

Simultaneous cooling of the internal and external degrees of freedom of a levitated micromagnet in a cavity magnomechanical system

Lei Chen¹, Yang Liu^{1,2}, Liang Bin¹, Sai-Yun Ye^{1,*} and Zhi-Rong Zhong^{1,†}

¹*Fujian Key Laboratory of Quantum Information and Quantum Optics (Fuzhou University), Fuzhou 350108, China*

²*Center for Joint Quantum Studies and Department of Physics, School of Science, Tianjin University, Tianjin 300350, China*



(Received 28 February 2025; accepted 23 July 2025; published 14 August 2025)

Cooling the internal and external degrees of freedom (DOFs) of a massive mechanical system close to the quantum ground state is a prerequisite for the quantum manipulation of macroscopic mechanical devices. In this paper, we propose a approach to simultaneously cool both the internal and external DOFs (phonon and center-of-mass modes) of a levitated micromagnet within a cavity magnomechanical (CMM) system. By driving the cavity mode and magnon mode on their respective red sidebands with microwave fields, the proposed scheme effectively suppresses Stokes scattering and significantly enhances the net cooling rate. Moreover, differing from conventional CMM systems, our proposal can achieve simultaneous cooling of the internal and external DOFs even in the unresolved-sideband regime. Our numerical simulations demonstrate the robustness and effectiveness of the scheme by analyzing average occupancies of the internal and external DOFs, as well as identifying the optimal cooling conditions. The proposed scheme establishes a foundation for probing and manipulating the internal DOFs by controlling the external DOFs, which may open additional possibilities for studying nonequilibrium quantum mesoscopic physics.

DOI: [10.1103/8pzl-6c5l](https://doi.org/10.1103/8pzl-6c5l)

I. INTRODUCTION

The cavity magnomechanical (CMM) system [1–4], which includes a yttrium iron garnet (YIG) sphere in a cavity, provides an ideal platform for quantum information processing [5–10]. Its exceptional performance stems from unique properties of the YIG sphere, including high spin density, enhanced nonlinearities [11], and rich degrees of freedom (DOFs). These DOFs are categorized into two types: internal DOFs, such as magnon modes (collective spin excitations) and phonon modes (quantized lattice vibrations) [12], and external DOFs [13], such as center-of-mass (CM) motion [14–16] and rotational modes [17,18]. Interactions between these DOFs, including strong magnon-photon interactions [19–21], magnon-phonon interactions [12], and acoustomechanical interactions [12,22], enable versatile quantum control. The coupling between the YIG sphere and a superconducting coplanar microwave resonator [23] and the bistability of cavity magnon polaritons have been investigated [24]. Furthermore, the YIG sphere holds significant potential for quantum precision sensing in the CMM system.

However, environmental noise poses a significant challenge to the practical applications in the CMM system. To

address this issue, magnetic levitation technology has been developed [25–29], which levitates the YIG sphere in vacuum to achieve exceptional environmental isolation. A recent report on superconducting levitation platforms enables quantum control of mesoscopic systems. In this work, Gieseler *et al.* demonstrated stable three-dimensional levitation of micromagnets via flux pinning in type-II superconductors, achieving exceptional quality factors $Q_c > 10^6$ at 1–5 kHz trapping frequencies [30]. Fuwa *et al.* experimentally demonstrated passive magnetic levitation and three-dimensional harmonic trapping of a 0.3-mg, 250- μm -radius YIG sphere at 4 K [31]. The technology combines the advantages of magnetism and levitation, facilitating free interactions between the internal and external DOFs of levitated micromagnets. Recent years have witnessed significant advances in magnetic levitation techniques for micron-scale particles [32]. Agarwal and coworkers demonstrated magnon entanglement in two YIG spheres with $d = 40\text{-}\mu\text{m}$ [33]. Hofer *et al.* demonstrated magnetic levitation of a 100- μm superconducting sphere, achieving a record CM quality factor of 2.6×10^7 [34]. Hybrid systems coupled to levitated micromagnets enable ultrasensitive sensing applications [35], including high-precision magnetometry [36], torque sensing [37], gravity measurements [38], and dark matter detection [39–42].

A critical prerequisite for realizing these quantum applications is cooling the massive system to its quantum ground state. Specifically, cooling the CM motion of large-mass levitated micromagnets is essential for exploring physics beyond the standard model [43], realizing the macroscopic quantum superpositions and studying the classical-quantum boundary [44–48]. Several methods for cooling the CM motion [43,49–52] have been proposed, including passive

*Contact author: yesaiyun@fzu.edu.cn

†Contact author: zhirz@fzu.edu.cn

cooling [53], continuous measurement cooling, and feedback cooling [54–58]. However, these methods [53–56] are often incompatible with massive oscillators exhibiting low frequencies and high quality factors. The development of sideband cooling techniques [59–61] has overcome the limitations, enabling efficient cooling of the CM motion of levitated micromagnets via precise laser control.

In addition, the internal DOF, also referred to the phonon mode, is the vibrational mode of the ferrimagnet and can couple to magnon mode via magnetostrictive interaction. Some theoretical studies primarily concentrate on the tripartite entanglement between phonon, magnon, and photon [62], as well as the generation of squeezed states [63]. In experiments, You and coworkers observed phonon bistability in CMM systems [64]. Zhang *et al.* observed the coherent magnon-phonon interaction, providing a versatile platform for coherent information processing [65]. Although experimental observation of the phonon mode in levitated systems remains challenging, rigorous theoretical work has demonstrated that the mode can be probed by measuring CM motion at experimentally attainable displacement sensitivities [12]. Cooling the phonon mode of large-mass levitated micromagnets to near the ground state is equally important for quantum precision sensing. Phonons of levitated micromagnets, due to their remarkable sensitivity, are critical for observing quantum effects in the macroscopic system [22] and performing ultrahigh-precision measuring [35]. To achieve ground-state cooling of phonon modes, various theoretical approaches [66–70] have been proposed, including pure cryogenic cooling [53], dissipation cooling [71,72], cavity-assisted backaction cooling [73], and sideband cooling [74].

Recently, some theoretical studies have focused on acoustomechanical interactions—the strong and tunable coupling between the CM mode and the phonon mode of a levitated micromagnet—enabling the phonon mode to be probed and manipulated through its CM motion [22]. This coupling establishes a correlation between the internal and external DOFs, enabling coherent quantum control between them, which includes quantum information transfer and quantum sensing. However, the simultaneous cooling of the internal and external DOFs of large-mass levitated micromagnets is a prerequisite for observing these quantum phenomena.

In this paper, we propose a different scheme for simultaneously cooling the internal DOF (the phonon mode) and the external DOF (the CM mode) of a levitated micromagnet in the CMM system. In the scheme, we employ the sideband cooling to suppress Stokes scattering by coherently driving the cavity mode and magnon mode on their respective red sidebands with microwave fields. We demonstrate that the internal and external DOFs of a levitated micromagnet can be simultaneously cooled to near their ground states, even in the unresolved-sideband regime. Additionally, the cooling processes of the internal and external DOFs in the levitated micromagnet can be controlled separately. Furthermore, we can identify the optimal conditions for effective cooling by appropriately selecting parameters of the system, i.e., coupling strength, effectively detuning, and damping rate. The numerical simulation results indicate that the average occupancy of the internal DOF can be cooled to a smaller value of

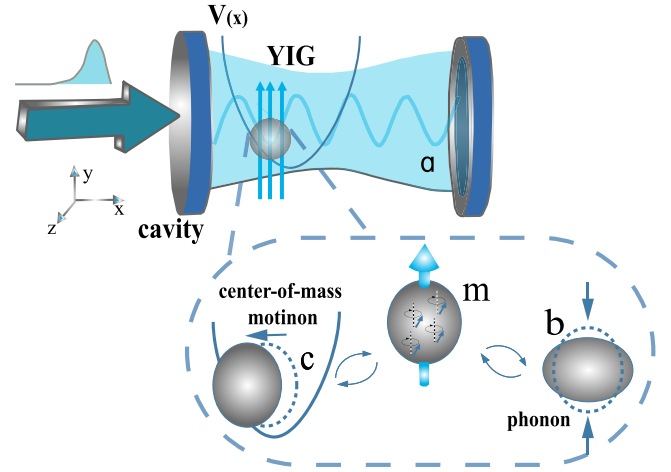


FIG. 1. Sketch of the considered CMM system: A YIG sphere is trapped at a node within a harmonic potential $V(x)$. The magnetic component of the driving field (in the y direction), the bias magnetic field (in the z direction), and the magnetic field (in the x direction) of the microwave cavity mode are all mutually perpendicular at the location of the YIG sphere.

$n_b = 0.018$, while the average occupancy of the external DOF is $n_c = 0.000\,608$.

The paper is organized as follows: In Sec. II, we propose the theoretical model. In Sec. III, we derive the quantum Langevin equations and the linearized Hamiltonian of the system. In Sec. IV, we calculate the noise spectral density and analyze the net cooling rate of the subsystem. In Sec. V, we further study occupancies of the internal and external DOFs of a levitated micromagnet within the CMM system, and identify the optimal parameters for simultaneously cooling the internal and external DOFs close to their ground states. In Sec. VI, we analyze the feasibility of the proposed scheme in specific experiments. Finally, we present a conclusion in Sec. VII.

II. THEORETICAL MODEL

As shown in Fig. 1, we consider a CMM system with a YIG sphere trapped in a harmonic oscillator potential $V(x)$ inside a microwave cavity. The YIG sphere is subjected to a static uniform bias magnetic field ($\mathbf{H}_B = H_{B0}\hat{z}$) aligned in the z direction, along with a driving magnetic field $\mathbf{B}_{\text{cav}} = \hat{y}B \cos(kx)$ in the y direction, where k represents the wave number of the microwave field [43]. Experimental studies have demonstrated that a YIG sphere can be levitated in a passive superconducting magnetic trap [31], where three-dimensional harmonic confinement is established through dual mechanisms: (i) vertical confinement via static magnetic field gradients and (ii) horizontal stabilization leveraging the diamagnetic response of type-II superconductors to magnetic moments [32]. Studies have demonstrated that systematic optimization of field gradients and size reduction enables kilohertz-frequency trapping [32]. Our scheme employs the YIG sphere with diameter $d \geq 20\,\mu\text{m}$, featuring kilohertz-scale trap frequencies that facilitate coherent quantum manipulation.

The magnon mode represents quantized collective spin excitations (the Kittel mode) in ferromagnetic systems and dominates the magnon dynamics in subwavelength YIG spheres under uniform magnetization conditions, while higher-order spin-wave modes are effectively suppressed [75]. The emergence of phonon modes primarily results from magnetic excitations within ferromagnetic crystals, which induce variations in magnetization intensity and lead to alterations in their geometric structure. There are various phonon modes that exist in a YIG sphere, and the typical profiles of the lowest-order spheroidal phonon modes are characterized as S_{1,l,m_a} (where l and m_a are the angular and azimuthal mode numbers, respectively). These modes each exhibit distinct displacement profiles and, consequently, possess varying coupling strengths with respect to the magnon mode. Among these phonon modes, the $S_{1,2,2}$ mode shows the highest coupling strength when the bias field is along the direction of maximum displacement, as reported by Zhang *et al.* [65].

