

Observation of a phase shift in nonlinear parity-time symmetric film bulk acoustic resonators

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(Received 22 August 2025; accepted 15 December 2025; published 31 December 2025)

An exceptional point (EP) in *PT* (parity-time)-symmetric systems featuring loss and nonlinear saturable gain has been recently utilized to enhance sensing, leading to the cube-root singularity in a two-level system. However, a shift in the frequency eigenvalue comes with a relatively small scaling factor, which could potentially reduce the sensitivity augmented by the cube-root singularity, and achieving the cube-root singularity necessitates sacrificing the sensing dynamic range. Instead of detecting frequency eigenvalue, this article explores the phase shift between both resonators in parity-time symmetric film bulk acoustic resonators (FBARs) with a nonlinear saturable gain. When the FBAR is linked to an external (inductor-capacitor loop with a nonlinear negative resistance, it forms *PT*-symmetric resonators with loss and nonlinear gain. Our findings show that the phase difference between the loss and gain resonators displays both a cube-root dependence on perturbation and an increased scaling factor. Experimental results from our FBAR humidity sensors indicate that the sensitivity of phase shift has increased by several orders of magnitude compared to the conventional frequency shift. These discoveries have potential applications for piezoelectric resonant sensors requiring ultrahigh sensitivity across various fields.

DOI: [10.1103/7vrg-tmys](#)

I. INTRODUCTION

A piezoelectric resonant sensor has a long history, with roots tracing back to the quartz crystal microbalance (QCM) proposed by Sauerbrey in 1959 [1]. This type of sensor, characterized as a resonator with a mass (m) and the resonant frequency (f_0), typically consists of a piezoelectric quartz plate sandwiched by two metallic electrodes with a sensing layer deposited on the electrode. As gravimetric detectors, the analyte to be measured is absorbed by the sensing layer, causing a mass variation (Δm) in the resonator and consequently resulting in a shift ($\Delta f = f - f_0$) in its resonance frequency. This phenomenon is usually described by the Sauerbrey equation, $\Delta f/f_0 = -\alpha\delta$, where $\delta = \Delta m/m$ is here referenced to as a perturbation to the resonator and α represents the scaling factor. The capability of the QCM for the real-time monitoring and label-free detection has led to extensive study and commercial development for applications in chemical and biological sensing, along with *in situ* interfacial mass detection [2–5]. For the detection of more minor changes in the analyte, it is imperative for the resonators to exhibit higher sensitivities. As indicated by the Sauerbrey equation, a reduction in the resonator size will give rise to a decrease in mass and an increase in resonant frequency,

thereby enhancing the sensitivity. Nevertheless, the upper operating frequency of the QCM resonator is limited by the thickness of quartz crystal and it typically operates in the 5–20 MHz frequency range. Accordingly, a film bulk acoustic resonator (FBAR) was proposed for chemical and biological sensing [6]. This technology employs piezoelectric thin films instead of bulk crystals as the transduction material. The piezoelectric films used in FBARs are significantly thinner than the quartz crystal plate used in QCMs, providing FBARs with a high resonant frequency up to GHz and small base mass. This translates into superior sensitivity compared to QCMs. In addition to this heightened sensitivity, FBARs offer other advantages, including cost efficiency, compactness, and the capability for both array fabrication and monolithic integration with processing circuits. As a result, FBARs have attracted increasing attention in the field of chemical and biological sensing in recent years [7–10]. In this study, we explore the consequence of exceptional point (EP) enhanced sensing for piezoelectric resonance sensors instead of further reducing the resonator size to amplify its sensitivity. When an external *LC* (inductor-capacitor) circuit with a nonlinear negative resistance is connected to the FBAR, it transforms into *PT* (parity-time)-symmetric resonators with linear loss and nonlinear gain. Our findings demonstrate that the phase difference between the loss and gain resonators shows the cube-root dependence of perturbation over a broad measurement range. Clearly, in cases of relatively small perturbations, this greatly enhances the sensitivity compared to a traditional linear response.

