

Offer of a reward does not always promote trust in spatial games

Haidong Zhang¹, Chaoqian Wang^{2,*}, Shuo Liu³, Charo I. del Genio^{3,4,5}, Stefano Boccaletti^{3,6,7} and Xin Lu^{1,†}¹College of Systems Engineering, National University of Defense Technology, Changsha 410073, China²School of Mathematics and Statistics, Nanjing University of Science and Technology, Nanjing 210094, China³School of Mathematics, North University of China, Taiyuan 030051, China⁴Institute of Interdisciplinary Intelligent Science, Ningbo University of Technology, Ningbo 315211, China⁵Institute of Smart Agriculture for Safe and Functional Foods and Supplements, Trakia University, Stara Zagora 6000, Bulgaria⁶CNR, Institute for Complex Systems, Florence 50019, Italy⁷Research Institute of Interdisciplinary Intelligent Science, Ningbo University of Technology, Ningbo 315104, China

(Received 21 November 2025; accepted 20 February 2026; published 13 March 2026)

Trust is one of the cornerstones of human society. One of the evolutionary pressure mechanisms that may have led to its emergence is the presence of incentives for trustworthy behavior. However, this type of reward has received relatively little attention in the context of spatial trust games, which are often used to build models in evolutionary game theory. To fill this gap, we introduce an inter-role reward mechanism in the spatial trust game, so that an investing trustor can choose to pay an extra cost to reward a trustworthy trustee. With extensive numerical simulations, we find that this type of reward does not always promote trust. Rather, while moderate rewards break the dominance of mistrust, thereby favoring investment, excessive rewards eventually stimulate a nonreturn strategy, ultimately suppressing the evolution of trust. Additionally, lower reward costs do not necessarily promote trust. Instead, more costly, but not excessive, rewards enhance the advantage of the original investment, consolidating the clusters of rewarders and improving trust. Our model thus provides evidence about the counterintuitive nature of the relationship between trust and rewards in a complex society.

DOI: [10.1103/8992-hzsq](https://doi.org/10.1103/8992-hzsq)

I. INTRODUCTION

Evolutionary game theory builds on classical game theory by adding mechanisms like selection, mutation, and imitation, to model the emergence and stabilization of cooperation and other complex behavioral strategies within groups [1–3]. The explicit incorporation of spatial structures in the models used, with agents interacting on complex networks, allowed researchers to investigate effects that cannot be captured by a mean-field, all-to-all description [4–6]. A key insight is that network topology may promote the emergence of cooperation, governed by a simple rule: Cooperation is favored when the benefit-to-cost ratio exceeds the network's average degree [7]. Building on this rule, recent studies have established general conditions for cooperation across a wide range of topologies, from heterogeneous networks to systems with higher-order interactions [8–11]. Furthermore, advances in temporal network theory have shown that time-varying interactions and dynamically evolving networks can fundamentally alter evolutionary trajectories compared to static settings [12,13]. Despite these

developments, lattice networks remain a fundamental benchmark for sustaining cooperation and give rise to rich emergent phenomena such as fixation, coexistence, and cyclic dynamics [1,14–18]. These theoretical insights have transcended biology, economics, and social sciences, increasingly inspiring algorithmic innovations in multiagent optimization, swarm intelligence, and evolutionary computation [19–23].

A paradigmatic model used to study interpersonal reciprocity in mathematical and computational social sciences is the trust game [24]. In its classic form, this asymmetric game is played between a trustor and a trustee, with the trustor sending a portion of an endowment to the trustee, and the trustee subsequently deciding how much to return to the trustor. Under purely selfish preferences, this setup creates a social paradox: The trustee's payoff-maximizing response is to keep all the transfer, but in practice trust is common and can easily diffuse across interactions [25]. The study of evolutionary games on networks explains this apparent contradiction. In fact, while exploitation commonly arises in well-mixed populations, introducing a nontrivial spatial structure can instead promote cooperation [26–30]. To model this, existing studies have focused on mechanisms such as reinforcement learning [31,32], belief-based preferences [33], adaptive reputation systems [34,35], second-order reputation [36], and trust dynamics [37]. However, the classic formulation of the trust game has one more main limitation, namely that, because of its asymmetric nature, it requires players to imitate strategies solely within the same roles [28,38–41]. Recognizing this limitation, some scholars have considered role alternation, so

*Contact author: CqWang814921147@outlook.com

†Contact author: xin.lu.lab@outlook.com

that each player occupies each of the two roles half of the time [42–44]. However, this approach does not fully capture settings in which the roles are fixed, but agents in one role can evolve their strategy using information originating from the agents in the other role. To bridge the two roles, the idea of reward has been extensively explored, in theoretical models as well as in behavioral experiments, with the overall result that third-party rewards effectively promote trust and cooperation [45–48]. In fact, rewards function more effectively than punishments, and despite the associated costs and diminishing returns, because they entice players to remain in the game and to adopt cooperative strategies, particularly when multiple cooperative options are concurrently available [49–52], and even though they may generate a “free-rider aversion” effect that can occasionally result in the reduction of individual contributions [53]. However, although reward mechanisms have been extensively studied, a fundamental question remains, namely, how costly third-party rewards affect trust dynamics in structured populations with fixed roles.

In this article, we fill this knowledge gap by investigating how inter-role rewards influence trust evolution in a spatial two-role trust game on a lattice network. Specifically, we extend the classical trust game by introducing a trustor-side reward strategy, where trustors can decide to bear some cost to reward trustworthy trustees. Our findings reveal a nonmonotonic, counterintuitive behavior of the system, so that rewards do not always promote trust and may even suppress it under certain conditions. Moreover, the effect of the cost of the reward incurred by the trustor is also opposite to what one would expect, as higher costs tend to improve trust, whereas lower ones do not necessarily promote it.

II. MODEL

A. Trust game with reward

As described above, in the trust game, trustors decide at every time step whether to transfer a unitary endowment to trustees. Following the classic setup, if the trustor invests, the transferred amount is tripled upon reaching the trustee [24,54,55]. The trustee then chooses between keeping all three units of wealth or returning a fraction r thereof to the trustor. For the trustor, investing is rational only when the trustee returns a fraction r such that $3r > 1$. Otherwise, keeping the initial endowment is more profitable than investing and receiving less than the invested amount in the best case, or nothing at all in the worst case. From the point of view of the trustee, if the trustor does not invest, they receive no payoff, making the question of their strategy moot. If instead the trustor invests, the trustee receives 3 by keeping everything, but only $3(1 - r)$ by returning a fraction r . This shows that the trustor has no dominant strategy, while the trustee’s dominant strategy is to keep the entire amount.

To increase the likelihood of trustors receiving some return, a third strategy can be introduced, namely that of trustors rewarding trustees who choose to return some of the invested wealth. Specifically, the reward strategy requires trustors to incur an additional cost $\gamma < 1$ when investing. This cost is converted into a reward β that the trustees receive if they return the investment. Thus, this strategy functions as an

incentive that fosters reciprocal behavior from the trustees. We set that reward cost $\gamma \in [0, 1]$ and reward value $\beta \in [0.1, 100]$. The reason for setting cost γ and reward benefit β to different values and orders of magnitude stems from numerous real-world examples. For instance, a low-cost action may generate a disproportionately large benefit, as in the case where a simple word of encouragement yields a substantial payoff to the recipient, particularly under adverse conditions. Conversely, a high-cost action may produce little or even negative benefit, as when a costly gift fails to align with the recipient’s preferences and is perceived unfavorably [56].

