

Nonlinear quantum evolution of a dissipative superconducting qubit

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Unitary and dissipative models of quantum dynamics are linear maps on the space of states or density matrices. This linearity encodes the superposition principle, a key feature of quantum theory. However, this principle can break down in effective non-Hermitian dynamics arising from postselected quantum evolution. We theoretically characterize and experimentally investigate this breakdown in a dissipative superconducting transmon circuit. Within the circuit's three-level manifold, no-jump postselection generates an effective non-Hermitian Hamiltonian governing the excited two-level subspace and an anti-Hermitian nonlinearity. We prepare different initial states and use quantum state tomography to track their evolution under this effective, nonlinear Hamiltonian. By comparing the evolution of a superposition state to a superposition of individually evolved basis states, we test linearity and observe clear violations, which we quantify across the exceptional-point degeneracy of the non-Hermitian Hamiltonian. We extend the analysis to density matrices, revealing a breakdown in linearity for the two-level subspace while demonstrating that linearity is preserved in the full three-level system. These results provide an operational demonstration of postselection-induced nonlinearity in a controlled superconducting platform. The observed nonlinearity, organized by the exceptional-point degeneracy of the non-Hermitian Hamiltonian, is absent in classical non-Hermitian systems, where norm change reflects real exchange with an environment rather than renormalization.

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I. INTRODUCTION

Linearity is a foundational property of quantum dynamics. Given two states $|a\rangle$, $|b\rangle$ and a Hermitian Hamiltonian H , the time evolution is given by $|a\rangle \rightarrow |a(t)\rangle \equiv G(t)|a\rangle$, $|b\rangle \rightarrow |b(t)\rangle \equiv G(t)|b\rangle$, where $G(t) \equiv \exp(-iHt)$ is determined by the Schrödinger equation. Linearity implies that a state $|c\rangle = \alpha|a\rangle + \beta|b\rangle$, unitarily evolved to $|c(t)\rangle$, is equal to the linear superposition of time-evolved $|a(t)\rangle$, $|b(t)\rangle$ states, i.e., $|c\rangle \rightarrow |c(t)\rangle = \alpha|a(t)\rangle + \beta|b(t)\rangle$ [1]. The same holds for convex combinations of density matrices that evolve unitarily via the von Neumann equation [2]. This assumption of linearity extends to open, dissipative quantum systems. When a quantum system interacts with its environment, the resultant dynamics of its reduced density matrix $\rho(t)$ is described by the Lindblad equation $\partial_t \rho(t) = \mathcal{L}\rho(t)$ [2] that is linear in the density matrix or, more generally, linear completely positive trace-preserving maps [3]. Thus, linear dynamics underpin

signatures of quantumness, such as spatiotemporal entanglement, and their degradation.

In contrast, exceptional classical phenomena such as rogue waves [4] result from a *nonlinear* Schrödinger equation that applies to shallow fluids and optics [5–7]. Here, nonlinearity encodes changes in the potential due to local wave intensity. In quantum systems, such state-dependent Hermitian nonlinearity leads to remarkable predictions [8], including arbitrarily fast signaling [9,10], closed timelike curves [11], and efficient solutions to classical, NP-complete problems [12]. Experiments have constrained the dimensionless strength of such nonlinearity to less than parts per trillion [13,14], bolstering the belief that Hermitian nonlinearities are absent in nature.

In recent years, classical and quantum systems governed by an effective non-Hermitian Hamiltonian $H_{\text{eff}} = H - i\Gamma$ have proliferated [15]. In linear models, their dynamics do not preserve the state norm since the corresponding time-evolution operator $G_{\text{eff}}(t)$ is not unitary. In linear classical non-Hermitian systems (e.g., photonic or mechanical gain-loss platforms), norm violation represents exchange with the environment, but the underlying equations of motion remain linear. In quantum cases, H_{eff} arises from postselection over no-quantum-jump trajectories [16,17], state of the ancilla [18], or no-photon-loss detection [19,20]. Here, the system *does not* undergo exchange with the environment. Thus, the postselected state $|\psi(t)\rangle$ is always normalized and satisfies a

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nonlinear Schrödinger equation [21]:

$$i\partial_t|\psi(t)\rangle = (H_{\text{eff}} + i\langle\psi|\Gamma|\psi\rangle)|\psi(t)\rangle. \quad (1)$$

Thus, in the quantum context, an anti-Hermitian nonlinearity $\langle\psi|H_{\text{eff}}^\dagger - H_{\text{eff}}|\psi\rangle/2$ always accompanies a non-Hermitian Hamiltonian H_{eff} . Similar nonlinearity considerations apply to density matrices in the postselected subspace. However, evolution under H_{eff} keeps pure states pure while changing the purity of mixed states, and we focus on the former. Importantly, non-Hermiticity by itself does not imply nonlinear dynamics: Evolution under a fixed non-Hermitian Hamiltonian is a linear, non-trace-preserving map. Nonlinearity arises only when the state is renormalized under postselection. These considerations motivate a systematic investigation of signatures of nonlinearity in such quantum systems.

Nonunitary quantum evolution conditioned on null measurement outcomes has previously been studied in superconducting phase qubits using the language of Bayesian conditioning and partial-collapse measurement [22–24]. In these works, the absence of a tunneling event continuously updated the conditional state estimate, producing nonlinear normalized evolution. In particular, Ref. [22] reported that a superposition state can evolve nontrivially under such conditioning even when its basis states do not. The same conditioned dynamics can equivalently be described using an effective non-Hermitian Hamiltonian governing the no-jump subspace [16,17]. In recent years, this framework has enabled experimental studies of parity-time ($\mathcal{PT}$) symmetry breaking, exceptional points, and topological non-Hermitian dynamics in superconducting circuits, trapped ions, and photonic systems [16,18–20,25–27]. These studies focused on spectral structure and mode evolution under an effective non-Hermitian generator. However, the implicit assumption that postselected evolution remains linear at the level of state vectors has not been directly tested.