EPs are non-Hermitian degeneracies characterized by the convergence of eigenvalues and the collapse of their corresponding eigenvectors [11–17]. Consequently, non-Hermitian systems tuned at EPs can exhibit significantly magnified

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responses to external perturbations. This presents opportunities for the development of ultrasensitive sensors leveraging the physics of EPs [18,19]. Specifically, in a PT -symmetric resonator that accommodates an N th-order linear EP, the frequency eigenvalue splitting ($\Delta\lambda$) resulting from a perturbation follows $\Delta\lambda = \alpha\delta^{1/N}$, where α represents the scaling factor that depends on the coupling between resonators. Clearly, for perturbations that are relatively small ($\delta \ll 1$), this demonstrates an enhanced sensing capability compared to the typical linear response in Hermitian systems, provided the scaling factor remains consistent in magnitude. Enhanced responsivity at EPs has been noted across various systems, including optical [20–24] and plasmonic [25] systems, LC systems [26–29], and microelectromechanical systems [30,31]. Nevertheless, there is ongoing debate on the potential nullification of this sensitivity advantage due to increased noise in the context of linear non-Hermitian theory [32–38]. PT -symmetric systems that incorporate a nonlinear saturable gain demonstrate significant differences compared to systems with balanced loss and gain. The presence of a gain saturation mechanism allows PT -symmetric systems to ultimately evolve into a steady state, even in the presence of initial instability induced by external perturbations or minor mismatches between two resonators [39]. Enhanced sensitivity, exhibiting both square-root singularity [40,41] and cube-root singularity [42–44], has been observed based on EPs in nonlinear PT -symmetric LC systems. However, such enhancement results in a small scaling factor ($\alpha \ll 1$), potentially diminishing the sensitivity boosted from the singularity. Moreover, it should be noted that the cube-root singularity is realized at the expense of the dynamic range [42–44], defined as the ratio between the maximum and minimum perturbation that a sensor can measure. In contrast to using frequency shift as a measure of sensitivity, this study investigates the phase difference (ϕ) between the loss and gain resonators in PT -symmetric FBARs with nonlinear saturable gain. We demonstrate that $\phi = \alpha\delta^{1/3}$ with $\alpha \gg 1$, implying a markedly enhanced sensitivity across an extensive measurement range.

II. THEORETICAL CONSIDERATIONS

As illustrated in Fig. 1(a), a piezoelectric resonant sensor in this study is composed of a top electrode, an aluminum nitride (AlN) piezoelectric film, a bottom electrode, and a support layer, with the top electrode coated by a sensing material [cellulose acetate (CA)/graphene oxide (GO)]. The Butterworth-van Dyke (BVD) equivalent circuit is commonly used to represent the piezoelectric resonator [4]. In this circuit, C_0 , L_m , C_m , and R_m correspond to the electrode capacitance, the motional inductance representing the vibrating mass (m), the motional capacitance representing the stored energy, and the motional resistance representing damping, respectively. These component values can be extracted from the S -parameter measurement (see Supplemental Material S3 [45]). The fundamental resonant frequency of the piezoelectric resonant sensor in the motional branch is denoted as $f_0 = \omega_0/2\pi$, where $\omega_0 = 1/\sqrt{L_m C_m}$ is the angular frequency. Accumulation of analyte in the sensing material results in a mass change from m to $m + \Delta m$, where Δm represents the absorbed mass. Correspondingly, the motional inductance

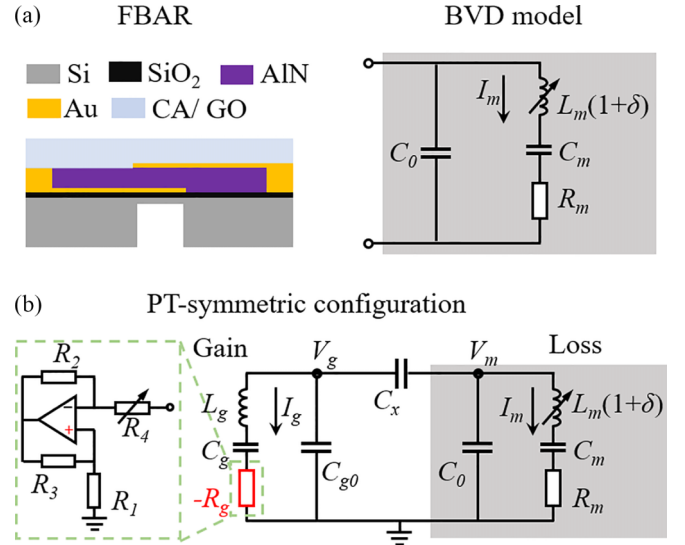


FIG. 1. A nonlinear PT -symmetric FBAR system for phase shift sensing. (a) Schematic diagram of a conventional FBAR sensor and its corresponding BVD equivalent circuit model. The adsorbed mass causes the motional inductance to increase from L_m to $L_m(1 + \delta)$. (b) Schematic diagram of an equivalent circuit for a nonlinear PT -symmetric FBAR system. An external LC circuit (L_g , C_g , C_{g0}) with a nonlinear negative resistance (R_g) is coupled to the FBAR sensor via coupling capacitance (C_x), which can be adjusted so as to support an EP in the PT -symmetric system. The negative resistance is realized by an operational amplifier.