Summarizing, we can categorize the trustors’ strategies into three types, namely, invest (I), not invest (N) and reward (R), and the trustees’ strategies into two types, namely, trustworthy (T) and untrustworthy (U), as illustrated in the game tree shown in Fig. 1(a). If the trustor chooses not to invest, the game step terminates, the trustor retains the initial capital of 1, and the trustee’s payoff is 0. If the trustor opts to invest or to reward, the trustee then decides between T and U. All the possible payoffs are conveniently represented in the matrix

$$\begin{array}{c|cc}
 & \text{T} & \text{U} \\
 \hline
 \text{I} & (3r, 3(1 - r)) & (0, 3) \\
 \text{N} & (1, 0) & (1, 0) \\
 \text{R} & (3r - \gamma, 3(1 - r) + \beta) & (-\gamma, 3)
 \end{array} \quad (1)$$

Specifically, each pair ([trustor], [trustee]) in the payoff matrix corresponds to the respective payoffs obtained by an individual from the trustor population adopting the row strategy (I, N, or R) and an individual from the trustee population adopting the column strategy (T or U) when they interact.

In the following, $\Gamma = (\mathcal{N}, (S_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$ models the game in normal form as a two-player noncooperative strategic game. Here, $\mathcal{N} = \{1, 2\}$ represents the trustor and the trustee, respectively. The trustor’s strategy set is $S_1 = \{I, N, R\}$, while the trustee’s strategy set is $S_2 = \{T, U\}$, resulting in an overall strategy space $S = S_1 \times S_2$. The payoffs are given by a valued function $u : S \rightarrow \mathbb{R}^2$, where $u(s_1, s_2) = (u_1(s_1, s_2), u_2(s_1, s_2))$, $s_1 \in S_1$, $s_2 \in S_2$.

With this setup, the previous globally dominant strategy of the trustee, namely that of not returning any fraction of the investment, disappears. In fact, when the trustor adopts the R strategy, the comparison of payoffs between returning and not returning depends on both r and β , so that no single strategy is favored for any choice of the game parameters. Similarly, for the trustor, not investing is no longer the preferred strategy, and, in addition, investing is at least not worse than paying for a reward. Notably, however, the introduction of a reward strategy only affects nontrivial structures, whereas it has no impact on the collapse of trust in a well-mixed population (see Appendix A for details).

B. Evolutionary dynamics on lattices

When the game interactions take place on a network, the separation of payoff calculation and strategy imitation can cause even a nondominant strategy to be evolutionarily stable.

In many practical domains, interactions between trustors and trustees can be represented as a two-dimensional (2D) grid structure with local connections, consisting of L rows and L columns of nodes with periodic boundary conditions.

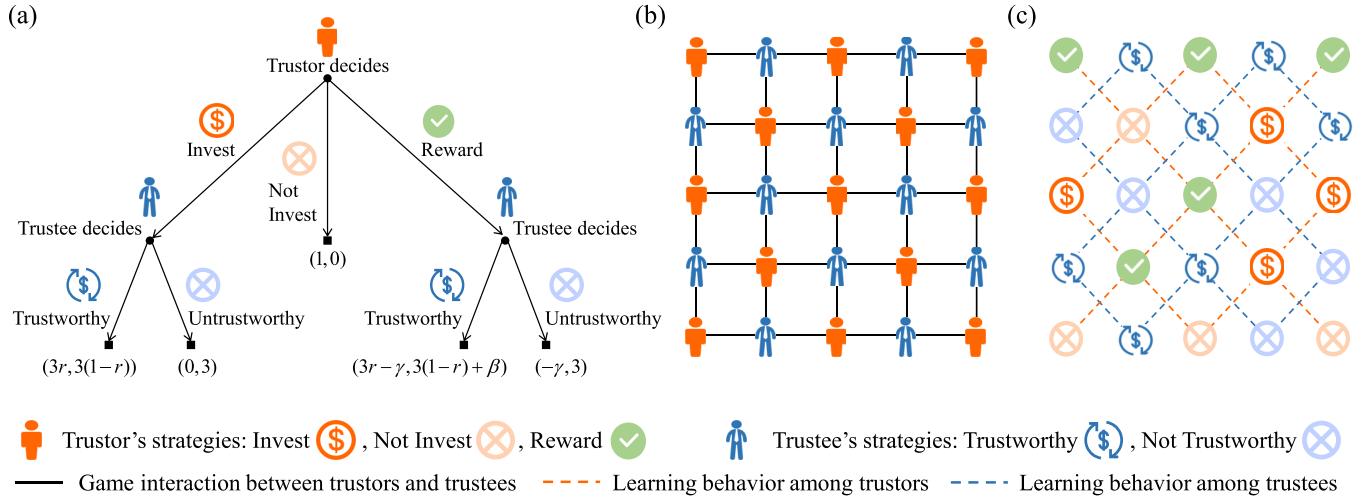


FIG. 1. Trust game model with rewards and spatial interaction structure. (a) The game tree for the trust game with a reward mechanism. The trustor first decides whether to invest (I), not invest (N), or reward (R). Then, the trustee decides whether to return (T) or not return (U), and a single round of the game ends after these two stages. (b) Individuals interact with their nearest neighbors, calculating the average payoff from interactions with multiple neighbors. (c) Individuals engage in strategy learning with their second nearest neighbors, ensuring that strategies are updated among individuals in the same role.

To distinguish between the two roles, we can categorize them according to the parity of the lattice coordinates. Thus, each node i has von Neumann neighbors (up, down, left, right), denoted as Ω_i^+ , that play a different role, and diagonal neighbors (top left, bottom left, top right, bottom right), denoted as $\Omega_i^\times$, that have the same role as i .

As a result, the game interactions of node i , which determine its payoff, occur with its Ω_i^+ neighbors. Conversely, the strategy of i is determined by imitating its $\Omega_i^\times$ neighbors. This is similar to the cross-learning framework of Ref. [55], with the difference that here the payoff interactions are inter-role, whereas the strategy learning is intrarole, as illustrated in Figs. 1(b) and 1(c).

Using the payoff matrix in Eq. (1), we define the actual payoff π_i of each node as the average payoff from all pairwise games with its Ω_i^+ neighbors. In formulas,

$$\pi_i = \frac{1}{|\Omega_i^+|} \sum_{k \in \Omega_i^+} u_{\text{role}(i)}(s_i, s_k), \quad (2)$$

where $\text{role}(i) = [(x_i + y_i) \bmod 2] + 1$ determines the role of node i and, consequently, its payoff function, from its lattice coordinates (x_i, y_i) , and s_i and s_k are the strategies of node i and node k , respectively. Note that, since we are modeling the game on a 2D lattice topology, the size of the interaction neighborhood is fixed, with $|\Omega_i^+| = 4$ for all nodes. The strategy evolution follows an asynchronous Fermi update rule. At each time step, a node f is selected uniformly at random from the network. A partner node m is then chosen from the learning neighborhood of f , $\Omega_f^\times$. Their average payoffs π_f and π_m are computed using Eq. (2), and subsequently node f adopts the strategy of node m with a probability given by the Fermi function,

$$P_{f \leftarrow m} = \frac{1}{1 + e^{(\pi_f - \pi_m)/\kappa}}, \quad (3)$$

where $\kappa = 0.1$ is a noise parameter.

To study the behavior of our model, we carried out each simulation experiment on a 300×300 grid. Each experiment involved 10 000 Monte Carlo sweeps. A single Monte Carlo sweep comprised L^2 time steps [with one asynchronous update between a node pair occurring at each time step using Eq. (3)]. In each simulation run, the first 5000 sweeps were used for system equilibration and subsequently discarded. The average ratio of trustor to trustee strategies was then computed over the subsequent 5000 sweeps.