Here, we investigate the breakdown of linearity in a superconducting transmon under an effective non-Hermitian Hamiltonian by comparing the evolution of prepared superposition states with the corresponding superposition of independently evolved basis-state trajectories under identical conditioning. Postselection on no-jump trajectories isolates the effective dynamics within the excited two-level subspace, where state-dependent renormalization induces nonlinear evolution. Using quantum state tomography [28–30], we quantify the resulting deviation from the superposition principle and thereby provide an operational demonstration of postselection-induced nonlinearity in a controlled superconducting platform.

II. SETUP

To introduce how we realize effective non-Hermitian evolution with superconducting circuits, consider the three lowest levels of a transmon circuit [31], where we label the three lowest-energy eigenstates as $\{|g\rangle, |e\rangle, |f\rangle\}$. By engineering the transmon's dissipative coupling to its electromagnetic environment, we realize a hierarchy of decay rates: The decay from $|f\rangle$ to $|e\rangle$, described by the dissipator $L_f = \sqrt{\Gamma_f}|e\rangle\langle f|$, is relatively slow ($\Gamma_f = 0.057\mu\text{s}^{-1}$), while the decay from

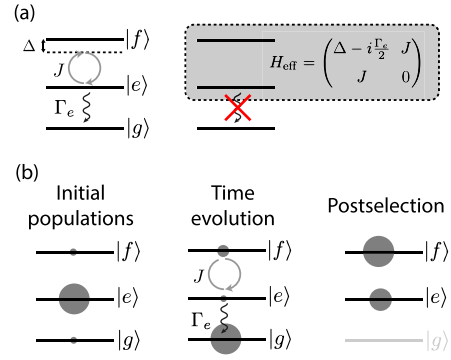


FIG. 1. Effective non-Hermitian evolution via postselection on no quantum jumps. (a) Three-level transmon manifold with states $|g\rangle$, $|e\rangle$, $|f\rangle$. The $|e\rangle$ state decays to $|g\rangle$ at rate Γ_e , while coherent coupling J hybridizes $|e\rangle$ and $|f\rangle$ with detuning Δ . (b) Illustration of non-Hermitian dynamics. Left: initial population distribution. Middle: coherent coupling and decay during time evolution. Right: postselection yielding the effective dynamics within the $|e\rangle, |f\rangle$ subspace.

$|e\rangle$ to $|g\rangle$, given by $L_e = \sqrt{\Gamma_e}|g\rangle\langle e|$, is comparatively fast ($\Gamma_e = 0.91\mu\text{s}^{-1}$). The evolution of the qutrit density matrix $\rho^{(3)}$ is governed by the Lindblad equation [3,32]:

$$\partial_t \rho^{(3)} = -i[H, \rho^{(3)}] + \sum_{j \in \{e,f\}} \left(L_j \rho^{(3)} L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho^{(3)}\} \right). \quad (2)$$

To access non-Hermitian dynamics, we consider the regime where the $|f\rangle$ decay is negligible, $\Gamma_f \ll \Gamma_e$, and quantum jumps to the $|g\rangle$ state do not occur [Fig. 1(a)]. The driving Hamiltonian H is given by a drive with amplitude J that is detuned by Δ from the $\{|e\rangle, |f\rangle\}$ transition. Under these approximations, the evolution of the system is described by an effective non-Hermitian Hamiltonian. Expressed in the rotating frame of the $\{|e\rangle, |f\rangle\}$ basis, the Hamiltonian is given by

$$H_{\text{eff}} = \begin{pmatrix} \Delta - i\frac{\Gamma_e}{2} & J \\ J & 0 \end{pmatrix}. \quad (3)$$

This effective Hamiltonian captures the evolution of the system conditioned on no quantum jumps and serves as the generator of dynamics for the postselected, two-level subspace as seen in Fig. 1(b). H_{eff} is closely related to the widely studied $\mathcal{PT}$ -dimer Hamiltonian [33,34], which features balanced gain and loss between the two states:

$$H_{\text{eff}} = \frac{-i\Gamma_e}{4}\mathbb{1} + \begin{pmatrix} \Delta - i\frac{\Gamma_e}{4} & J \\ J & +i\frac{\Gamma_e}{4} \end{pmatrix} = \frac{-i\Gamma_e}{4}\mathbb{1} + H_{\mathcal{PT}}. \quad (4)$$

Of particular interest is the exceptional-point (EP) degeneracy, occurring at $\Delta = 0$ and $J = \Gamma_e/4$, where the eigenvectors coalesce.

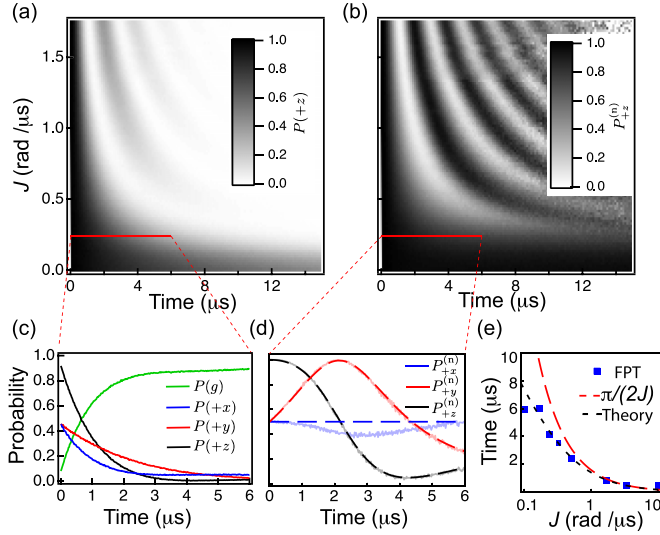


FIG. 2. Characterization of the system dynamics before and after postselection. (a) $P(+z)$ versus J and time. (b) The normalized $P_{+z}^{(n)}$ after postselection. (c) Measurement probabilities from tomographic projections along different axes in the $\{|e\rangle, |f\rangle\}$ manifold along with the ground-state population. (d) Postselected data from panel (c). The opaque curves are experimental data, and the dotted curves are simulation. (e) First passage time (FPT) vs J (blue squares) compared to the expected FPT for Hermitian evolution ($\pi/(2J)$, red dashed line) and the theoretical non-Hermitian FPT (black dashed line).

