

Topological defects organize thermally excited vortical phonons in two-dimensional crystals

Wei-Shuo Lo , Yi-Jie Pang , Yi-Cheng Zhao, and Lin I *

Department of Physics and Center for Complex Systems, *National Central University*, Zhongli, Taiwan 32001, Republic of China



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Thermal phonons in crystals are typically described in Fourier space, leaving open how disordered thermal vibrations are organized in real space-time. Recent work on two-dimensional crystals identified thermally excited vortical phonons—vorticity waves composed of traveling vortices with varying shapes and alternating-sign vorticity. Whether and how the dynamics of thermal vibrations can be organized by topological defects of vortex cores and saddles remain unclear. Here, using direct experiments on dusty plasma crystals and molecular-dynamics simulations of Yukawa crystals, we show that topological defects in the instantaneous velocity field organize thermally excited vortical phonons. In the Eulerian frame, slow-mode vorticity fluctuations are locally Gaussian yet develop coherent, intermittent, multiscale clusters of large vorticity-wave amplitude in space-time. In the Lagrangian frame, vortex cores display persistent superdiffusivelike transport together with concurrent evolution of core vorticity. Their kinetics obey robust topological rules, including charge-conserving pair creation and annihilation, exponential lifetime statistics, charge-dependent short-range correlations, and saddle-mediated splitting and merging of corotating vortices. These results move the description of thermal lattice vibrations beyond spectra alone and establish topological defects as organizing centers of thermally driven multiscale wave dynamics.

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I. INTRODUCTION

Complex spatiotemporal dynamics often look overwhelmingly disordered, yet across many nonequilibrium systems, disorder coexists with a sparse set of localized coherent entities (CEs). Topological defects—singular points where the orientations of a vector field or the phases of a wave field acquire quantized winding—have become a unifying language for characterizing CEs [1]. Examples include vortex cores and saddles in hydrodynamic turbulence [2–4]; screw-type phase-dislocation filaments wound around by helical wave fronts in water, optical, acoustic, and plasma waves [5–14], as well as spiral and scroll waves in excitable media [15,16]; and nematic defects in active and biological soft matter [17–19]. They direct collective dynamics and material transport [4,5,7,18,19]. Across diverse systems, sparse defects act as robust classifiers of local coherence within global disorder, providing a Lagrangian description that complements traditional Eulerian spectral analyses [1,3,18,19].

Microscopically, thermal phonons are a paradigm of multiscale disordered waves, continuously excited by stochastic thermal agitation [20]. Beyond Fourier-based spectrum analysis, whether and how thermal vibration dynamics can be organized by topological defects are open fundamental issues.

Recent studies using Hessian-matrix and dynamical-matrix decompositions of vibrations in amorphous solids and glasses have revealed that low-frequency eigenmodes exhibit swirl patterns hosting vortex cores and saddle points as topological defects [21–25], whose spatial distribution correlates with soft spots and plastic events [26–29]. However, in this static picture, the time-independent basis patterns are unable to characterize defect kinetics and phonon dynamics, even in crystals without intrinsic structural disorder. Moreover, in the commonly used colloidal systems for experimental studies, the highly viscous background liquid overdamps particle vibrations [22,29–31]. The dynamical behaviors of CEs and their associated topological defects in thermal phonons of ordered crystals supporting underdamped vibrations remain elusive.

Dusty plasma crystals composed of charged micrometer-sized particles in low-pressure discharges [32–34] uniquely combine two requirements for addressing this question: underdamped vibrational dynamics and individual-particle real-space resolution. However, previous works mainly addressed thermal phonon behaviors using spectral analyses [35–38]. Recent work showed that slow vibrational modes, obtained by empirical mode decomposition of particle position evolutions in ordered two-dimensional (2D) crystals, manifest as *vortical phonons*—traveling *vorticity waves* composed of propagating vortices with alternating-sign vorticity [39]. The phase singularities, screw-dislocation defect filaments wound around by helical vorticity-wave fronts in *xyt*, were demonstrated. While those interweaving defect filaments capture the topological architecture of vorticity waves, they only act as a proxy derived from the *scalar phases of vorticity waves*. It does not explicitly reveal the topological skeleton

*Contact author: lini@phy.ncu.edu.tw

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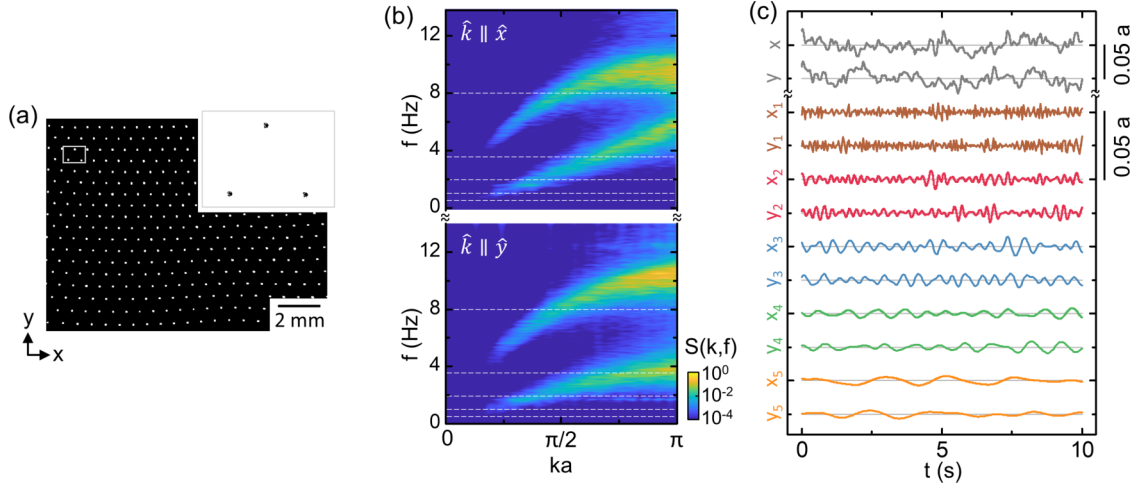


FIG. 1. Multiscale spatiotemporal signatures of thermal vibrations. (a) Typical image of the 2D monolayer dusty plasma crystal. The inset displays the 10 s particle trajectories in the region enlarged from the gray box region of panel (a). (b) Power spectra $S(k, f)$ of the spatiotemporal particle velocity fluctuations for the wave vector $\mathbf{k}$ along the x (upper panel) and y (lower panel) axes. The white horizontal lines (from top to bottom) indicate the averaged frequency positions of modes 1–5. (c) Example of the temporal fluctuations of $x(t)$ and $y(t)$ of a typical particle from its equilibrium position, and the decomposed $x_m(t)$ and $y_m(t)$ for modes 1–5. Panel (c) is reproduced from Ref. [39].

derived from the *vector orientations* of the instantaneous velocity fields of particle vibrations.

