

Heat operator approach to quantum stochastic thermodynamics in the strong-coupling regime

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Heat exchanged between an open quantum system and its environment exhibits fluctuations that carry crucial signatures of the underlying dynamics. Within the well-established two-point measurement scheme, we identify a “heat operator,” whose moments with respect to the vacuum state of a thermofield-doubled Hilbert space correspond to the stochastic moments of the heat exchanged with a bath. This recasts heat statistics as a unitary time evolution problem, which we solve by combining chain-mapped reservoirs with tensor-network propagation. In a multibath setup, all total and bath-resolved heat moments then follow from a single pure state evolution. We employ this approach to compute transient and steady-state heat fluctuations in Ohmic spin-boson models in and out of equilibrium, accessing the challenging low-temperature and long memory time regimes of the environment. In the nonequilibrium case, we show a crossover in the Fano factor from super-Poissonian to nearly Poissonian statistics under strong coupling asymmetry, corresponding to thermal rectification behavior. The method applies to noninteracting (bosonic or fermionic) nonequilibrium environments with arbitrary spectral densities, offering a powerful, nonperturbative framework for understanding heat transfer in open quantum systems.

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I. INTRODUCTION

In open quantum systems, heat exchanged with the environment not only is a fundamental source of decoherence [1] and dissipation [2,3] but can also be exploited for technological benefit, e.g., to drive useful autonomous machines [4,5] or characterize unknown noise sources [6]. Moreover, quantum coherence can induce nonclassical heat fluctuations [7,8], whose important consequences range from dramatically increased dissipation during the erasure of quantum information [9] to reduced power fluctuations in quantum thermal machines [10–15]. However, in general, open quantum systems are significantly affected by their surroundings due to their small size, leading to complex phenomena including non-Markovian dynamics [16,17], level broadening [18,19], and modified equilibrium states [20,21], which are absent from conventional thermodynamic frameworks [20]. Predicting fluctuating heat exchange in open quantum systems with strong system-environment coupling is thus an important yet challenging problem, driving the recent development of advanced computational methods [22–27].

Tensor-network algorithms provide powerful tools to simulate open quantum system dynamics in the strong-coupling regime. Various strategies exist, including mapping the environment onto a chain that is amenable to exact simulation [28–31], representing the environment’s influence by a process tensor [32–35], or approximating the environment by a collection of damped pseudomodes [36–41]. However, none of the aforementioned methods can be straightforwardly used to predict heat fluctuations, because heat is not a state function (i.e., an observable). Rather, heat is a process-dependent quantity that depends on the outcomes of (at least) two measurements at different times [20,42,43]. Recent works have sought to solve this problem in the strong-coupling regime, either by targeting the characteristic function while sacrificing the trace-preserving character of the evolution [44–47] or by exploiting a Markovian embedding [48,49] to perform stochastic unraveling of the dynamics [50,51], at the cost of sampling a large number of trajectories. These difficulties are compounded for environments with complex spectral features or low temperatures, since more pseudomodes or higher bond dimensions are required to capture the associated non-Markovian effects.

In this work, we calculate quantum heat statistics by introducing a “heat operator,” $\hat{Q}$, which acts on an extended Hilbert space (Fig. 1) that furnishes a purification of the environment’s initial thermal state. Remarkably, our main results [Eqs. (10) and (12)] recast two-point measurement (TPM) of heat statistics as single-time expectation values, albeit with a modified (yet still unitary) time evolution starting from a vacuum state of the extended environment. We use the proposed framework to capture rectification of heat current and

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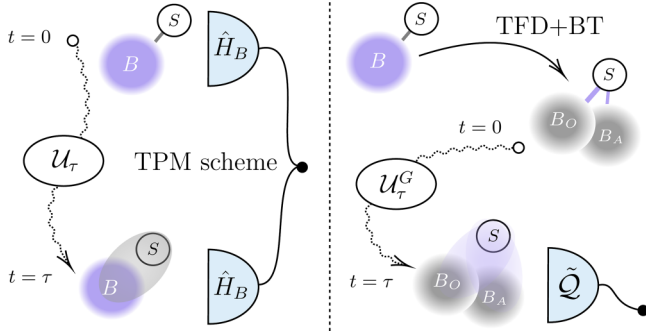