The magnon mode simultaneously couples to a microwave cavity mode via the magnetic dipole interaction [19] and to a phonon mode of the YIG sphere via the magnetostrictive interaction [12]. The total Hamiltonian of the system can be expressed as

$$H = H_0 + H_{\text{int}} + H_d. \quad (1)$$

H_0 corresponds to the free Hamiltonian of the system, which can be expressed as

$$H_0/\hbar = \omega_a a^\dagger a + \omega_m m^\dagger m + \omega_b b^\dagger b, \quad (2)$$

where a ($a^\dagger$), m ($m^\dagger$), and b ($b^\dagger$) are the annihilation (creation) operators of the cavity mode, the magnon mode, and the phonon mode at frequencies ω_a , ω_m , and ω_b , respectively. The frequency of the magnon mode ω_m in the YIG sphere is proportional to the external bias magnetic field $\mathbf{H}_B$, i.e., $\omega_m = \gamma H_{B0}$ with the gyromagnetic ratio $\gamma/2\pi = 28$ GHz. The frequency of phonon mode ω_b in the YIG sphere generally decreases with an increase of the sphere size [65].

The second term H_{int} in Eq. (1) describes the interactions between different components, which is

$$H_{\text{int}}/\hbar = g_{ma}(am^\dagger + a^\dagger m)\cos(kx) + g_{mb}m^\dagger m(b + b^\dagger). \quad (3)$$

Here, the first term refers to the magnon-photon interaction between the cavity mode and the magnon mode [43]. The corresponding coupling strength g_{ma} is given by [19]

$$g_{ma} = \frac{\gamma}{2} \sqrt{\frac{\hbar\omega_a\mu_0}{V_a}} \sqrt{2\rho_s s V}, \quad (4)$$

where V and V_a are the volume of the YIG sphere and the cavity mode, respectively. The parameters μ_0 , ρ_s , and $s = 5/2$ represent the vacuum permeability, the spin density of the YIG sphere, and the spin number of the YIG sphere's ground state, respectively. Even for large-mass YIG spheres ($R > 100\mu\text{m}$) [20], their high spin density can enhance the strong magnon-photon coupling, as demonstrated in Eq. (4). The coupling strength of magnon-photon interaction can be tuned by adjusting the position of the YIG sphere [19]. Meanwhile, the second term in Eq. (3) represents the

magnon-phonon interaction between the magnon mode and the phonon mode, which resembles a radiation pressurelike interaction [76]. Here, g_{mb} characterizes the bare magnomechanical coupling strength. Although the coupling strength g_{mb} in the single-magnon level is typically weak, it can be significantly enhanced by imposing a strong driving field on the magnon mode [24].

In Eq. (1), H_d refers to the driving field of the cavity mode and the magnon mode:

$$H_d/\hbar = \varepsilon(ae^{i\omega_d t} + a^\dagger e^{-i\omega_d t}) + i\Omega_m(me^{i\omega_l t} - m^\dagger e^{-i\omega_l t}), \quad (5)$$

where ω_d and ω_l are the frequencies of the applied laser and the driving magnetic field, respectively. Here, we apply a weak drive field to the cavity mode; $\varepsilon = \sqrt{2P\gamma_a/\omega_d}$ is the amplitude of the external laser, where P is the input field power and γ_a is the cavity damping rate. Ω_m is the coupling rate between the magnon and the driving magnetic field, which is given by

$$\Omega_m = \frac{\sqrt{5}}{4} \gamma \sqrt{N} B_0,$$

where γ is the gyromagnetic ratio, B_0 is amplitude, and $N = \rho_s V$ is the total number of spins. It should be noted that directly driving magnons with a microwave field, e.g., using a loop antenna [24], achieves higher pump efficiency compared to indirect driving with the microwave cavity field.

The YIG sphere, with a diameter d larger than $20\mu\text{m}$ [20], is placed in a low-frequency trap (~ 50 kHz). Therefore, we must take into account the DOF associated with the CM motion of the micromagnet. The zero-point fluctuations of the magnon in the trapping potential is $x_{\text{ZPM}} = \sqrt{\hbar/(2\rho_m V \omega_c)}$, where ω_c is the CM mechanical oscillation frequency and $\rho_m = 5.170\text{ kg/m}^3$ is the mass density of the YIG sphere. The thermal variance in the particle's position, which is much smaller than the microwave wavelength, is given by $\Delta_x = \sqrt{k_B T/(m\omega_c^2)}$, where k_B is the Boltzmann constant and T is the environment temperature. Therefore, we first expand $\cos(kx)$ in Eq. (3) in a Taylor series up to the first order in particle position around the position of the trap minimum x_0 , i.e., $\cos(kx) \rightarrow \cos(kx_0) - k \sin(kx_0)x$. The displacement operator is quantized as follows: $x \equiv \sqrt{\hbar/(2\rho_m V \omega_c)}(c + c^\dagger)$, where c ($c^\dagger$) is the annihilation (creation) operator of the CM oscillation at the trap frequency ω_c [13,43]. This expansion yields a photon-magnon-CM interaction Hamiltonian. Then, the total resultant Hamiltonian reads as follows:

$$\begin{aligned} H_1/\hbar = & \omega_a a^\dagger a + \omega_m m^\dagger m + \omega_c c^\dagger c + \omega_b b^\dagger b \\ & + g_{mb}m^\dagger m(b + b^\dagger) + g_{ma}\cos(kx_0)(am^\dagger + a^\dagger m) \\ & - g_{mac}\sin(kx_0)(am^\dagger + a^\dagger m)(c^\dagger + c) \\ & + \varepsilon(ae^{i\omega_d t} + a^\dagger e^{-i\omega_d t}) \\ & + i\Omega_m(me^{i\omega_l t} - m^\dagger e^{-i\omega_l t}). \end{aligned} \quad (6)$$

The photon-magnon-CM coupling strength g_{mac} is given by [19]

$$g_{mac} = g_{ma}kx_{\text{ZPM}} = \frac{1}{2}\hbar k\gamma \sqrt{\frac{\omega_a\mu_0\rho_s s}{\rho_m\omega_c V_a}}, \quad (7)$$

which describes the interaction among the cavity mode, the magnon mode, and the CM mode. It should be noted that g_{mac} remains independent of particle size. Suppose that the YIG sphere is trapped at a node of the cavity magnetic field [$kx_0 = (2n + 1)\pi/2$], where the micromagnet field gradient is maximal [19,21]. Therefore, we could achieve $g_{\text{mac}}\sin(kx_0) = J_{\text{amc}}$ and $g_{\text{ma}}\cos(kx_0) = 0$ via controlling the particle position. This position controlling can be achieved through the levitation technology [19–21]. Supposing $\omega_d = \omega_l$, the Hamiltonian in Eq. (6) in the rotating frame at this frequency can be written as

$$\begin{aligned} H_2/\hbar = & \Delta_a a^\dagger a + \Delta_m m^\dagger m + \omega_c c^\dagger c + \omega_b b^\dagger b \\ & + g_{mb} m^\dagger m (b + b^\dagger) \\ & - J_{\text{amc}} (a m^\dagger + a^\dagger m) (c^\dagger + c) \\ & + \varepsilon (a + a^\dagger) + i\Omega_m (m - m^\dagger), \end{aligned} \quad (8)$$

where $\Delta_a = \omega_a - \omega_l$ ($\Delta_m = \omega_m - \omega_l$) is the detuning of the cavity (magnon) mode from the driving field.

III. QUANTUM LANGEVIN EQUATIONS

In this section, we describe the dynamic behavior of the system, which is governed by Lindblad master equation formulated as follows:

$$\dot{\rho} = \frac{-i}{\hbar} [H_2, \rho] - \sum_{o=a,m,b,c} \frac{\gamma_o}{2} [(\bar{n}_o + 1)\mathcal{L}(o)\rho + \bar{n}_o\mathcal{L}(o^\dagger)\rho], \quad (9)$$

where the Lindblad superoperator is defined as $\mathcal{L}(o)\rho = (o^\dagger o \rho + \rho o^\dagger o - 2o\rho o^\dagger)/2$ and $\bar{n}_o = 1/[\exp(\hbar\omega_o/k_B T) - 1]$ is the mean thermal occupation of the bosonic mode o ($o \in \{a, m, b, c\}$). Here, k_B denotes the Boltzmann constant, T is the environment temperature, and ρ is the density operator. ω_o and γ_o are frequency and damping rates of the bosonic mode o , respectively. The quality factors of the bosonic mode are defined as $Q_o = \omega_o/\gamma_o$. An experimental value reported by Gieseler *et al.*, the nanomechanical oscillator with a quality factor of $Q_c = 10^8$, has been observed at a pressure of $P = 0.5 \times 10^{-6}$ mbar [77–79]. It can be estimated that in an ultrahigh vacuum ($P = 0.5 \times 10^{-10}$ mbar), the quality factor Q_c will exceed 10^{12} [80] and it is even more favorable for a macroscopic particle, as the quality factor scales linearly with particle size [81]. To examine the impact of quality factors under experimental conditions, we present in Appendix A a systematic discussion of the Q_c parameter space spanning 10^6 – 10^{11} . Besides, millimeter-scale YIG spheres exhibit phononic quality factors $Q_b \in [10^5, 10^7]$ [65,82]. Although magnon-phonon coupling is well studied, phonon mode characterization remains scarce for levitated systems. For levitated microresonators, Q_b is constrained by interactions with other acoustic modes, yet achieves values exceeding 10^{10} [22]. Furthermore, Appendix A presents a detailed discussion of the systemic implications for quality factors $Q_b \in [10^4, 10^6]$.

Moreover, it is necessary to consider various plausible sources of decoherence in this system, such as magnetic noise, internal magnon losses, current damping, gas damping, and so on. Magnetic noise is a fundamental decoherence mechanism for levitated ferromagnets and can be mitigated through

effective magnetic shielding [83]. Current damping represents a significant loss mechanism in magnetomechanical systems, which is proportional to the material's conductivity. This damping can be reduced to negligible levels through three complementary approaches: utilizing the Meissner effect in superconductors, optimizing structural geometry to minimize conductive components, and employing dielectric substrates such as silica or rutile resonators [32]. Gas damping can be considered negligible under ultrahigh vacuum conditions [25,84]. Magnon damping, which refers to internal magnon losses in YIG sphere due to phonon damping, can be effectively suppressed when the quality factor is sufficiently high, specifically greater than 10^8 [22]. Detailed discussion on these topics can be found in Appendix B.

Additionally, experimentally measured cavity linewidths span kHz to GHz scales [85], including Fabry-Pérot cavities (80–200 kHz [86]) and nanophotonic crystals (5 GHz) [87], with superconducting coplanar waveguides achieving ultranarrow linewidths of 7.0 kHz [88]. Correspondingly, diverse schemes for reducing cavity linewidths have been developed.