Two questions therefore remain open: How the coherence of vortical phonons is organized in space-time, and whether vortex cores and saddle points of the instantaneous velocity field constitute their topological skeleton with generic, time-resolved kinetic rules.

Here, we address these questions by developing a unified Eulerian-Lagrangian description of thermally excited vortical phonons, experimentally in 2D dusty plasma crystals and numerically in 2D Yukawa crystals. We decompose multiscale particle vibrations into modes of different temporal scales and track the motions of vortex cores and saddle points in the instantaneous velocity fields of the decomposed modes. In the Eulerian frame, slow-mode vorticity fluctuations are locally Gaussian but form intermittent multiscale clusters of large vorticity-wave amplitude in space-time. In the Lagrangian frame, the defects form trackable worldlines and obey robust kinetic rules across scales, including superdiffusive-like transport accompanied by concurrent evolution of core vorticity and surrounding vortex shape, vortex-saddle pair creation/annihilation and higher-order saddle-mediated vortex merging and splitting under topological-charge conservation, exponential lifetime statistics, and charge-dependent short-range correlations. These results promote topological defects of vibrational modes from static descriptors of disordered solids to dynamical organizers of multiscale phonons and establish ordered crystals as a clean platform for the time-resolved kinetics of vibrational topological defects.

II. METHODS

The dusty plasma crystal experiment is conducted in a cylindrical symmetric rf dusty plasma system, as described elsewhere [39]. The weakly ionized Ar glow discharge ($n_e \sim 10^9 \text{ cm}^{-3}$) in a hollow glass cylinder 13.5 cm in inner diameter and between two parallel capacitively coupled

electrodes with 3 cm in vertical separation is generated by a 13.56-MHz rf power system operated at 5 W. The 1.3 Pa Ar pressure provides a low viscous damping rate $\nu = 1.63 \text{ s}^{-1}$, much smaller than the characteristic lattice vibration frequency, ensuring underdamped vibrational dynamics. The monolayer dusty plasma crystal composed of 2000 negatively charged MF (melamine resin) particles ($9.3 \mu\text{m}$ in diameter) is levitated by the sheath field above the bottom grounded electrode and horizontally confined by the sheath field above a concentric copper ring, 2 mm in height, and 60 and 80 mm in inner and outer diameters, respectively, on the bottom electrode surface. The particles self-organize into a triangular lattice with mean lattice constant $a = 0.65 \text{ mm}$. The estimated screening parameter κ and the particle charge are 0.6 and $1.7 \times 10^4 e$ per particle, respectively. Here, κ is defined as the ratio of Wigner-Seitz radius r_{ws} to Debye length λ_D . These parameters provide good fits between the dispersion relations in the spatiotemporal power spectra of the transverse and longitudinal phonons of our dusty plasma crystal experiment [Fig. 1(b)] and our Yukawa crystal simulation [see Fig. S1(a) in the Supplemental Material [40]]. Through an optical microscope, particle images illuminated by a thin horizontal laser sheet are recorded by a charge-coupled device at 50 Hz frame rate, mounted above the discharge system.

The molecular-dynamics simulation using 16 384 particles in a rectangular box with periodic boundary conditions is conducted using the LAMMPS (Large-scale Atomic/Molecular Massively Parallel Simulator) [41]. The $2/\sqrt{3}$ box aspect ratio allows the initialization of a perfect triangular lattice at zero temperature. The coupled Langevin equation of motion of particle i with mass m is

$$m\ddot{\mathbf{r}}_i = - \sum_{j \neq i} \nabla U_{ij} - \nu m \dot{\mathbf{r}}_i + \boldsymbol{\zeta}_i(t),$$

where U_{ij} , ν , and $\boldsymbol{\zeta}$ are the interaction potential between particles i and j , the friction coefficient, and the spatially

and temporally uncorrelated thermal (Gaussian) noise, respectively. Here, $\langle \zeta^2 \rangle / (m\nu)$ is the effective temperature T .

For the 2D Yukawa crystal, the Yukawa potential, $U_{ij} = \frac{Q^2 e^{-r_{ij}/\lambda_D}}{4\pi\epsilon_0 r_{ij}}$, between particles i and j with separation r_{ij} is employed. Here, Q , λ_D , and ϵ_0 are the particle charge, Debye length, and vacuum permittivity, respectively. The potential is truncated at $r_{ij} = 25 r_{ws}$, and $Q^2/(4\pi\epsilon_0)$ is normalized to 1. The screening parameter $\kappa = 0.6$, the damping rate $\nu/\omega_0 = 0.02$, and the temperature $T/T_m = 0.003$, where T_m is the melting temperature of the 2D Yukawa crystal at this κ , are set to match the experiment. In our simulation, $1/\omega_0$ is chosen as the time unit τ_0 . For the Yukawa crystal, the one-component plasma frequency, $\omega_0 = (\frac{Q^2}{2\pi\epsilon_0 m r_{ws}^3})^{1/2}$, is used. The time step for integration is $0.01 \tau_0$.

