = \begin{pmatrix} 0 & 0 & t & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & -t & \cdots & 0 & 0 \\ -t & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & t & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}.$$

In this case, the unitary transformation matrices are $U_{\text{OBC}} = U \otimes I_n \otimes U$. By performing the unitary transformation $U_{\text{OBC}}^\dagger H_{\mathbb{Z}_2}^{\text{OBC}} U_{\text{OBC}}$, the Hamiltonian is transformed into

$$U_{\text{OBC}}^\dagger H_{\mathbb{Z}_2}^{\text{OBC}} U_{\text{OBC}} = \begin{pmatrix} h_1 & 0 \\ 0 & h_2 \end{pmatrix},$$

where the diagonal blocks are

$$h_1 = \begin{pmatrix} 0 & t_+ & 0 & 2t & \cdots & 0 & 0 \\ t_- & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & t_+ & \cdots & 0 & 0 \\ 2t & 0 & t_- & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & t_+ \\ 0 & 0 & 0 & 0 & \cdots & t_- & 0 \end{pmatrix}$$

and

$$h_2 = \begin{pmatrix} 0 & t_+ & 0 & 0 & \cdots & 0 & 0 \\ t_- & 0 & 2t & 0 & \cdots & 0 & 0 \\ 0 & 2t & 0 & t_+ & \cdots & 0 & 0 \\ 0 & 0 & t_- & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & t_+ \\ 0 & 0 & 0 & 0 & \cdots & t_- & 0 \end{pmatrix},$$

with the hopping amplitudes being $t_+ = v + i(\gamma + \delta)$ and $t_- = v + i(\gamma - \delta)$.

With a unit cell added to the left end, the finite-size Hamiltonian is modified accordingly. After applying the unitary transformation $U_{O1} = U_{OBC} \oplus U$, the explicit form of the Hamiltonian H_{O1} takes the form

$$H_{O1} = \begin{pmatrix} 0 & t_+ & 0 & 2t & \dots & 0 & 0 \\ t_- & 0 & 0 & 0 & \dots & \sqrt{2}t & 0 \\ 0 & 0 & 0 & t_+ & \dots & 0 & 0 \\ 2t & 0 & t_- & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ & & & 0 & t_+ & 0 & 0 & \dots \\ & & & t_- & 0 & 2t & 0 & \dots \\ & & 0 & 0 & 2t & 0 & t_+ & \dots \\ & & & 0 & 0 & t_- & 0 & \dots \\ & & & \vdots & \vdots & \vdots & \vdots & \ddots \\ 0 & \sqrt{2}t & 0 & 0 & \dots & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots & -i\sqrt{2}t & 0 & 0 & \dots \end{pmatrix}, \quad (B1)$$

where the upper-left $4n \times 4n$ block remains unchanged. However, the additional sites at the boundaries break the independence between the h_1 and h_2 sectors, effectively coupling them through the hoppings of strength $\sqrt{2}t$. This coupling is explicitly reflected in the last two columns and rows of H_{O1} in Eq. (B1).

After adding an additional unit cell to the opposite end and applying the unitary transformation $U_{O2} = U_{OBC} \oplus U \oplus U$, the Hamiltonian H_{O2} is obtained as

$$H_{O2} = \begin{pmatrix} 0 & t_+ & 0 & 2t & \dots & 0 & 0 & 0 & 0 \\ t_- & 0 & 0 & 0 & \dots & 0 & 0 & \sqrt{2}t & 0 \\ 0 & 0 & 0 & t_+ & \dots & 0 & 0 & 0 & 0 \\ 2t & 0 & t_- & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 & t_+ & 0 & \sqrt{2}t \\ 0 & 0 & 0 & 0 & \dots & t_- & 0 & 0 & 0 \\ & & & 0 & t_+ & 0 & 0 & 0 & 0 \\ & & & t_- & 0 & 2t & 0 & 0 & 0 \\ & & 0 & 2t & 0 & t_+ & \dots & 0 & 0 \\ & & & 0 & 0 & t_- & 0 & \dots & 0 \\ & & & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ & & 0 & 0 & 0 & 0 & \dots & 0 & t_+ \\ & & 0 & 0 & 0 & 0 & \dots & t_- & 0 \\ 0 & \sqrt{2}t & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & i\sqrt{2}t & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & i\sqrt{2}t \\ 0 & 0 & 0 & 0 & \dots & \sqrt{2}t & 0 & 0 & 0 \end{pmatrix}, \quad (B2)$$

which differs from H_{O1} by the inclusion of additional columns and rows. The newly introduced boundary sites connect the respective ends of h_1 and h_2 , thereby creating a closed loop that couples the two subsystems.

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