

Restoring thermalization in long-range quantum magnets with staggered magnetic fields

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 (Received 4 April 2025; revised 28 October 2025; accepted 25 February 2026; published 4 May 2026)

Quantum systems with strong long-range interactions are thought to robustly resist thermalization because their energy spectra organize into a discrete number of multiplets. We show that this nonthermal behavior is surprisingly fragile to an experimentally ubiquitous perturbation: a staggered magnetic field. Using self-consistent mean-field theory and exact diagonalization, we reveal that the energy spectrum, while composed of discrete subspaces or multiplets, collectively forms a dense spectrum. The equilibration time is independent of system size and depends only on the fluctuations in the initial state. For initial states at low to intermediate energies, the long-time average aligns with the S^z sector-weighted microcanonical ensemble. However, for states in the middle of the spectrum, the long-time average depends on the initial state due to quantum scarlike eigenstates localized at unstable points in classical phase space. Our results can be readily tested on a range of experimental platforms, including Rydberg atoms or optical cavities.

DOI: [10.1103/8nqf-xjnh](https://doi.org/10.1103/8nqf-xjnh)

I. INTRODUCTION

A central challenge in nonequilibrium quantum physics is to understand the mechanisms by which isolated quantum systems approach thermal equilibrium. In many instances, the eigenstate thermalization hypothesis (ETH) provides a successful framework for explaining how systems thermalize [1–3]. However, there are exceptions to ETH where thermalization in many-body systems fails, for example, due to many-body localization [4–9], Hilbert space fragmentation [10,11], and integrability [12–14].

Quantum systems with strong long-range (LR) interactions—where the interaction strength decays as $1/r^\alpha$ with $\alpha < d$ (d is the dimension) [15,16]—have been shown to violate several principles valid in short-ranged systems, such as locality, ensemble equivalence, and additivity [17–20]. This physics is directly relevant to quantum-simulation platforms that realize strong long-range interactions, including trapped ions [21–24], Rydberg atom arrays [25,26], and atoms coupled to optical cavities [27,28].

In the strong long-range regime, absence of equilibration is believed to originate from a generic reorganization of the many-body spectrum into discrete multiplets: Intermultiplet gaps remain finite even in the thermodynamic limit, yielding long-lived metastability and equilibration times that diverge as $N \rightarrow \infty$ [29–33]. Crucially, this discrete multiplet structure is believed to be robust, i.e., not confined to fine-tuned, high-symmetry points, making the barrier to equilibration qualitatively different from short-ranged systems [16,34–36].

Previous work has predominantly focused on how the range of interactions α affects thermalization [30,32–40]. More recently, it has been shown that reducing the symmetry of initial states can lead to long-time coherent oscillations [41] and superexponential scrambling dynamics [42]. However, whether thermalization can be restored—independent of interaction range α —by reducing the symmetry at the level of the Hamiltonian remains an exciting and experimentally relevant open question.

In this article, we demonstrate that nonthermal behavior in strong long-ranged systems is surprisingly fragile and can be overcome by a minimal, experimentally ubiquitous perturbation: a staggered magnetic field. Using self-consistent mean-field theory and exact diagonalization, we show the spectrum consists of N^2 discrete subspaces (each corresponding to a multiplet in the spectrum), where N is the system size. Collectively, they form a dense spectrum. Equilibration progresses through three distinct dynamical stages: (1) oscillatory mean-field dynamics resembling classical pendulum motion, (2) intermediate-time equilibration with periodic revivals, and (3) long-time equilibration on average. In stark contrast with permutationally invariant LR systems, the equilibration time is independent of system size and depends only on the fluctuations in the initial state. For initial states at low to intermediate energies, the long-time average aligns with the microcanonical ensemble, indicating thermalization. However, for states close to the energy dividing bound and free trajectories in classical phase space, ergodicity breaks down, and the long-time average depends on the initial state. This behavior stems from quantum scarlike eigenstates localized at the unstable point.

II. THE MODEL

We consider a long-range quantum Heisenberg antiferromagnet in a staggered magnetic field:

$$H = \frac{J}{4Z_\alpha} \sum_{i,j} \frac{\sigma_i \cdot \sigma_j}{r_{ij}^\alpha} + \frac{h}{2} \sum_j (\sigma_{A,j}^z - \sigma_{B,j}^z). \quad (1)$$