Similar to previous studies [14,39,42–45], we decompose the multiscale vibration of each particle into different temporal-scale modes using multidimensional complementary ensemble empirical mode decomposition (MCEEMD) [46,47]. Complementary ensemble empirical mode decomposition (CEEMD) is an extension of empirical mode decomposition (EMD), which adaptively decomposes a non-stationary and nonlinear time series into a set of intrinsic mode functions (IMFs) without prescribing basis functions. Compared with conventional EMD, CEEMD reduces the problem of mode mixing, namely, the appearance of oscillations with different characteristic timescales in the same mode [46,47]. This is achieved by ensemble averaging the decomposed signals obtained after adding pairs of complementary noise sequences. The use of complementary noise pairs further suppresses artificial noisy fluctuations in the resulting IMFs. In the present analysis, each single-particle displacement time series is decomposed using 500 sets of complementary noise and 20 siftings for each mode.

For each particle i , the displacement from its equilibrium position on the xy plane is denoted by $\mathbf{x}_i(t) = [x_i(t), y_i(t)]$. The temporal evolution of $\mathbf{x}_i(t)$ is adaptively decomposed into a set of amplitude- and frequency-modulated oscillatory intrinsic mode functions $\mathbf{x}_{im}(t)$, with mode number m , and a slowly varying residual trend $\mathbf{r}_i(t)$, i.e., $\mathbf{x}_i(t) = \sum_m \mathbf{x}_{im}(t) + \mathbf{r}_i(t)$. Each mode $\mathbf{x}_{im}(t)$ has approximately zero local mean and represents particle vibration at a characteristic temporal scale. Formally, each mode can be represented in terms of an instantaneous amplitude and phase, $\mathbf{x}_{im}(t) = \mathbf{a}_{im}(t) \cos \phi_{im}(t)$, where $\mathbf{a}_{im}(t)$ and $\phi_{im}(t)$ characterize the varying amplitude and phase of the oscillatory mode. The extension of CEEMD from a single time series to the full set of particle displacement trajectories gives the MCEEMD analysis used here. This decomposition is nearly complete and orthogonal, allowing the original particle motion to be reconstructed from the sum of the decomposed modes and the residual trend. More importantly, it enables us to monitor the spatiotemporal evolution of particle vibrations at different temporal scales. For each mode m , the corresponding particle velocity is obtained from $\mathbf{v}_{im}(t) = \frac{d\mathbf{x}_{im}(t)}{dt}$.

For computing the vorticity field $\Omega_m(\mathbf{r}, t)$, we first obtain the instantaneous local velocity field $\mathbf{v}_m(\mathbf{r}, t)$ from a two-dimensional spline interpolation of the decomposed particle velocities $\{\mathbf{v}_{im}(t)\}$ to grid points with $a/3$ grid size. The local vorticity is then calculated as $\Omega_m(\mathbf{r}, t) = [\nabla \times \mathbf{v}_m(\mathbf{r}, t)]_z$.

To suppress particle-scale noise, $\Omega_m(\mathbf{r}, t)$ is coarse grained using a Gaussian window with half width $1a$, where a is the mean lattice constant. The normalized vorticity field is $\omega_m(\mathbf{r}, t) = \frac{\Omega_m(\mathbf{r}, t)}{\langle |\Omega_m| \rangle_{xyt}}$, where $\langle \cdots \rangle_{xyt}$ denotes the spatiotemporal average over the analyzed field of view and time interval.

III. EXPERIMENTAL RESULTS

A. Multiscale spatiotemporal signatures of thermal vibrations

Figure 1(a) shows a typical image of the monolayer dusty plasma crystal with a triangular lattice structure. The inset displays the 10 s trajectories of particles in the gray boxed region, illustrating small-amplitude vibrations around their equilibrium positions. Figure 1(b) shows the power spectra $S(k, f)$ of the spatiotemporal particle velocity fluctuations for the wave vector $\mathbf{k}$ along the x (upper panel) and y (lower panel) axes, respectively. The upper and lower yellow branches in each power-spectrum panel correspond to longitudinal and transverse phonons, respectively, with their ridges following the corresponding dispersion relations.

Figure 1(c) shows an example of the temporal fluctuations of $x(t)$ and $y(t)$, the displacements along x and y axes of a typical particle from its equilibrium position, and the corresponding decomposed x_m and y_m for modes 1–5. All modes exhibit amplitude and frequency modulations. The white horizontal lines in Fig. 1(b) indicate the averaged frequency positions of the first five decomposed modes shown in Fig. 1(c).

B. Multiscale vortical phonons with topological defects

We next visualize the real-space structures of these multiscale vibrations. Figure 2(a) presents representative snapshots for slow modes $m = 3$ –5, separated by a 0.4 s interval, in which the decomposed particle velocities $\mathbf{v}_{im}(t)$ are plotted as vectors with their ends located at the corresponding particle positions, superposed on the color map of the normalized local vorticity $\omega_m(\mathbf{r}, t)$ obtained from the interpolated velocity field $\mathbf{v}_m(\mathbf{r}, t)$ (see Sec. II). For each slow mode, the vibrational excitations manifest as *vortical phonons* (vorticity waves) in the form of traveling vortices with varying shapes and alternating signs of vorticity [39], whose characteristic length scale increases with mode number m . These are the modes for which well-resolved coherent vortical structures emerge. In contrast, the faster modes $m = 1$ and 2 contain shorter-wavelength fluctuations and a relatively stronger longitudinal-wave contribution, producing more disordered instantaneous velocity fields that do not exhibit the clearly resolved coherent vortical structures required for the present defect analysis [39].

To extract the underlying topological skeleton of slow modes, we examine the local topology of the instantaneous velocity-orientation field $\theta(\mathbf{r}, t) = \arctan[v_y(\mathbf{r}, t)/v_x(\mathbf{r}, t)]$. Using the standard winding-number criterion [1], we identify a singular point as a topological defect when the accumulated orientation change along a closed loop surrounding it equals $2\pi q$. As shown in Fig. 2(a), we distinguish two types of topological defects with opposite topological charge q : vortex cores and saddle points. The red and green dots are the cores of the counterclockwise and clockwise vortices, respectively; they both have the same charge, $q = +1$, regardless of the