FIG. 1. In the two-point measurement scheme for calculating heat moments, the bath Hamiltonian $\hat{H}_B$ is measured at times $t = 0$ and τ . The joint system-bath (S - B) state evolves under $\mathcal{U}_t$. In the proposed method (right), thermofield doubling and Bogoliubov transformation (denoted together in this figure as “TFD+BT”) take a Gaussian state on B to a vacuum state on B_O - B_A . Measuring the “heat operator” $\tilde{Q}$ at $t = \tau$ on the unitarily evolved joint B_A - S - B_O state then yields the heat moments of B .

its fluctuations on an equal footing, in both transient and nonequilibrium steady states of a two-level system coupled to low-temperature and highly non-Markovian bosonic environments. These results are directly applicable to describe contemporary quantum physics experiments, for example, in superconducting circuits, trapped ions, solid-state spins, and other platforms featuring strong coupling to structured environments.

II. SETUP

We consider an open quantum system S with Hamiltonian $\hat{H}_S(t)$, whose time dependence takes into account any nondissipative control or driving on S by an external agent. The environment comprises some number N of baths, B_j , where $j = 1, \dots, N$. These baths, which can be either bosonic or fermionic, are described by free Hamiltonians $\hat{H}_{B,j} = \sum_v \omega_{jv} \hat{a}_{jv}^\dagger \hat{a}_{jv}$. We will take $\hbar = 1$. The mode operators $\hat{a}_{jv}$ satisfy the standard canonical bosonic or fermionic algebra. We assume that S couples independently to each B_j through a linear interaction Hamiltonian

$$\hat{H}_{I,j} = \sum_v g_{jv} \hat{L}_j^\dagger \hat{a}_{jv} + \text{H.c.}, \quad (1)$$

with coupling constants g_{jv} and arbitrary system operators $\hat{L}_j$, leading to the total system-bath Hamiltonian

$$\hat{H}(t) = \hat{H}_S(t) + \sum_j \hat{H}_{I,j} + \hat{H}_{B,j}. \quad (2)$$

In this setting, the effect of each bath on the system’s evolution is characterized by the spectral density, $J_j(\omega) = 2\pi \sum_v |g_{jv}|^2 \delta(\omega - \omega_{jv})$, which becomes a continuous function in the limit of an infinite bath and a smooth density of states [52,53]. We assume that the baths are initially in thermal equilibrium and uncorrelated with the system, leading to the initial global density matrix $\hat{\rho}(0) = \hat{\rho}_S(0) \otimes \hat{\pi}_{\beta_1} \otimes \dots \otimes \hat{\pi}_{\beta_N}$, where $\hat{\pi}_{\beta_j}$ is a thermal state of bath j at inverse temperature β_j . The state then evolves as $\hat{\rho}(t) = \mathcal{U}_t[\hat{\rho}(0)] \equiv \hat{U}(t)\hat{\rho}(0)\hat{U}^\dagger(t)$, where $\hat{U}(t)$ is the unitary operator generated

by Eq. (2). Heat, however, is not a state function, being defined by the two-point measurement described below.