III. RESULTS

According to our model definition, the system employs five strategies: I (trustor, invest), R (trustor, reward), N (trustor, not invest), T (trustworthy trustee), and U (untrustworthy trustee). Their respective proportions within each role are denoted as $\rho_I, \rho_R, \rho_N = 1 - \rho_I - \rho_R, \rho_T$, and $\rho_U = 1 - \rho_T$. To characterize the reciprocity within the population, we define the level of trust as the proportion of I and R, on the premise that the proportion of T is positive, i.e., $\rho_I + \rho_R$ and $\rho_T > 0$ [24]. Since the invest strategy and reward strategy are inherently altruistic, their very existence implies that some trustors will invest funds, and trustees have sufficient motivation to return a portion of those funds. Thus, trust has been generated.

Unless stated otherwise, the system is initialized on a square lattice with a bipartite topology [Fig. 1(b)]. While agent roles are spatially fixed, their initial strategies are assigned stochastically following a uniform distribution within each subpopulation (ensuring equal initial probability for all competing strategies). On a periodic lattice network of $L = 300$, a population size of 90 000 agents is sufficiently large to ensure global convergence. For each parameter set, we conduct a simulation spanning 10 000 Monte Carlo sweeps (MCSs). Due to the large population size, independent runs yield identical results, making a single run representative (see Appendix B for details).

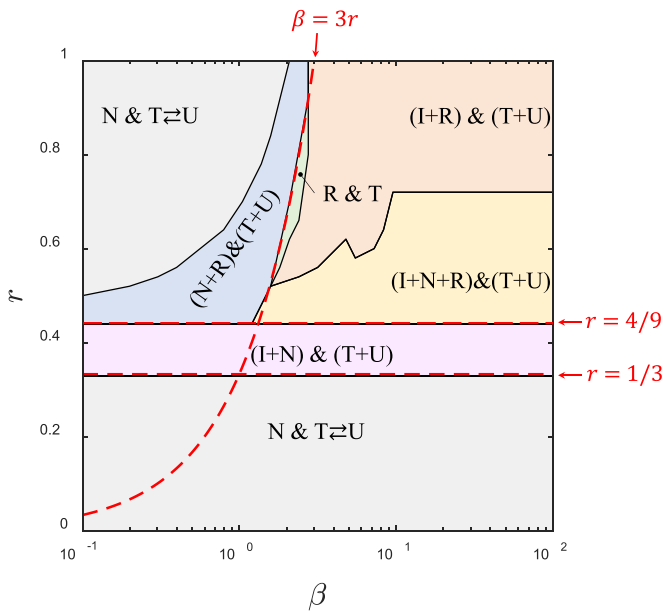


FIG. 2. Trust mainly emerges when returns promote investments and rewards offset the cost of reciprocation. The (β, r) phase diagram for fixed $\gamma = 0.1$ shows that the system admits trust as a stable strategy when the return ratio r is moderate, consistently with the conclusions of the classic two-strategy trust game. However, unlike what happens in the classic setting of the game, in which large return ratios suppress trust, our reward mechanism can support it for large enough rewards. The regions of different colors represent the stable strategy populations found for the corresponding parameters. Note that when the only strategy of the trustors is not to invest (N), the two strategies of the trustees are completely equivalent.

A. Inter-role rewards can promote trust

To explore the role of rewards on the emergence of stable strategies, we first build the phase diagram of the system in the (β, r) plane for fixed reward cost $\gamma = 0.1$.

The diagram, reported in Fig. 2, shows that, especially for rewards roughly smaller than 3, trust emerges as a stable strategy only when the return ratio r is moderate. This is consistent with the general conclusions of the classic two-strategy trust game, in which trust requires $r > 1/3$ to emerge, but excessively high values of r eventually suppress it [55]. The underlying mechanism can be described as follows: First, the return fraction must be at least $1/3$ for the trustor to have any incentive to invest. But then, the trustee has a net advantage of $3r$ (the payoff of strategy U minus the payoff of strategy T) when not reciprocating. Thus, as r increases, the U strategy is increasingly profitable, thereby undermining the sustainability of investment.

Conversely, for fixed values of r greater than $4/9$, the mechanism induces the reemergence of trust as the reward value β increases. In fact, in regions where both β and r are high, the noninvesting strategy N completely disappears from the stable population, driven by the incentives offered to the trustees. This can be understood by noting that, when the trustor adopts the reward strategy, the trustee ends up with a profit of $3(1-r) + \beta$ by reciprocating, but only 3 by defecting. Therefore, reciprocation becomes advantageous

when $3(1-r) + \beta > 3$, i.e., when $\beta > 3r$. Only then does the reward sufficiently offset the cost of reciprocation, motivating the trustee to adopt the T strategy. Thus, both for fixed β and for fixed r , the trust primarily emerges in the parameter regions satisfying $r > 1/3$ and $\beta > 3r$. To better characterize the evolution of trust for different reward values, we study how the fractions of strategies at steady state change as a function of the return ratio r at $\beta = 0.1$ and $\beta = 10$. The results, reported in Figs. 3(a) and 3(b), shed light on the individual regions of the phase diagram.

For $\beta = 1$, under the trust emergence condition $r > 1/3$, the inequality $\beta > 3r$ never holds. Thus, as r increases, the trustees lose any incentive to choose the T strategy. Eventually, the trustors completely abandon the I strategy and split between noninvesting and offering a reward. While this causes a discrete jump in the adoption of trust, the reward mechanism is ultimately too weak, and the fraction of trustors choosing the R strategy monotonically decreases with r , eventually leading to the complete suppression of investment.

In contrast, for $\beta = 10$, the inequality $\beta > 3r$ always holds for any value of r , so that, so long as trustors invest, the reward makes the payoff for trustees choosing T greater than that for trustees choosing U. Then, as r increases, the returns obtained by trustors improve, while trustees still maintain a relative advantage after receiving rewards. These factors synergistically drive an increase in the proportion of I and the adoption of R, strengthening trust overall. In turn, this causes a decrease of noninvestors (N), which completely disappears at $r \approx 0.74$, and the stabilization of a strong majority of trustworthy trustees.

It is worth noting that regardless of how β varies, $r = 4/9$ serves as the threshold for the emergence of the R strategy, as evident from Figs. 2, 3(a), and 3(b). In fact, $r = 4/9$ represents the threshold for the extinction of the I strategy. When $r > 4/9$, the I strategy becomes insensitive to U strategy invasions (failing to immediately convert to N). Comparing Figs. 4(a) and 4(c), when a trustor's T-strategy environment is invaded by just one U strategy, if the payoff $\pi_I = 9r$ from choosing the I strategy exceeds the payoff $\pi_N = 4$ from the N strategy (i.e., r is excessively large: $r > 4/9$), the trustor will ignore the U invasion and persist with the I strategy. At this point, the payoff difference between T and U strategies surrounding the trustor is $3r$, which will eventually drive all neighbors to switch to the U strategy. Once all T strategies in the vicinity are eliminated, the I strategy will also perish. When strategy I dies out, its suppression of strategy R ceases. As shown in Fig. 4(b), the payoff difference between T and U surrounding R is $3r - \beta$. This reduced payoff difference delays U's invasion of T, favoring R's survival. Thus, when $r > 4/9$, strategy R emerges.

To better elucidate the suppression effect of I on R, we removed the I strategy from the population of trustors. We set the initial distribution conditions to $\rho_R, \rho_T \approx 50\%$. This yielded the proportion changes of the R, N, and T strategies as a function of r , as shown in Figs. 3(c) and 3(d).