changes from L_m to $L_m(1 + \delta)$, where $\delta = \Delta L_m/L_m$ is referred to as a perturbation. The typical operation of the piezoelectric resonant sensor involves monitoring the shift in the resonant frequency due to absorbed analyte, which is approximately linear with respect to the perturbation [46]. As illustrated in Fig. 1(b), unlike the traditional approach, the FBAR sensor is coupled to an external LC circuit with a nonlinear negative resistance via a series capacitance C_x , and the phase difference across this series capacitance is then observed. This configuration also distinguishes it from the preceding PT -symmetric piezoelectric resonant sensor [44], wherein the coupling capacitance is configured in parallel and the resonant frequency is monitored. To elucidate how this configuration results in enhanced sensing, we provide an analysis based on a coupled-mode theory model for two coupled resonators. As illustrated in Fig. 1(b), an external LC circuit leverages the nonlinear negative resistance (R_g) instead of the motional resistance (R_m). The negative resistance is implemented by an operational amplifier (see Figs. S7 and S16 of the Supplemental Material [45]). The series loop ($L_m C_m R_m$) for the individual FBAR can be regarded as a loss resonator with the loss factor $\gamma = R_m\sqrt{C/L}$, while the external series loop ($L_g C_g R_g$) can be regarded as a gain resonator with the gain factor $g(|V_g|) = R_g\sqrt{C/L}$. It is imperative to note that g is contingent upon the voltage amplitude of the gain resonator. For simplicity, it is assumed that $L_g = L_m = L$, $C_g = C_m = C$, and $C_{g0} = C_0$. The coupling capacitance (C_x) can be adjusted experimentally. Applying Kirchhoff's laws in Fig. 1(b), the complex voltage across the gain resonator (V_g) and the loss

resonator (V_m) satisfies (see Supplemental Material S1 [45])

$$\begin{bmatrix} 2(\lambda - 1) - \mu_c + ig & -\mu_x[2(\lambda - 1) + ig] \\ -\mu_x[2(\lambda - 1) - i\gamma] & 2(\lambda - 1) - \mu_c - i\gamma \end{bmatrix} \begin{bmatrix} V_g \\ V_m \end{bmatrix} = 0, \quad (1)$$

where $\lambda = \omega/\omega_0$ is referred to here as the frequency eigenvalue, ω is the operation angular frequency, $\mu_c = C/(C_x + C_0)$, and $\mu_x = C_x/(C_x + C_0)$, respectively. The system exhibits PT symmetry when $g = \gamma$. For a nonzero solution of voltages in Eq. (1), it is imperative that the characteristic equation equals zero. This study primarily focuses on steady-state oscillations; hence, the frequency eigenvalues must be real. Notably, due to the gain saturation mechanism, PT -symmetric systems ultimately evolve into a steady state [39,43,44] even when confronted with either initial instabilities from external perturbations or minor discrepancies between both resonators (see Fig. S10 of the Supplemental Material [45]). The gain associated with this steady state establishes the saturated gain level (g_s). Depending upon the coupling strength (κ) relative to the loss (γ), there exist two circumstances for solutions. The frequency eigenvalue (λ) and the phase difference (ϕ) between the loss and gain resonators turn out to be, respectively (see Supplemental Material S1 [45])

$$\begin{aligned} g_s &= \gamma \\ \lambda_{\mp} &= 1 + \frac{\kappa}{2\mu_x} \mp \frac{1}{2}\sqrt{\kappa^2 - \gamma^2} \\ \left| \frac{V_m}{V_g} \right|_{\mp} &= 1 \quad (\kappa \geq \gamma). \end{aligned} \quad (2a)$$

$$\begin{aligned} \phi_{\mp} &= \tan^{-1} \left[\frac{(1 - \mu_x^2)(\frac{\gamma}{\kappa})}{2\mu_x \mp (1 + \mu_x^2)\sqrt{1 - (\frac{\gamma}{\kappa})^2}} \right] \\ g_s &= \kappa^2/\gamma \\ \lambda_b &= 1 + \frac{\kappa}{2\mu_x} \\ \left| \frac{V_m}{V_g} \right|_b &= \frac{1 + \mu_x^2(\frac{\gamma}{\kappa})^2}{(\frac{\gamma}{\kappa})^2 + \mu_x^2} \quad (\kappa < \gamma) \\ \phi_b &= \tan^{-1} \left[\frac{(1 - \mu_x^2)(\frac{\gamma}{\kappa})}{\mu_x(1 + (\frac{\gamma}{\kappa})^2)} \right] \end{aligned} \quad (2b)$$