The dynamics of the system with dissipation can be governed by the quantum Langevin equations (QLEs):

$$\begin{aligned} \dot{a} &= \Delta'_a a + iJ_{\text{amc}} m(c^\dagger + c) - i\varepsilon + \sqrt{\gamma_a} a_{\text{in}}, \\ \dot{m} &= \Delta'_m m + iJ_{\text{amc}} a(c + c^\dagger) - ig_{mb} m(b + b^\dagger) \\ &\quad - \Omega_m + \sqrt{\gamma_m} m_{\text{in}}, \\ \dot{b} &= \omega'_b b - ig_{mb} m^\dagger m + \sqrt{\gamma_b} b_{\text{in}}, \\ \dot{c} &= \omega'_c c + iJ_{\text{amc}} (a^\dagger m + am^\dagger) + \sqrt{\gamma_c} c_{\text{in}}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} \Delta'_a &= -\left(\frac{\gamma_a}{2} + i\Delta_a\right), & \Delta'_m &= -\left(\frac{\gamma_m}{2} + i\Delta_m\right), \\ \omega'_c &= -\left(\frac{\gamma_c}{2} + i\omega_c\right), & \omega'_b &= -\left(\frac{\gamma_b}{2} + i\omega_b\right). \end{aligned}$$

The operators $o_{\text{in}} (o \in \{a, m, b, c\})$ are the input noise operators affecting the cavity mode, the magnon mode, the phonon mode, and the CM mode, respectively. These noise operators have zero mean values and are fully characterized by the following correlation functions:

$$\begin{aligned} \langle o_{\text{in}}(t) \rangle &= 0, \\ \langle o_{\text{in}}(t) o_{\text{in}}^\dagger(t') \rangle &= (\bar{n}_o + 1)\delta(t - t'), \\ \langle o_{\text{in}}^\dagger(t) o_{\text{in}}(t') \rangle &= \bar{n}_o\delta(t - t'). \end{aligned} \quad (11)$$

We assume that the magnon mode is driven by a strong microwave field, allowing us to linearize this system. Note that standard sideband cooling of the oscillator requires a strong driving field, primarily due to the constraints imposed by internal heating resulting from absorption. Therefore, the linearization approximation can be used, namely, $o = o_0 + \bar{o}$ ($o \in \{a, m, b, c\}$). Here, o_0 and $\bar{o}$ refer to the steady-state value and the quantum fluctuation, respectively. By inserting the linearization approximation into Eq. (10), we can obtain a set of equations for the steady-state values and quantum fluctuation operators. Subsequently, the equations for the

steady-state mean values can then be obtained as follows:

$$\begin{aligned} \dot{a}_0 &= \Delta'_a a_0 + 2iJ_{\text{amc}} \text{Re}[c_0] m_0 - i\varepsilon = 0, \\ \dot{m}_0 &= (\Delta'_m - 2ig_{mb} \text{Re}[b_0]) m_0 + 2iJ_{\text{amc}} \text{Re}[c_0] a_0 - \Omega_m = 0, \\ \dot{b}_0 &= \omega'_b b_0 - ig_{mb} m_0^* = 0, \\ \dot{c}_0 &= \omega'_c c_0 + iJ_{\text{amc}} (a_0^* m_0 + a_0 m_0^*) = 0. \end{aligned} \quad (12)$$

Also, we assume that $|\varepsilon| \ll |\Omega_m|$ and $|\Delta'_a|$, with the magnon driven by a strong microwave field while the system is not in the multistable regime. Based on these assumptions, we can obtain the stable solutions. From Eq. (12), the steady-state values of each bosonic mode are

$$\begin{aligned} a_0 &\approx 0, \quad m_0 = \frac{-\Omega_m}{i\Delta_m + ig_{mb} 2\text{Re}[b_0] + (\kappa_m/2)}, \\ b_0 &= \frac{-ig_{mb}|m_0|^2}{i\omega_b + (\kappa_b/2)}, \quad c_0 \approx 0. \end{aligned} \quad (13)$$

Besides, the linearized QLEs describing the system are given by

$$\begin{aligned} \dot{\tilde{a}} &= \Delta'_a \tilde{a} + iG_{ac}(\tilde{c}^\dagger + \tilde{c}) + \sqrt{\gamma_a} a_{\text{in}}, \\ \dot{\tilde{m}} &= \Delta'_{m_1} \tilde{m} - iG_{mb}(\tilde{b}^\dagger + \tilde{b}) + \sqrt{\gamma_m} m_{\text{in}}, \\ \dot{\tilde{b}} &= \omega'_b \tilde{b} - iG_{mb}(\tilde{m}^\dagger + \tilde{m}) + \sqrt{\gamma_b} b_{\text{in}}, \\ \dot{\tilde{c}} &= \omega'_c \tilde{c} + iG_{ac}(\tilde{a}^\dagger + \tilde{a}) + \sqrt{\gamma_c} c_{\text{in}}, \end{aligned} \quad (14)$$

where

$$\Delta'_{m_1} = -\frac{\gamma_m}{2} - i(\Delta_m + 2g_{mb} \text{Re}[b_0]). \quad (15)$$

Here, $G_{ac} = |m_0|J_{\text{amc}}$ denotes the effective coupling among the cavity mode, the magnon mode, and the CM mode, and $G_{mb} = |m_0|g_{mb}$ is the effective magnetomechanical coupling strength between the magnon mode and the phonon mode. The linearized QLEs in Eq. (14) correspond to a Hamiltonian of the form

$$\begin{aligned} \tilde{H}/\hbar &= \Delta_a \tilde{a}^\dagger \tilde{a} + \Delta_{m_1} \tilde{m}^\dagger \tilde{m} + \omega'_b \tilde{b}^\dagger \tilde{b} + \omega'_c \tilde{c}^\dagger \tilde{c} \\ &\quad - G_{ac}(\tilde{a}^\dagger + \tilde{a})(\tilde{c}^\dagger + \tilde{c}) \\ &\quad + G_{mb}(\tilde{m}^\dagger + \tilde{m})(\tilde{b}^\dagger + \tilde{b}), \end{aligned} \quad (16)$$

where

$$\Delta_{m_1} = \Delta_m + 2ig_{mb} \text{Re}[b_0].$$

As shown in Eq. (16), the internal DOF (the phonon mode) of the magnon is coupled solely to the magnon mode, while the external DOF (the CM mode) is only exclusively coupled to the cavity mode. This indicates that the internal and external DOFs of a levitated micromagnet are decoupled, allowing us to cool both the internal and external DOFs simultaneously without relying on other media.

IV. NOISE SPECTRAL DENSITY AND NET COOLING RATES

In this section, we analyze the cooling dynamics of the internal DOF and the external DOF by examining the average occupancies of both DOFs. The steady-state average occupancy of the bosonic mode o [73,89–91] in the weak coupling

limit ($G_{mb} \ll \omega_b$, $G_{ac} \ll \omega_c$) is given by

$$n_o \approx \frac{\gamma_o \bar{n}_o + A_+}{\gamma_o + \Gamma_o}, \quad (17)$$

where $\Gamma_o = A_- - A_+$ is the net cooling rate of the bosonic mode o . Here, $A_- = G^2 S_{F_o}(\omega)$ represents the cooling rate (corresponding to the anti-Stokes scattering), while $A_+ = G^2 S_{F_o}(-\omega)$ represents the heating rate (corresponding to the Stokes scattering) of the bosonic mode o , where

$$S_{F_o}(\omega) = \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle F_o(t) F_o(0) \rangle \quad (18)$$

is the power spectral density and G is the coupling strength. According to Eq. (18), the spectra $S_{F_o}(\omega)$ of fluctuations of the cavity field quadrature ($F_a = a + a^\dagger$) and the magnon field quadrature ($F_m = m + m^\dagger$) are shown in Figs. 2(a) and 2(b), respectively.

To achieve simultaneous cooling of both the internal and external DOFs of a levitated micromagnet, it is essential to maximally suppress the Stokes scattering processes of the phonon mode and the CM mode. Both the cavity mode and the magnon mode should work on their respective red detunings, i.e., $\Delta_a = \omega_a - \omega_l = \omega_c$ and $\Delta_{m_1} = \omega_m - \omega_l = \omega_b$. In this case, the cooling rate A_- is greater than the heating rate A_+ in the system, leading to the net cooling rates $\Gamma_b > 0$ and $\Gamma_c > 0$. Therefore, the internal DOF and external DOF of the levitated micromagnet are both cooled to close to the quantum ground state, and this process is termed sideband cooling [73].

As shown in Figs. 2(a) and 2(b), the orange solid line represents the red-sideband detuning condition ($\Delta_a = \omega_c$ and $\Delta_{m_1} = \omega_b$), while the green solid line represents the blue-detuning condition ($\Delta_a = -\omega_c$ and $\Delta_{m_1} = -\omega_b$), with the spectrum varying as a function of the normalized frequency. The orange solid line indicates that the spectrum $S_{F_a}(\omega)$ exhibits a cooling peak at $\Delta_a = \omega_c$ in Fig. 2(a), and the spectrum $S_{F_m}(\omega)$ exhibits a cooling peak at $\Delta_{m_1} = \omega_b$ in Fig. 2(b). Therefore, the Stokes scattering can be significantly suppressed under the conditions $\Delta_a = \omega_c$ and $\Delta_{m_1} = \omega_b$. In this case, it results in a significant enhancement of the net cooling rate.

The net cooling rates of the external DOF and the internal DOF in a levitated micromagnet, denoted as Γ_c and Γ_b , respectively, are illustrated in Figs. 2(d) and 2(e). It is clear that the net cooling rate is maximum when the system is at $\Delta_a = \omega_c$ and $\Delta_{m_1} = \omega_b$, as indicated by the green dashed line in Fig. 2(d) and the orange dashed line in Fig. 2(e). In contrast, for blue-sideband detuning, where $\Delta_a = -\omega_c$ and $\Delta_{m_1} = -\omega_b$, the behavior is the opposite. Note that under red-sideband detuning, the net cooling rate for the external DOF is significantly higher than that of the internal DOF in a levitated micromagnet. Specifically, the maximum net cooling rate for the external DOF is $\Gamma_c/\omega_c = 0.31$, whereas the maximum net cooling rate for the internal DOF reaches $\Gamma_b/\omega_b = 0.05$. Next, we plot the exact numerical results of the average occupancy of the external DOF n_c in steady-state as a function of the effective detuning Δ_a in Fig. 2(c). Similarly, Fig. 2(f) shows that the average occupancy of the internal DOF n_b in steady-state varies as a function of the effective detuning Δ_{m_1} . The significant suppression of Stokes scattering leads to an enhancement of the net cooling rates Γ_c and Γ_b ,