We find that the condition for the emergence of strategy R in panels (c) and (d) is $3r - \gamma > 1$ (i.e., $r > (1 + \gamma)/3 = 0.3\bar{6}$), whereas this condition does not hold in panels (a) and (b). This is because the critical value $0.3\bar{6}$ still falls within the emergence range of strategy I, $1/3 \leq r \leq 4/9$, where strategy

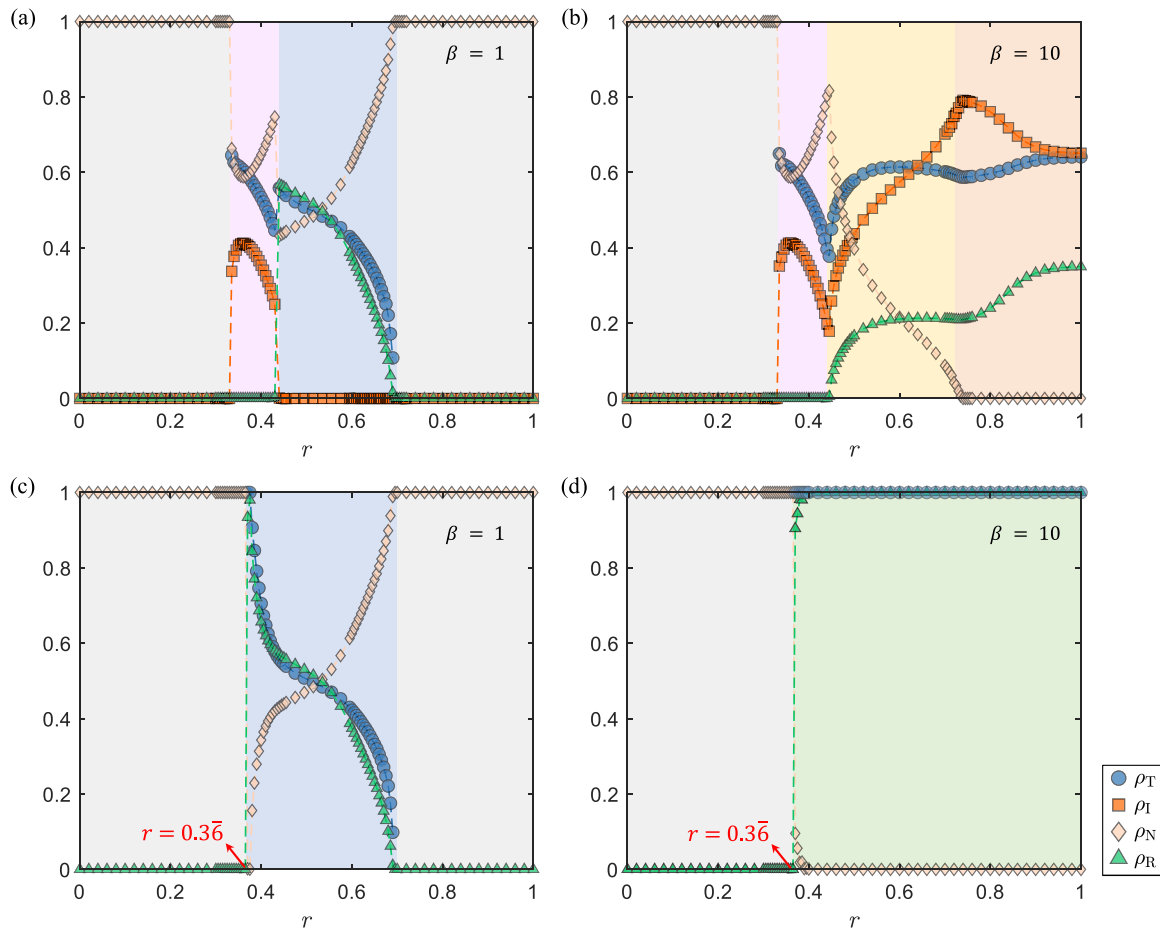


FIG. 3. A sufficiently high reward can promote trust even at high returns. (a) If $\beta = 1$, the reward value is insufficient, as the two inequalities $\beta > 3r$ and $r > 1/3$ cannot be satisfied simultaneously, stopping trustees from adopting the reciprocation strategy T, and thus not promoting the emergence of trust as r increases. (b) If $\beta = 10$, the reward is sufficiently strong, satisfying $\beta > 3r$ for all values of r . Thus, even in high- r regions, the mechanism drives trustees to reciprocate and trustors to choose investment or reward strategies, thereby promoting and stabilizing trust. (c) Except for removing strategy I, all other settings remain consistent with panel (a). Without strategy I suppressing strategy R, the reward strategy emerges at $r > 0.3\bar{6}$. Subsequently, due to $\beta < 3r$, the trust level decreases as r increases. (d) Except for excluding strategy I, all other settings remain consistent with panel (b). The reward strategy also emerges at $r > 0.3\bar{6}$. Due to the reciprocal condition $\beta > 3r$, the population subsequently remains in the R&T phase regardless of how r varies. In both panels, the reward cost for trustors is fixed at $\gamma = 0.1$, and the color code is the same as in Fig. 2.

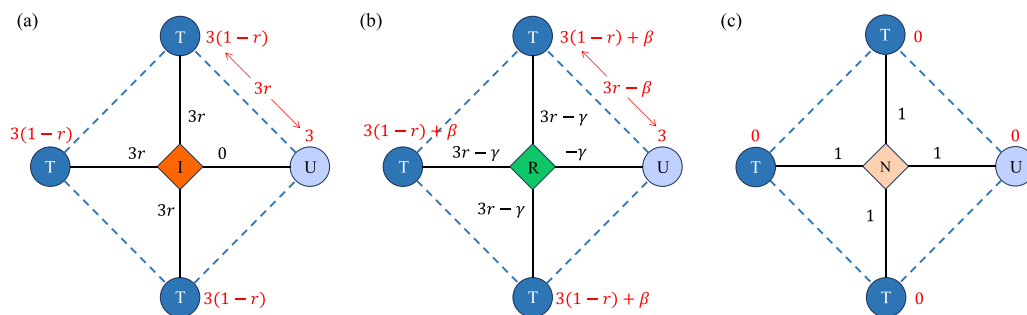


FIG. 4. The sensitivity of I to U's invasion is crucial for maintaining I's survival, while the reward benefit of R is key to determining U's transformation into T. (a) When there is one U strategy among the trustee neighbors surrounding I, I's payoff is $\pi_I = 9r/4$. The payoffs for a single T or U are $3(1-r)$ or 3, respectively (with a net payoff gap of $3r$). (b) When there is one U strategy among the trustee neighbors surrounding R, R's payoff is $\pi_R = (9r - 4\gamma)/4$. The payoffs for a single T or U are $3(1-r) + \beta$ or 3, respectively (the magnitude of β relative to $3f$ determines the conversion direction between T and U). (c) Regardless of the strategy distribution among the trustor's neighbors, the payoff for N is always 1, with a zero payoff difference between T and U among its neighbors.

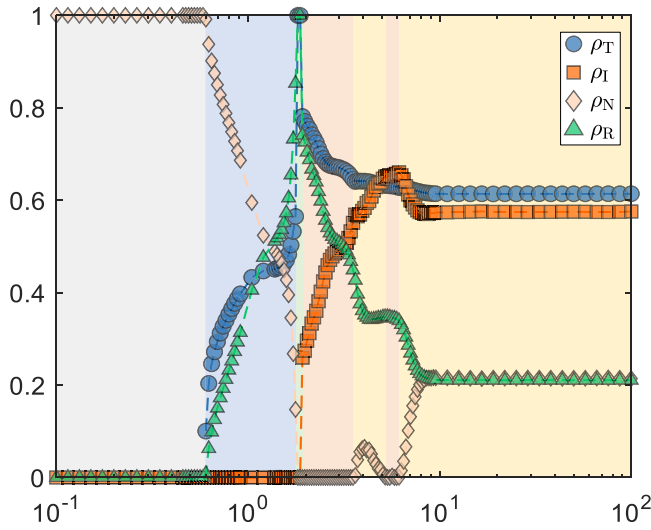


FIG. 5. Greater rewards do not promote trust if the return ratios are not sufficiently large. For $\gamma = 0.1$ and $r = 0.6$, the adoption of reciprocation by the trustees reaches its maximum for $\beta = 2$, decreasing afterward.