with

$$\kappa = \frac{\mu_c \mu_x}{1 - \mu_x^2} = \frac{CC_x}{C_0^2 + 2C_0C_x}.$$

One can see that the system has two modes with λ_+ and λ_- for $\kappa \geq \gamma$, where both modes have the same saturated gain ($g_s = \gamma$) and the same amplitude ($|V_m/V_g| = 1$). Note that Eq. (2a) for a nonlinear PT -symmetric system has the same results as those of linear PT system [28,29,47], but their physical mechanisms are entirely different. In the linear PT system with linear gain and loss, one can strictly keep the PT symmetry by choosing a system Hamiltonian with PT symmetry, where g is set to equal γ . While in the nonlinear PT system with linear loss and nonlinear gain, the PT symmetry appears automatically from the gain saturation mechanism

and thus the system shows PT symmetry naturally at steady state. Especially, the linear PT -symmetric system does not have the steady-state oscillation for $\kappa < \gamma$, but the nonlinear PT system has a steady-state oscillation with frequency eigenvalue λ_b due to its gain saturation mechanism. When $\kappa = \gamma$, both frequency eigenvalues and phase difference coalesce, resulting in the emergence of an exceptional point, that is,

$$\begin{aligned} \lambda_{EP} &= 1 + \frac{\kappa}{2\mu_x}, \\ \phi_{EP} &= \tan^{-1} \left[\frac{1 - \mu_x^2}{2\mu_x} \right]. \end{aligned} \quad (3)$$

When the accumulation of analyte on sensitive materials causes the motional inductance of the loss resonator to shift from L_m to $L_m(1 + \delta)$, the voltage equation corresponding to Eq. (1) becomes

$$\begin{bmatrix} 2\lambda - 2 - \mu_c + ig & -\mu_x(2\lambda - 2 + ig) \\ -\mu_x(2\lambda - 2 + \delta - i\gamma) & 2\lambda - 2 - \mu_c + \delta - i\gamma \end{bmatrix} \begin{bmatrix} V_g \\ V_m \end{bmatrix} = 0. \quad (4)$$

The characteristic equation is found to be

$$\begin{aligned} y^2 + y \left(\delta - 2\frac{\kappa}{\mu_x} \right) + g\gamma + \left[(1 - \mu_x^2) \frac{\kappa}{\mu_x} - \delta \right] \frac{\kappa}{\mu_x} \\ + i \left[g \left(y - \frac{\kappa}{\mu_x} + \delta \right) - \gamma \left(y - \frac{\kappa}{\mu_x} \right) \right] = 0, \end{aligned} \quad (5)$$

where $y = 2(\lambda - 1)$. A perturbation induces unsteady-state oscillations in the system; however, due to the saturation mechanism of nonlinear gain, the system eventually evolves into a steady state. This steady state subsequently defines the saturated gain level (g_s). To ensure that a frequency eigenvalue is real, one sets the imaginary part of Eq. (5) to equal zero, leading to

$$g_s = \gamma \frac{y - \frac{\kappa}{\mu_x}}{y - \frac{\kappa}{\mu_x} + \delta}. \quad (6)$$

Inserting Eq. (6) into Eq. (5), one has

$$\begin{aligned} y^2 + y \left(\delta - 2\frac{\kappa}{\mu_x} \right) + \gamma^2 \left(\frac{y - \frac{\kappa}{\mu_x}}{y - \frac{\kappa}{\mu_x} + \delta} \right) \\ + \left[(1 - \mu_x^2) \frac{\kappa}{\mu_x} - \delta \right] \frac{\kappa}{\mu_x} = 0. \end{aligned} \quad (7)$$

This is a cubic equation concerning the frequency eigenvalue λ . An analytical solution to Eq. (7) at an EP ($\kappa = \gamma$) can be derived using the Cardano formula (see Supplemental Material S1 [45]). Assuming $\kappa \ll 1$ and $\delta \ll 1$, the frequency eigenvalues can be approximately expressed as

$$\lambda \approx 1 + \frac{\kappa}{2\mu_x} + \frac{1}{2}\kappa^{2/3}\delta^{1/3} - \frac{1}{3}\delta. \quad (8)$$

By inserting Eqs. (6) and (8) into Eq. (4), the phase difference between the loss and gain resonators turns out to be

$$\phi \approx \tan^{-1} \left[\frac{(1 - \mu_x^2)}{2\mu_x} \right] - \left(\frac{1 - \mu_x^2}{1 + \mu_x^2} \right) \left(\frac{\delta}{\kappa} \right)^{1/3}, \quad (9)$$

where the Taylor series expansion is used under the conditions that $\kappa = \gamma$ and $\delta > 0$. If the system is initially biased at an