III. EXPERIMENTAL IMPLEMENTATION AND CHARACTERIZATION

Our experimental platform utilizes two superconducting transmons Q_1 and Q_2 coupled via a tunable coupler C_1 , with the dissipative qutrit realized in the $\{|g\rangle, |e\rangle, |f\rangle\}$ manifold of Q_1 . High-fidelity SWAP-assisted readout combined with rotations in the $\{|e\rangle, |f\rangle\}$ subspace enables quantum state tomography (see Appendix for details). We define Pauli operators for this subspace: $\sigma_z \equiv |e\rangle\langle e| - |f\rangle\langle f|$, $\sigma_x \equiv |e\rangle\langle f| + |f\rangle\langle e|$, and $\sigma_y \equiv -i|e\rangle\langle f| + i|f\rangle\langle e|$. After correcting for readout infidelities, we obtain probabilities for the relevant eigenstates of the qutrit, $P(g)$, $P(+i)$ for $i \in \{x, y, z\}$. To isolate the effective non-Hermitian dynamics, we postselect on trajectories with no quantum jumps to $|g\rangle$ and renormalize: $P^{(n)}(\pm i) = P(\pm i)/[P(+i) + P(-i)]$. Because postselection relies only on identifying trajectories ending in $|g\rangle$, and because the measured assignment fidelity for $|g\rangle$ is close to unity (see Appendix), modest variations within calibration uncertainty produce negligible shifts in observed dynamics.

Figure 2 displays the characterization of the system dynamics with quantum state tomography. We prepare Q_1 in the initial state $|e\rangle$ and apply microwave driving for varying durations followed by tomography. We repeat such experiments for different values of J with $\Delta \simeq 0$ fixed. Figure 2(a) displays the corrected probability $P(+z)$ versus time and J . Postselection on the no-jump evolution yields the renormalized subensemble probabilities; $P_{+z}^{(n)}$ is shown in Fig. 2(b). The plot of $P_{+z}^{(n)}$ versus time and J reveals two clear regions with different dynamics: For large J , the dynamics are oscillatory; for smaller J , the oscillations abruptly stop. This transition cor-

responds to the $\mathcal{PT}$ -symmetry breaking transition, with $J > \Gamma_e/4$ corresponding to the $\mathcal{PT}$ -symmetry unbroken region and $J < \Gamma_e/4$ corresponding to the $\mathcal{PT}$ -symmetry broken regime. $J = \Gamma_e/4$ marks the EP degeneracy. Figures 2(c) and 2(d) display the tomography components versus time for a selected region [red line in panels (a) and (b)] near the EP. In Fig. 2(d), the dotted lines show the theoretical prediction and the opaque curves show the experimental data. We note that for $P_{+x}^{(n)}$ there is slight deviation between the data and the theory. We attribute this to a small nonzero detuning $\Delta \neq 0$.

Figure 2(d) highlights how dynamics under an effective non-Hermitian Hamiltonian can accelerate quantum dynamics near the EP. Here, the drive strength is $J = 0.24$ rad/ μ s. In the Hermitian limit, the first passage time (FPT) (or π -pulse time) would be $T_\pi = \pi/2J \simeq 6.5$ μ s. Instead, the dynamics show oscillation from $|e\rangle$ to $|f\rangle$ in $T_\pi \approx 4$ μ s. This accelerated dynamics occurs because the dissipation and drive conspire to drive the system rapidly from $|e\rangle$ to $|f\rangle$ [35]. This accelerated dynamics can also be understood in a Bayesian sense [24]: The absence of a quantum jump updates the conditional state toward $|f\rangle$, because trajectories occupying $|e\rangle$ are more likely to decay to $|g\rangle$ and fail the postselection. When this conditional update reinforces the coherent drive, the evolution is accelerated. Figure 2(e) further explores the first passage time for different drive strengths; we compare it to the Hermitian FPT (dashed red), and a simulation of the FPT from the nonlinear Schrödinger equation in Eq. (1) (dashed black). We see clear agreement with the dynamics from the nonlinear Schrödinger equation, and clear deviation from the expected Hermitian FPT.

IV. TESTING LINEARITY OF NON-HERMITIAN QUANTUM EVOLUTION

We now turn to testing whether the non-Hermitian evolution realized through postselection on no quantum jumps preserves linearity of quantum evolution. As sketched in the introduction, our test will be based on comparing the time evolution of a superposition of states to the time evolution of a superposition of the individual states. If the postselected evolution were linear at the level of state vectors, then the time-evolved state obtained from preparing a superposition (e.g., $|+x\rangle$) would equal the corresponding superposition of the independently evolved basis states (e.g., $|e\rangle$ and $|f\rangle$), provided all trajectories are subjected to the same postselection criterion. The comparison of these two procedures forms the basis of our linearity test. This notion of linearity applies to state vectors, but not to arbitrary density matrices. In particular, a pure superposition, such as $|+x\rangle$ expressed as a density operator $|+x\rangle\langle +x|$, cannot be written as a linear combination of the projectors $|e\rangle\langle e|$ and $|f\rangle\langle f|$. Because the superposition principle is defined at the level of kets, a meaningful test of its breakdown under postselected evolution must therefore be performed at the state-vector level. For this reason, we construct pure-state representatives from the reconstructed density matrices via projection onto the dominant eigenvector. We will revisit a special case of classical mixtures in a later section, but for now our first step is to construct pure-state kets from the measured density operators.