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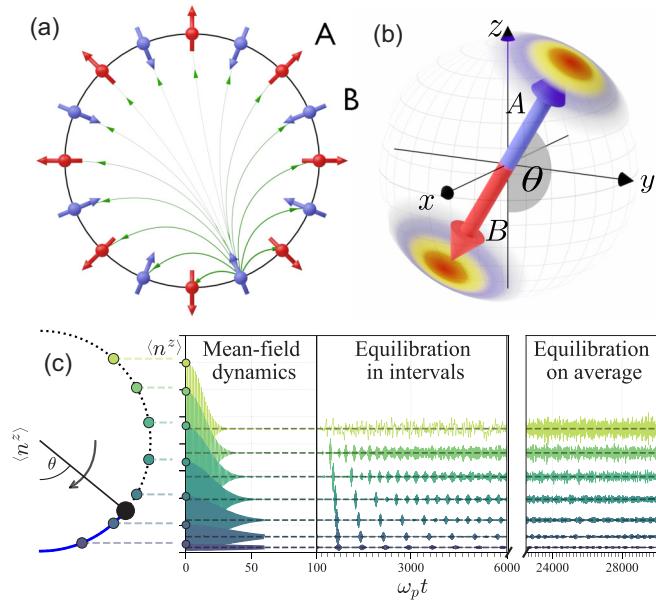


FIG. 1. Dynamics of an LR Heisenberg antiferromagnet. (a) Illustration of the system (1); a spin chain with two sublattices A (red) and B (blue) coupled via power-law interactions. (b) Bloch sphere representation of a mixed Néel state (2), parametrized by angle θ and uncertainty σ in the orientation (increasing from red to blue). (c) Time evolution of the observable $\langle n^z \rangle = (\langle S_A^z \rangle - \langle S_B^z \rangle)/N$, for mixed Néel states (2), features three distinct dynamical stages: mean-field pendulumlike oscillations at short times, periodic revivals, and eventual equilibration on average to the S^z sector-weighted microcanonical ensemble (dashed lines). Stage (1) admits a semiclassical phase-space (pendulum) description, whereas stages (2) and (3) are intrinsically quantum. Results are obtained via exact diagonalization of Eq. (3) with $J/h = 5$, $N = 500$, $\alpha = 0$, and $\sigma = 10^{-2}$.

Specifically, the geometry is that of a one-dimensional spin chain with sublattices A and B and periodic boundary conditions [Fig. 1(a)]. The operators $\sigma_{A,i}$ and $\sigma_{B,i}$ are spin-1/2 Pauli operators on site i and obey standard commutation relations. The first term represents an antiferromagnetic exchange interaction ($J > 0$) between all spins. Here, $\sigma_i = \sigma_{A,i} + \sigma_{B,i}$ is the composite spin at site i . Crucially, the interaction strength decays with distance as a power law, $1/r_{ij}^\alpha$. We consider the strong long-range interaction regime with $0 \leq \alpha < 1$. The normalization factor $Z_\alpha = (1 - \alpha)2^{1-\alpha}N^{\alpha-1}$ ensures that the total energy is extensive [15,43]. The second term introduces a staggered magnetic field of strength h along the z direction. This field breaks the inherent symmetry between the A and B sublattices and the rotation symmetry of the Heisenberg interactions.

We investigate system dynamics starting from mixed Néel states, $\rho(\Omega_0, \sigma)$. These states represent different orientations, $\Omega_0 = (\theta_0, \phi_0)$, of the Néel vector defined as $n^z = \frac{1}{N} \sum_j (\sigma_{A,j}^z - \sigma_{B,j}^z)$. Crucially, this allows for tuning the fluctuations $\sigma_{n^z}^2 = \langle (n^z)^2 \rangle - \langle n^z \rangle^2$ independent of system size [Fig. 1(b)]. Specifically, the mixed Néel state is a statistical ensemble of pure Néel states, $|\uparrow\rangle_A \otimes |\downarrow\rangle_B$, rotated by angles θ and ϕ according to $|\theta, \phi\rangle = e^{i\phi S^x} e^{i\theta S^y} |\uparrow\rangle_A \otimes |\downarrow\rangle_B$. Here, $S^y = \frac{1}{2} \sum_j (\sigma_{A,j}^y + \sigma_{B,j}^y)$ is the collective spin operator. In the mixed Néel state, these pure states are weighted by a Gaussian

factor according to

$$\rho(\Omega_0, \sigma) = \frac{1}{Z} \int d\Omega \exp\left(-\frac{(1 - \mathbf{r}(\Omega) \cdot \mathbf{r}(\Omega_0))^2}{2\sigma^2}\right) |\Omega\rangle\langle\Omega|. \quad (2)$$

Here, Z is the normalization constant and $\mathbf{r}(\Omega)$ is the unit vector in spherical coordinates. We note that even for $\sigma \rightarrow 0$, the mixed Néel state still exhibits quantum fluctuations $\sigma_{n^z}^2 \propto 1/N$ (see the Supplemental Material [44]).