EP, the frequency shift and phase shift due to perturbation are represented by, respectively,

$$\Delta\lambda_{PT} = \lambda - \lambda_{EP} \approx \frac{1}{2}\kappa^{2/3}\delta^{1/3} - \frac{1}{3}\delta, \quad (10a)$$

$$\Delta\phi = \frac{\phi_{EP} - \phi}{\phi_{EP}} \approx \left(\frac{2}{\pi} + \frac{8\mu_x}{\pi^2}\right)\left(\frac{\delta}{\kappa}\right)^{1/3}, \quad (10b)$$

where the phase shift is normalized to the phase at an EP for comparative analysis with the frequency shift. The sensitivity of the frequency shift and phase shift with respect to perturbation is given by, respectively,

$$\chi_{\lambda PT} = \frac{\partial|\Delta\lambda|}{\partial\delta} \approx \frac{1}{6}\kappa^{2/3}\delta^{-2/3} - \frac{1}{3}, \quad (11a)$$

$$\chi_{\phi} = \frac{\partial|\Delta\phi|}{\partial\delta} \approx \frac{1}{3}\left(\frac{2}{\pi} + \frac{8\mu_x}{\pi^2}\right)\kappa^{-1/3}\delta^{-2/3}. \quad (11b)$$

The frequency shift and phase shift as a function of perturbation, as well as their sensitivities with respect to perturbation, are simulated and plotted (see Fig. S11 of the Supplemental Material [45]). For comparison, results corresponding to an individual FBAR are also provided. It is obvious from Eqs. (8)–(11) that the frequency shift in the small perturbation limit exhibits a cube-root singularity in the nonlinear *PT*-symmetric resonator rather than a square-root singularity in the linear *PT*-symmetric resonator. Since $\delta \ll 1$, $\delta^{1/3} \gg \delta^{1/2} \gg \delta$, which leads to an enhanced sensitivity of the nonlinear *PT*-symmetric system [42–44], compared to that of the linear response in a conventional system or the second-order EP response observed previously in the linear *PT*-symmetric system [26–31]. However, the cube-root singularity is valid only in the limit of small perturbations, indicating that the dynamic range of a sensor is reduced considerably. When the perturbation is increased, the sensitivity of the frequency shift in the nonlinear *PT*-symmetric resonator becomes even lower than that in the traditional resonator. From Eq. (10a), solving $\partial|\Delta\lambda_{PT}|/\partial\delta = 0$ yields the perturbation $\delta_T = \kappa/2\sqrt{2} \approx 0.354\kappa$, where δ_T is referred to as the turning point. Specifically, when $\delta < \delta_T$, $\Delta\lambda_{PT}$ increases with increasing perturbations (an approximate cube-root singularity). In contrast, when $\delta > \delta_T$, $\Delta\lambda_{PT}$ decreases with increasing perturbations (an approximate linearity, worse than a single resonator). Theoretically, this restricts the maximum measurable perturbation to $\delta_T = \kappa/2\sqrt{2}$. Meanwhile, Eq. (10b) is obtained under conditions that $\delta < \kappa$; that is, the cube-root singularity of the phase shift approximately holds as long as $\delta < \kappa$. Theoretically, this restricts the maximum measurable perturbation to κ , leading to an enlarged dynamic range.

Also noteworthy is that the scaling factor for the phase shift is given by $\kappa^{-1/3}(2/\pi + 8\mu_x/\pi^2)$, while it is $\kappa^{2/3}/2$ for the frequency shift in the small perturbation limit. The scaling factor of the phase shift is equal to $(2/\kappa)(2/\pi + 8\mu_x/\pi^2)$ times that of the frequency shift. Noting that $\kappa \ll 1$, this means an enhancement in the sensitivity of the phase shift by several orders of magnitude compared to that of the frequency shift.

A time-domain simulation of the equivalent circuit representing a nonlinear *PT*-symmetric configuration for an FBAR sensor was executed using the Advanced Design System (ADS) software (see Supplemental Material S2 [45]). The