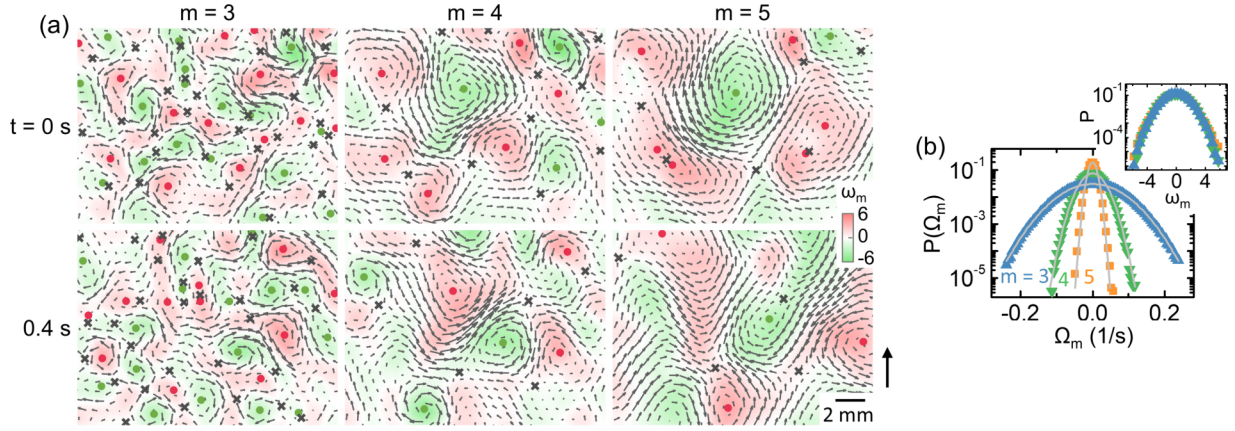


FIG. 2. Multiscale vortical phonons with topological defects and their Gaussian statistics. (a) Snapshots of the velocity fields of slow modes 3–5, represented by vectors with their ends located at particle positions, separated by a 0.4 s interval, each color coded by the normalized vorticity ω_m . Red/green circles: vortex cores ($q = +1$, counterclockwise/clockwise rotation). Black crosses: saddle points ($q = -1$). Defects are singularities of the instantaneous velocity field. The length of the vector on the right corresponds to the particle speed of each mode (normalized by the mean particle speeds of each mode) and equals 6. (b) Probability distribution functions of the local vorticity of different modes, showing the Gaussian distribution for modes 3–5. The statistics are collected from the vorticity at all interpolated grid points within a region containing ~ 600 particles, over 60 s. The gray lines are the curves for the best Gaussian fits. Inset: collapse of those curves after rescaling by $\langle |\Omega_m| \rangle_{xyt}$. Also see Video S1 of the Supplemental Material [40] for the dynamical behaviors of vortical phonons and defects.

opposite swirling directions. The crosses are the saddle points with $q = -1$. These vortex cores and saddle points constitute the fundamental particlelike excitations embedded within the fluctuating vortical phonon field. Crucially, unlike the defects of static eigenmode patterns, these defects are defined on the instantaneous velocity field; as shown in Video S1 of the Supplemental Material [40], they appear, migrate, and disappear as the field fluctuates.

Before turning to the dynamics of these defects (Sec. III D), we first characterize the statistical nature of the underlying vorticity field. The probability distribution functions of the local vorticities of all slow modes, shown in Fig. 2(b), exhibit Gaussian distributions that collapse onto a master curve after rescaling by their mean absolute values $\langle |\Omega_m| \rangle_{xyt}$. This Gaussian nature reflects the random thermal agitations driving the lattice vibrations.

C. Eulerian coherence and intermittency of vorticity waves

Despite the random nature of local vorticity fluctuations, these vortical excitations are not merely spatially uncorrelated, independent vortices. To quantify their spatiotemporal correlation, Fig. 3(a) shows the power spectra $S_m(k, f)$ of the spatiotemporal vorticity fluctuations of different slow modes. The emergence of mode-dependent ridges directly demonstrates that these are genuine *vorticity waves* possessing characteristic length scales and timescales. The spatial scale of vorticity waves systematically increases with mode number m , consistent with the larger vortex sizes observed in Fig. 2(a). Furthermore, the weaker intensity of the longitudinal phonons compared to transverse phonons at low frequencies [Fig. 1(b)] implies that the vorticity waves for these slow modes are predominantly contributed by transverse lattice deformations due to the lower shear modulus than the compressional modulus. Therefore, the ridges of slow

modes in the power spectra of Fig. 3(a) coincide with the ridges of the power spectra of particle velocity fluctuations of transverse waves at the corresponding averaged mode frequencies, shown in Fig. 1(b).

While the f - k spectra capture averaged coherence, the vorticity-wave excitations exhibit pronounced spatiotemporal heterogeneity in real space and time. To visualize this intermittency of large vorticity-wave amplitude domains, Fig. 3(b) shows the plots of the isoamplitude surfaces enclosing the large wave-amplitude sites with instantaneous local vorticity-wave amplitude $A_m(\mathbf{r}, t) \geq 3.1$ (2.5 times the standard deviation of ω_m) in the xyt space. Here, the local vorticity-wave amplitude $A_m(\mathbf{r}, t)$ of ω_m is computed through applying the Hilbert transform in time on the local vorticity, $\omega_m(\mathbf{r}, t)$ [i.e., $\omega_m(\mathbf{r}, t) = A_m(\mathbf{r}, t) \cos \Phi_m(\mathbf{r}, t)$]. These plots indicate that the large amplitude events intermittently emerge in the form of clusters with various sizes (the clusters are color coded by their size N), each representing a spatiotemporally coherent burst of large-amplitude vortical phonons. The cluster size N denotes the total number of large-amplitude event sites connected through nearest-neighbor interpolated grid points in the three-dimensional xyt dataset; a site is a sampled spatial grid location at a particular time frame, rather than a distinct particle. Thus, the same spatial region can contribute event sites over many successive frames, and N is effectively a space-time volume measured in discrete xyt sites, so that it can greatly exceed the number of particles in the crystal.