III. DYNAMICS

Let us first focus on the fully connected limit ($\alpha = 0$). This limit shares many of the qualitative features with the finite $\alpha > 0$ case but crucially allows for comparison of analytical results with large-scale numerical diagonalization. The Hamiltonian depends on the collective spin operators $S_{A/B} = \frac{1}{2} \sum_j \sigma_{A/B,j}$. To further simplify the analysis, we introduce the total spin $S = S_A + S_B$ and the staggered spin $N = S_A - S_B$. The Hamiltonian (1) reduces to

$$H = \frac{J}{N} S^2 + hN^z. \quad (3)$$

Using the well-established Schwinger boson mapping, the Hamiltonian (3) can be diagonalized for systems up to hundreds of spins (see the Supplemental Material [44]).

For short times, the mean-field equations of motion are expected to accurately describe the dynamics of long-ranged interacting systems. Here, the phase-space variables are (θ, p_θ) , with the normalized Néel coordinate $n^z = \cos \theta$ acting as the position and the conserved magnetization S^z serving as the conjugate momentum p_θ . In our system, when the total spin S^2 is small, the angle θ of the Néel parameter behaves like a simple pendulum, $\ddot{\theta} + \omega_p^2 \sin(\theta) = 0$ [45], where $\omega_p = \sqrt{Jh}$. At low energies, the system exhibits bound oscillations, while at high energies, it transitions to free rotations. These regimes are divided by a separatrix at energy E_s , at which the pendulum approaches the unstable point $\theta = \pi$ exponentially slowly.

We compare this mean-field dynamics to the exact quantum evolution of mixed Néel states (2) for the Hamiltonian (3) [Fig. 1(c)]. Initially, $\langle n^z \rangle$ follows the simple pendulum mean-field dynamics. At longer times, however, the pendulumlike oscillations decay, give way to a complex series of revivals, and ultimately equilibrate to the microcanonical ensemble. The microcanonical average is computed in each S^z sector individually and then weighted by the sector overlaps (see the Supplemental Material [44]). In the Supplemental Material [44], we provide general evidence of thermalization for initial states beyond Eq. (2) and for observables other than N^z .

IV. SPECTRUM

To understand the long-time dynamics, we examine the spectrum. Figure 2(a) shows the expectation value $\langle n^z \rangle$ in eigenstates as a function of their energy. At low energies, the mean-field equation predicts bound (libration) pendulum oscillations. In this regime, the spectrum is single-valued. With increasing energy, the dependence of $\langle n^z \rangle$ on the magnetization sector S^z (red to blue) increases reaching

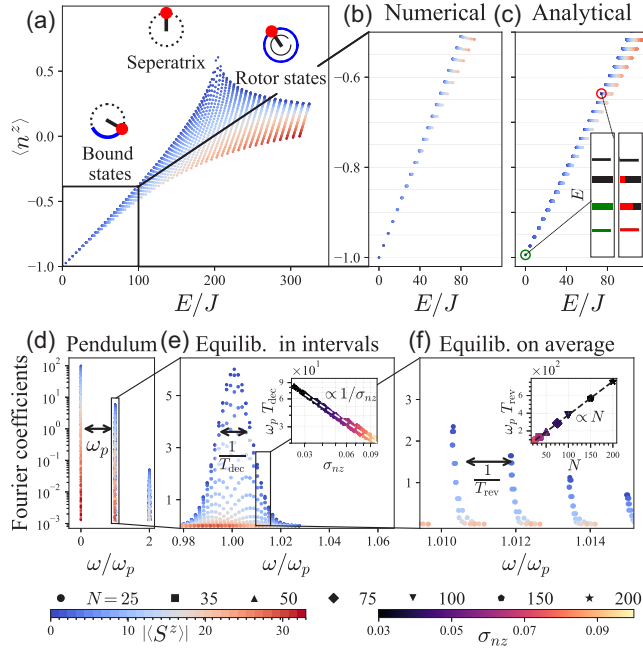


FIG. 2. Eigenspectrum and Néel vector dynamics. (a) Eigenstate expectation value $\langle n^z \rangle$ as a function of energy E/J , color coded by $\langle S^z \rangle$ (blue to red) according to exact diagonalization of Eq. (3). Bound, separatrix, and rotor states reflect classical trajectories in phase space. Note that in the $\alpha = 0$ case the system is integrable and ETH is not expected to apply. Nevertheless, away from the separatrix, $\langle n^z \rangle$ within single $\langle S^z \rangle$ sectors form a single-valued curve. Close to the ground state, numerical results [panel (b)] agree well with analytical many-body energies [panel (c)]. The inset of PANEL (c) shows how single-particle levels are occupied to form representative many-body eigenstates. (d)–(f) Fourier analysis of $\langle n^z(t) \rangle$ mirroring the real-time dynamics in Fig. 1(c). (d) A dominant peak at ω_p corresponds to mean-field pendulum oscillations. (e) Gaussian broadening of each peak induces periodic decay and revivals, with the decay time $T_{\text{dec}} \propto 1/\sigma_{n_z}$ (inset). (f) Closer inspection reveals equally spaced level splittings setting the revival period $T_{\text{rev}} \propto N$ (inset); a smaller quadratic splitting leads to equilibration on average. The system parameters are $J/h = 2$, $N = 50$, and $\alpha = 0$ and the initial state is a mixed Néel state (2) with $\theta_0 = \pi/4$ and $\sigma = 10^{-2}$.

a maximum at the separatrix energy E_s , separating bound oscillations and free rotations in classical phase space. At very high energy, there are eigenstates corresponding to free pendulum rotations.