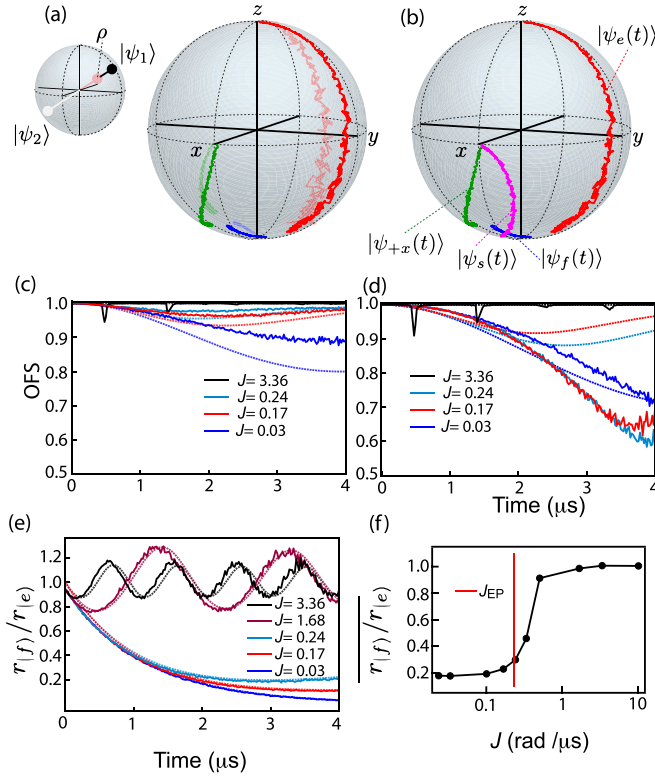


FIG. 3. Testing linearity of non-Hermitian quantum evolution. (a) The pure-state projection of the tomographically reconstructed density matrix ρ is given by its eigenvector with largest eigenvalue $|\psi_1\rangle$ (inset). The main panel displays three trajectories within the Bloch ball (transparent red, blue, green) reconstructed over time $t \in (0, 6)\mu\text{s}$ from different initial states (respectively, $|e\rangle$, $|f\rangle$, $|+x\rangle$). The pure-state projections are shown in solid colors. (b) We compare the trajectory originating in $|+x\rangle$, denoted by $|\psi_{+x}(t)\rangle$, to the trajectory reconstructed from a superposition of $|\psi_e(t)\rangle$ and $|\psi_f(t)\rangle$ trajectories, denoted by $|\psi_s(t)\rangle$ and displayed in magenta. (c) Fubini-Study-metric-inspired complex overlap (OFS) vs time for different values of J and initial state $|+x\rangle$ with theory curves displayed with dotted lines. (d) OFS vs time for initial state $|+y\rangle$ with theory curves displayed with dotted lines. (e) The postselection ratios as a function of time showing oscillations for values above the exceptional point and decay for those below with theory curves displayed with dotted lines. (f) The time-averaged value of the postselection ratio.

Figure 3(a) displays the procedure; given a state ρ within the Bloch ball, take the eigenvectors of ρ : $\{|\psi_1\rangle, |\psi_2\rangle\}$ and choose the eigenvector with greatest eigenvalue as the pure-state projection. We note that this dominant-eigenvector projection is itself a nonlinear map on the reconstructed density matrix. We therefore restrict the analysis to time windows where the measured states remain close to pure (see Fig. 7 in the Appendix), ensuring that the leading eigenvalue is well separated and the projected state is robust to small reconstruction errors. In Fig. 3(a), we display the measured trajectories within the Bloch ball and compare to the pure-state projections to the Bloch sphere. We consider trajectories originating from different initial states $|e\rangle$ (red), $|f\rangle$ (blue), and $|+x\rangle$ (green). The different dynamics for these three states again highlight the nonunitary evolution generated by H_{eff} ; the trajectory originating from $|e\rangle$ evolves along the y - z longitude

from $|e\rangle$ to $|f\rangle$, while the trajectory originating in $|f\rangle$ barely deviates from its initial state.

To assess whether the measured dynamics preserve linearity, we compare the trajectory originating in $|+x\rangle$, denoted by $|\psi_{+x}(t)\rangle$, to a trajectory constructed from the superposition $|\psi_s(t)\rangle \equiv A_s(|\psi_e(t)\rangle + |\psi_f(t)\rangle)/\sqrt{2}$. Here, A_s is a normalization factor to account for nonorthogonality of $|\psi_e(t)\rangle$ and $|\psi_f(t)\rangle$. The trajectories are compared in Fig. 3(b); the trajectory for $|\psi_s(t)\rangle$ is displayed in magenta. Clearly, $|\psi_{+x}(t)\rangle \neq |\psi_s(t)\rangle$. We choose to quantify the deviation from linearity in terms of a Fubini-Study-metric-inspired complex overlap (OFS) as a function of time, given by

$$\text{OFS}(t) = \frac{|\langle \Phi(t) | \psi(t) \rangle|}{\sqrt{\langle \Phi(t) | \Phi(t) \rangle}}, \quad (5)$$

where $\Phi(t)$ is the trajectory from the constructed superposition and $\psi(t)$ is the pure-state projection of the trajectory initialized in the $|+x\rangle$ state. We expect when linearity is preserved, this value will be 1; reductions from 1 constitute a measure of the system evolution's nonlinearity. Figures 3(c) and 3(d) display the time evolution of the OFS for the initial states $|+x\rangle$ and $|+y\rangle$, respectively, for different values of J . In both cases, large values of J yield an $\text{OFS} \approx 1$ (the small deviations from 1 arise from imperfections in the state purification process). For $J < J_{EP}$, both cases show $\text{OFS} < 1$; the deviation from linearity is more pronounced in Fig. 3(d), where the initial state $|+y\rangle$ is orthogonal to the lone eigenstate at the exceptional point: $|-y\rangle$. We note that the two theory curves in Fig. 3(d) for $J = 0.24, 0.17$ differ from the experiment more than the rest. We attribute this to a known nonzero detuning in the Hamiltonian [as explained in Fig. 2(d)], in addition to these trajectories being at an unstable point due to the orthogonality to the lone eigenstate at the EP.