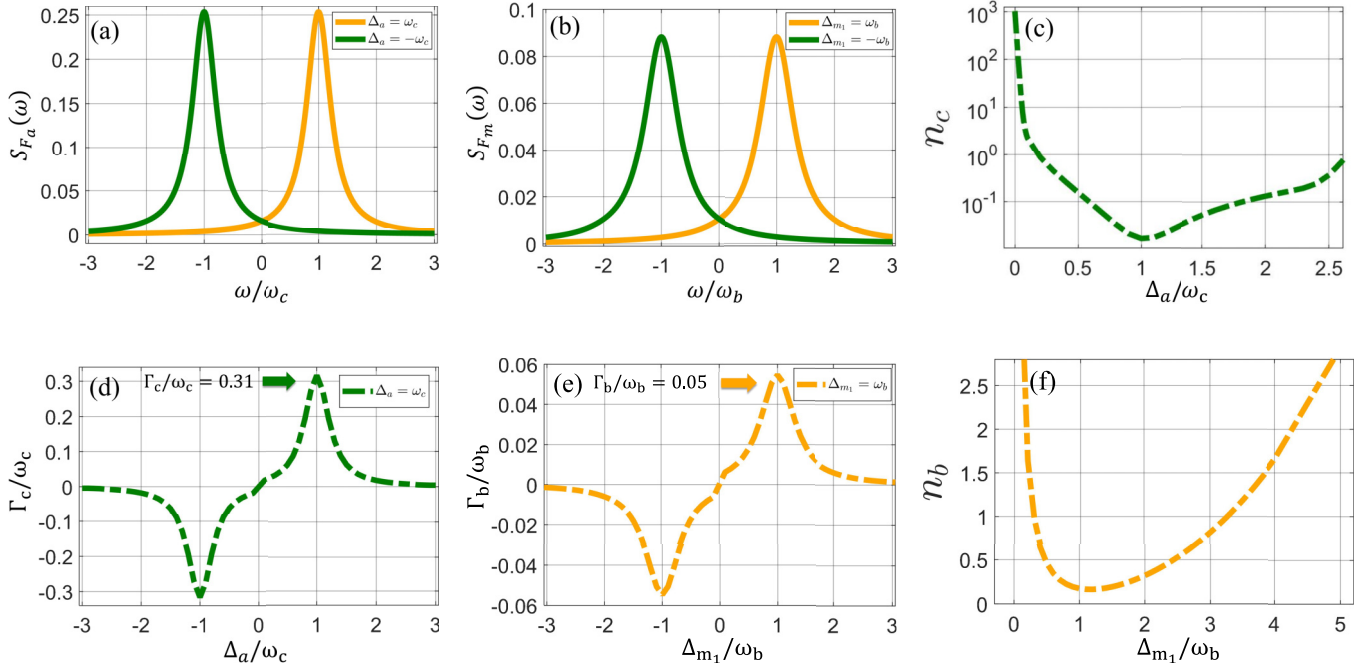


FIG. 2. (a) The spectrum density $S_{F_a}(\omega)$ vs the normalized frequency ω/ω_c . (b) The spectrum density $S_{F_m}(\omega)$ vs the normalized frequency ω/ω_b . (c) The average occupancy of the external DOF n_c vs the normalized detuning Δ_a/ω_c in steady-state. (d) The net cooling rate Γ_c of the occupancy of the external DOF with $\Delta_a = \omega_c$. (e) The net cooling rate Γ_b of the internal DOF with $\Delta_{m_1} = \omega_b$. (f) The average occupancy of the internal DOF n_b vs the normalized detuning Δ_{m_1}/ω_b in steady-state. The other parameters are taken as follows: $\omega_c/2\pi = 50$ kHz, $G_{ac} = 0.05 \omega_c$, $\gamma_a = 0.5 \omega_c$, $\gamma_c = 2 \times 10^{-11} \omega_c$, $\omega_b = 9.4$ MHz, $\Delta_{m_1} = 1.49 \omega_b$, $G_{mb} = 0.1 \omega_b$, $\gamma_m = 1.429 \omega_b$, and $\gamma_b = 10^{-6} \omega_b$. The background temperature is 450 mK, with the average thermal magnon and cavity occupancies as $\bar{n}_m = 0$ and $\bar{n}_a = 0$, and the average thermal occupancies of the internal DOF and the external DOF are $\bar{n}_b = 1000$ and $\bar{n}_c = 1.87 \times 10^5$, respectively.

which results in markedly reduced n_c and n_b at the optimal detuning conditions in steady-state, as displayed in Figs. 2(c) and 2(f). We find that the smallest average occupancy of the external DOF is $n_c = 0.0169$ at $\Delta_a = \omega_c$, and the smallest average occupancy of the internal DOF is $n_b = 0.157$ at $\Delta_{m_1} = \omega_b$ in steady-state.

V. SIMULTANEOUS COOLING OF THE INTERNAL AND EXTERNAL DOFS

To study the cooling performance of the internal DOF and the external DOF, we need to calculate the average occupancy of external DOF n_c and the average occupancy of the internal DOF n_b by solving the mean values of all second-order moments. The differential equations are given by

$$\begin{aligned}
 \langle \dot{\tilde{m}} \tilde{m} \rangle &= -\gamma_m \langle \tilde{m}^\dagger \tilde{m} \rangle + iG_{mb}(\langle \tilde{m} \tilde{b} \rangle + \langle \tilde{m} \tilde{b}^\dagger \rangle - \text{c.c.}) + \gamma_m \bar{n}_m, \\
 \langle \dot{\tilde{b}}^\dagger \tilde{b} \rangle &= -\gamma_b \langle \tilde{b}^\dagger \tilde{b} \rangle + iG_{mb}(\langle \tilde{m} \tilde{b} \rangle + \langle \tilde{m}^\dagger \tilde{b} \rangle - \text{c.c.}) + \gamma_b \bar{n}_b, \\
 \langle \dot{\tilde{m}} \tilde{b} \rangle &= -(\frac{1}{2}(\gamma_m + \gamma_b) + i(\Delta_{m_1} + \omega_b)) \langle \tilde{m} \tilde{b} \rangle \\
 &\quad - iG_{mb}(\langle \tilde{m}^\dagger \tilde{m} \rangle + \langle \tilde{m} \tilde{m} \rangle + \langle \tilde{b}^\dagger \tilde{b} \rangle + \langle \tilde{b} \tilde{b} \rangle + 1), \\
 \langle \dot{\tilde{m}} \tilde{b}^\dagger \rangle &= -(\frac{1}{2}(\gamma_m + \gamma_b) + i(\Delta_{m_1} - \omega_b)) \langle \tilde{m} \tilde{b}^\dagger \rangle \\
 &\quad + iG_{mb}(\langle \tilde{m}^\dagger \tilde{m} \rangle + \langle \tilde{m} \tilde{m} \rangle - \langle \tilde{b}^\dagger \tilde{b} \rangle - \langle \tilde{b} \tilde{b}^\dagger \rangle), \\
 \langle \dot{\tilde{m}} \tilde{m} \rangle &= -(\gamma_m + 2i\Delta_{m_1}) \langle \tilde{m} \tilde{m} \rangle - 2iG_{mb}(\langle \tilde{m} \tilde{b}^\dagger \rangle + \langle \tilde{m} \tilde{b} \rangle),
 \end{aligned}$$

$$\begin{aligned}
 \langle \dot{\tilde{b}} \tilde{b} \rangle &= -(\gamma_b + 2i\omega_b) \langle \tilde{b} \tilde{b} \rangle - 2iG_{mb}(\langle \tilde{m}^\dagger \tilde{b} \rangle + \langle \tilde{m} \tilde{b} \rangle), \\
 \langle \dot{\tilde{a}}^\dagger \tilde{a} \rangle &= -\gamma_a \langle \tilde{a}^\dagger \tilde{a} \rangle - iG_{ac}(\langle \tilde{a} \tilde{c} \rangle + \langle \tilde{a} \tilde{c}^\dagger \rangle - \text{c.c.}) + \gamma_a \bar{n}_a, \\
 \langle \dot{\tilde{c}}^\dagger \tilde{c} \rangle &= -\gamma_c \langle \tilde{c}^\dagger \tilde{c} \rangle - iG_{ac}(\langle \tilde{a} \tilde{c} \rangle + \langle \tilde{a}^\dagger \tilde{c} \rangle - \text{c.c.}) + \gamma_c \bar{n}_c, \\
 \langle \dot{\tilde{a}} \tilde{c} \rangle &= -(\frac{1}{2}(\gamma_a + \gamma_c) + i(\Delta_a + \omega_c)) \langle \tilde{a} \tilde{c} \rangle \\
 &\quad + iG_{ac}(\langle \tilde{a}^\dagger \tilde{a} \rangle + \langle \tilde{a} \tilde{a} \rangle + \langle \tilde{c}^\dagger \tilde{c} \rangle + \langle \tilde{c} \tilde{c} \rangle + 1), \\
 \langle \dot{\tilde{a}} \tilde{c}^\dagger \rangle &= -(\frac{1}{2}(\gamma_a + \gamma_c) + i(\Delta_a - \omega_c)) \langle \tilde{a} \tilde{c}^\dagger \rangle \\
 &\quad - iG_{ac}(\langle \tilde{a}^\dagger \tilde{a} \rangle + \langle \tilde{a} \tilde{a} \rangle - \langle \tilde{c}^\dagger \tilde{c} \rangle - \langle \tilde{c} \tilde{c}^\dagger \rangle), \\
 \langle \dot{\tilde{a}} \tilde{a} \rangle &= -(\gamma_a + 2i\Delta_a) \langle \tilde{a} \tilde{a} \rangle + 2iG_{ac}(\langle \tilde{a} \tilde{c}^\dagger \rangle + \langle \tilde{a} \tilde{c} \rangle), \\
 \langle \dot{\tilde{c}} \tilde{c} \rangle &= -(\gamma_c + 2i\omega_c) \langle \tilde{c} \tilde{c} \rangle + 2iG_{ac}(\langle \tilde{a}^\dagger \tilde{c} \rangle + \langle \tilde{a} \tilde{c} \rangle), \quad (19)
 \end{aligned}$$

where the above equations are numerically solved with the initial conditions $\bar{n}_m \approx 0$, $\bar{n}_a \approx 0$, $\bar{n}_b \approx 1000$, and $\bar{n}_c \approx 1.87 \times 10^5$.

First, we investigate the cooling dynamics of the CMM system as functions of the coupling strengths as described in Eq. (19). The average occupancy of the external DOF n_c and the average occupancy of the internal DOF n_b in a levitated micromagnet can be numerically simulated, as illustrated in Fig. 3. In Figs. 3(a) and 3(b), we plot the time evolution of n_c and n_b for different coupling strengths G_{ac} and G_{mb} ,

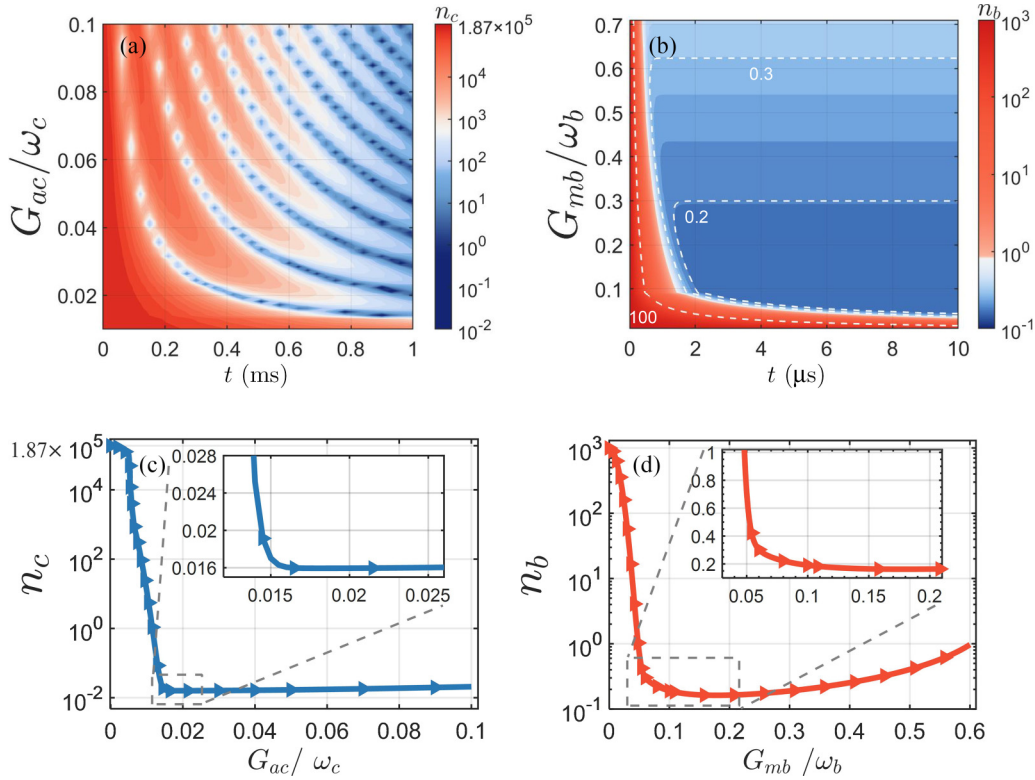


FIG. 3. (a) Contour plot of the average occupancy of the external DOF n_c vs time t and normalized coupling strength G_{ac}/ω_c with $\Delta_a = \omega_c$. Color bar represents $\log_{10} n_c$ values ranging from 10^{-2} to 1.87×10^5 . (b) Contour plot of the average occupancy of the internal DOF n_b vs time t and normalized coupling strength G_{mb}/ω_b with $\Delta_{m1} = \omega_b$. Color bar represents $\log_{10} n_b$ values ranging from 10^{-1} to 10^3 and white dashed contours mark n_b . (c) steady-state n_c (log scale) as a function of G_{ac}/ω_c , as indicated by the blue solid line with triangular markers. Inset of panel (c): The magnified view of n_c vs G_{ac}/ω_c ($G_{ac}/\omega_c < 0.025$). (d) steady-state n_b (log scale) vs G_{mb}/ω_b , as indicated by the red solid line with triangular markers. Inset of panel (d): The magnified view of n_b vs G_{mb}/ω_b ($G_{mb}/\omega_b < 0.2$). Other parameters are identical to those in Fig. 2.