For characterizing the spatial extent of a cluster in the xyt space and its relation with the temporal span, we measure the spatial radius R of gyration of a cluster with lifetime Δt , defined as $R = \sqrt{\frac{1}{N} \sum_j |r_j - r_c|^2}$, where $r_j = (x_j, y_j)$ is the spatial position of site j and $r_c = \frac{1}{N} \sum_j r_j$ is the spatial centroid of the cluster. Figure 3(c) shows the radius of gyration $\langle R \rangle$ averaged over clusters sharing the same

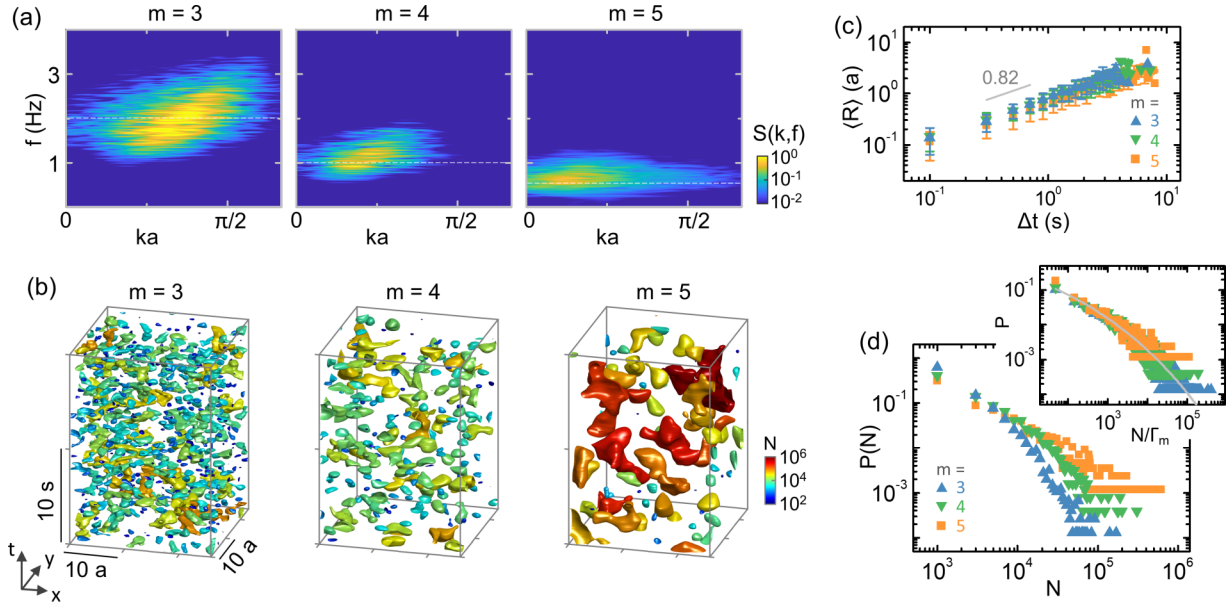


FIG. 3. Eulerian coherence and intermittency of vorticity waves. (a) Power spectra $S_m(k, f)$ of the spatiotemporal vorticity fluctuations of slow modes 3–5, for the wave vector $\mathbf{k}$ along the x axis, showing the characteristic spatial and temporal scales of vorticity waves of different modes. The white horizontal dashed lines indicate the averaged frequency of each mode. (b) Isoamplitude surfaces enclosing the large-amplitude sites with instantaneous local vorticity-wave amplitude $A_m(\mathbf{r}, t) \geq 3.1$ (2.5 times the standard deviation of ω_m) in the xyt space. The isoamplitude surfaces are color coded by cluster size N (the number of sites in the cluster) of modes 3–5 in the xyt space. (c) Mean radius of gyration $\langle R \rangle$ vs lifetime Δt of the large-amplitude clusters, obeying the scaling behavior $\langle R \rangle \propto \Delta t^{0.82}$. (d) Probability distribution functions $P(N)$ of cluster size N for different modes. The stretched-exponential distribution indicates the multiscale nature of intermittent vortical bursts. Inset: Collapse of $P(N)$ into a single curve. The gray curve is the best fit, $P(N/\Gamma) \propto \exp[-2.62(N/\Gamma)^{0.12}]$, after rescaling by $\Gamma_m = f_m^{-1} k_m^{-2}$. Despite Gaussian local vorticity fluctuations, vortical phonons exhibit spatiotemporal coherence and intermittent large-amplitude clustering across modes.

lifetime Δt , as a function of Δt . The power law increase indicates that the long-lived clusters tend to have larger spatial extent. The probability distribution functions $P(N)$ of cluster size N exhibit a heavy-tailed behavior for all slow modes [Fig. 3(d)]. To uncover the underlying universality, we introduce a scaling factor $\Gamma_m = f_m^{-1} k_m^{-2}$, where f_m and k_m represent the characteristic frequency and wave number extracted from a 2D Gaussian fit to the $S_m(k, f)$ spectrum in Fig. 3(a). Notably, the distributions $P(N)$ for different modes collapse when rescaled by Γ_m , following stretched-exponential distribution $P(N/\Gamma) \propto \exp[-2.62(N/\Gamma)^{0.12}]$ [Fig. 3(d), inset]. This self-similar collapse suggests a common scaling behavior underlying the intermittency of vortical phonons across the resolved modes.

D. Lagrangian dynamics of topological defects and transport

We next switch to a Lagrangian perspective by tracking defect trajectories. Figure 4(a) shows representative vortex-core worldlines in the xyt space for slow modes $m = 3-5$, color coded by red and green for the counterclockwise and clockwise rotating vortices, respectively. The wiggling, finite-length filaments directly visualize the continuous creation, transport, and annihilation of vortex cores. We overlay, at discrete times, the areas (shown as red and green flat surfaces) with vorticity magnitude exceeding a threshold, $|\omega_m| = 2$ (twice $\langle |\omega_m| \rangle$), around the vortex-core filaments, as shown in Fig. 4(b), blown up from the box region of Fig. 4(a). These areas provide a geometric proxy for the instantaneous shape and strength of the vortex surrounding the core, revealing that

core strength and vortex shape evolve concurrently with core transport.