III. QUANTUM HEAT STATISTICS

Following standard nomenclature [2,3], we identify the energy change of each bath as heat; however, we note that other approaches to heat at strong coupling [20,54] have been proposed in the recent literature [55]. To simplify the notation, we consider a single bath (i.e., $N = 1$) unless stated otherwise, and omit the index j where it will not cause confusion. The generalization to multiple baths is straightforward. The heat $Q(t)$ is a stochastic variable defined by a projective two-point measurement of the bath Hamiltonian $\hat{H}_B$ at the initial time 0 and final time t [20,42]. We decompose $\hat{H}_B$ as $\sum_n E_n \hat{\Pi}_n$, where $\{\hat{\Pi}_n\}$ is the unique family of projection operators corresponding to the eigenvalues $\{E_n\}$ of $\hat{H}_B$. Born’s rule gives us the joint probability of fluctuation from E_m to E_n to be $P[E_n, E_m] = \text{Tr}\{\hat{\Pi}_n \mathcal{U}_t[\hat{\Pi}_m \hat{\rho}(0) \hat{\Pi}_m]\}$. The probability distribution of heat fluctuations is then given by $P(Q, t) = \sum_{m,n} P[E_n, E_m] \delta(Q + E_m - E_n)$. Using this distribution, we determine the n th moment of $Q(t)$ as

$$\langle Q^n(t) \rangle = \int dQ P(Q, t) Q^n = (-i)^n \partial_\lambda^n \chi(\lambda, t) \big|_{\lambda=0}, \quad (3)$$

where $\chi(\lambda, t) = \int dQ P(Q, t) e^{i\lambda Q}$ is called the characteristic function of $P(Q, t)$ and λ is the counting field. The characteristic function uniquely determines the probability distribution $P(Q, t)$, and can be expressed as follows (see Appendix A in the Supplemental Material [56]):

$$\chi(\lambda, t) = \text{Tr}\{e^{i\lambda \hat{H}_B} \mathcal{U}_t[e^{-i\lambda \hat{H}_B} \hat{\rho}(0)]\}. \quad (4)$$

Thus, we recover the first moment as the mean energy change of the bath, $\langle Q(t) \rangle = \text{Tr}\{\hat{H}_B(\hat{\rho}(t) - \hat{\rho}(0))\}$. The second moment $\langle Q^2(t) \rangle$ is related to the variance of heat as $\langle Q^2(t) \rangle = \langle Q(t) \rangle^2 + \langle Q^2(t) \rangle - \langle Q(t) \rangle^2$, and so on. Unlike the first moment, however, higher moments do not have a simple expression in terms of quantum expectation values. Therefore, an efficient procedure to evaluate $\chi(\lambda, t)$ or its derivatives is crucial, and we describe our method below.

IV. HEAT OPERATOR APPROACH

The form of Eq. (4) obtained using the TPM scheme poses two apparent obstacles to a direct numerical evaluation of $\chi(\lambda, t)$. The first stems from initialization of the bath’s thermal state $\hat{\pi}_\beta$, which can be achieved by imaginary-time evolution starting from a maximally mixed state. This procedure is costly for certain representations of the bath modes, including the one-dimensional chain geometry pertinent for the chain mapping of baths using orthogonal polynomials [28]. In the language of tensor networks, imaginary-time evolution generates larger bond dimension (which is a measure of correlations) of the matrix-product operator [57–60] representing the bath state, especially while targeting low, nonzero temperatures. Second, the propagator in Eq. (4) is dressed by the counting field via the operators $e^{\pm i\lambda \hat{H}_B}$, leading to a nonunitary (trace-decreasing) overall time evolution, which can be unstable for standard tensor-network methods, especially for higher heat moments.

We sidestep both these issues using the thermofield approach [61,62] that introduces a decoupled (otherwise identical) auxiliary environment (denoted by A) to the original system-bath Hamiltonian (denoted as S and O , respectively) [63,64]; see Fig. 1 for an illustration. Bath A consists of modes $\hat{b}_v$ of frequency $-\omega_v$ that correspond to every mode $\hat{a}_v$ of frequency ω_v in bath O . This effectively doubles the Hilbert space of the bath, akin to a purification scheme. The auxiliary bath can be interpreted as a quantum register that coherently stores the outcome of the first measurement, in accordance with the principle of deferred measurement [65]. The total Hamiltonian in Eq. (2) is then transformed to

$$\tilde{H}(t) = \hat{H}(t) - \hat{H}_{B,A}, \quad \hat{H}_{B,A} = \sum_v \omega_v \hat{b}_v^\dagger \hat{b}_v. \quad (5)$$