respectively. Within approximately 1 ms, both n_c and n_b can be simultaneously cooled to stable values below unity. The cooling time for the internal DOF to reach its ground state is shorter than that of the external DOF. Specifically, the internal DOF can be cooled to its ground state within 10 μ s. We plot the exact numerical results of the average occupancy of the external DOF n_c in steady-state as a function of the effective coupling G_{ac} in Fig. 3(c) and the average occupancy of the internal DOF n_b in steady-state as a function of the effective coupling G_{mb} in Fig. 3(d). Both n_c and n_b do not change monotonically with their respective coupling strengths, G_{ac} and G_{mb} , in steady-state. As shown in Figs. 3(c) and 3(d), we find that the lowest average occupancy $n_c = 0.016$ is obtained with $G_{ac} = 0.02\omega_c$, while the lowest average occupancy $n_b = 0.162$ is achieved at $G_{mb} = 0.18\omega_b$ in steady-state. The optimal coupling parameters G_{ac} and G_{mb} are well within the weak coupling regime. This phenomenon is due to the fact that the steady-state occupancy is governed by the net cooling rate, which arises from the competition between cooling and heating processes. The large net cooling rate has the capacity to reduce the average occupancies n_b and n_c . However, this also introduces additional noise from magnon and cavity scattering, which heats the phonon motion and the CM motion, thereby increasing n_b and n_c . As a result of these two competing effects, there exist optimal coupling values of G_{ac} and G_{mb} , respectively.

Next, we study the cooling dynamics of the system as functions of the dissipation rates, as described in Eq. (19). To account for practical experimental constraints, we investigate the influence of decreasing the CM trap frequency ω_c and increasing the phonon damping rate γ_b by 2 orders of magnitude on system dynamics. The time evolution of the average occupancy of the external DOF n_c for different cavity damping rates γ_a and internal DOF n_b for different magnon damping rates γ_m is presented in Figs. 4(a) and 4(b), respectively. The average occupancies of both the internal and external DOFs in a levitated micromagnet can be simultaneously cooled to below 1 within approximately 1 s. In addition, we plot the exact numerical results of the average occupancy of the external DOF n_c in steady-state as a function of the cavity damping rate γ_a in the inset of Fig. 4(a) and the average occupancy of the internal DOF n_b in steady-state as a function of the magnon damping rate γ_m in the inset of Fig. 4(b). The cavity and magnon damping rates range from the resolved-sideband regime ($\gamma_a \ll \omega_c$, $\gamma_m \ll \omega_b$) to the unresolved-sideband regime ($\gamma_a \gg \omega_c$ and $\gamma_m \gg \omega_b$). Notably, even in the unresolved-sideband regime, the occupancies of both internal DOF and external DOF can still be simultaneously cooled to near their ground states, which cannot be achieved by conventional sideband cooling [59]. Increased cavity dissipation γ_a enables cooling of n_c but reduces cooling efficiency, as illustrated in the inset of

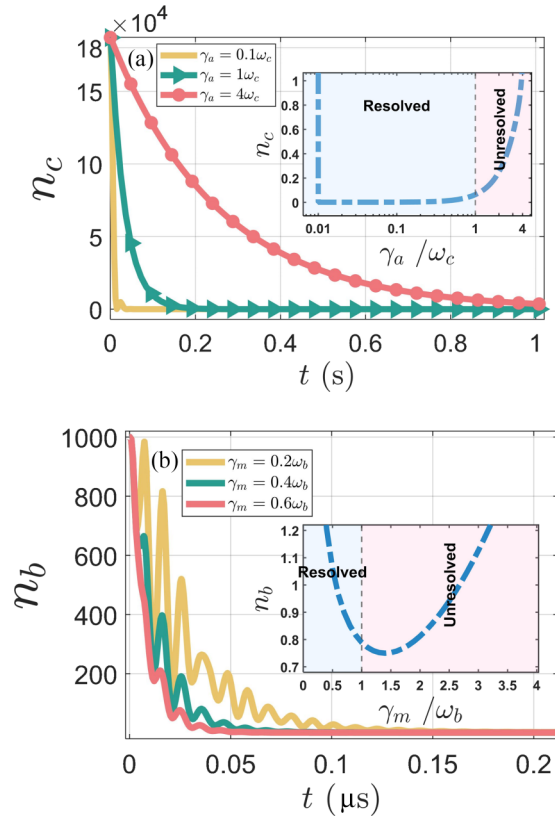


FIG. 4. (a) Time evolution of the average occupancy of the external DOF n_c for different cavity damping rates γ_a with $\Delta_a = \omega_c$, $G_{ac} = 0.32\omega_c$, and $\omega_c = 0.5$ kHz. The cavity damping rates are $\gamma_a = 0.1\omega_c$ (the yellow solid line), $\gamma_a = 1\omega_c$ (the green solid line with triangle), and $\gamma_a = 4\omega_c$ (the red solid line with circle). Inset of panel (a): The average occupancy of the external DOF n_c vs γ_a (log scale) in steady-state. The parameter range $\gamma_a/\omega_c \in [0, 4]$ was selected to theoretically study the cooling performance across both resolved- and unresolved-sideband regimes, extending beyond typical experimental values. (b) Time evolution of the average occupancy of the internal DOF n_b for different magnon damping rates γ_m with $\Delta_{m1} = 1.49\omega_b$ and $G_{mb} = 0.5\omega_b$, $\gamma_b = 10^{-4}\omega_b$. The magnon damping rates are $\gamma_m = 0.2\omega_b$ (the yellow solid line), $\gamma_m = 0.4\omega_b$ (the green solid line), and $r_m = 0.6\omega_b$ (the red solid line). Inset of panel (b): The average occupancy of the internal DOF n_b vs γ_m (log scale) in steady-state. Other parameters are identical to those in Fig. 2.

Fig. 4(a). Reducing the CM trap frequency ω_c prolongs the cooling time while leaving the steady-state occupancy of the external DOF unchanged, as illustrated in the inset of Fig. 4(a). Increased phonon dissipation elevates the steady-state occupancy of the internal DOF and reduces cooling efficiency, yet sustains ground-state cooling ($n_b < 1$), as illustrated in the inset of Fig. 4(b). We find that the smallest occupation number of external DOF $n_c = 0.00126$ is achieved at $\gamma_a = 0.01\omega_c$, and the smallest occupancy of internal DOF $n_b = 0.749$ is achieved at $\gamma_m = 1.4\omega_b$ in steady-state.

In Fig. 5(a), the final average occupancy of external DOF n_c is plotted as functions of coupling strength G_{ac} and cavity damping rate γ_a . There exists an optimal parameter region (represented by the dark blue region in the contour plot) for

G_{ac} and γ_a , which results in a lower final average occupancy of the external DOF n_c . Figure 5(b) provides a zoomed-in view of the optimal parameter region from Fig. 5(a). As illustrated by the red dot in Fig. 5(b), the parameters $G_{ac} = 0.02\omega_c$ and $\gamma_a = 0.01\omega_c$ achieve a lower final average occupancy of external DOF $n_c = 0.000608$. In Fig. 5(c), the final average occupancy of internal DOF n_b is plotted as functions of coupling strength G_{mb} and magnon damping rate γ_m . Similarly, an optimal parameter region (represented by the dark blue region in the contour plot) exists for G_{mb} and γ_m , which results in a lower final average occupancy of the internal DOF n_b . Figure 5(d) shows a zoomed-in view of this optimal parameter region from Fig. 5(c). As shown by the red dot in Fig. 5(d), the parameters $G_{mb} = 0.10\omega_b$ and $\gamma_m = 0.2\omega_b$ achieve a lower final average occupancy of internal DOF $n_b = 0.018$. Additionally, we find that the final average occupancy of external DOF n_c is unaffected by the parameters G_{mb} and γ_b , while the final average occupancy of internal DOF n_b remains unaffected by the parameters G_{ac} and γ_c .

From Eq. (7), it can be observed that, due to the collective spin compensation effect in the YIG sphere, the coupling strength $g_{mac} = (1/2)\hbar k \gamma \sqrt{(\omega_a \mu_0 \rho_s S)/(\rho_m \omega_c V_a)}$ is independent of the volume of the YIG sphere. Consequently, the coupling strength G_{ac} is also independent of the sphere's volume. The CM frequency ω_c scales as $\omega_c \propto R^{-1}$ with the YIG sphere radius R , while other parameters (e.g., dissipation rates) scale linearly with ω_c . Our numerical implementation incorporates these dependencies by normalizing ω_c to unity and proportionally scaling all parameters relative to ω_c . Under these conditions, the steady-state population n_c of external DOF demonstrates size invariance, consistent with Kani *et al.*'s results [43]. Figure 6 displays the steady-state occupancy of external DOF n_c as a function of the YIG sphere radius R , confirming its intensive property independent of size. Additionally, when the frequency of the phonon mode $\omega_b \ll \omega_m$, the coupling strength g_{mb} is dependent on the volume of the YIG sphere, and thus the coupling strength G_{mb} is related to the volume of the YIG sphere. Furthermore, the coupling strength G_{ac} can be adjusted by changing the position of the YIG sphere, which does not affect the coupling strength G_{mb} . Therefore, by adjusting appropriate parameters, the cooling processes of the internal and external degrees of freedom in a levitated micromagnet can be controlled separately.