To quantify these Lagrangian fluctuations, we measure, over time interval Δt , (1) the mean-square displacement (MSD) of vortex cores, $\text{MSD}(\Delta t) = \langle |\mathbf{r}(t + \Delta t) - \mathbf{r}(t)|^2 \rangle$, and (2) the mean-square vorticity change (MSVC) of vortex cores, $\text{MSVC}(\Delta t) = \langle [\omega_c(t + \Delta t) - \omega_c(t)]^2 \rangle$, where ω_c is the normalized local vorticity evaluated at the instantaneous core position. Both MSD and MSVC display persistent *superdiffusivelike* growth over the intermediate- Δt regime [Fig. 4(c)], with scaling exponents ~ 1.65 (MSD) and ~ 1.55 (MSVC). The similar exponents across modes indicate self-similar dynamical rules across scales.

Rescaling time by the individual lifetime τ of each core and averaging the core vorticity conditioned on rotation sense reveals a reproducible *birth-growth-decay* profile [Fig. 4(d), with the gray regions showing the error bars]: Cores are born with small vorticity, reach a maximum near mid-life, and decay before annihilation. The stereotyped life cycle, together with the behaviors of MSD and MSVC evaluated along defect worldlines, reveals that the vortex cores are Lagrangian coherent entities whose internal vorticity evolves concurrently with their motion.

E. Topological rearrangements, lifetime statistics, and spatial correlations of defects

The Lagrangian tracking reveals rich dynamical behaviors of the topological defects. Figure 5(a) displays the

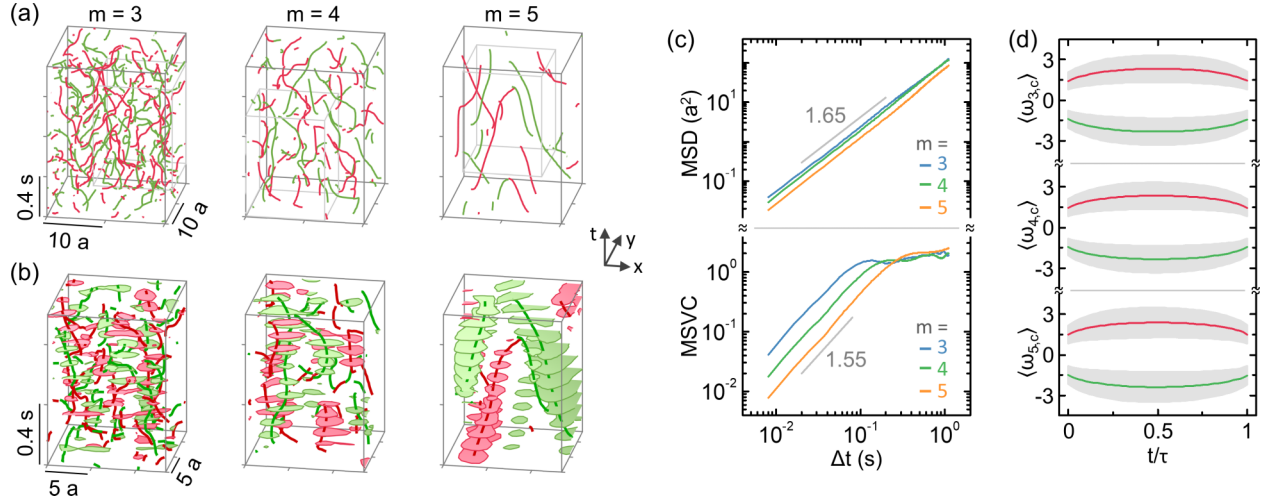


FIG. 4. Lagrangian dynamics of topological defects and transport. (a) Trajectories of vortex cores, color coded by red and green for vortices with positive and negative vorticity, respectively, in the xyt space, for modes 3–5. (b) The corresponding vortex-core trajectories magnified from the gray box regions of panel (a). Flat areas enclosed by contour lines surrounding the vortex-core filaments are the regions with vorticity magnitude larger than a certain threshold ($|\omega_m| = 2$, twice $\langle |\omega_m| \rangle$). They reflect the shape and strength evolutions of the surrounding vortex. (c) Top: Mean-square displacement of vortex cores. Bottom: mean-square vorticity changes of the vortex cores. (d) Temporal evolution of vortex-core vorticity rescaled by the lifetime of each vortex core, averaged over 1000 vortex-core trajectories of each mode, with the gray regions showing the error bar. Vortex cores behave as mobile coherent entities, showing persistent superdiffusivelike transport together with concurrent evolution of core vorticity over their lifetime.

worldlines of vortex cores (red/green) and saddle points (gray) for mode $m = 4$ in xyt space. The leftmost closed vortex core-saddle loop demonstrates the spontaneous generation of a vortex-saddle pair from the fluctuating background, propagating through the lattice,

followed by their pair annihilation, with conserved topological charges.

Beyond the above closed-loop pair dynamics, the dense defect environment facilitates higher-order topological rearrangements. The examples in Fig. 5(b) illustrate splitting and