We will now analytically diagonalize the Hamiltonian in the region corresponding to bound oscillations. Similar to the approach used for permutationally invariant LR systems in Ref. [30], we apply the Jordan-Wigner transformation to map the spin operators to fermions. A detailed step-by-step derivation is presented in the Supplemental Material [44]. Crucially, the long-ranged nature of the interactions allows us to replace string operators with their mean-field expectation values. In contrast to the permutationally invariant case, where the string operator expectation values are fixed to one [30], here they depend on the expectation values of the collective operators $\langle s^z \rangle = \langle S^z \rangle/N$ and $\langle n^z \rangle = \langle N^z \rangle/N$. To evaluate these string operator expectation values, we make three key assumptions. (1) We apply a mean-field decoupling $\langle S_A^z S_B^z \rangle = \langle S^z \rangle^2 - \langle N^z \rangle^2$.

(2) We consider the spectrum close to the ground state where $n^z \approx -1$ and $s^z \approx 0$. (3) We assume permutational invariance of the two-point spin correlation function $\langle \sigma_{L,i}^z \sigma_{M,j}^z \rangle = \langle \sigma_{L,i'}^z \sigma_{M,j'}^z \rangle$, where sites i and i' belong to the same sublattice (L), and sites j and j' belong to the same sublattice (M). This reflects the bipartite nature of the system. After obtaining a quadratic Hamiltonian, we transform to Fourier space,

$$H = -2 \sum_n \begin{pmatrix} c_{A,n}^\dagger \\ c_{B,n}^\dagger \end{pmatrix} \begin{pmatrix} h_{N,n} & \Gamma_n \\ \Gamma_n^* & -h_{N,n} \end{pmatrix} \begin{pmatrix} c_{A,n} \\ c_{B,n} \end{pmatrix} + JN \langle s^z \rangle^2, \quad (4)$$

where $c_{A/B,n}$ are fermionic operators with standard anticommutation relations. The coefficients $h_{N,n}$ and Γ_n depend on the collective variables $\langle s^z \rangle$ and $\langle n^z \rangle$ (see the Supplemental Material [44]) and on the form factor

$$F_\alpha(n) = (1 - \alpha) 2^{1-\alpha} \int_0^{1/2} ds \frac{\cos(2\pi sn)}{s^\alpha}. \quad (5)$$

Unlike in short-ranged systems, where the form factor is typically a smooth function of a continuous wave vector k , here $F_\alpha(n)$ depends on a discrete index n . This means even in the thermodynamic limit, $|F_\alpha(n+1)| - |F_\alpha(n)|$ is finite.

Diagonalizing the Hamiltonian (4) reveals an energy spectrum composed of N^2 discrete subspaces or multiplets. This arises from the energy levels $E_{A/B,n} = \pm \sqrt{h_{N,n}^2 + |\Gamma_n|^2} + JN \langle s^z \rangle^2$ [inset of Fig. 2(c)] inheriting the properties of the discrete form factor (5). Thus, the level spacings in each subspace stay finite $E_{A/B,n+1} - E_{A/B,n} > 0$, even in the thermodynamic limit $N \rightarrow \infty$. The energy also depends on the magnetization sector $\langle s^z \rangle$ and Néel parameter $\langle n^z \rangle$, with each of the N^2 pairings defining a unique subspace. Note that here $\langle n^z \rangle$ labels the subspaces, but it is not a quantum number as N^z does generally not commute with H . Crucially, the number of subspaces $\mathcal{O}(N^2)$ scales faster than that in permutationally invariant LR systems $\mathcal{O}(N)$. This is a consequence of the staggered magnetic field reducing full permutational symmetry to bipartite symmetry and therefore lifting the degeneracy in $\langle n^z \rangle$.

Although each individual subspace is discrete, the scaling with the number of subspaces leads to a dense spectrum. As the system is extensive, the energy range scales as $\mathcal{O}(N)$. However, there are N^2 subspaces, meaning that the average energy spacing must scale as $\mathcal{O}(N)/N^2 = \mathcal{O}(1/N)$. Therefore, in the thermodynamic limit $N \rightarrow \infty$, the average energy spacing vanishes and the spectrum becomes dense. This contrasts sharply with permutationally invariant LR systems, where only N subspaces exist, meaning that the average level spacing $\mathcal{O}(N)/N = \mathcal{O}(1)$ can remain finite [30,46] and the total spectrum is discrete. Note that this argument generally implies that any long-range system in which breaking of permutational symmetry produces at least N^2 nondegenerate energy levels will exhibit a dense spectrum, provided these levels remain nondegenerate in the thermodynamic limit.