VI. DISCUSSION OF EXPERIMENT IMPLEMENTATION

In this section, we present a discussion of the experimental implementation of this cooling scheme. The CMM system involves the magnon-photon coupling and magnon-phonon coupling, which are experimentally accessible in current CMM systems [20]. In the simulations, we consider a model that takes the linearized magnon-photon coupling strengths as $G_{ac} = 0.05\omega_c$, which have been realized in many CMM systems [19,20]. The magnon-photon coupling strength can be adjusted by manipulating the position of the YIG sphere. It is maximized when the YIG sphere is placed at the location of the maximum microwave magnetic field, as demonstrated by Hou and Liu [20]. Additionally, the magnon-phonon interaction is significantly enhanced in magnetic materials by

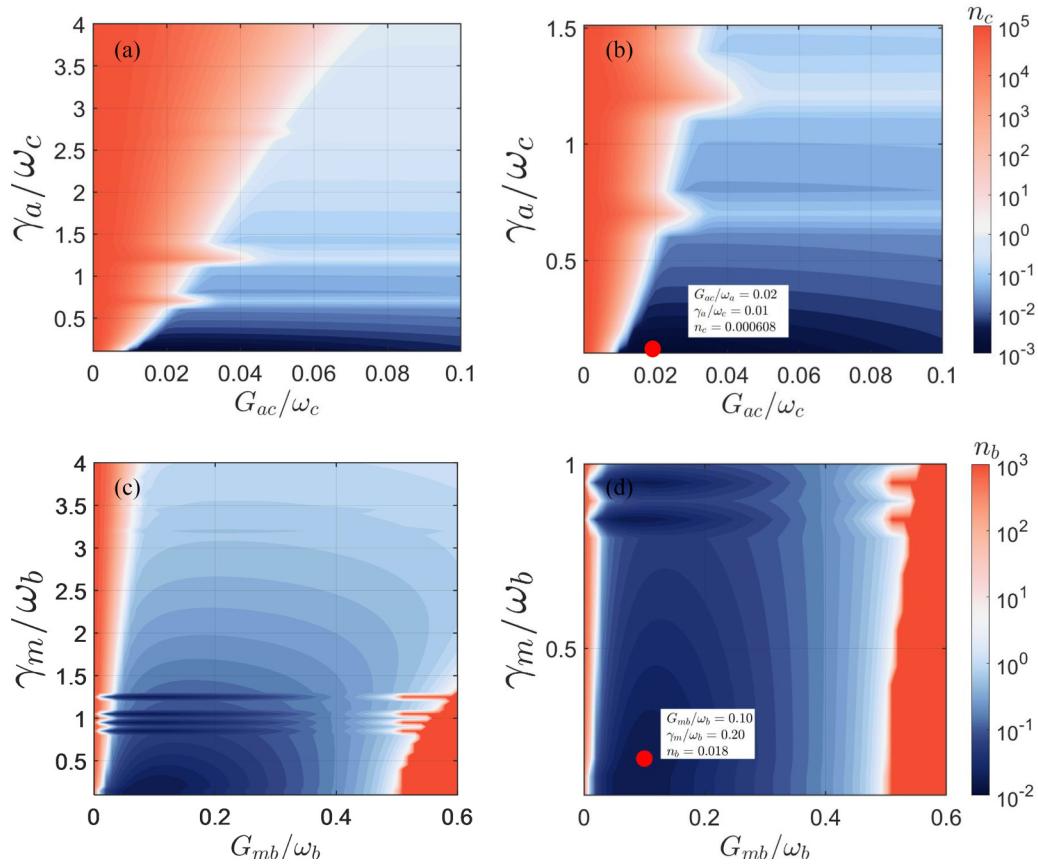


FIG. 5. (a) Contour plot of the final occupancy of external DOF n_c vs normalized coupling strength G_{ac}/ω_c and cavity damping rate γ_a/ω_c . Color bar shows $\log_{10} n_c$ values, with the color gradient ranging from dark blue (low values) to red (high values). (b) Zoomed-in view of panel (a), focusing on the region with lower final average occupancy n_c . (c) Contour plot of the final occupancy of internal DOF n_b vs normalized coupling strength G_{mb}/ω_b and magnon damping rates γ_m/ω_b . Color bar shows $\log_{10} n_b$ values, with the color gradient ranging from dark blue (low values) to red (high values). (d) Zoomed-in view of panel (c), focusing on the region with lower final occupancy n_b . Other parameters are identical to those in Fig. 2.

directly driving magnons with a microwave field, e.g., via a loop antenna [24]. All these advances confirm the experimental feasibility of the proposed scheme.

Next, we present an analysis of the experimental parameters based on CMM systems, where a low-dissipation YIG

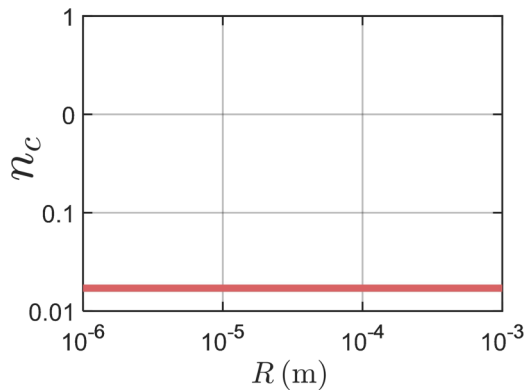


FIG. 6. steady-state occupancy of external DOF n_c vs YIG sphere radius R (log scale). Parameter configuration: CM frequency $\omega_c \propto R^{-1}$, $\gamma_c = 2 \times 10^{-11} \omega_c$, and $\Delta_a = \omega_c$. All other parameters are consistent with Fig. 2.

sphere is placed in a high- Q microwave cavity. The magnon frequency can be tuned by adjusting the applied bias magnetic field within a range 0–3 T, corresponding to the magnon frequency ranging from a few hundred MHz to about 50 GHz [19]. The magnon mode is close to resonance with the cavity mode, when the bias field strength is 281 mT. Here, the cavity mode and the magnon mode have the same resonance frequency $\omega_a = \omega_m = 50$ GHz. In addition, it has been reported that the superconducting microwave cavity requires only a small input microwave power of $P = -22$ dBm [92], which enables an extremely high quality factor exceeding $Q_c = 10^{11}$.

Furthermore, the damping of the mechanical motion of levitated particles is primarily caused by friction from the background gas. It can be adjusted by varying the environmental pressure. We take the damping rates of the cavity mode, the magnon mode, the CM mode, and the phonon mode to be $\gamma_a = 0.5 \omega_c$, $\gamma_m = 1.429 \omega_b$, $\gamma_c = 2 \times 10^{-11} \omega_c$, and $\gamma_b = 10^{-6} \omega_b$, respectively.

Finally, we discuss how to detect the cooling of the internal and external DOFs of a levitated macromagnet. The phonon states can be directly measured by coupling the displacement deformation to an optical cavity, using the phase of the cavity field to achieve high-sensitivity measurements [62,76]

of mechanical displacement. The elusive phonon mode can also be probed by indirectly measuring the CM displacement [22]. Thus, our scheme is highly feasible under the current experimental conditions.

VII. CONCLUSION

In summary, we have proposed a cooling scheme that simultaneously cools both the internal and external DOFs of a levitated macromagnet. We show that the Stokes process for both the internal and external DOFs is significantly suppressed, leading to an accelerated cooling process by driving both the cavity mode and the magnon mode with red-detuning microwave fields. Furthermore, the cooling process of both the internal and external DOFs in the levitated micromagnet can be easily manipulated by adjusting the parameters, such as coupling strength, effective detuning, and damping rate. As an alternative to traditional approaches, our scheme is not constrained by the limitations of sideband conditions. In the unresolved-sideband regime, the simultaneous cooling of both the internal and external DOFs of a levitated macromagnet can be realized. Finally, the optimal cooling parameters are obtained through numerical simulations.

In the context of large-mass levitated particles, our work highlights the important fact that large-mass particles are not pointlike objects with only external DOF, but complex systems with internal DOF that can be exploited in the quantum regime. Furthermore, the CM motion of the levitated macromagnet can be easily manipulated, allowing for the probing of its internal DOF by manipulating the external DOF [22]. Therefore, it is crucial to simultaneously cool both the internal and external DOFs of a levitated macromagnet for studying the quantum phenomena. Our scheme lays the foundation for exploring the quantum correlation between macroscopic and microscopic physics.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grant No. 12074070 and the Natural Science Foundation of Fujian Province under Grant No. 2020J01471.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: QUALITY FACTOR ANALYSIS

To further illustrate the effectiveness of our parameter selection, we present exact numerical results for the steady-state average occupancies of both external and internal DOFs over broader quality factor ranges. As illustrated in Fig. 7(a), it is evident that even when Q_c varies across a wide range $Q_c \in [5 \times 10^6, 10^{11}]$, the change in n_c remains minimal. Similarly, n_b exhibits comparable results, as shown in Fig. 7(b). Thus, our protocol maintains a robust cooling capability ($n_c < 1$, $n_b < 1$) in experimentally accessible regimes, such as $Q_c \in [10^7, 10^{11}]$ and $Q_b \in [10^4, 10^7]$.

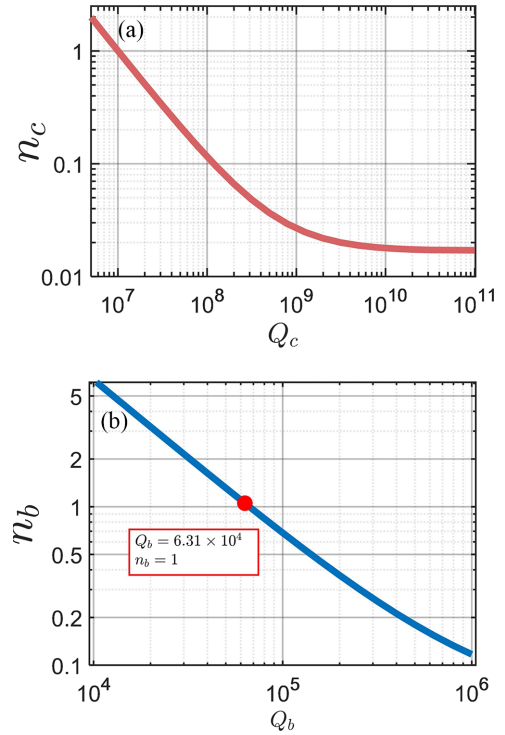


FIG. 7. (a) steady-state average occupancy of the external DOF n_c vs Q_c (log scale). (b) steady-state average occupancy of the internal DOF n_b vs Q_b (log scale). The other parameters are taken as follows: $G_{ac} = 0.05 \omega_c$, $\gamma_a = 0.5 \omega_c$, $\gamma_c = \omega_c/Q_c$, $\Delta_{m1} = 1.49 \omega_b$, $G_{mb} = 0.15 \omega_b$, $\gamma_m = 1.429 \omega_b$, and $\gamma_b = \omega_b/Q_b$.

APPENDIX B: OTHER DISSIPATIVE SOURCES

In realistic magnetically levitated systems, decoherence channels beyond mechanical damping—including magnetic field fluctuations (δB), CM trap frequency fluctuations, gas damping, and force perturbations—persist even under cryogenic conditions [31,32]. We systematically characterize these nondissipative decoherence mechanisms to establish experimental feasibility.

Magnetic field fluctuations induce two distinct effects: (i) force noise $\delta F/F = 2\delta B/B$ and (ii) parametric modulation of the trap frequency ω_c scaling as $\delta\omega_c \propto \delta B/(B^{1/2}R)$ [32], arising from δB -induced alterations to the trapping potential curvature. Magnetic noise can be mitigated through effective magnetic shielding [83]. Under cryogenic conditions, the magnetic shielding and ultralow-noise current drivers ($\delta I/I < 10^{-10}$) enable suppression of magnetic noise fluctuations $\delta B/B$ to levels below 10^{-10} in YIG spheres with radius $R < 0.2$ mm. Eddy current damping is suppressed, achieving quality factors $Q_{\text{eddy}} > 10^8$ via superconducting components and ≥ 10 mm dielectric isolation from metallic structures [32].

Squeeze-film damping becomes significant when the gap between the YIG sphere and superconductor narrows below the gas mean free path, but can be suppressed to negligible levels under ultrahigh vacuum ($P < 10^{-5}$ Pa) at cryogenic temperatures [25,84].