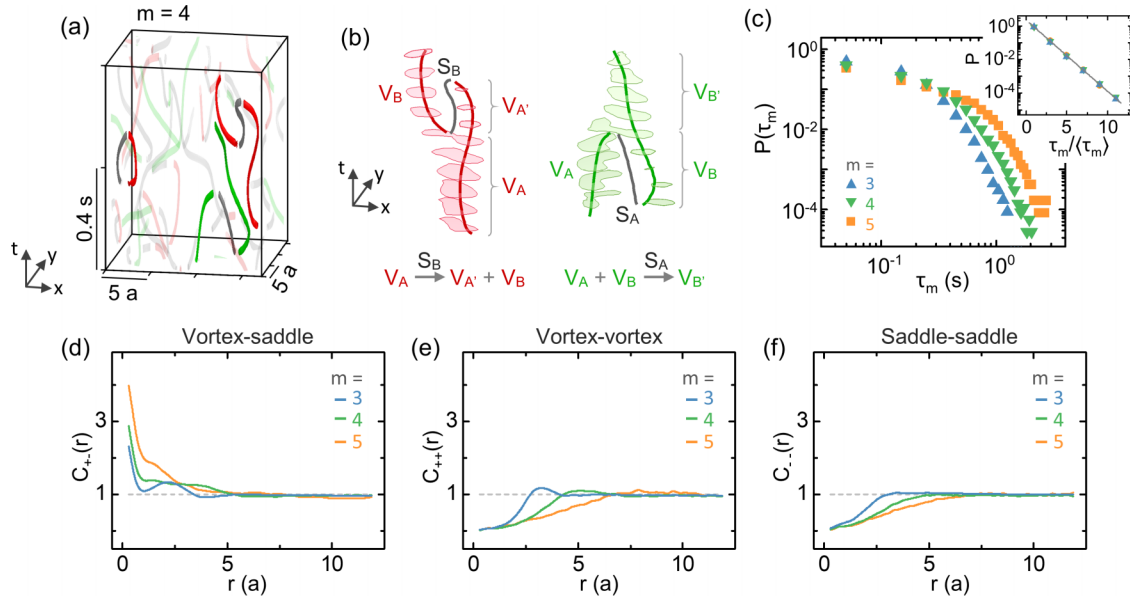


FIG. 5. Topological rearrangements, lifetime statistics, and spatial correlations of defects. (a) Worldlines of vortex cores (red/green) and saddle points (gray) in xyt space for mode $m = 4$. The closed defect loop (the leftmost loop) highlights an elementary vortex-saddle pair creation/annihilation event with conserved topological charge. (b) Saddle-mediated splitting (left, $V_A \xrightarrow{S_B} V'_A + V_B$) and merging (right, $V_A + V_B \xrightarrow{S_A} V'_B$) of corotating-vortex cores; shaded contours denote vorticity envelopes $|\omega_m| \geq 2$. (c) Lifetime distributions $P(\tau_m)$ for modes $m = 3$ –5. Inset: Distributions rescaled by $\langle \tau_m \rangle$ collapse onto a single exponentially decaying curve (gray line). Radial pair-correlation functions $C(r)$ for vortex-saddle [panel (d)], vortex-vortex [panel (e)], and saddle-saddle [panel (f)] pairs, showing short-range vortex-saddle clustering, liquidlike vortex-vortex ordering, and gaslike saddle-saddle distribution, respectively.

merging of vortices with the same swirling direction, which are fundamentally linked to the fate of saddle defects. In the splitting process [Fig. 5(b), left], the vorticity contour surrounding a single vortex core V_A undergoes significant elongation and is eventually pinched off, associated with the emergence of saddle S_B at the pinched-off point, and a detached new vortex V_B . This topological transition manifests as the coherent structure of the original vortex being effectively pierced by a nucleating saddle point, resulting in the fission into two daughter vortices V'_A and V_B , with the same swirling direction, but each with the smaller vorticity magnitude than that of V_A before splitting (see the change of the surrounding red surface area). Conversely, vortex merging [Fig. 5(b), right] exhibits the reverse dynamic: The vorticity contours of two corotating-vortex cores V_A and V_B are observed to connect and fuse into a single larger vortex V'_B with larger vorticity magnitude (see the change of the surrounding green surface area), enabled by the annihilation of a mediating saddle point S_A with V_A , thereby preserving the net topological charge. Namely, saddles can act as transient catalysts that enable defect-number changes while maintaining local topological-charge conservation.

The pair-process picture implies finite defect lifetimes. Figure 5(c) shows the lifetime distributions $P(\tau_m)$ of defects for modes $m = 3-5$. The mean lifetime $\langle\tau_m\rangle$ increases with m , consistent with the slower characteristic timescales of higher modes. When rescaled by $\langle\tau_m\rangle$, the distributions across modes collapse onto a single *exponential* curve [Fig. 5(c), inset], indicating self-similar memoryless termination statistics. This collapse indicates that defect termination in the statistically steady state is approximately a Poisson process at the mode-dependent rate $\langle\tau_m\rangle^{-1}$.

To examine spatial organization, we compute the radial pair-correlation function $C(r)$ for the three combinations of charges, represented by $+$ and $-$. For *opposite charges* (vortex-saddle), $C_{+-}(r)$ shows a strong short-range peak at $r < a$ [Fig. 5(d)], dominated by vortex-saddle pairs immediately after creation or just before annihilation—a direct signature of the closed-loop pair kinetics. For *same-charge vortex pairs*, $C_{++}(r)$ exhibits a short-range bump at r of several a [Fig. 5(e)], indicating *liquidlike* local ordering of vortex cores at the intervortex scale set by the dominant vortical-phonon wavelength. In contrast, $C_{--}(r)$ for *saddle-saddle* pairs lacks short-range structure and remains close to unity across r [Fig. 5(f)], indicating a weakly correlated *gaslike* distribution. Similar $C_{++}(r)$ and $C_{--}(r)$ behaviors were demonstrated in static eigenmode analyses of 2D glasses [27]. The hierarchy is suggestive: Vortex cores carry the dominant spatial organization of the wave field, while saddles populate the interstitial regions as a more diffuse background—yet, as the splitting and merging events demonstrate, this background plays an active dynamical role.

IV. NUMERICAL RESULTS

Complementing the experiment, we also conduct molecular-dynamics simulations of a 2D Yukawa crystal under low damping (see Sec. II); the corresponding numerical results are presented in Figs. S1 and S2 of the Supplemental Material [40]. These findings reveal

generic Eulerian-Lagrangian dynamical behaviors of vortical phonons, vortex cores, and saddle points consistent with the experimental results. The Yukawa crystal simulations reproduce the key experimental observations, including multiscale vortical phonons with vortex/saddle defects and the robust kinetic rules of defects. Note that thermally excited vortical phonons in the form of multiscale vortices with varying shapes and alternating signs of vorticity were also observed in a numerical study of 2D Lennard-Jones crystals [39]. Our study suggests that the defect-based description is not specific to dusty plasma interactions but may apply broadly to thermally excited vortical motions in general 2D crystals with different interaction forms.

V. DISCUSSION

Our results support a unified picture in which thermally excited vortical phonons in a 2D crystal can be described at two complementary levels: (1) an Eulerian wave-field level, where vorticity waves exhibit mode-dependent spatiotemporal coherence and intermittent high-amplitude excitation events in the xyt space, and (2) a defect level, where sparse topological singularities of the decomposed velocity-orientation field act as Lagrangian coherent entities whose interactions and correlations provide a coarse-grained description of the evolving wave field. This dual description bridges Fourier-space characterizations of thermal phonons with a real space-time description based on coherent excitations and topological defects.