R is suppressed. The distinction between Figs. 3(a) and 3(c) lies in the interval $1/3 \leq r \leq 4/9$ where strategy I is present. Upon the extinction of strategy I, strategy R begins to exert its influence: It narrows the payoff gap between neighboring T and U, and even causes T's payoff to exceed U's ($\beta > 3r$), as illustrated in Fig. 4(b).

It can be observed that Figs. 3(a) and 3(b) exhibit a discrete jump and a relatively continuous change, respectively, at $r > 4/9$. This primarily depends on whether $\beta < 3r$ or $\beta > 3r$. When $\beta < 3r$ [Fig. 3(a)], the I strategy and R strategy play similar roles, both relying on the rapid transition of U invaders (I/R $\rightarrow$ N) to maintain trust. Therefore, the I strategy curve in Fig. 3(a) resembles the R strategy curve in Fig. 3(c). When I disappears, R emerges, exhibiting a discrete jump. However, when $\beta > 3r$ [Fig. 3(b)], the emergence of the R strategy causes surrounding U to tend toward T. Thus, when $r > 4/9$, the survival logic of the I strategy shifts from sensitivity to U to reliance on R. Once the I strategy ceases to suppress R, the emergence of R introduces more T individuals into the population, thereby stimulating the growth of the I strategy again. Since the environment consistently favors the generation of the T strategy, the proportion of R changes relatively slowly.

B. Inter-role rewards not always promote trust

To more clearly characterize the impact of the reward value β on the emergence of trust, we fix the return ratio at $r = 0.6$, which crosses the most phase boundaries in the (β, r) phase diagram, as seen in Fig. 2. Then, we vary β , measuring the steady-state fractions of each strategy. The results, reported in Fig. 5, show that the proportion of I and R strategy does not increase monotonically with the reward value. Rather, under the moderate return ratio chosen, trust is maximized by a moderate reward value, becoming the only chosen strategy at $\beta = 2$, when the noninvestors disappear and the trustors only adopt the reward strategy. Conversely, the fraction of trustors choosing to invest is highest at $\beta = 6$, where the noninvestors

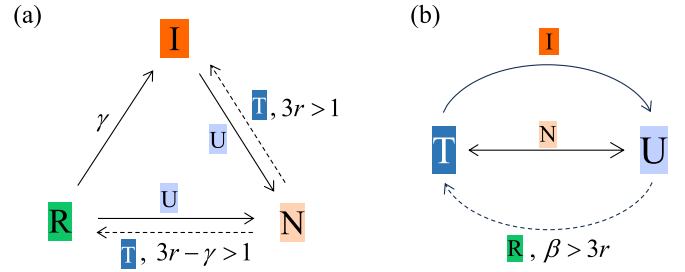


FIG. 6. Strategy transition relationships. (a) Transition pathways among the three trustor strategies I, N, and R as influenced by trustee behavior. (b) Transition pathways between the two trustee strategies T and U as influenced by trustor behavior. In both panels, arrows indicate shifts toward strategies yielding higher payoffs. Solid lines denote unconditional transitions, whereas dashed lines represent conditional transitions that require specific threshold criteria to be satisfied.

are still absent and the fraction of reciprocating trustees is approximately 0.63. However, when $\beta > 6$, as β increases, some trustors start to choose the noninvestment strategy, leading to a decline in the trust level.

An analysis of the core mechanisms of the rewarded trust model explains why excessively large rewards can counterintuitively hinder the emergence of trust. From the point of view of the trustors,

(1) The I strategy strictly dominates R across all parameter regimes, as players invariably have an incentive to select I over R, especially as the reward cost γ increases;

(2) The preference between I and N hinges on whether trustees reciprocate, so that trustors shift from N to I only when trustees select T and $3r > 1$; otherwise, if the trustees choose U or $3r \leq 1$, N dominates I;

(3) Trustors prefer R to N only when trustees choose T and $3r - \gamma > 1$; otherwise, if trustees adopt U or $3r - \gamma \leq 1$, N dominates R.

For what concerns the trustees, instead,

(1) When trustors opt for N, there is no difference between choosing T or U, as the payoff vanishes anyway;

(2) When trustors select I, trustees shift from T to U;

(3) When trustors adopt R, trustees transition from U to T only if $\beta > 3r$; otherwise, if $\beta \leq 3r$, U strictly dominates T.

These relations are visually summarized in Fig. 6, where dashed lines denote transitions that require meeting their respective thresholds. These considerations make the mechanism underlying the suppression of trust at large values of β evident: Increasing β increases the relative payoff of adopting a trustworthy strategy only when the trustor offers a reward, but at the same time it does not provide any direct benefit to the trustor. Concurrently, for the trustors, if the strategy distribution of surrounding trustees is identical, I is always preferable to R, so the cluster of nodes adopting R that may be temporarily induced by a higher β is ultimately replaced by I under selective pressure. Subsequently, neighboring trustees revert from T to U due to the absence of rewards, thereby undermining the reciprocity framework. The net result is that large values of β counterintuitively amplify a cascading sequence, causing trust to be lower than at moderate values.

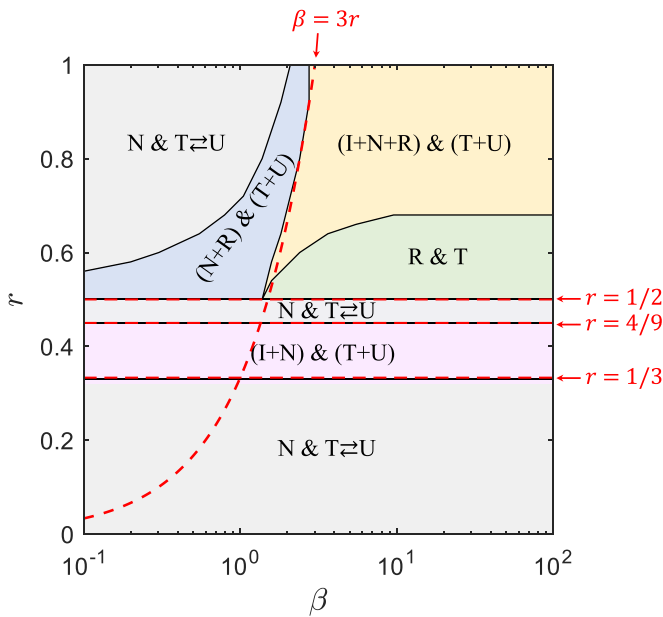


FIG. 7. The features of the emergence of trust are stable with respect to the reward cost. The (β, r) phase diagram for $\gamma = 0.5$ remains broadly the same as that for $\gamma = 0.1$, illustrated in Fig. 2. The main difference is that the four-strategy coexistence phase, shown in red in Fig. 2, is replaced by the yellow five-strategy coexistence phase, which, in turn, leaves the place to the complete-trust phase (green).

The results discussed above are qualitatively robust with respect to changes in the reward cost γ . To show this, we computed the phase diagram of the system in the (β, r) plane for $\gamma = 0.5$. The results, illustrated in Fig. 7, show that, while the stable adoption of the reward strategy now requires a higher return fraction, slightly greater than 0.5, the region where trust reaches the theoretical maximum (green) becomes much larger than at $\gamma = 0.1$. This indicates that the reward cost itself plays a complex role in determining the stable states of the system.

Furthermore, Fig. 7 indicates that the emergence conditions for strategies I and R are $1/3 < r < 4/9$ and $r > 0.5$, respectively. Notably, the threshold for the emergence of R ($3r - \gamma > 1$, yielding $r > 0.5$) falls strictly outside the survival interval of strategy I. Consequently, the emergence of R is decoupled from the competitive suppression of I. This leads to a distinct sequential transition as r increases: Strategy I first emerges and then vanishes; following an intermediate phase dominated by strategy N, strategy R emerges. This reappearance fosters the selection of T by trustees, ultimately promoting the evolution of trust.