Force perturbations during operational phases (readout or control) are maintained below the Brownian force limit ($\delta F \ll 4k_B T \gamma_c$) using pulse-tube coolers in cryogen-free

cryostats that generate vibrations below 10 Hz [93]. This effect is negligible for kHz frequency for micron-scale YIG spheres. Multistage active vibration control further suppresses

displacement fluctuations below the thermal noise floor at 4 K, achieving CM quality factors $Q_c > 10^8$ and trap frequency stability [22,32,94].

-
- [1] Y. P. Wang, J. W. Rao, Y. Yang, P. C. Xu, Y. S. Gui, B. M. Yao, J. Q. You, and C. M. Hu, Nonreciprocity and unidirectional invisibility in cavity magnonics, *Phys. Rev. Lett.* **123**, 127202 (2019).
 - [2] W. Li, J. Cheng, W. J. Gong, and J. Li, Nonlinear self-sustaining dynamics in cavity magnomechanics, *Phys. Rev. A* **108**, 033518 (2023).
 - [3] Y. Xu, J. Y. Liu, W. Liu, and Y. F. Xiao, Nonreciprocal phonon laser in a spinning microwave magnomechanical system, *Phys. Rev. A* **103**, 053501 (2021).
 - [4] J. X. Peng, A. Kundu, Z. X. Liu, A. ur Rahman, N. Akhtar, and M. Asjad, Vector photon-magnon-phonon coherence in a polarized microwave driven cavity magnomechanical system, *Phys. Rev. B* **109**, 064412 (2024).
 - [5] Z. L. Xiang, S. Ashhab, J. Q. You, and F. Nori, Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems, *Rev. Mod. Phys.* **85**, 623 (2013).
 - [6] L. Bai, M. Harder, Y. P. Chen, X. Fan, J. Q. Xiao, and C.-M. Hu, Spin pumping in electro-dynamically coupled magnon-photon systems, *Phys. Rev. Lett.* **114**, 227201 (2015).
 - [7] R. C. Shen, Y. P. Wang, J. Li, S. Y. Zhu, G. S. Agarwal, and J. Q. You, Long-time memory and ternary logic gate using a multistable cavity magnonic system, *Phys. Rev. Lett.* **127**, 183202 (2021).
 - [8] W. Zhang, T. Wang, X. Han, S. Zhang, and H.-F. Wang, Quantum entanglement and one-way steering in a cavity magnomechanical system via a squeezed vacuum field, *Opt. Express* **30**, 10969 (2022).
 - [9] Y. P. Gao, X. F. Liu, T. J. Wang, C. Cao, and C. Wang, Photon excitation and photon-blockade effects in optomagnonic microcavities, *Phys. Rev. A* **100**, 043831 (2019).
 - [10] J. K. Xie, S. L. Ma, and F. L. Li, Quantum-interference-enhanced magnon blockade in an yttrium-iron-garnet sphere coupled to superconducting circuits, *Phys. Rev. A* **101**, 042331 (2020).
 - [11] P. K. Gupta, S. Chakraborty, S. Kalita, and A. K. Sarma, Quantum signatures of bistability and limit cycle in Kerr-modified cavity magnomechanics, *Phys. Rev. A* **110**, 063504 (2024).
 - [12] C. Gonzalez-Ballester, D. Hümmer, J. Gieseler, and O. Romero-Isart, Theory of quantum acoustomagnonics and acoustomechanics with a micromagnet, *Phys. Rev. B* **101**, 125404 (2020).
 - [13] S. Bayati, A. Mahdifar, and M. Bagheri Harouni, Entanglement formation between magnon and center-of-mass motion of a levitated particle in a magnomechanical system, *Phys. Rev. A* **110**, 013710 (2024).
 - [14] D. Lachance-Quirion, Y. Tabuchi, S. Ishino, A. Noguchi, I. Ishikawa, R. Yamazaki, and Y. Nakamura, Resolving quanta of collective spin excitations in a millimeter-sized ferromagnet, *Sci. Adv.* **3**, e1603150 (2017).
 - [15] R. Hisatomi, A. Osada, Y. Tabuchi, T. Ishikawa, A. Noguchi, R. Yamazaki, K. Usami, and Y. Nakamura, Bidirectional conversion between microwave and light via ferromagnetic magnons, *Phys. Rev. B* **93**, 174427 (2016).
 - [16] U. Delić, M. Reisenbauer, K. Dare, D. Grass, V. Vuletić, N. Kiesel, and M. Aspelmeyer, Cooling of a levitated nanoparticle to the motional quantum ground state, *Science* **367**, 892 (2020).
 - [17] C. C. Rusconi and O. Romero-Isart, Magnetic rigid rotor in the quantum regime: Theoretical toolbox, *Phys. Rev. B* **93**, 054427 (2016).
 - [18] B. A. Stickler, K. Hornberger, and M. S. Kim, Quantum rotations of nanoparticles, *Nat. Rev. Phys.* **3**, 589 (2021).
 - [19] X. Zhang, C. L. Zou, L. Jiang, and H. X. Tang, Strongly coupled magnons and cavity microwave photons, *Phys. Rev. Lett.* **113**, 156401 (2014).
 - [20] J. T. Hou and L. Liu, Strong coupling between microwave photons and nanomagnet magnons, *Phys. Rev. Lett.* **123**, 107702 (2019).
 - [21] Y. Tabuchi, S. Ishino, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Hybridizing ferromagnetic magnons and microwave photons in the quantum limit, *Phys. Rev. Lett.* **113**, 083603 (2014).
 - [22] C. Gonzalez-Ballester, J. Gieseler, and O. Romero Isart, Quantum acoustomechanics with a micromagnet, *Phys. Rev. Lett.* **124**, 093602 (2020).
 - [23] H. Huebl, C. W. Zollitsch, J. Lotze, F. Hocke, M. Greifenstein, A. Marx, R. Gross, and S. T. B. Goennenwein, High cooperativity in coupled microwave resonator ferrimagnetic insulator hybrids, *Phys. Rev. Lett.* **111**, 127003 (2013).
 - [24] Y. P. Wang, G. Q. Zhang, D. Zhang, T. F. Li, C. Hu, and J. Q. You, Bistability of cavity magnon polaritons, *Phys. Rev. Lett.* **120**, 057202 (2018).
 - [25] T. Wang, S. Lourette, S. R. O'Kelley, M. Kayci, Y. Band, D. F. J. Kimball, A. O. Sushkov, and D. Budker, Dynamics of a ferromagnetic particle levitated over a superconductor, *Phys. Rev. Appl.* **11**, 044041 (2019).
 - [26] C. C. Rusconi, V. Pöschhacker, J. I. Cirac, and O. Romero-Isart, Linear stability analysis of a levitated nanomagnet in a static magnetic field: Quantum spin stabilized magnetic levitation, *Phys. Rev. B* **96**, 134419 (2017).
 - [27] T. Delord, L. Nicolas, Y. Chassagneux, and G. Hétet, Strong coupling between a single nitrogen-vacancy spin and the rotational mode of diamonds levitating in an ion trap, *Phys. Rev. A* **96**, 063810 (2017).
 - [28] C. Gonzalez-Ballester, M. Aspelmeyer, L. Novotny, R. Quidant, and O. Romero-Isart, Spin-controlled quantum interference of levitated nanorotors, *Science* **374**, eabg3027 (2021).
 - [29] A. Pontin, H. Fu, M. Toroš, T. S. Monteiro, and P. F. Barker, Simultaneous cavity cooling of all six degrees of freedom of a levitated nanoparticle, *Nat. Phys.* **19**, 1003 (2023).
 - [30] J. Gieseler, A. Kabcenell, E. Rosenfeld, J. D. Schaefer, A. Safira, M. J. A. Schuetz, C. Gonzalez-Ballester, C. C. Rusconi, O. Romero-Isart, and M. D. Lukin, Single-spin