Since the slow modes ($m \geq 3$) are dominated by transverse phonons (Sec. III C), the superposition of multiscale transverse waves with fluctuating amplitudes, phases, and propagation directions naturally generates swirl-like velocity patterns and vorticity waves. Unlike vortices in incompressible hydrodynamic turbulence that can persist for long times [2–4], vortical patterns in a solid are continuously rebuilt by the interplay of stochastic thermal driving and elastic restoring forces: Shear deformations accumulate around saddle regions and are released by the rebound of the lattice, providing an oscillatory origin for vorticity-wave excitations. Establishing a minimal theory that links transverse-wave superposition, elasticity, and defect kinetics to the measured f - k signatures and cluster statistics is an important direction for future work.

The multiscale clusters of large vorticity-wave amplitude sites in the xyt space of each slow mode provide an operational, space-time definition of coherent-excitation events for vortical phonons. Their stretched-exponential size distribution and the growth of lifetime with spatial extent indicate that the excitations are intrinsically multiscale. Such intermittency is not captured by f - k spectra alone. Indeed, tracking the worldlines of topological defects, vortex cores, and saddle points establishes robust reaction rules dominated by local vortex-saddle pair generation and annihilation, consistent with topological-charge conservation. The collapse of the rescaled defect-lifetime distributions onto an exponential form suggests that defect termination is approximately memoryless in the statistically steady state. Importantly, this does not imply the absence of correlations; rather, defect interactions and saddle-mediated rearrangements may primarily renormalize the effective mode-dependent reaction rate, while the age

dependence of defect survival remains weak. The microscopic origin of this effective memoryless behavior remains to be clarified. Together, these results establish vortex and saddle cores not merely as geometric signatures but as dynamical degrees of freedom that capture how local motional order persists within thermally driven disorder.

The Lagrangian viewpoint connects defect motion to the evolution of the vorticity carried by the core. The observed superdiffusivelike scaling of both core displacement and core-vorticity change indicates persistent dynamics rather than simple Brownian wandering. A vortex core is not a material particle subjected to independent random kicks, but a moving singularity embedded in a propagating and continuously deforming vortical wave field. Over times shorter than the decorrelation time of the surrounding wave structure, translation and deformation of the vortical pattern advect the core with temporally correlated displacements, producing an MSD exponent larger than the normal diffusive value of 1. However, stochastic thermal forcing, wave interference, and vortex-saddle reactions reduce this persistence, preventing purely ballistic motion with exponent 2. We therefore regard the measured MSD exponent of ~ 1.65 as characterizing their persistent (super) diffusion rather than a universal critical exponent.

The distinct MSD and MSVC exponents further indicate that core transport and core-strength evolution are coupled but not identical: The former mainly reflects persistent defect advection, whereas the latter is additionally affected by local wave-amplitude fluctuations and superposition near the singularity. The lifetime-rescaled core-vorticity evolution further reveals a reproducible birth-growth-decay scenario, consistent with saddle-mediated creation and annihilation. Qualitatively, similar persistent superdiffusive motion of multiscale vortex cores was also observed in electromagnetically driven two-dimensional turbulence [4], suggesting a broader connection to the Lagrangian vortex singularity dynamics in different evolving disordered multiscale fields.

Beyond pair reactions, we also observe saddle-mediated vortex merging and splitting events, providing a concrete topological pathway for higher-order rearrangements. They suggest that saddles can act as transient *topological catalysts* that enable defect-number changes while maintaining local topological constraints. The corotating-vortex selection rule is nontrivial, since direct merger of corotating cores would violate the orientation continuity of $\theta(\mathbf{r}, t)$ without a mediating -1 defect. The saddle population is recycled across multiple higher-order events: A saddle need not pair annihilate with its original vortex-core partner from generation.

VI. CONCLUSION

Employing MCEMD, which allows monitoring the spatiotemporal evolutions of the velocity and vorticity fields of decomposed modes, we have established a unified Eulerian-Lagrangian framework to unveil the multiscale dynamics

of spatiotemporally disordered thermal phonons in two-dimensional dusty plasma crystals, complemented by Yukawa crystal simulations. In the Eulerian frame, vortical phonons of different slow modes display similar behaviors of vorticity-wave coherence, exhibiting characteristic frequency-wave number correlations of vorticity fluctuations and intermittent coherent-excitation clusters of large vorticity-wave amplitude sites in the xyt space. In the Lagrangian framework, vortex and saddle cores obey robust kinetic rules including (1) persistent superdiffusivelike transport of vortex cores accompanied by concurrent evolution of core vorticity and surrounding vortex size, (2) vortex-saddle pair generation/annihilation and higher-order saddle-mediated merging and splitting of corotating vortices, with topological-charge conservation, (3) memoryless exponential defect lifetime statistics, and (4) liquid- and gaslike short-range spatial correlations of vortex pairs and saddle pairs, respectively. The above rules are self-similar over different slow scales. These findings establish vortex cores (and saddle points as more diffusive complementary partners) as Lagrangian coherent entities that carry the organization of disordered thermal phonons. Saddles, in particular, act as topological catalysts that mediate higher-order vortex rearrangements.

Without the confounding effects of intrinsic structural heterogeneity, our study utilizing an ordered lattice provides a general, experimentally accessible defect-centric paradigm to quantify coherence, intermittency, and effective interactions in stochastically driven linear waves, complementing traditional Fourier-space analyses. Beyond the 2D ordered crystal platform and conventional spectrum analyses, the defect-based Lagrangian framework developed here can be applied to any time-resolved velocity field in solids, glasses, or active matter. It opens a path to compare the kinetics of vibrational topological defects across systems with different structural disorder. Of particular interest is the extension to amorphous solids, where the spatial distribution of static eigenmode defects has been linked to plastic events [26–29]; whether *time-resolved* defect kinetics of the instantaneous velocity field also encode precursors of plastic rearrangements is a question inaccessible to static eigenmode analyses alone, and one that the present framework is well suited to address. More broadly, this framework provides a real-space dynamical handle for different stochastically driven elastic wave systems.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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