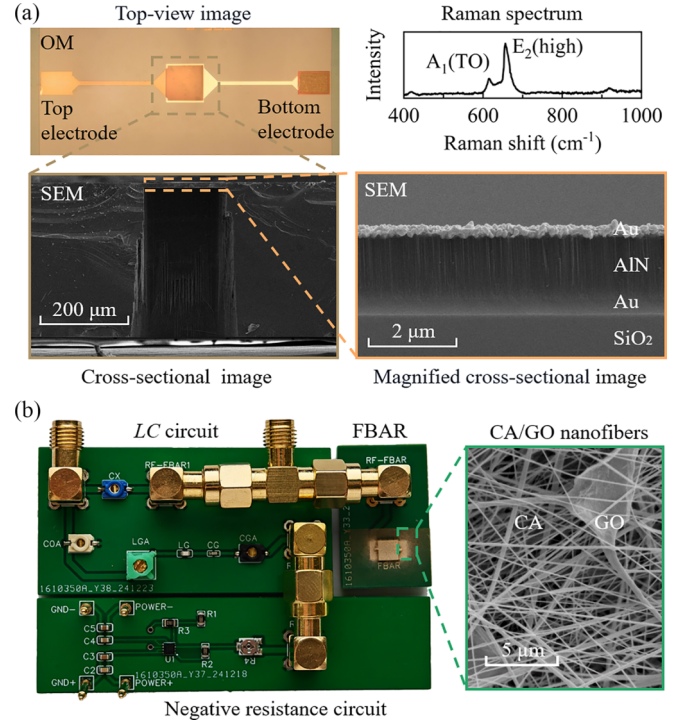


FIG. 2. Fabrication of a *PT*-symmetric FBAR sensor. (a) The picture of the fabricated AlN FBAR. The top view is given by the OM (optical microscopy). The cross-sectional view of the FBAR is given by the SEM (scanning electron microscopy). The inset provides the measured Raman spectrum of the AlN film, displaying characteristic peaks at 615.27 cm^{-1} (A_1 mode) and 656.29 cm^{-1} (E_2 mode). (b) Photo of experimental setup. A FBAR was covered with electrospun CA/GO nanofibers for humidity sensing. The *PT*-symmetric system including an external gain resonator was fabricated on a PCB.

parameters for the operational amplifier, designed for the negative resistance circuit, were derived from the OPA855 model (Texas Instruments). The parameters for the circuit components, namely, C_0 , L_m , C_m , and R_m , were experimentally extracted from a fabricated FBAR. The frequency and phase in the resonators were extracted by performing a fast Fourier transform operation on the obtained time-domain voltage signal (built-in simulators). Simulation results (dots) are provided (see Fig. S11 of the Supplemental Material [45]). These simulation outcomes generally align with the theoretical results.

III. EXPERIMENTS

Based on the circuit simulation, we developed a prototype of an FBAR humidity sensor (see Supplemental Material S3 [45]), as illustrated in Fig. 2(a). Humidity sensors with heightened sensitivity are imperative in the medical field for applications such as respiration monitoring, infusion pumps, ventilators, and diagnostic instruments [48,49]. The FBAR humidity sensor consists of a top and a bottom electrode (200 nm thick Au and 15 nm thick Cr), an AlN piezoelectric film (1.5 μm thick) with a vertical *c*-axis orientation, and a supporting layer (10 μm Si and 1 μm SiO₂). The top electrode (240 $\mu\text{m} \times 240 \mu\text{m}$) was covered with CA/GO nanofibers for humidity sensing [50,51].

The S_{11} parameters for the individual FBAR humidity sensor were characterized using a vector network analyzer at a humidity level close to 0% RH (actually 0.54% RH) (see Fig. S15 of the Supplemental Material [45]). The extracted circuit parameters are $C_0 = 12.52$ pF, $L_m = 148.85$ nH, $C_m = 597.96$ fF, and $R_m = 4.71$ Ω , respectively, yielding the fundamental resonant frequency $f_0 = 533.470$ MHz and loss factor $\gamma = 9.44 \times 10^{-3}$. Based on the specified parameters, a gain resonator was designed and implemented on a printed circuit board (PCB), as shown in Fig. 2(b). The negative resistance (R_g) was achieved using an operational amplifier (see Figs. S7 and S16 of the Supplemental Material [45]). The coupling capacitance (C_x) was fine-tuned such that the PT -symmetric system was initially set close to an EP, with $C_x = 4.222$ pF, equating to a coupling coefficient $\kappa = 9.62 \times 10^{-3}$. Even in the absence of external perturbations, minor mismatches between the gain and loss resonators or slight component offsets render the system marginally unstable. This instability, combined with the nonlinear gain, allows the signal to increase until the gain approaches saturation. As a result, the operation of the PT -symmetric system will be close to the EP. This system was placed within a controlled environment chamber, with humidity levels ranging from 0% RH to 90% RH. An oscilloscope was utilized to monitor the time-domain voltages of both the gain and loss resonators across various humidity levels. The frequency and phase difference were subsequently extracted from the recorded voltages using the oscilloscope built-in modules.