V. EQUILIBRATION

The organization of the energy spectrum into discrete subspaces, which collectively form a dense spectrum, shapes the equilibration dynamics. Intuitively, states exhibiting finite

fluctuations $\sigma > 0$ equilibrate, as they span multiple subspaces collectively forming a dense spectrum.

Using the analytical spectrum, we can now understand the dynamics of $\langle N^z \rangle(t)$ in the case $\alpha = 0$ in Fig. 1(c). Motivated by numerical results (see the Supplemental Material [44]), we (1) assume that N^z is predominantly diagonal in the energy eigenbasis, with nonzero matrix elements for transitions to adjacent eigenstates $N^z \rightarrow N^z \pm 1$, and (2) approximate the mixed Néel states (2) as a statistical mixture of energy eigenstates $|s^z, n^z, j\rangle$ with Gaussian coefficients. The n^z distribution is centered at $n_0^z = \cos(\theta_0)$ with width σ_{n^z} , while the s^z distribution is centered at zero with width σ_{s^z} . Assuming the fluctuations of the initial state are small $\sigma_{n^z}, \sigma_{s^z} \ll 1$, we obtain

$$\langle N^z \rangle(t) = N_{\text{stat}}^z + \mathcal{N}_{\text{pend}}^z(t) \mathcal{N}_{\text{comb}}^z(t) \mathcal{N}_{\text{eq}}^z(t). \quad (6)$$

Each term corresponds to a behavior at different timescales, manifesting as progressively finer structures in the Fourier spectrum [Figs. 2(d)–2(f)]. At the shortest timescales, a prominent peak at frequency ω_p [Fig. 2(d)] corresponds to pendulumlike oscillations $\mathcal{N}_{\text{pend}}^z(t) = (N_0^z - N_{\text{stat}}^z) \cos(\omega_p t)$ [Fig. 1(c)]. Here, the dynamics match the mean-field equations discussed at the beginning. At intermediate timescales, this peak splits into a Gaussian frequency comb [Fig. 2(e)]. This leads to periodic decay and revival of pendulum oscillations, described by $\mathcal{N}_{\text{comb}}^z(t) = \sum_n \exp(-(t - nT_{\text{rev}})^2/T_{\text{dec}}^2)$. The decay time, T_{dec} , is determined by the frequency comb's width [Fig. 2(e)]. It is inversely proportional to initial-state fluctuations $T_{\text{dec}} \propto 1/(\sigma_{n^z})$ but importantly not system size N , as confirmed by numerical simulations [Fig. 2(e), inset]. Conversely, the revival time, T_{rev} , depends on the frequency comb's spacing [Fig. 2(f)] scaling with system size $T_{\text{rev}} \propto N$ [Fig. 2(f), inset]. Finally, over extended timescales the finest structures in the spectrum are resolved and equilibration is characterized by $\mathcal{N}_{\text{eq}}^z(t)$. Antiferromagnetic fluctuations impose a quadratic frequency spacing $[\propto (s^z)^2]$, evident in Fig. 2(f) (bottom). This breaks the regularity of the frequency comb, causing the oscillation revivals to decay according to the envelope $|\mathcal{N}_{\text{eq}}^z(t)| \propto 1/\sqrt{1 + (t/T_{\text{eq}})^2}$. Again, the equilibration time is inversely proportional to fluctuations in the initial state $T_{\text{eq}} \propto 1/\sigma_{s^z}^2$.

The analytical and numerical results confirm that the equilibration timescales of N^z ($T_{\text{dec}} \propto 1/\sigma_{n^z}$, and $T_{\text{eq}} \propto 1/\sigma_{s^z}^2$) depend on the fluctuations in the initial state. For a state with finite fluctuations, the system always equilibrates in finite time. In the case $\sigma \rightarrow 0$, instead, the equilibration timescales diverge in the thermodynamic limit, $T_{\text{dec}} \propto \sqrt{N}$ and $T_{\text{eq}} \propto N$ (as quantum fluctuations vanish with $\sigma_{n^z}^2, \sigma_{s^z}^2 \propto 1/N$). While this resembles metastability in other strong LR systems [28,30], the effect here hinges on diminishing fluctuations. The equilibration time does not explicitly scale with system size, implying that the dynamics are not protected by energy gaps in the spectrum. We note that the reduced density matrix $\text{Tr}_{S^z}(\rho) = \sum_{S^z=-N}^N \langle S^z | \rho | S^z \rangle$ also equilibrates, in the time T_{eq} , to the diagonal ensemble determining the long-time averages. Crucially, this generalizes equilibration to all observables commuting with S^z (see the Supplemental Material [44]).