The breakdown of linearity in our system arises from the requirement that quantum states remain normalized under non-Hermitian evolution. A non-Hermitian Hamiltonian by itself produces nonunitary dynamics, but this evolution remains linear. Nonlinearity emerges because the non-norm-preserving evolution generated by H_{eff} causes different initial states to accumulate population in $|g\rangle$ at different rates. This leads to trajectory-dependent postselection success probabilities and corresponding trajectory-dependent renormalization factors that break the superposition principle. We define the renormalization factor $r_{|i\rangle}(t) = \frac{1}{1-P(g)}$ based on the postselection success probability $(1 - P(g))$ for initialization in state $|i\rangle$. Figure 3(e) displays the ratio $r_{|f\rangle}/r_{|e\rangle}$ versus time for different values of J . In the $\mathcal{PT}$ -unbroken regime ($J > J_{EP}$), this ratio oscillates around unity, indicating that the two trajectories spend comparable time in the lossy $|e\rangle$ state and receive similar renormalization. In the $\mathcal{PT}$ -broken regime ($J < J_{EP}$), the ratio decays, reflecting asymmetric population dynamics. The time-averaged ratio [Fig. 3(f)] approaches unity as J increases well beyond J_{EP} , where the trajectories become increasingly symmetric. This demonstrates that the exceptional point roughly demarcates the boundary between predominantly nonlinear and increasingly linear behavior over long timescales.

Having studied the breakdown of linear quantum evolution for superpositions of kets, we now turn to a special case:

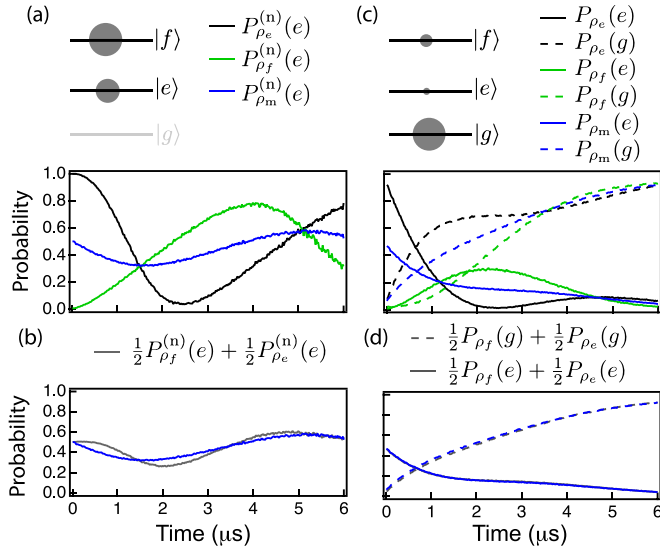


FIG. 4. Linearity of classical mixtures. (a) We show the postselected probability data vs time for different initial states, ρ_e , ρ_f , and ρ_m , evolved with $J = 0.5 \text{ rad}/\mu\text{s}$. (b) We compare $P^{(n)}(e)$ to the classical superposition $\frac{1}{2}P^{(n)}_{\rho_f}(e) + \frac{1}{2}P^{(n)}_{\rho_e}(e)$. The curves are in disagreement arising from the breakdown of linearity for classical mixtures. (c) We display the probabilities in the unpostselected three-state system for the same initial preparations. (d) Comparing the evolution from the ρ_m preparation to the classical superposition shows excellent agreement, verifying linearity for the three-state dissipative qubit.

superpositions of classical mixtures. We previously noted that the superposition principle does not extend in general to density operators. However, it can be expressed in terms of classical mixtures (i.e., with no coherence). This case is particularly interesting because it allows us to investigate linearity both in the postselected non-Hermitian qubit and for the dissipative three-level system spanned by $\{|g\rangle, |e\rangle, |f\rangle\}$.

As shown in Fig. 4(a), we first display the time evolution of populations in the $\{|e\rangle, |f\rangle\}$ manifold, simply displayed as $P^{(n)}(e)$. We consider three different initial states: $\rho_e = |e\rangle\langle e|$ (with $P(e) = 1$), ρ_f (with $P(e) = 0$), and a classical mixture ρ_m with $P(e) = 0.5$. The mixture is composed of equal parts preparations in $|+x\rangle$ and $|-x\rangle$. The dynamics for the populations are characteristic of the non-Hermitian evolution in this manifold with asymmetric oscillation favoring population in $|f\rangle$ over $|e\rangle$. The lower panel compares the evolution of the initial mixture to a superposition of the curves for the $|e\rangle$ and $|f\rangle$ preparations. While the curves are close, there is clear disagreement—a breakdown of linearity for classical mixtures.

Figure 4(d) extends this analysis to the three-state system. Here, we retain all measurement outcomes after the iterative Bayesian update, $P(g)$, $P(e)$, and $P(f)$, which can be arranged in a diagonal density matrix,

$$\rho^{(3)} = \begin{pmatrix} P(g) & 0 & 0 \\ 0 & P(e) & 0 \\ 0 & 0 & P(f) \end{pmatrix}. \quad (6)$$

Figure 4(b) displays $P(e)$ and $P(g)$ for the same three preparations. The lower panel tests whether the superposition of the density matrices preserves linearity: it does.