Figure 3(a) depicts the measured frequency shift (left y-axis) and phase shift (right y-axis) of PT -symmetric systems as a function of humidity level (bottom x-axis) and the corresponding perturbation (top x-axis) for a humidity range spanning 0.54% RH to 90.03% RH. For comparison, the measured results for an individual FBAR sensor are also provided. The inset of Fig. 3(a) recalibrates these results in a log-log scale, revealing a slope of 1/3 for the frequency shift (within small perturbations) and phase shift in the PT -symmetric system, and a slope of 1 for the frequency shift in the individual FBAR sensor. This supports the theoretical predictions outlined by Eq. (10). According to the intercept of the fitting curves in the inset of Fig. 3(a), the scaling factor (α) for the phase shift and frequency shift in the PT -symmetric system and the frequency shift in the individual FBAR sensor is estimated as 3.374, 0.021, and 0.5, respectively. As far as the PT -symmetric system is concerned, this indicates that the scaling factor for the phase shift is roughly 161 times greater than that of the frequency shift. Despite both exhibiting a cube-root singularity in the limit of small perturbations, the phase shift shows a markedly higher responsivity due to its larger scaling factor. Furthermore, such a behavior for the phase shift expands across a broader range of perturbations compared to the frequency shift.

Based on the data in Fig. 3(a), the sensitivity of the frequency shift and phase shift in PT -symmetric systems is shown in Fig. 3(b) as a function of humidity levels and the associated perturbation, presented in a double-logarithmic format. For comparative purposes, the frequency shift of an individual FBAR sensor is included. Notably, the sensitivity of the phase shift in the PT -symmetric system, due to both an increased scaling factor and a cube-root singularity across

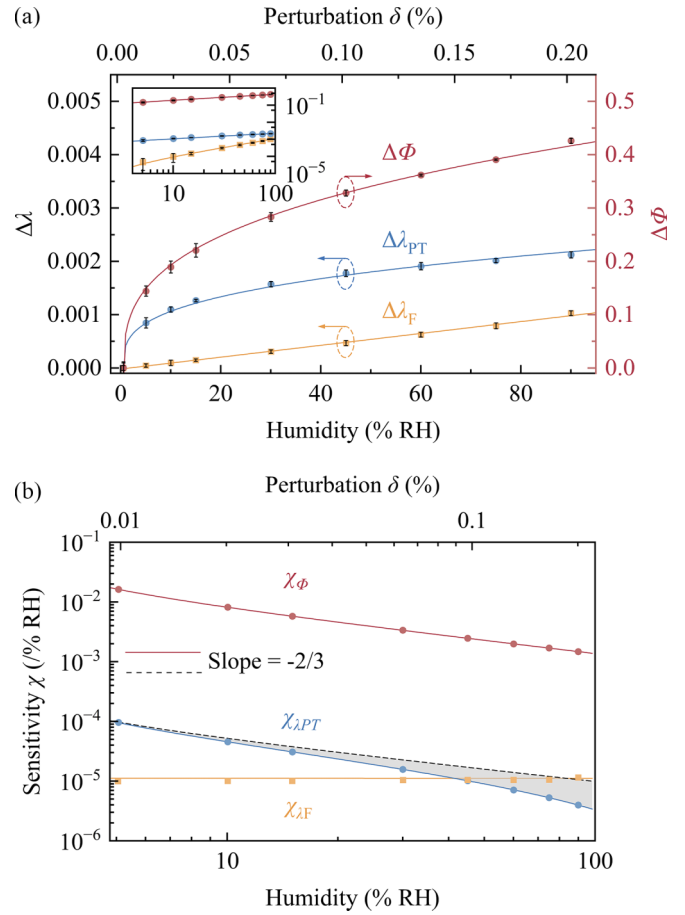


FIG. 3. The measured results. (a) The measured frequency shift and phase shift as a function of relative humidity (% RH, bottom x axis) and corresponding perturbation (δ , top x axis). The frequency shift $\Delta\lambda_F$ denotes the individual FBAR sensor and $\Delta\lambda_{PT}$ denotes the PT -symmetric system, and phase shift ($\Delta\phi$) denotes the PT -symmetric system. The inset shows the same data in a double-logarithmic scale, revealing a slope of 1/3 for the frequency shift and phase shift in the PT -symmetric system, and a slope of 1 for the frequency shift in the individual FBAR sensor. (b) The estimated sensitivity (χ) according to panel (a), plotted in double-logarithmic scale, as a function of relative humidity (% RH, bottom x axis) and corresponding perturbation (δ , top x axis). The highlighted gray area between the black dashed line and blue solid curve represents the deviations of the frequency shift sensitivity in the PT -symmetric system from the cube-root singularity. Error bars in panel (a) represent ± 1 standard deviation obtained from five independent measurements. As the perturbation value goes toward zero, the increase of error bars is mainly due to the uncertainty of the perturbation. The experimental results are denoted by dots, while the solid curves correspond to the fitting results.