For finite-range interactions $\alpha > 0$, a timescale T_{spin} emerges controlling the decay of the collective spins, $\langle S_A^2 \rangle$

and $\langle S_B^2 \rangle$ (see the Supplemental Material [44]). If $\alpha = 0$, all N^2 subspaces are fully degenerate. For $\alpha > 0$, the energy levels of each subspace split, and the width of the resulting multiplets determines T_{spin} . However, as system size increases, the energy levels within each subspace accumulate at a single point, effectively reducing the multiplet's width. This implies the lifetime of the spin lengths increases with system size, contrasting with the Néel parameter where it depends on the fluctuations. Consequently, in the thermodynamic limit, the lifetime of the collective spins $\langle S_{A/B}^2 \rangle$ diverges, so that the dynamics for systems with $0 < \alpha < 1$ approximately match those of systems with $\alpha = 0$.

VI. ERGODICITY OF OBSERVABLES

Thermalization demands not only the equilibration of observables but also that long-time averages match those in the microcanonical ensemble within the conserved S^z sector(s) [47]. We study the evolution of initial states with varying angles θ_0 at fixed energy E_0 . To achieve this, in the pendulum analogy, finite initial momentum is required. For the spin system, this corresponds to tilting the spins S_A and S_B by an angle γ_0 , yielding the state $|\theta_0, \gamma_0\rangle = e^{i\theta_0 S^z} (e^{i\gamma_0 S^x} |\downarrow\rangle_A \otimes e^{-i\gamma_0 S^x} |\uparrow\rangle_B)$. Here, $\gamma_0 = 0$ recovers the Néel state. Note at any finite system size this state equilibrates due to its finite quantum fluctuations. We observe two distinct dynamical regimes: (1) Away from the separatrix $E_0 = E_p < E_s$, all trajectories converge to the microcanonical average [Fig. 3(a)] and (2) at the separatrix energy $E_0 = E_s$, the long-time averages depend on the initial angle and deviate from microcanonical ensemble [Fig. 3(b)], indicating that ergodicity is broken.

The discrepancy at the separatrix stems from the broad range of $\langle n^z \rangle$ and from the uneven distribution of the initial states across the eigenstates. These determine the long-time average through the diagonal ensemble, which weights each eigenstate $|j\rangle$ expectation value $\langle j | n^z | j \rangle$ by the overlap with the initial state $|\langle j | \theta_0, \gamma_0 \rangle|^2$. Conversely, microcanonical averaging gives equal weight to all eigenstates within a narrow energy window. A discrepancy between ensembles occurs when a small subset of eigenstates exhibits an anomalously large overlap with $|\theta_0, \gamma_0\rangle$ and the distribution of expectation values at that energy is wide. Away from the separatrix [Fig. 3(a), top right], the overlap distribution is roughly homogeneous. However, at the separatrix [Fig. 3(a), bottom right] rare high-overlap eigenstates dominate the diagonal ensemble and hence determine the long-time average.

We further investigate the ergodicity breaking eigenstates at the separatrix energy E_s .

Analyzing the entanglement entropy, $S_N = -\text{Tr } \rho_A \ln \rho_A$, where $\rho_A = \text{Tr}_B |\psi\rangle\langle\psi|$, reveals a significant reduction for these eigenstates [see Fig. 3(b)]. While this is reminiscent of the reduced entanglement entropy in quantum many-body scars [48], here the ergodicity breaking does not manifest as persistent oscillations but as nonthermal long-time averages. To explore the structure of these special eigenstates, we examine the Husimi Q -distribution, $Q(\theta, \gamma) = |\langle \theta, \gamma | \psi \rangle|^2$, over the classical pendulum phase space (θ, γ) [Fig. 3(b), inset].

Away from the separatrix, eigenstates spread across the entire phase-space segment accessible at that energy (white dashed line). At the separatrix, however, they localize in a