The initial state of bath O that is a Gaussian state, $\hat{\pi}_\beta$, then gets mapped to a pure state in the extended Hilbert space of OA (original + auxiliary), referred to as the “thermofield double” or “thermal vacuum” state: $\tilde{\tau}_\beta = |\psi(\beta)\rangle_{OA} \langle\psi(\beta)|_{OA}$. The thermofield transformation ensures that

$$\text{Tr}_A\{\tilde{\tau}_\beta\} = \hat{\pi}_\beta = \frac{e^{-\beta(\hat{H}_B - \mu\hat{N})}}{\text{Tr}\{e^{-\beta(\hat{H}_B - \mu\hat{N})}\}}. \quad (6)$$

Here, for generality, we have included a chemical potential μ coupled to the particle number $\hat{N}$ of the bath. Purification, which is a linear operation, preserves the Gaussianity of the state $\tilde{\tau}_\beta$. Now, there always exists a Bogoliubov transformation [66–68], $\mathcal{G}_\beta$, which squeezes $\tilde{\tau}_\beta$ to the vacuum state of OA :

$$\tilde{\Phi} = \mathcal{G}_\beta[\tilde{\tau}_\beta] = \bigotimes_v \phi_{v,O} \otimes \phi_{v,A} = |\tilde{\Phi}\rangle \langle\tilde{\Phi}|, \quad (7)$$

where we denote the zero occupation state of mode v as $\phi_v = |0_v\rangle \langle 0_v|$ and $|\tilde{\Phi}\rangle = \bigotimes_v |0_{v,O}\rangle |0_{v,A}\rangle$ is the initial vacuum state of baths OA . The transformation $\mathcal{G}_\beta$ is canonical; i.e., it preserves the bosonic or fermionic (anti)commutation relations, and it can be shown [69,70] that $\mathcal{G}_\beta[\bullet] = e^{-i\hat{G}_\beta} \bullet e^{i\hat{G}_\beta}$ is generated by the Hermitian operator $\hat{G}_\beta = -i \sum_v \theta_{\beta,v} (\hat{a}_v^\dagger \hat{b}_v^\dagger - \hat{b}_v \hat{a}_v)$, with $\theta_{\beta,v}$ depending on the thermal occupation of the v th mode, $n_{\beta,v} = [e^{\beta(\omega_v - \mu)} + (-1)^\eta]^{-1}$ as shown below. This transformation mixes the modes of the baths O and A as follows:

$$\begin{aligned} \mathcal{G}_\beta[\hat{a}_v] &= u_{\beta,v} \hat{a}_v + v_{\beta,v} \hat{b}_v^\dagger, \\ \mathcal{G}_\beta[\hat{b}_v] &= u_{\beta,v} \hat{b}_v + (-1)^{\eta+1} v_{\beta,v} \hat{a}_v^\dagger, \end{aligned} \quad (8)$$

with $v_{\beta,v} = \sin((-i)^\eta \theta_{\beta,v}) = \sqrt{n_{\beta,v}}$ and $u_{\beta,v}^2 + (-1)^\eta v_{\beta,v}^2 = 1$, $u_{\beta,v} \geq 0$. Here, $\eta = 0$ or $\eta = 1$ for fermions or bosons, respectively.