V. OUTLOOK

Our results extend prior observations of postselection-induced nonlinearity by directly testing the superposition principle at the level of independently evolved, independently renormalized basis states and by characterizing the resulting violation across the EP degeneracy of the non-Hermitian Hamiltonian. This demonstration of nonlinearity in postselected quantum evolution represents a departure from both Bender's original formulation of $\mathcal{PT}$ -symmetric quantum theory [36], which sought to preserve unitarity through modified inner products, and subsequent classical realizations of non-Hermitian dynamics in photonic and mechanical systems. While the mathematical isomorphism between classical dynamical matrices and quantum Hamiltonians has enabled extensive exploration of non-Hermitian phenomena across diverse platforms, the quantum implementation through postselection on no-jump trajectories introduces a new element: Unlike a single, discrete conditioning event, the renormalization here is continuously fed back into the generator of motion along the coherent drive, and although similar dynamics have been studied theoretically, whether these dynamics are nonlinear has not been previously tested. This nonlinearity is not a generic consequence of non-Hermiticity itself, but rather arises from postselected quantum dynamics, where state-dependent renormalization breaks linearity at the level of state vectors. Our characterization of where nonlinear regimes become prominent—particularly near exceptional points but extending throughout the parameter space—elucidates the trade-offs inherent in accessing these dynamics. While nonlinear quantum evolution has been theoretically predicted to offer computational advantages for certain problems, our results highlight that these benefits come at the cost of reduced postselection success rates and probabilistic operation. Future work exploring applications of controlled nonlinearity in quantum information processing must therefore balance the potential for enhanced computational power against the practical limitations imposed by postselection overhead.

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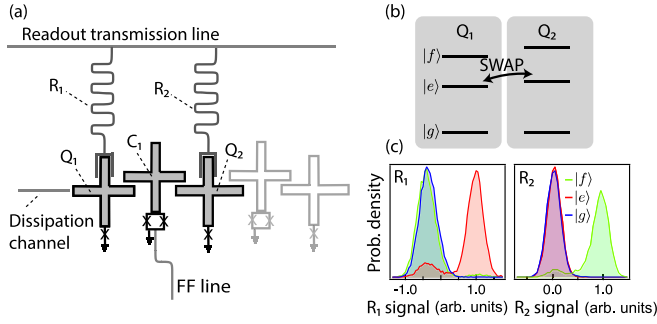


FIG. 5. Setup. (a) Schematic of the superconducting transmon processor including qubits Q_1 and Q_2 , coupler C_1 , and readout resonators R_1 and R_2 . Q_1 is coupled to a dissipation channel leading to energy decay of the state $|e\rangle$. (b) Parametrically activated SWAP readout: Population in the $|e\rangle$ state on Q_1 is transferred to Q_2 via a parametric modulation of the coupler C_1 . (c) Readout signal histograms display how joint readout of R_1 and R_2 yields high-fidelity state assignment.

DATA AVAILABILITY

The data that support the findings of this article are openly available [37].

APPENDIX

1. Quantum state tomography

The experimental platform utilizes a three-transmon subsection of a multitransmon chip [Fig. 5 (a)]. Two transmons, Q_1 and Q_2 , are dispersively coupled to dedicated microwave readout resonators, and a flux-tunable transmon, C_1 , mediates coupling between the two. The readout resonators enable state measurement of the transmons in the energy basis. A microwave tone near resonance with the readout resonator will acquire a transmon-state-dependent phase shift, which is detected with heterodyne demodulation. A traveling-wave parametric amplifier (TWPA) is used to enhance the measurement signal-to-noise ratio [38]. When the demodulated measurement tone is integrated for a time $t_{\text{meas}} = 2 \mu\text{s}$, the signal-to-noise ratio is sufficient to distinguish between energy states with about 88% fidelity.

We use the $\{|g\rangle, |e\rangle, |f\rangle\}$ manifold of states of Q_1 to realize the dissipative qutrit captured by Eq. (2). The rapid decay of $|e\rangle$ for Q_1 (with energy decay time $1/\Gamma_e = 1.1 \mu\text{s} < t_{\text{meas}}$) would severely limit the measurement fidelity of this state. To overcome this, we implement a SWAP operation between Q_1 and Q_2 , transferring population to a transmon with slower energy decay [Fig. 5 (b)]. Specifically, the SWAP is implemented by parametric modulation of C_1 's frequency [39,40]. When the modulation frequency is equal to half the detuning between the $|e\rangle_{Q_1} \otimes |g\rangle_{Q_2} \equiv |eg\rangle$ and $|ge\rangle$ states, it brings the two states into parametric resonance. This parametric resonance results in a SWAP gate time $t_{\text{SWAP}} = 156 \text{ ns}$. Because $t_{\text{SWAP}} \ll 1/\Gamma_e$, population is transferred with reasonable fidelity. Following the SWAP, we perform simultaneous readout on both qubits.