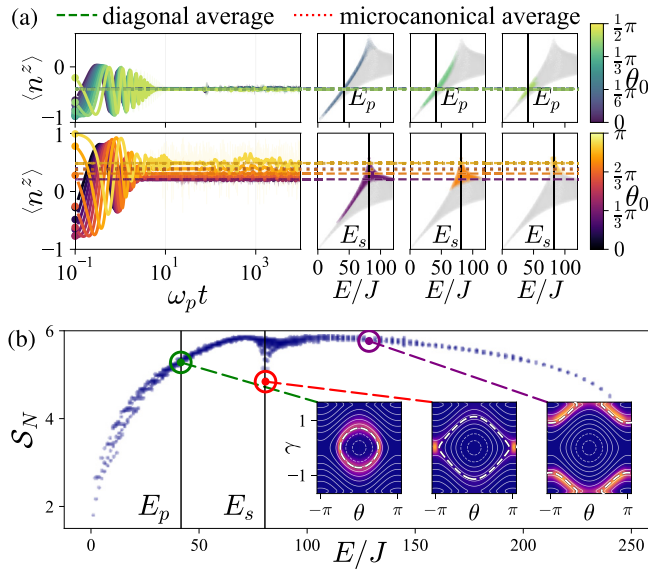


FIG. 3. Breakdown of ergodicity at the separatrix. (a) Left: Long-time evolution of $\langle n^z \rangle$ for initial states at energy E_0 with varying initial angles θ_0 (different shading). Right: Energy spectrum; eigenstates are colored by their overlap with the corresponding initial state on the left. Dotted line: S^z sector-weighted microcanonical at initial state energy. Dashed lines: diagonal averages of initial states. Top row ($E_0 = E_p < E_s$): The long-term average of $\langle n^z \rangle$ is independent of θ_0 . Both ensembles agree. Bottom row ($E_0 = E_s$): The long-term average of $\langle n^z \rangle$ clearly shows a dependence on θ_0 and the diagonal ensemble average deviates from the microcanonical average. (b) Entanglement entropy S_N of energy eigenstates for the $A-B$ partition for eigenstates with $\langle S^2 \rangle / N^2 < 0.2$. Insets: Husimi- Q distributions for selected eigenstates; white dashed line shows accessible phase space at that eigenenergy. Near the separatrix, eigenstates exhibit low S_N and are localized in an unstable phase space region. System parameters: $J/h = 5$, $N = 50$, and $\alpha = 0$.

small region near the unstable fixed point $\theta = \pi$, corresponding to the upright pendulum. This can be understood from the classical dynamics, as the Néel parameter exponentially slows down at the unstable point. This phenomenon is reminiscent of quantum single-particle scars [49–52]. However, in this case the eigenfunctions are predominantly localized at a single unstable point rather than along an unstable periodic orbit.

VII. CONCLUSION

We showed that a staggered magnetic field can restore equilibration in a strong long-range antiferromagnet for initial states with finite fluctuations. The magnetic field leads to a dense spectrum in the thermodynamic limit. While the long-time averages of low-energy initial states align with those in the microcanonical ensemble, they become initial-state dependent in the middle of the spectrum, reflecting quantum scarlike eigenstates arising from an unstable classical phase-space point.

We presented a minimal example demonstrating that thermalization can be restored for collective observables independently of the interaction range. Note that the staggered magnetic field leaves permutation symmetry within each sub-

lattice approximately conserved implying that thermalization cannot be expected for arbitrary operators in this setting. This invites further research into the role of symmetries in the dynamics of LR systems beyond the bipartite symmetry considered here.

In addition to long-ranged Heisenberg interactions, our proposal requires only local staggered magnetic fields, making it readily implementable in existing experimental platforms such as trapped ions [21–24], Rydberg atom arrays [25,26], and atoms coupled to cavities [27,28].

ACKNOWLEDGMENTS

This research was funded in whole or in part by the Austrian Science Fund (FWF) [10.55776/COE1, 10.55776/ESP9057324, 10.55776/F101200].

DATA AVAILABILITY

The data that support the findings of this article are openly available in Zenodo, version v1, at [53].

APPENDIX: DETAILS ON THERMALIZATION FOR FINITE $\alpha > 0$

For $\alpha = 0$, the collective spin lengths $\langle s_A^2 \rangle$ and $\langle s_B^2 \rangle$ are constants of motion. At finite $\alpha > 0$, they are not: Exact