V. MAIN RESULT

We now introduce the “heat operator” defined on OA ,

$$\tilde{\mathcal{Q}} := \hat{H}_{B,O} - \hat{H}_{B,A}. \quad (9)$$

For any Gaussian initial bath state,

$$\chi(\lambda, t) = \text{Tr}\{e^{i\lambda\tilde{\mathcal{Q}}} \mathcal{U}_t^\dagger[\hat{\rho}_S(0) \otimes \tilde{\Phi}]\}. \quad (10)$$

Here, $\mathcal{U}_t^\dagger[\bullet] = \tilde{U}_G(t) \bullet \tilde{U}_G^\dagger(t)$ and $\tilde{U}_G(t)$ is the unitary time evolution operator generated by $\tilde{H}_G(t) = \mathcal{G}_\beta[\hat{H}(t)]$, which is

given explicitly by

$$\tilde{H}_G(t) = \tilde{H}(t) + \sum_v (u_{\beta,v} g_v \hat{L}^\dagger \hat{a}_v + v_{\beta,v} g_v \hat{b}_v^\dagger \hat{L} + \text{H.c.}). \quad (11)$$

VI. PROOF SKETCH

Starting from Eq. (4), we purify the thermal bath state $\hat{\pi}_\beta$ by performing the thermofield transformation that introduces bath Hamiltonian $\hat{H}_{B,A}$ [Eq. (5)], the pure state $\tilde{\tau}_\beta$ [Eq. (6)], and the heat operator $\tilde{\mathcal{Q}}$ [Eq. (9)]. Using the facts that $\tilde{\mathcal{Q}}$ commutes with $\hat{G}_\beta$ and that it also annihilates $\hat{\rho}_S(0) \otimes \tilde{\Phi}$, we show that the $\chi(\lambda, t)$ reduces to the single-time expectation value in Eq. (10), with evolution generated by $\tilde{H}_G(t)$. The details of the proof are presented in Appendix B in the Supplemental Material [56].

It follows from Eq. (10) that

$$\langle Q^n(t) \rangle = \langle \tilde{\mathcal{Q}}^n(t) \rangle \equiv \text{Tr}\{\tilde{\mathcal{Q}}^n \mathcal{U}_t^\dagger[\hat{\rho}_S(0) \otimes \tilde{\Phi}]\}, \quad (12)$$

which states that each moment of stochastic heat equals the corresponding moment of the heat operator $\tilde{\mathcal{Q}}(t)$, where the time evolution is generated by the transformed Hamiltonian $\tilde{H}_G(t)$ and the expectation value is taken with respect to an initial vacuum state $\tilde{\Phi}$ of the extended bath OA . Thus, the requirement of nonunitary preparation and evolution steps with a single bath O in the conventional scheme [Eq. (4)] is replaced by pure unitary evolution of an extended zero temperature bath OA . This is particularly advantageous for low temperatures and multiple bath settings, and mitigates error accumulation in the higher-order heat cumulants by avoiding nonunitary evolution.

VII. CALCULATION OF HEAT MOMENTS

In case of macroscopic baths, direct simulation of the unitary evolution in Eq. (12) is a formidable task; however, it can be achieved efficiently using standard tensor-network wavefunction methods, e.g., the time evolving density matrix using orthogonal polynomials algorithm (TEDOPA) [28,31], or the multilayer multiconfiguration time-dependent Hartree [71,72] method. We choose the orthogonal polynomial approach (details in Appendix C in the Supplemental Material [56]) to represent thermodynamic operators and perform unitary time evolution of the global state using the time-dependent variational principle [73]. A tensor-network diagram illustrating the evaluation of heat moments using Eq. (12) is shown in Fig. 2.

For a pure initial system state $|\psi_S(0)\rangle$, Eq. (12) becomes $\langle Q^n(t) \rangle = \langle \Psi_G(t) | \tilde{\mathcal{Q}}^n | \Psi_G(t) \rangle$, where we prepare $|\Psi_G(t)\rangle = \tilde{U}_G(t) |\psi_S(0)\rangle \otimes |\tilde{\Phi}\rangle$ using standard time propagation schemes mentioned above. After the chain mapping of OA baths into L_B bosonic sites each, the state $|\Psi_G(t)\rangle$ is defined on L_S sites of the system and $2L_B$ of the baths. In case of a mixed initial state of the system, we can perform a spectral decomposition of $\hat{\rho}_S(0) = \sum_{k=1}^{d_S} p_k |k\rangle \langle k|$ and evaluate $\langle Q^n(t) \rangle = \sum_{k=1}^{d_S} p_k \langle \Psi_{G,k}(t) | \tilde{\mathcal{Q}}^n | \Psi_{G,k}(t) \rangle$ with $|\Psi_{G,k}(t)\rangle = \tilde{U}_G(t) |k\rangle \otimes |\tilde{\Phi}\rangle$. If we denote by C_n the computational complexity of evaluating the n th heat moment of the pure system initial state (see Appendix E in the Supplemental Material [56]), the same for mixed states using the decomposition above is