the entire measurement range, is significantly greater than the frequency shift of either the PT -symmetric system or the individual FBAR sensor. In the limit of small perturbations ($\leq 5\%$ RH), the estimated sensitivity for the phase shift and frequency shift of the PT -symmetric system and the frequency shift of the individual FBAR sensor is found to be $\chi_\phi = 1.62 \times 10^{-2}/\%$ RH, $\chi_{\lambda_{PT}} = 9.57 \times 10^{-5}/\%$ RH, and $\chi_{\lambda_F} = 9.98 \times 10^{-6}/\%$ RH, respectively. In terms of frequency shifts for the limit of small perturbations, the frequency shift in the

PT -symmetric system, attributed to the cube-root singularity [42–44], is significantly more pronounced than that in the individual FBAR sensor. However, the sensitivity of the frequency shift in the PT -symmetric system diminishes relative to that of the individual FBAR sensor when the humidity level surpasses approximately 40% RH. This implies a substantial reduction in the dynamic range of the PT -symmetric system using the frequency shift as a measure of sensitivity.

In a PT -symmetric system, noise originates from the loss and gain resonators as well as from its interaction with the environment. Phase noise is crucial in phase shift as a measure of sensitivity, since it determines the precision of the measurement. The phase noise spectra of an individual FBAR sensor and the PT -symmetric system in either the absence or presence of humidity perturbations were measured using an oscilloscope (DSOS204A). During phase noise measurements, the sweeping frequency bandwidth was adjusted according to the resonant frequency of the system to be detected. The power spectrum bandwidth was set at 500 kHz, while the phase noise spectrum was defined at 10 kHz. The power of the driving signal was consistently set to 0.01 mW for all these measurements.

The single-sideband phase noise spectra ranging from 0 to 5 kHz frequency offset were recorded. The phase noise spectrum for the individual FBAR sensor without humidity perturbation (actually at 1.03% RH), the PT -symmetric FBAR without humidity perturbation (actually at 0.54% RH, which corresponds to an EP), and the PT -symmetric FBAR with humidity perturbation (at 90.27% RH) was measured, respectively (see Fig. S17 of the Supplemental Material [45]). These results indicate that all phase noises remain below -100 dBc/Hz at a 5-kHz offset from the resonant frequency. The measured power spectra can be quantitatively transformed into the signal to noise ratio (SNR). Corresponding to Fig. 3(a), the SNR for the PT -symmetric FBAR without humidity perturbation (actually at 0.54% RH) and with humidity perturbation (at 90.27% RH) is estimated as 68.408 and 69.236 dB, respectively. It is evident that the SNR remains basically unchanged irrespective of the perturbation. In particular, when the perturbation is reduced toward an EP, no corresponding increase in noise is observed. Notably, the phase noise for the individual FBAR sensor is slightly higher than that of the PT -symmetric FBAR. This suggests that even with the incorporation of gain, the steady-state oscillation in the PT -symmetric system, attributed to nonlinear gain saturation, possesses a self-regulatory mechanism that filters out noise.

Experimentally, minor mismatches influence the initial phase difference (ϕ_{EP}). An external perturbation will result in a shift in the phase difference (ϕ). The outputs for EP sensing are the phase difference under perturbation compared to the initial phase difference. This is inherently self-referenced, thereby negating the effect of mismatches on perturbation measurements.

In addition, for a conventional piezoelectric resonant sensor, the sensitivity increases with the resonant frequency, which can be achieved by decreasing the geometric dimensions of the resonators. However, in this case, the performance of the FBAR is significantly influenced by the adsorbed analyte because of the higher frequency and smaller size of the resonator, and, it is not easy to accurately monitor the continuous frequency shift at these high frequencies. The monitoring of the phase shift, however, is promising at high frequencies.

IV. CONCLUSIONS

In summary, we have shown that by designing a piezoelectric resonant sensor as a PT -symmetric system with nonlinear gain, its sensitivity, as measured by phase shift, can be significantly amplified when the system is biased at an EP. The phase difference in the nonlinear PT system demonstrates a cubic-root singularity of perturbation across a broad measurement range. While FBARs for humidity sensing have been validated experimentally, our approach suggests potential applications of EP phase sensing in other chemical or biological sensors by using diverse sensing materials. Moreover, the gain resonator and coupling capacitors can be integrated on chip with FBARs. This work sets the stage for the development of piezoelectric sensors with ultrahigh sensitivity.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (Grants No. 62271144 and No. 61727812).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available because they contain commercially sensitive information. The data are available from the authors upon reasonable request.

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