C. Low costs do not always favor the offer of a reward

To evaluate the effect of γ in more detail, we examine the (β, γ) phase diagram of the system at $r = 0.5$, reported in Fig. 8. The clear division of the diagram into an upper part with no investment and a complex lower part shows that trust emerges only when $\gamma < 0.5$, confirming the accuracy of the criterion $\gamma < 3r - 1$, which we discussed in the previous subsection. This indicates that, to effectively facilitate trusting relationships, reward costs cannot be excessively high.

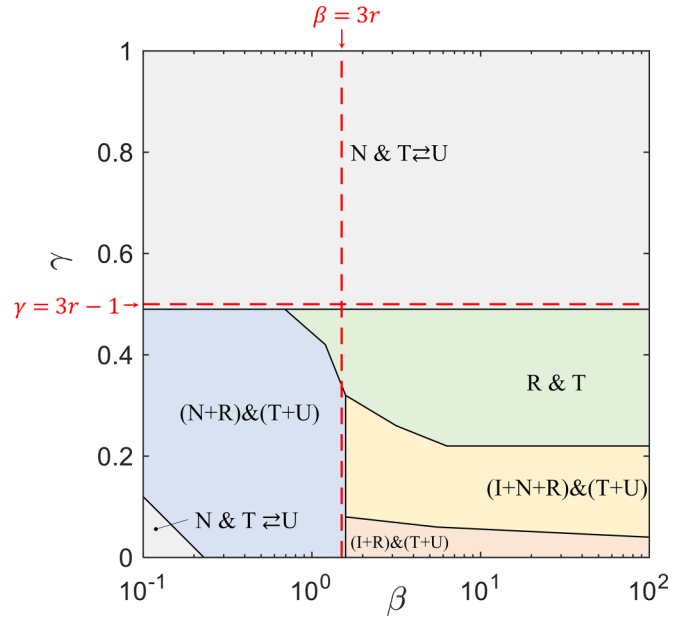


FIG. 8. Reducing the cost of rewards does not always foster trust. The (β, γ) phase diagram for fixed return ratio $r = 0.5$ shows that the phase where the adoption of a reward strategy and trust are both maximum does not occur for the lowest reward cost γ , but rather for intermediate values thereof, closer to the threshold $3r - 1$.

Similarly, a further division of the diagram can be observed, with trust emerging only when $\beta > 3r$, consistently with our previous analysis.

However, in the parameter region where both criteria are satisfied (bottom right in Fig. 8), a counterintuitive behavior can be observed. In principle, one would expect that a lower reward cost would promote the adoption of a rewarding strategy on the part of the trustors, but the opposite effect is found, namely that the R strategy is maximum in the area closest to the threshold $\gamma = 0.5$, and its adoption actually decreases if the cost becomes lower. To explain this observation, consider the mesoscopic evolutionary rules summarized in Fig. 6. Assuming that the reward is greater than $3r$, a very low value of γ will initially result in trustors that have adopted the N strategy switching in roughly equal fractions to R and I. In turn, the trustees that interact with investors offering no reward are subject to pressure to switch from T to U. But then, nonreciprocating trustees tend to go back to trustworthy behavior when offered a reward, because of its advantageous value. This results in the red region in the phase diagram, where the trustors always invest and sometimes offer a reward, while the trustees partially return the fixed fraction of the investment.

As the reward cost increases, however, the rate at which investing trustors stop offering rewards while keeping investing increases, while that of noninvesting trustors that switch to offering a reward decreases. The net effect of this is the appearance of N as a stable strategy, resulting in the total coexistence phase (yellow). Note that, in this phase, there is a dynamic equilibrium between N and I, as well as one between N and R, because the inequality $3r > 1$ is satisfied.

When γ increases even further, the faster rate at which trustors switch from R to I causes a transient adoption of the U

strategy by a large number of trustees, which, in turn, converts the trustors from I to N. However, the effective lack of I causes a progressive depletion of the U strategy among the trustees, which shifts the balance between N and R in favour of the latter. Ultimately, this nucleates a growing phase in which all the trustors offer a reward, and all the trustees return the investment, resulting in the phase where both the offer of a reward and trust are maximum.

IV. DISCUSSION

In summary, we have established a spatial trust game to model and analyze how the introduction of a reward mechanism promotes the emergence of trust in structured populations. Our findings show several counterintuitive phenomena. First, trust emerges only under moderate rewards. This observation can be explained by considering that the cyclical structure formed by the three strategies of the trustors coevolves with the strategies of the trustees. Thus, increases in the reward eventually cause a disbalance between the trustors, who do not receive any direct benefit from them, and the trustees, who only profit if they choose a trustworthy strategy and actually receive a reward. Ultimately, the overall trust becomes lower for higher rewards than it is for moderate ones.

Previous studies on trust games without reward mechanisms have shown that strong relations of trust are only found for moderate values of the return ratio, which has been recognized as the most critical factor influencing their emergence. In our model, we find that higher rewards can indeed promote trust under high return rates. However, this relationship is non-monotonic, and the maximum trust occurs for moderate levels of both reward and cost, whereas excessively high rewards cause an increase of the number of noninvestors and nonreciprocating trustees, as does an excessive reduction of the reward. Similarly, higher reward costs require correspondingly greater return rates or reward values to sustain the reward strategy. To explain how the coevolution mechanism of strategies causes the rich phase diagram observed, we carried out extensive numerical simulations of the model. These allowed us to describe how the system reaches each of its possible steady states, and to elucidate the interplay of the conversion rates between the N strategy and the I one, and the role played by the model parameters in balancing them. Given the latest theoretical advances in evolutionary game theory, future research may explore how the inherent asymmetry between trustors and trustees influences trust evolution, while also examining scenarios where network structures dynamically change over time. Incorporating spatial and temporal complexity information could enhance the robustness of the mechanisms identified in this study [9,13].

In conclusion, by employing a minimal trust model augmented with a reward mechanism, we have uncovered complex emergent patterns in strategic transitions. Our work offers a framework for understanding multistrategy evolutionary dynamics, especially in the presence of complex transitional relationships, highlighting the subtle role of spatial structure in social dilemmas and providing insights into why reward mechanisms do not always promote trust in real-world scenarios. Additionally, our setup has the potential to be applied also to other complex systems, such as chemical

reaction networks with catalytic processes involving multiple species or complex food webs in ecology, both of which exhibit dynamical changes in chemical or biological species composition.

ACKNOWLEDGMENTS

This work was supported by the National Science and Technology Major Project for Brain Science and Brain-like Intelligence Technology (Grant No. 2025ZD0215700), the National Natural Science Foundation of China (Grants No. 72421002, No. 72401287, No. 72401289), the Hunan Provincial Department of Science and Technology (Grant No. 2025JJ60447), and the Major Program of Xiangjiang Laboratory (24XJJCXYJ01001). C.I.d.G. acknowledges funding from the Bulgarian Ministry of Education and Science (Grant No. BG-RRP-2.004-0006-C02).

The authors declare that they have no conflict of interest.

DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: REPLICATOR EQUATIONS IN A WELL-MIXED POPULATION

Here, we analyze the behavior of our model in the thermodynamic limit and under the assumption of a well-mixed population. We denote the frequencies of trustor strategies as ρ_I , ρ_N , and ρ_R , and the frequencies of trustee strategies as ρ_T and ρ_U .