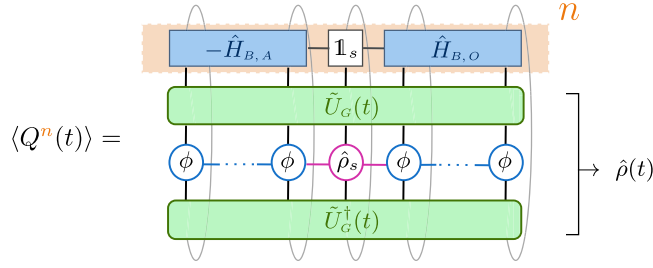


FIG. 2. Diagram illustrating the evaluation of heat moments in Eq. (12). The heat operator (shaded in orange) defined in Eq. (9) is applied n times to the time-evolved state $\hat{\rho}(t)$ (denoted with circles) defined on S and the extended bath OA .

approximately $d_S C_n$. Alternatively, we can purify the mixed initial system state leading to a single global pure state defined on $2L_B + 2L_S$ sites. Then, the complexity is slightly higher than C_n , but generally better than the scaling with d_S in the spectral decomposition approach.

The characteristic function from Eq. (10) can be cast as a survival amplitude between the time-evolved state $|\Psi_G(t)\rangle$ and its λ -tilted counterpart $e^{i\lambda\hat{Q}}|\Psi_G(t)\rangle$. Since it is defined via a thermofield purification, this survival amplitude differs from the overlap that appears when measuring heat by an ancilla-assisted interferometric protocol [74]. Such ancilla-based protocols have previously been exploited in tensor-network simulations of characteristic functions for work [75] or charge [76] fluctuations in many-body lattice systems.

VIII. EXAMPLES

We implement our method to calculate the heat statistics in the paradigmatic example of a spin embedded in a bosonic environment, considering both equilibrium and nonequilibrium settings of the same. The equilibrium spin-boson model describes a two-level system (spin) interacting with a harmonic oscillator bath and stands as a cornerstone of quantum dissipation theory [52]. It is described by the Hamiltonian $\hat{H} = \epsilon_0 \hat{S}_z + \Delta \hat{S}_x + \sum_v \omega_v \hat{a}_v^\dagger \hat{a}_v + \hat{S}_x \otimes \sum_v g_v (\hat{a}_v + \hat{a}_v^\dagger)$, with $\hat{S}_{x,z}$ the spin-1/2 angular momentum operators and $\hat{a}_v$ the bosonic annihilation operator for the bath mode v . The two-level system can be understood as representing the two lowest-energy states of a particle confined in a double-well potential. The energy splitting, ϵ_0 , corresponds to an asymmetry between the wells, while the tunneling amplitude, Δ , quantifies the coupling between the two localized states due to tunneling through the central barrier. We assume the bath spectral density to be of Ohmic type, $J(\omega) = 2\alpha\omega \exp(-\omega/\omega_C)$, with α a dimensionless coupling strength and ω_C a cutoff frequency. As a benchmark, we first study the exactly solvable independent-boson limit ($\epsilon_0 = 0$, $\Delta = 1$) of this model. As shown in Appendix D in the Supplemental Material [56], we see excellent agreement with the exact solution at all times for both zero and finite temperatures, and both weak ($\alpha < 1$) and strong ($\alpha \gtrsim 1$) bath couplings. In particular, we verify that the Fano factor, $F(t) = \langle\langle Q^2(t) \rangle\rangle / \langle Q(t) \rangle$, is independent of α , in agreement with the exact solution that predicts that all heat cumulants are proportional to α [44].