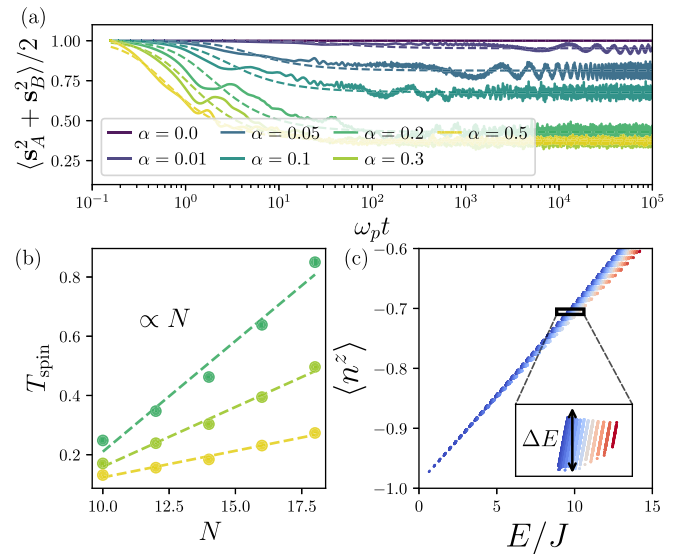


FIG. 4. Thermalization for $\alpha > 0$. (a) Time evolution of the total sublattice spin length, $\langle s_A^2 + s_B^2 \rangle / 2$, for several $\alpha > 0$, obtained by exact diagonalization of Eq. (1). For $\alpha = 0$, the quantity $s_A^2 + s_B^2$ is conserved; for $\alpha > 0$, it relaxes to a stationary value. (b) Decay time T_{spin} vs system size N . The lifetime grows linearly with N , indicating that $s_A^2 + s_B^2$ is quasiconserved in the thermodynamic limit even for $\alpha > 0$. Parameters: $J/h = 2.5$ and $\theta_0 = \pi/2$. (c) Many-body spectrum from the analytic theory, Eq. (4), at $\alpha = 0.1$ and $N = 100$. Relative to Fig. 2(c), each level splits into a multiplet; the multiplet width $\Delta E > 0$ sets the decay time of the collective spins, $T_{\text{spin}} \sim 1/\Delta E$.

diagonalization (ED) up to $N = 18$ [Fig. 4(a)] shows that $\langle s_A^2 + s_B^2 \rangle$ relaxes to a stationary value, defining a timescale T_{spin} (see main text). Extracted from the same data, T_{spin} increases approximately linearly with N [Fig. 4(b)], within the limitations of small- N numerics that do not resolve the precise exponent. In contrast to the Néel parameter n^z , the collective spin lengths are metastable for $\alpha > 0$ and become quasiconserved as $N \rightarrow \infty$.

Origin of T_{spin} . The analytic spectrum [Eqs. (4) and (5)] shows that, for $\alpha > 0$, each highly degenerate level at $\alpha = 0$ [cf. Fig. 2(c)] splits into a multiplet; Fig. 4(c) displays the multiplet structure for $N = 100$ and $\alpha = 0.1$. To leading order, the multiplet width fixes the decay time of the collective spins, where larger width implies faster decay.

We now show that the effective width ΔE of these multiplets decreases with N . At $\alpha = 0$, the single-particle energies $E_{A/B,n} = \pm \sqrt{h_{N,n}^2 + \Gamma_n^2}$ [Eq. (5)] are degenerate for $n \geq 2$. Turning on $\alpha > 0$ lifts this degeneracy, and for large n one finds $E_{A/B,n} \simeq \pm h_{N,n} - \tilde{J} n^{\alpha-1}$ with $\tilde{J} = J \langle n^z \rangle c^\alpha \Gamma(1 - \alpha) \cos(\frac{\pi\alpha}{2})$ so that levels approach accumulation points at $\pm h_N$ with spacing $\propto n^{-(1-\alpha)}$. Many-body energies at fixed occupations $N_A = \sum_n \langle d_{A,n}^\dagger d_{A,n} \rangle$ and $N_B = \sum_n \langle d_{B,n}^\dagger d_{B,n} \rangle$ are sums of occupied single-particle levels, producing a cluster whose finite width sets T_{spin} . Any two states within a given cluster differ by flips $d_{A,n}^\dagger d_{A,m}$ or $d_{B,n}^\dagger d_{B,m}$ that change the energy by $\Delta_{nm} = \tilde{J} (n^{\alpha-1} - m^{\alpha-1})$. Averaging uniformly over

$n, m \in \{2, \dots, N\}$ gives the variance

$$(\Delta E)^2 \propto \frac{4\tilde{J}^2}{N} \left[\sum_{n=2}^N n^{-2(1-\alpha)} - \frac{1}{N} \left(\sum_{n=2}^N n^{-(1-\alpha)} \right)^2 \right] \quad (\text{A1})$$

$$\approx 4\tilde{J}^2 \left(\frac{\pi^2}{6N} - \frac{\ln^2 N}{N^2} \right) \xrightarrow{N \rightarrow \infty} 0, \quad (\text{A2})$$

where the last line is valid for $\alpha \simeq 0$. This means $\Delta E \sim N^{-1/2}$ implying T_{spin} increases with system size.

Thus, the multiplet widths vanish and the associated decay time of the collective spins diverges for all $0 < \alpha < 1$. Importantly, this long lifetime is a property of the spectrum, stemming from accumulation points in the single-particle levels, rather than of initial-state fluctuations, which explains why T_{spin} depends on system size rather than on initial-state details.

Combining ED with the analytic spectrum shows that, for $0 < \alpha < 1$, multiplet widths vanish with system size, so the decay time T_{spin} of the collective-spin lengths $s_{A/B}^2$ grows with N and diverges as $N \rightarrow \infty$. Thus, $s_{A/B}^2$ are metastable at finite N but become quasiconserved in the thermodynamic limit, continuously connecting to the exactly conserved $\alpha = 0$ case. For $t \ll T_{\text{spin}}$, the T_{dec} and T_{eq} analysis applies unchanged: The system rapidly equilibrates within fixed (s_A^2, s_B^2) shells on size-independent times set by initial-state fluctuations.

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