Based on the payoff matrix in Eq. (1), the expected payoffs for a trustor adopting each of the possible strategies are

$$\bar{\pi}_I = 3\rho_T r, \quad (A1)$$

$$\bar{\pi}_N = 1, \quad (A2)$$

$$\bar{\pi}_R = 3\rho_T r - \gamma. \quad (A3)$$

Similarly, the expected payoffs for a trustee adopting either of the available strategies are

$$\bar{\pi}_T = 3(\rho_I + \rho_R)(1 - r) + \rho_R \beta, \quad (A4)$$

$$\bar{\pi}_U = 3(\rho_I + \rho_R). \quad (A5)$$

Then, let $\bar{\pi}^{[1]} = \rho_I \bar{\pi}_I + \rho_N \bar{\pi}_N + \rho_R \bar{\pi}_R$ and $\bar{\pi}^{[2]} = \rho_T \bar{\pi}_T + \rho_U \bar{\pi}_U$ be the average payoffs of the trustor population and of the trustee population, respectively.

Using replicator dynamics [57], the evolution of ρ_I , ρ_R , and ρ_T can be expressed as

$$\begin{aligned} \dot{\rho}_I &= \rho_I(\bar{\pi}_I - \bar{\pi}^{[1]}) \\ &= \rho_I[3(1 - \rho_I - \rho_R)\rho_T r - 1 + \rho_I + \rho_R(1 + \gamma)], \end{aligned} \quad (A6)$$

$$\begin{aligned} \dot{\rho}_R &= \rho_R(\bar{\pi}_R - \bar{\pi}^{[1]}) \\ &= \rho_R[3(1 - \rho_I - \rho_R)\rho_T r - 1 + \rho_I + \rho_R(1 + \gamma) - \gamma], \end{aligned} \quad (A7)$$

$$\begin{aligned} \dot{\rho}_T &= \rho_T(\bar{\pi}_T - \bar{\pi}^{[2]}) \\ &= \rho_T(1 - \rho_T)[-3(\rho_I + \rho_R)r + \rho_R \beta]. \end{aligned} \quad (A8)$$

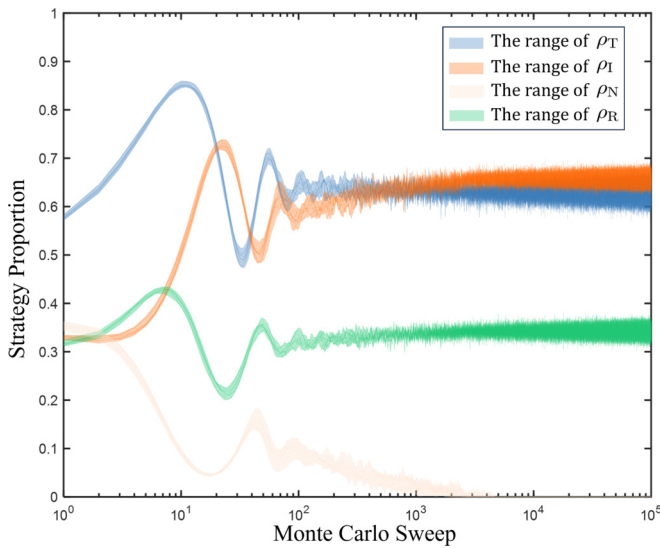


FIG. 9. The strategy proportion within the population as it evolves through Monte Carlo sweeps. Each curve in the figure represents the temporal variation in strategy proportions formed during a single Monte Carlo run, with shaded areas indicating the range of variation across ten runs.

From the equations above, it follows that the system has no interior equilibrium or isolated boundary equilibria. However, $\rho_N = 1$ forms a stable line $(0, 0, \rho_T^*)$. On this line, $\rho_T = 0$, and

the value of ρ_T is subject to neutral drift. The Jacobian matrix of this stable line is

$$J = \begin{pmatrix} 3\rho_T^*r - 1 & 0 & 0 \\ 0 & 3\rho_T^*r - 1 - \gamma & 0 \\ -3\rho_T^*(1 - \rho_T^*) & \rho_T^*(1 - \rho_T^*)(-3r + \beta) & 0 \end{pmatrix}, \quad (\text{A9})$$

whose eigenvalues are $\lambda_1 = 3\rho_T^*r - 1$, $\lambda_2 = 3\rho_T^*r - 1 - \gamma$, and $\lambda_3 = 0$. When $3\rho_T^*r < 1$, it is $\lambda_1 < 0$ and $\lambda_2 < 0$, and the system evolves toward this stable line, so that, for a large population, the mean-field solution is that all trustors adopt the noninvesting strategy, and trust collapses.

APPENDIX B: EVIDENCE OF CONVERGENCE ACROSS MULTIPLE RUNS

To verify the convergence of the simulation, we performed ten independent Monte Carlo runs with the parameter set $r = 0.6$, $\beta = 6$, $\gamma = 0.1$, which corresponds to a representative data point in Figs. 2 and 5. Figure 9 illustrates the temporal evolution of the fraction of each strategy within the population. To validate that the standard 10 000 MCSs are sufficient for the system to reach a steady state, we extended the simulation duration to 100 000 MCSs for this analysis.

The results from ten independent Monte Carlo runs exhibit strong convergence throughout the entire evolutionary process. This consistency demonstrates that for large populations, the simulation outcomes are robust and representative. Consequently, the results presented in the figures are derived from a single representative simulation run.

- [1] M. A. Nowak and R. M. May, Evolutionary games and spatial chaos, *Nature (London)* **359**, 826 (1992).
- [2] A. Traulsen and C. Hauert, Stochastic evolutionary game dynamics, *Rev. Nonlinear Dyn. Complex.* **2**, 25 (2009).
- [3] R. Cressman and Y. Tao, The replicator equation and other game dynamics, *Proc. Natl. Acad. Sci. USA* **111**, 10810 (2014).
- [4] C. Hauert, M. Holmes, and M. Doebeli, Spatial dynamics of ecological public goods, *Nature (London)* **428**, 643 (2004).
- [5] C. I. del Genio and T. Gross, Emergent bipartiteness in a society of knights and knaves, *New J. Phys.* **13**, 103038 (2011).
- [6] Z.-H. Rong, Q. Zhao, Z.-X. Wu, T. Zhou, and C. K. Tse, Proper aspiration level promotes generous behavior in the spatial prisoner's dilemma game, *Eur. Phys. J. B* **89**, 166 (2016).
- [7] H. Ohtsuki, C. Hauert, E. Lieberman, and M. A. Nowak, A simple rule for the evolution of cooperation on graphs and social networks, *Nature (London)* **441**, 502 (2006).
- [8] B. Allen, G. Lippner, Y.-T. Chen, B. Fotouhi, N. Momeni, S.-T. Yau, and M. A. Nowak, Evolutionary dynamics on any population structure, *Nature (London)* **544**, 227 (2017).
- [9] Q. Su, B. Allen, and J. B. Plotkin, Evolution of cooperation with asymmetric social interactions, *Proc. Natl. Acad. Sci. USA* **119**, e2113468118 (2022).
- [10] Q. Su, A. McAvoy, Y. Mori, and J. B. Plotkin, Evolution of prosocial behaviours in multilayer populations, *Nat. Hum. Behav.* **6**, 338 (2022).
- [11] A. Sheng, Q. Su, L. Wang, and J. B. Plotkin, Strategy evolution on higher-order networks, *Nat. Comput. Sci.* **4**, 274 (2024).
- [12] A. Li, L. Zhou, Q. Su, S. P. Cornelius, Y.-Y. Liu, L. Wang, and S. A. Levin, Evolution of cooperation on temporal networks, *Nat. Commun.* **11**, 2259 (2020).
- [13] Q. Su, A. McAvoy, and J. B. Plotkin, Strategy evolution on dynamic networks, *Nat. Comput. Sci.* **3**, 763 (2023).
- [14] G. Szabó and G. Fáth, Evolutionary games on graphs, *Phys. Rep.* **446**, 97 (2007).
- [15] M. Perc, J. J. Jordan, D. G. Rand, Z. Wang, S. Boccaletti, and A. Szolnoki, Statistical physics of human cooperation, *Phys. Rep.* **687**, 1 (2017).
- [16] A. Szolnoki, M. Mobilia, L.-L. Jiang, B. Szczesny, A. M. Rucklidge, and M. Perc, Cyclic dominance in evolutionary games a review, *J. R. Soc. Interface* **11**, 20140735 (2014).
- [17] M. A. Nowak, S. Bonhoeffer, and R. M. May, Spatial games and the maintenance of cooperation, *Proc. Natl. Acad. Sci. USA* **91**, 4877 (1994).
- [18] C. Wang, M. Perc, and A. Szolnoki, Evolutionary dynamics of any multiplayer game on regular graphs, *Nat. Commun.* **15**, 5349 (2024).
- [19] S. M. Lucas and G. Kendall, Evolutionary computation and games, *IEEE Comput. Intell. Mag.* **1**, 10 (2006).
- [20] H. Duan and C. Sun, Swarm intelligence inspired skills and the evolution of cooperation, *Sci. Rep.* **4**, 5210 (2014).
- [21] Y. Meng, S. P. Cornelius, Y.-Y. Liu, and A. Li, Dynamics of collective cooperation under personalised strategy updates, *Nat. Commun.* **15**, 3125 (2024).
- [22] Y. Meng, A. McAvoy, and A. Li, Promoting collective cooperation through temporal interactions, *Proc. Natl. Acad. Sci. USA* **122**, e2509575122 (2025).