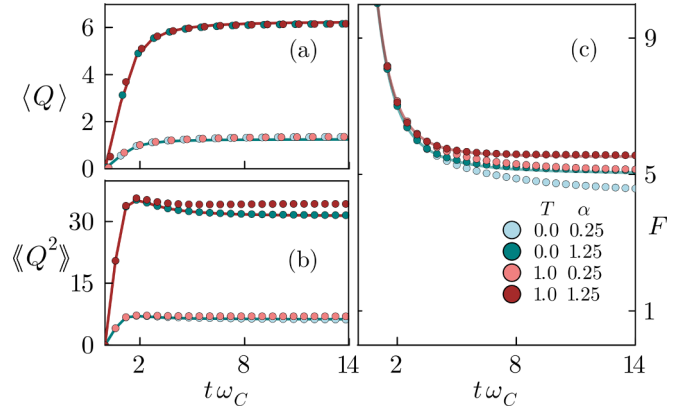


FIG. 3. Evolution of (a) mean, (b) variance, and (c) Fano factor of heat fluctuations in the spin-boson model ($\epsilon_0 = 1$, $\Delta = 0$, $\omega_C = 5$). The initial state of the spin is the $|+\rangle$ eigenstate of $\hat{S}_x$. Curves corresponding to each T - α pair for the independent-boson model (see Appendix D in the Supplemental Material [56]) are shown as solid colored lines for reference.

In the nonintegrable regime ($\epsilon_0 = 1$, $\Delta = 0$) of this model, there exists a quantum phase transition [77,78] from the delocalized to the localized regime of the spin in the $\hat{S}_x$ eigenbasis when the bath coupling constant α exceeds the critical value $\alpha_c = 1 + \mathcal{O}(\epsilon_0/\omega_C)$. In Fig. 3, we examine the heat statistics for the coupling strengths on either side of this critical point. For both zero and finite temperatures, the mean and variance of the exchanged heat increases with coupling. For $\alpha > \alpha_c$, the heat statistics approach the predictions of the independent-boson model, since the spin dynamics is effectively frozen and the energetics are dominated by the system-bath interaction. For weaker couplings, the mean heat quickly equilibrates, while the variance exhibits two well-separated timescales: a fast rise followed by a slow relaxation dictated by spin equilibration. These features are most clearly evident in the Fano factor, which depends significantly on both temperature and coupling strength, in contrast to the coupling-independent behavior of the independent-boson model in Fig. 3.

We further extend our analysis to the nonequilibrium spin-boson model, where the spin is coupled to two bosonic baths at different temperatures, $T_1 = \epsilon_0$ and $T_2 = 0$ ($\epsilon_0 = 1$, $\Delta = 0$). We compute the statistics of the heat current $J(t) = \delta_Q(t)/t$ by defining the integrated current $\delta_Q(t) = Q_2(t) - Q_1(t)$ flowing from bath 1 to bath 2 over time t . The scaled cumulants of the current are obtained as $\langle\langle J^n(t) \rangle\rangle = \langle\langle \tilde{\Delta}_Q^n(t) \rangle\rangle / t$ with $\tilde{\Delta}_Q(t) = \tilde{Q}_2(t) - \tilde{Q}_1(t)$, since $\tilde{Q}_1(t)$ commutes with $\tilde{Q}_2(t)$. Figure 4 summarizes how the time-dependent mean and fluctuations of the current depend on the coupling strengths (α_1 , α_2) and cutoff frequencies ω_C . For large coupling asymmetry, we observe thermal rectification; i.e., the mean heat current for $\alpha_1 > \alpha_2$ is suppressed by an order of magnitude compared to when $\alpha_2 > \alpha_1$. However, the low-current configuration ($\alpha_1 \gg \alpha_2$) is characterized by significant current fluctuations and a very large Fano factor, indicating that this thermal diode blocks the current quite unreliably. Conversely, the reverse configuration ($\alpha_2 \gg \alpha_1$) exhibits a more regular current, with the long-time Fano factor approaching $F \approx 1$ for extreme coupling asymmetry,