- magnetomechanics with levitated micromagnets, *Phys. Rev. Lett.* **124**, 163604 (2020).
- [31] M. Fuwa, R. Sakagami, and T. Tamegai, Ferromagnetic levitation and harmonic trapping of a milligram-scale yttrium iron garnet sphere, *Phys. Rev. A* **108**, 063511 (2023).
- [32] M. Fuwa, Stable magnetic levitation of soft ferromagnets for macroscopic quantum mechanics, *Phys. Rev. A* **108**, 023523 (2023).
- [33] Z. Zhang, M. O. Scully, and G. S. Agarwal, Quantum entanglement between two magnon modes via Kerr nonlinearity driven far from equilibrium, *Phys. Rev. Res.* **1**, 023021 (2019).
- [34] J. Hofer, R. Gross, G. Higgins, H. Huebl, O. F. Kieler, R. Kleiner, D. Koelle, P. Schmidt, J. A. Slater, M. Trupke, K. Uhl, T. Weimann, W. Wiczorek, and M. Aspelmeyer, High- q magnetic levitation and control of superconducting microspheres at millikelvin temperatures, *Phys. Rev. Lett.* **131**, 043603 (2023).
- [35] J. Prat-Camps, C. Teo, C. C. Rusconi, W. Wiczorek, and O. Romero-Isart, Ultrasensitive inertial and force sensors with diamagnetically levitated magnets, *Phys. Rev. Appl.* **8**, 034002 (2017).
- [36] S. P. Wolski, D. Lachance Quirion, Y. Tabuchi, S. Kono, A. Noguchi, K. Usami, and Y. Nakamura, Dissipation-based quantum sensing of magnons with a superconducting qubit, *Phys. Rev. Lett.* **125**, 117701 (2020).
- [37] A. Vinante, C. Timberlake, D. Budker, D. F. J. Kimball, A. O. Sushkov, and H. Ulbricht, Surpassing the energy resolution limit with ferromagnetic torque sensors, *Phys. Rev. Lett.* **127**, 070801 (2021).
- [38] C. Timberlake, A. Vinante, F. Shankar, A. Lapi, and H. Ulbricht, Probing modified gravity with magnetically levitated resonators, *Phys. Rev. D* **104**, L101101 (2021).
- [39] S. Kalia, D. Budker, D. F. J. Kimball, W. Ji, Z. Liu, A. O. Sushkov, C. Timberlake, H. Ulbricht, A. Vinante, and T. Wang, Ultralight dark matter detection with levitated ferromagnets, *Phys. Rev. D* **110**, 115029 (2024).
- [40] D. C. Moore and A. A. Geraci, Searching for new physics using optically levitated sensors, *Quantum Sci. Technol.* **6**, 014008 (2021).
- [41] F. Monteiro, G. Afek, D. Carney, G. Krnjaic, J. Wang, and D. C. Moore, Search for composite dark matter with optically levitated sensors, *Phys. Rev. Lett.* **125**, 181102 (2020).
- [42] B. A. Stickler, B. Papendell, S. Kuhn, J. Millen, M. Arndt, and K. Hornberger, Probing macroscopic quantum superpositions with nanorotors, *New J. Phys.* **20**, 122001 (2008).
- [43] A. Kani, B. Sarma, and J. Twamley, Intensive cavity-magnomechanical cooling of a levitated macromagnet, *Phys. Rev. Lett.* **128**, 013602 (2022).
- [44] C. Wan, M. Scala, G. W. Morley, A. A. Rahman, H. Ulbricht, J. Bateman, P. F. Barker, S. Bose, and M. S. Kim, Free nano-object Ramsey interferometry for large quantum superpositions, *Phys. Rev. Lett.* **117**, 143003 (2016).
- [45] C. C. Rusconi, M. Perdriat, G. H  t  t, O. Romero-Isart, and B. A. Stickler, Spin-controlled quantum interference of levitated nanorotors, *Phys. Rev. Lett.* **129**, 093605 (2022).
- [46] O. Romero-Isart, A. C. Pflanzer, F. Blaser, R. Kaltenbaek, N. Kiesel, M. Aspelmeyer, and J. I. Cirac, Large quantum superpositions and interference of massive nanometer-sized objects, *Phys. Rev. Lett.* **107**, 020405 (2011).
- [47] M. Roda-Llordes, A. Riera-Campen  , D. Candoli, P. T. Grochowski, and O. Romero-Isart, Macroscopic quantum superpositions via dynamics in a wide double-well potential, *Phys. Rev. Lett.* **132**, 023601 (2024).
- [48] J. Bateman, S. Nimmrichter, K. Hornberger, and H. Ulbricht, Near-field interferometry of a free-falling nanoparticle from a point-like source, *Nat. Commun.* **5**, 4788 (2014).
- [49] K. Streltsov, J. S. Pedernales, and M. B. Plenio, Ground-state cooling of levitated magnets in low-frequency traps, *Phys. Rev. Lett.* **126**, 193602 (2021).
- [50] T. Delord, P. Huillery, L. Nicolas, and G. Hetet, Spin-cooling of the motion of a trapped diamond, *Nature (London)* **580**, 56 (2020).
- [51] J. Piotrowski, D. Windey, and J. Vijayan, Spin-cooling of the motion of a trapped diamond, *Nature (London)* **19**, 1038 (2023).
- [52] D. Windey, C. Gonzalez Ballesteros, P. Maurer, L. Novotny, O. Romero Isart, and R. Reimann, Cavity-based 3D cooling of a levitated nanoparticle via coherent scattering, *Phys. Rev. Lett.* **122**, 123601 (2019).
- [53] A. D. O'Connell, M. Hofheinz, M. Ansmann, R. C. Bialczak, and A. N. Cleland, Quantum ground state and single-phonon control of a mechanical resonator, *Nature (London)* **464**, 697 (2010).
- [54] R. Dum, P. Zoller, and H. Ritsch, Monte Carlo simulation of the atomic master equation for spontaneous emission, *Phys. Rev. A* **45**, 4879 (1992).
- [55] C. Timberlake, E. Simcox, and H. Ulbricht, Linear cooling of a levitated micromagnetic cylinder by vibration, *Phys. Rev. Res.* **6**, 033345 (2024).
- [56] M. Kamba, H. Kiuchi, T. Yotsuya, and K. Aikawa, Recoil-limited feedback cooling of single nanoparticles near the ground state in an optical lattice, *Phys. Rev. A* **103**, L051701 (2021).
- [57] L. Magrini, P. Rosenzweig, C. Bach, A. Deutschmann-Olek, S. G. Hofer, S. Hong, N. Kiesel, A. Kugi, and M. Aspelmeyer, Real-time optimal quantum control of mechanical motion at room temperature, *Nature (London)* **595**, 373 (2021).
- [58] F. Tebbenjohanns, M. Luisa Mattana, M. Rossi, M. Frimmer, and L. Novotny, Quantum control of a nanoparticle optically levitated in cryogenic free space, *Nature (London)* **595**, 378 (2021).
- [59] Y. C. Liu, Y. F. Xiao, X. Luan, and C. W. Wong, Dynamic dissipative cooling of a mechanical resonator in strong coupling optomechanics, *Phys. Rev. Lett.* **110**, 153606 (2013).
- [60] N. Meyer, A. de los Rios Sommer, P. Mestres, J. Gieseler, V. Jain, L. Novotny, and R. Quidant, Resolved-sideband cooling of a levitated nanoparticle in the presence of laser phase noise, *Phys. Rev. Lett.* **123**, 153601 (2019).
- [61] P. Z. G. Fonseca, E. B. Aranas, J. Millen, T. S. Monteiro, and P. F. Barker, Nonlinear dynamics and strong cavity cooling of levitated nanoparticles, *Phys. Rev. Lett.* **117**, 173602 (2016).
- [62] J. Li, S. Y. Zhu, and G. S. Agarwal, Magnon-photon-phonon entanglement in cavity magnomechanics, *Phys. Rev. Lett.* **121**, 203601 (2018).
- [63] J. Li, S. Y. Zhu, and G. S. Agarwal, Squeezed states of magnons and phonons in cavity magnomechanics, *Phys. Rev. A* **99**, 021801 (2019).

- [64] R.-C. Shen, J. Li, Z.-Y. Fan, Y.-P. Wang, and J. Q. You, Mechanical bistability in Kerr-modified cavity magnomechanics, *Phys. Rev. Lett.* **129**, 123601 (2022).
- [65] X. Zhang, C.-L. Zou, L. Jiang, and H. X. Tang, Cavity magnomechanics, *Sci. Adv.* **2**, e1501286 (2016).
- [66] M. Frimmer, J. Gieseler, and L. Novotny, Cooling mechanical oscillators by coherent control, *Phys. Rev. Lett.* **117**, 163601 (2016).
- [67] A. Xuereb, T. Freearge, P. Horak, and P. Domokos, Optomechanical cooling with generalized interferometers, *Phys. Rev. Lett.* **105**, 013602 (2010).
- [68] Y. Li, L. A. Wu, and Z. D. Wang, Fast ground-state cooling of mechanical resonators with time-dependent optical cavities, *Phys. Rev. A* **83**, 043804 (2011).
- [69] J. Q. Liao and C. K. Law, Cooling of a mirror in cavity optomechanics with a chirped pulse, *Phys. Rev. A* **84**, 053838 (2011).
- [70] S. Machnes, J. Cerrillo, M. Aspelmeyer, W. Wieczorek, M. B. Plenio, and A. Retzker, Pulsed laser cooling for cavity optomechanical resonators, *Phys. Rev. Lett.* **108**, 153601 (2012).
- [71] F. Elste, S. M. Girvin, and A. A. Clerk, Quantum noise interference and backaction cooling in cavity nanomechanics, *Phys. Rev. Lett.* **102**, 207209 (2009).
- [72] A. Xuereb, R. Schnabel, and K. Hammerer, Dissipative optomechanics in a Michelson-Sagnac interferometer, *Phys. Rev. Lett.* **107**, 213604 (2011).
- [73] C. Genes, D. Vitali, P. Tombesi, S. Gigan, and M. Aspelmeyer, Ground-state cooling of a micromechanical oscillator: Comparing cold damping and cavity-assisted cooling schemes, *Phys. Rev. A* **77**, 033804 (2008).
- [74] X. Wang, S. Vinjanampathy, F. W. Strauch, and K. Jacobs, Ultraefficient cooling of resonators: Beating sideband cooling with quantum control, *Phys. Rev. Lett.* **107**, 177204 (2011).
- [75] Y.-P. Wang, G.-Q. Zhang, D. Zhang, X.-Q. Luo, W. Xiong, S.-P. Wang, T.-F. Li, C.-M. Hu, and J. Q. You, Magnon Kerr effect in a strongly coupled cavity-magnon system, *Phys. Rev. B* **94**, 224410 (2016).
- [76] Z. Y. Fan, R. C. Shen, Y. P. Wang, J. Li, and J. Q. You, Optical sensing of magnons via the magnetoelastic displacement, *Phys. Rev. A* **105**, 033507 (2022).
- [77] R. A. Norte, J. P. Moura, and S. Gröblacher, Mechanical resonators for quantum optomechanics experiments at room temperature, *Phys. Rev. Lett.* **116**, 147202 (2016).
- [78] C. Reinhardt, T. Müller, A. Bourassa, and J. C. Sankey, Ultralow-noise SiN trampoline resonators for sensing and optomechanics, *Phys. Rev. X* **6**, 021001 (2016).
- [79] J. Gieseler, L. Novotny, and R. Quidant, Thermal nonlinearities in a nanomechanical oscillator, *Nat. Phys.* **9**, 806 (2013).
- [80] D. E. Chang, C. A. Regal, S. B. Papp, D. J. Wilson, J. Ye, O. Painter, H. J. Kimble, and P. Zoller, Cavity opto-mechanics using an optically levitated nanosphere, *Proc. Natl. Acad. Sci. USA* **107**, 1005 (2009).
- [81] M. Imboden and P. Mohanty, Dissipation in nanoelectromechanical systems, *Phys. Rep.* **534**, 89 (2014).
- [82] R. C. LeCraw, E. G. Spencer, and E. I. Gordon, Extremely low loss acoustic resonance in single-crystal garnet spheres, *Phys. Rev. Lett.* **6**, 620 (1961).
- [83] D. Wurm *et al.*, The Panedm neutron electric dipole moment experiment at the ILL, *EPJ Web Conf.* **219**, 02006 (2019).
- [84] M. Bao, H. Yang, H. Yin, and Y. Sun, Energy transfer model for squeeze-film air damping in low vacuum, *J. Micromech. Microeng.* **12**, 341 (2002).
- [85] R. P. James Millen, T. S. Monteiro, and A. N. Vamivakas, Optomechanics with levitated particles, *Rep. Prog. Phys.* **83**, 1361 (2013).
- [86] P. Asenbaum, S. Kuhn, S. Nimmrichter, U. Sezer, and M. Arndt, Cavity cooling of free silicon nanoparticles in high vacuum, *Nat. Commun.* **4**, 2743 (2013).
- [87] L. Magrini, R. A. Norte, R. Riedinger, I. Marinković, D. Grass, U. DeliĆ, S. Gröblacher, S. Hong, and M. Aspelmeyer, Near-field coupling of a levitated nanoparticle to a photonic crystal cavity, *Optica* **5**, 1597 (2018).
- [88] J. Verdú, H. Zoubi, C. Koller, J. Majer, H. Ritsch, and J. Schmiedmayer, Strong magnetic coupling of an ultracold gas to a superconducting waveguide cavity, *Phys. Rev. Lett.* **103**, 043603 (2009).
- [89] I. Wilson-Rae, N. Nooshi, W. Zwerger, and T. J. Kippenberg, Theory of ground state cooling of a mechanical oscillator using dynamical backaction, *Phys. Rev. Lett.* **99**, 093901 (2007).
- [90] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, *Rev. Mod. Phys.* **86**, 1391 (2014).
- [91] M. Asjad, J. Li, S. Y. Zhu, and J. Q. You, Magnon squeezing enhanced ground-state cooling in cavity magnomechanics, *Fundam. Res.* **3**, 3 (2023).
- [92] S. Kuhr, S. Gleyzes, C. Guerlin, J. Bernu, U. B. Hoff, S. Deleglise, S. Osnaghi, M. Brune, J. M. Raimond, S. Haroche, E. Jacques, P. Bosland, and B. Visentin, Ultrahigh finesse Fabry-Perot superconducting resonator, *Appl. Phys. Lett.* **90**, 697 (2010).
- [93] S. M. Fleischer, M. P. Ross, K. Venkateswara, C. A. Hagedorn, E. A. Shaw, E. Swanson, B. R. Heckel, and J. H. Gundlach, A cryogenic torsion balance using a liquid-cryogen free, ultra-low vibration cryostat, *Rev. Sci. Instrum.* **595**, 064505 (2021).
- [94] A. Buikema, C. Cahillane, G. L. Mansell, and Blair, Sensitivity and performance of the advanced LIGO detectors in the third observing run, *Phys. Rev. D* **102**, 062003 (2020).