We calibrate the readout by alternately preparing the states $|g\rangle$, $|e\rangle$, and $|f\rangle$ and perform the SWAP joint readout. Figure 5(c) displays histograms of the demodulated measurement

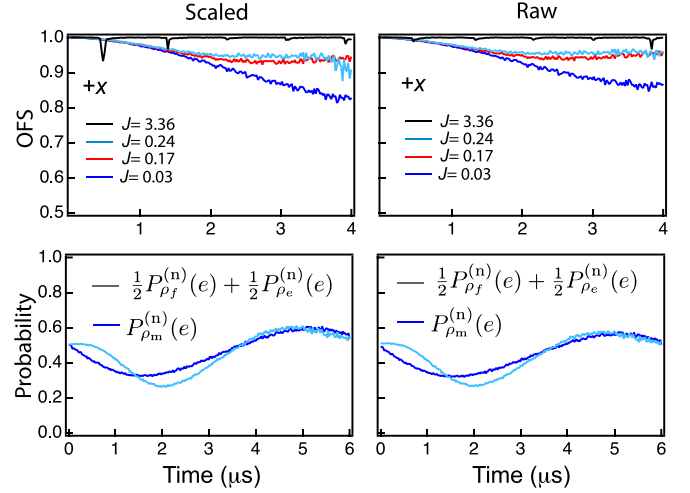


FIG. 6. Effect of measurement fidelity scaling. Comparison of the results with and without the bounded-minimization measurement fidelity correction. The top two panels display the overlap results between the scaled and raw cases corresponding to the bounded-minimized and the non-bounded-minimized cases. We can see that the difference between the two is minimal. We also look at the classical mixture comparison in the bottom two panels. In this comparison as well, there is very little difference between the results confirming that the source of nonlinearity is not the scaling update.

signal from two readout resonators, highlighting how the three states of Q_1 are resolved with high fidelity. A single measurement yields a 4-tuple that needs to be classified into a single state. We use a gradient boosting classifier from the LIGHTGBM package [41] to assign measurement outcomes. The success of the SWAP-assisted readout and classifier can be assessed by the normalized three-state classification (confusion matrix):

$$\beta = \begin{pmatrix} 0.993 & 0.003 & 0.005 \\ 0.123 & 0.871 & 0.006 \\ 0.056 & 0.018 & 0.925 \end{pmatrix}, \quad (\text{A1})$$

where β_{ij} is the probability of assigning the qubit to the j th state after preparing it in the i th state. To mitigate the measurement infidelities, we use a bounded-minimization approach solving $\|\beta \vec{P}_{\text{ideal}} - \vec{P}_{\text{noisy}}\|^2$, where $\vec{P}_{\text{ideal}}$ is constrained in $[0, 1]$ and $\vec{P}_{\text{noisy}}$ is the probability vector measured by our three-state readout [42].

Figure 6 shows the effect of this bounded-minimization approach. We call the data that have undergone these process the “scaled” data, and we compare this to the “raw” data. We can see that the effect of this bounded minimization is small but effective in cleaning up the probabilities. For example, if we look at the $t = 0$ probability data, we can see that in the scaled data, both curves start at $P = 0.5$, where it deviates slightly in the raw data. The minimal difference between the two cases allows us to conclusively rule out this process as the source of our nonlinearity.

To realize the effective non-Hermitian Hamiltonian [Eq. (3)], we focus on dynamics within the $\{|e\rangle, |f\rangle\}$ manifold in the absence of quantum jumps to $|g\rangle$. By only keeping trajectories that preserve this manifold of states, our post-selected ensemble of data is effectively renormalized. We define

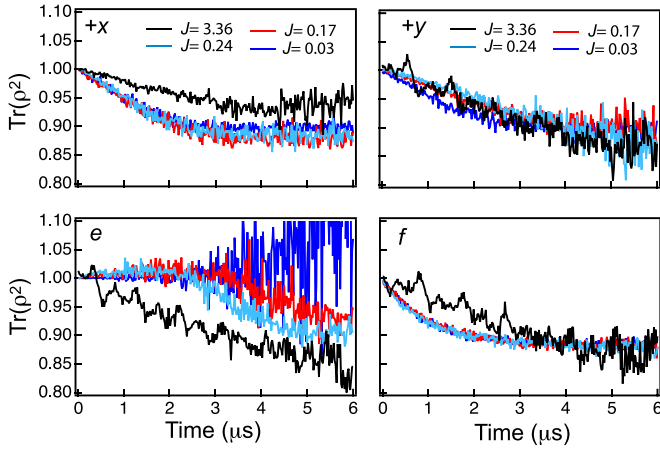


FIG. 7. State purity. We use $\text{Tr}(\rho^2)$ as the purity of reconstructed density from different initial states $|e\rangle$, $|f\rangle$, $|+x\rangle$, $|+y\rangle$. Each panel shows the purity of different initial state versus time for different J values. We can see that the purity decreases past ~ 0.9 after $4 \mu\text{s}$. Therefore, we chose to only look at the overlap up to $4 \mu\text{s}$, where the pure-state projection is most reliable.

Pauli operators on the $\{|e\rangle, |f\rangle\}$ manifold as $\sigma_z \equiv |e\rangle\langle e| - |f\rangle\langle f|$, $\sigma_x \equiv |e\rangle\langle f| + |f\rangle\langle e|$, and $\sigma_y \equiv -i|e\rangle\langle f| + i|f\rangle\langle e|$. We measure these operators' expectation values with quantum state tomography. The x -axis tomography is performed as follows. We apply a $\pi/2$ rotation in the $\{|e\rangle, |f\rangle\}$ manifold followed by the SWAP-based three-state readout. For each experimental trial, the classifier yields measurement outcomes “ g ”, “ e ”, or “ f ”, which given the $\pi/2$ rotation correspond to measurement results “ g ”, “ $+x$ ”, or “ $-x$ ”. By performing several trials, we obtain raw probabilities $\tilde{P}(g)$, $\tilde{P}(+x)$, $\tilde{P}(-x)$. We then correct these probabilities using the iterative bayesian update to bounded minimization approach, yielding corrected probabilities, $P(g)$, $P(+x)$, $P(-x)$; finally, we look at the renormalized subensemble: $P^{(n)}(+x) \equiv \frac{P(+x)}{P(+x)+P(-x)}$, $P^{(n)}(-x) \equiv \frac{P(-x)}{P(+x)+P(-x)}$. We perform tomography about the y and z axes similarly.