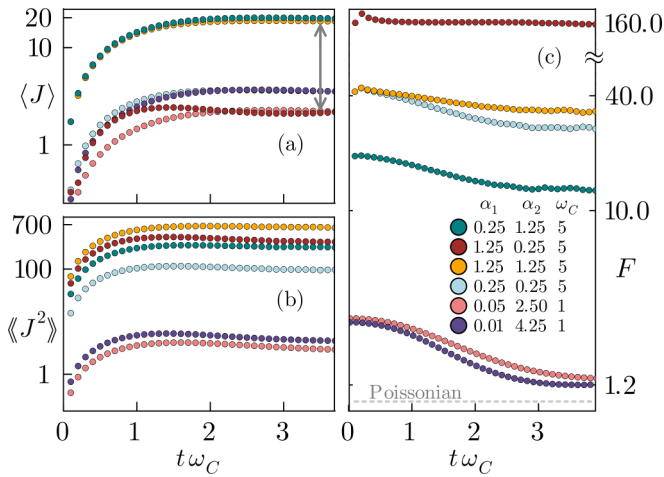


FIG. 4. Current fluctuations in the nonequilibrium spin-boson model. Time evolution of the (a) mean, (b) variance, and (c) Fano factor of the heat current for two bosonic baths with cutoff frequency ω_C at temperatures $T_1 = 1$ and $T_2 = 0$. We set $\epsilon_0 = 1$, $\Delta = 0$, and initialize the spin in the excited eigenstate of $\hat{S}_z$. The double-headed vertical arrow in panel (a) highlights the difference between the rectified currents (green and brown dots). A gray dashed line in panel (c) marks $F = 1$ corresponding to Poissonian statistics. All vertical axes are plotted on a logarithmic scale.

indicating nearly Poissonian heat statistics. Crucially, this tendency in this conducting configuration survives down to $\omega_C = 1$, where long bath memory ($\tau_B \sim \omega_C^{-1}$) usually renders the evaluation of current fluctuations particularly challenging. While previous work has studied thermal rectification in spin-boson junctions at the level of average currents [79–81], our results demonstrate the importance of heat fluctuations for characterizing the performance of quantum devices in the strong-coupling regime.

IX. CONCLUSIONS

Heat exchange is a crucial aspect of open quantum dynamics, but its characterization in the strong-coupling regime has proved elusive, partly because of the two-time measurement implicit in its definition. We bypass this issue by defining the heat operator (9), in terms of which the heat statistics can be extracted from dynamical pure state expectation values on an extended Hilbert space, as in Eq. (12). Our framework provides direct access to time-resolved, accurate heat-current statistics and complements previous analyses of

the equilibrium and nonequilibrium spin-boson models beyond perturbative, high-temperature, or steady-state limits [82–85]. In the equilibrium model, we uncover nontrivial dynamics of heat fluctuations at moderately weak coupling strengths, due to competition between the spin energy splitting and the system-bath interaction. In the nonequilibrium case, we characterize the reliability of a thermal diode and uncover a regime of current noise suppression, where the Fano factor approaches the Poissonian limit under extreme imbalance in coupling with the baths. The technique can also be extended to other quantities such as particle currents. Our work thus substantially expands the toolkit of quantum stochastic thermodynamics to encompass both steady-state and transient dynamics in the strong-coupling regime.

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DATA AVAILABILITY

The open-source code used in our simulations is based on the ITensor library [86] and the data that support the findings of this article are openly available [87].

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