- [23] J. Li and G. Kendall, Evolutionary stability of discriminating behaviors with the presence of kin cheaters, *IEEE Trans. Cybern.* **43**, 2044 (2013).
- [24] J. Berg, J. Dickhaut, and K. McCabe, Trust, reciprocity, and social history, *Games Econ. Behav.* **10**, 122 (1995).
- [25] M. A. Cohen and M. S. Isaac, Trust does beget trustworthiness and also begets trust in others, *Soc. Psychol. Q.* **84**, 189 (2020).
- [26] J. Iranzo, L. M. Floría, Y. Moreno, and A. Sánchez, Empathy emerges spontaneously in the ultimatum game: Small groups and networks, *PLoS One* **7**, e43781 (2012).
- [27] M. Chica, R. Chiong, M. Kirley, and H. Ishibuchi, A networked N -player trust game and its evolutionary dynamics, *IEEE Trans. Evol. Comput.* **22**, 866 (2018).
- [28] C. Wang, Evolution of trust in structured populations, *Appl. Math. Comput.* **471**, 128595 (2024).
- [29] Z. Liu, J. Wang, and X. Li, Evolutionary dynamics of networked N -player trust games with exclusion strategy, *Chaos Solit. Fractals* **186**, 115214 (2024).
- [30] J. Shihui and W. Zhiyi, An evolutionary game model with reputation threshold and reputation score to promote trust in the sharing economy, *Sci. Rep.* **15**, 14635 (2025).
- [31] T. Ren and X.-J. Zeng, Reputation-based interaction promotes cooperation with reinforcement learning, *IEEE Trans. Evol. Comput.* **28**, 1177 (2023).
- [32] J. Zhang, Z. Rong, G. Zheng, J. Zhang, and L. Chen, The emergence of cooperation via Q-learning in spatial donation game, *J. Phys. Complex.* **5**, 025006 (2024).
- [33] G. Attanasi, P. Battigalli, E. Manzoni, and R. Nagel, Disclosure of belief-dependent preferences in a trust game, *Econ. Theory* **1** (2025).
- [34] Z. Hu, X. Li, J. Wang, C. Xia, Z. Wang, and M. Perc, Adaptive reputation promotes trust in social networks, *IEEE Trans. Network Sci. Eng.* **8**, 3087 (2021).
- [35] C. Tang, A. Li, and X. Li, When reputation enforces evolutionary cooperation in unreliable manets, *IEEE Trans. Cybern.* **45**, 2190 (2014).
- [36] X. Li, M. Feng, W. Han, and C. Xia, N -player trust game with second-order reputation evaluation in the networked population, *IEEE Syst. J.* **17**, 2982 (2022).
- [37] T. A. Han, C. Perret, and S. T. Powers, When to (or not to) trust intelligent machines: Insights from an evolutionary game theory analysis of trust in repeated games, *Cogn. Syst. Res.* **68**, 111 (2021).
- [38] N. Masuda and M. Nakamura, Coevolution of trustful buyers and cooperative sellers in the trust game, *PLoS One* **7**, e44169 (2012).
- [39] N. Masuda, Evolution via imitation among like-minded individuals, *J. Theor. Biol.* **349**, 100 (2014).
- [40] I. S. Lim and N. Masuda, To trust or not to trust: Evolutionary dynamics of an asymmetric N -player trust game, *IEEE Trans. Evol. Comput.* **28**, 117 (2023).
- [41] A. Liu, L. Wang, G. Chen, and X. Guan, Heterogeneously networked evolutionary games with intergroup conflicts, *IEEE Trans. Cybern.* **54**, 5684 (2024).
- [42] M. A. Nowak, K. M. Page, and K. Sigmund, Fairness versus reason in the ultimatum game, *Science* **289**, 1773 (2000).
- [43] J. Hofbauer and K. Sigmund, Evolutionary game dynamics, *Bull. Am. Math. Soc.* **40**, 479 (2003).
- [44] K. Sigmund, *The Calculus of Selfishness* (Princeton University Press, Princeton, NJ, 2010).
- [45] F. Bolle, Rewarding trust: An experimental study, *Theory Decis.* **45**, 83 (1998).
- [46] M. Fiedler and E. Haruvy, The effect of third party intervention in the trust game, *J. Behav. Exp. Econ.* **67**, 65 (2017).
- [47] Y. Ohtsubo, S. Sasaki, D. Nakanishi, and J. Igawa, Within-individual associations among third-party intervention strategies: Third-party helpers, but not punishers, reward generosity, *Evol. Behav. Sci.* **12**, 113 (2018).
- [48] X. Fang and X. Chen, Evolutionary dynamics of trust in the N -player trust game with individual reward and punishment, *Eur. Phys. J. B* **94**, 176 (2021).
- [49] Z. Wang, A. Szolnoki, and M. Perc, Rewarding evolutionary fitness with links between populations promotes cooperation, *J. Theor. Biol.* **349**, 50 (2014).
- [50] M. H. Duong and T. A. Han, Cost efficiency of institutional incentives for promoting cooperation in finite populations, *Proc. R. Soc. A* **477**, 20210568 (2021).
- [51] T. A. Han, L. M. Pereira, F. C. Santos, and T. Lenaerts, Institutional incentives for the evolution of committed cooperation: Ensuring participation is as important as enhancing compliance, *J. R. Soc. Interface* **19**, 20220036 (2022).
- [52] J. Xiao, L. Liu, X. Chen, and A. Szolnoki, Evolution of cooperation driven by sampling reward, *J. Phys. Complex.* **4**, 045003 (2023).
- [53] A. Alventosa, A. Antonioni, and P. Hernández, Pool punishment in public goods games: How do sanctioners' incentives affect us? *J. Econ. Behav. Organ.* **185**, 513 (2021).
- [54] A. Kumar, V. Capraro, and M. Perc, The evolution of trust and trustworthiness, *J. R. Soc. Interface* **17**, 20200491 (2020).
- [55] C. Wang, W. Zhang, X. Wang, and A. Szolnoki, Inter-role reciprocity in evolutionary trust game on square lattices, *Chaos* **35**, 083117 (2025).
- [56] A. Szolnoki and M. Perc, Reward and cooperation in the spatial public goods game, *Europhys. Lett.* **92**, 38003 (2010).
- [57] P. D. Taylor and L. B. Jonker, Evolutionary stable strategies and game dynamics, *Math. Biosci.* **40**, 145 (1978).