The corrected probabilities allow us to reconstruct the 2×2 density matrix in the $\{|e\rangle, |f\rangle\}$ basis as

$$\rho_{ef} = \frac{1}{2}(\mathbb{I} + x\sigma_x + y\sigma_y + z\sigma_z), \quad (\text{A2})$$

where $x = 2P^{(n)}(+x) - 1$, $y = 2P^{(n)}(+y) - 1$, and $z = 2P^{(n)}(+z) - 1$ are the experimentally determined expectation values of σ_x , σ_y , and σ_z . This tomographic reconstruction enables us to track the evolution of quantum states in the $\{|e\rangle, |f\rangle\}$ manifold under the effective non-Hermitian Hamiltonian, as presented in the main text.

Figure 7 shows the purity of the reconstructed density matrix for the results presented in Fig. 3. We observe statistical artifacts arising from the postselection and renormalization process at later times ($T > 4 \mu\text{s}$). In our non-Hermitian scheme, the density matrix is reconstructed conditional on the system remaining in the $\{|e\rangle, |f\rangle\}$ manifold. As the population decays over time, the postselection success probability decreases. This effectively amplifies the statistical shot noise in the measured expectation values. Since we employ linear inversion tomography bounded-minimization approach, this amplified noise can lead to reconstructed matrices with

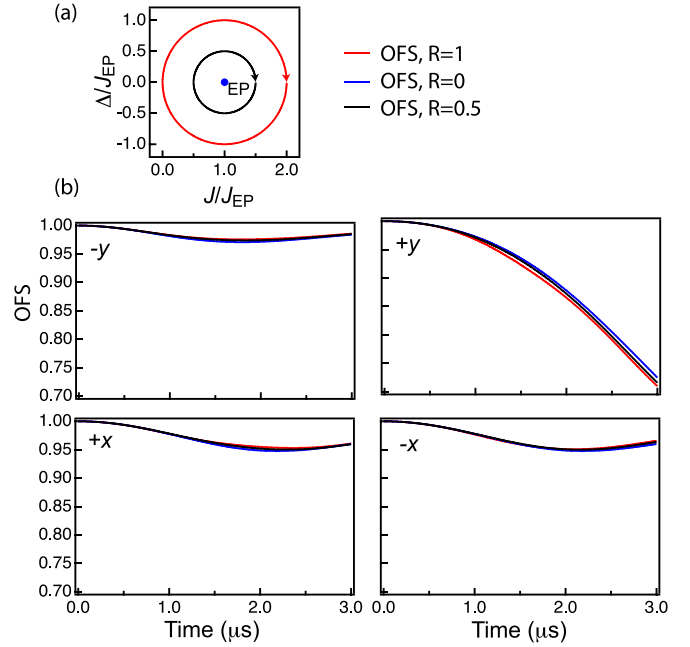


FIG. 8. Fubini-Study-metric-inspired overlap for EP encircling trajectories. (a) Trajectories in the (J, Δ) parameter space encircling the exceptional point. The paths correspond to radii $R = 1$ (red), $R = 0.5$ (black), and $R = 0$ (blue dot). (b) Simulated time evolution of the OFS for four different initial states ($\pm x$, $\pm y$).

slightly negative eigenvalues. Mathematically, the sum of squared eigenvalues can then exceed unity ($\sum \lambda_i^2 > 1$). Before $4 \mu\text{s}$, the purity of initial state $|e\rangle$, $|f\rangle$, $|+x\rangle$, $|+y\rangle$ is always larger than 0.845 and after $4 \mu\text{s}$ the density matrix of $T > 4 \mu\text{s}$ is not pure enough for reliable state projection.

2. Setup

The experimental setup comprises a multiqubit superconducting chip fabricated and provided by the SQUILL Foundry at MIT Lincoln Laboratory. The device is packaged, shielded, and mounted in a dilution refrigerator in a setup similar to Ref. [43]. The fast flux lines for the couplers are filtered with 30 dB attenuation and 1300 MHz low-pass filters (MiniCircuits 1300 VLFX). We use the readout input line to drive the qubit and send readout pulses. The input readout line has total 60 dB attenuation, an 8 GHz low-pass filter, and dissipative filters using eccosorb epoxy. For the readout output line, we use a TWPA and a high-electron-mobility transistor to amplify the output signal. The pump frequency of the TWPA is 5.1 GHz.

The device features five qubits and four tunable couplers. This work utilizes two of those qubits with resonant frequencies of 4.484 and 4.445 GHz. Each qubit is coupled to a microwave readout resonator with respective frequencies of 6.727 and 6.655 GHz.

3. Simulated overlap for EP encircling trajectories

Numerical simulations were performed by integrating the Lindblad master equation for a three-level system ($|g\rangle$, $|e\rangle$, $|f\rangle$) using a fourth-order Runge-Kutta method. The system was initialized in each of the six cardinal Bloch sphere states

and evolved under the non-Hermitian Hamiltonian with experimentally measured decay rates γ_e , γ_f , γ_{de} , and γ_{ef} . At each time step, the density matrix was projected onto the $|e\rangle\text{--}|f\rangle$ subspace to replicate the postselection applied in the experiment, and the Bloch vector components were extracted to directly compare with the measured tomography data. We also extend this framework to simulate the dynamics of a time-

dependent Hamiltonian encircling the EP, parametrized by $J(t)/J_{\text{EP}} = 1 + R \cos(2\pi t/T)$ and $\Delta(t)/J_{\text{EP}} = R \sin(2\pi t/T)$, with period $T = 3.0 \mu\text{s}$. As shown in Fig. 8, the OFS for finite radii ($R = 0.5$ and $R = 1$) deviates only slightly from the EP-driven case ($R = 0$), indicating that the nonlinear behavior characterized in the static regime persists during EP encirclement without introducing new qualitative features.

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