

Growth and spreading of quantum resources under random circuit dynamics

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Quantum many-body dynamics generate nonclassical correlations naturally described by quantum resource theories. Quantum magic resources (or nonstabilizerness) capture the deviation from classically simulable stabilizer states, while coherence and fermionic non-Gaussianity measure departure from the computational basis and from fermionic Gaussian states, respectively. We track these resources in a subsystem of a one-dimensional qubit chain evolved by random brickwall circuits. For resource-generating gates, evolution from low-resource states exhibits a universal rise-peak-fall behavior, with the peak time scaling logarithmically with subsystem size and the resource eventually decaying as the subsystem approaches a maximally mixed state. Circuits whose gates do not create the resource but entangle neighboring qubits give rise to a ballistic spreading of quantum resources initially confined to a region of the chain. Our results provide a unified picture of spatiotemporal resource dynamics in local circuits and a baseline for more structured quantum many-body systems.

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I. INTRODUCTION

Nonclassical features of quantum states such as entanglement [1,2], anticoncentration [3–7], and the formation of unitary designs [8–14] have become central diagnostics of quantum computational power beyond classical simulability [15–17]. Quantum resource theories [18] of entanglement [1,2], nonstabilizerness [19–24], coherence [25–27], and bosonic [28,29] or fermionic [30–39] non-Gaussianity provide a unified framework to quantify such nonclassicality, which underpins fault-tolerant quantum computation as well as metrological and thermodynamic advantages. In many-body systems, quantum resources track the complexity of states generated by time evolution, providing a lens on quantum chaos [40–44], information scrambling [45–52], thermalization [53–62], or probe the properties of ground states [63–67], conformal field theories [68–70], monitored systems [71–75], and metrological responses [76,77].

Capturing the dynamical behavior of quantum resources in many-body settings typically requires computing nonlinear resource measures (or monotones) [18,78]. Random quantum circuits [79–82] provide minimally structured models of local quantum dynamics and have yielded a detailed understanding of entanglement growth via the minimal-membrane

picture [83–88] and information scrambling [80,89]. Recent work has extended this analysis to coherence [90–96] and nonstabilizerness [97], showing that these resources saturate on timescales scaling only logarithmically with system size, much faster than the system-size-linear saturation of entanglement entropy. While ergodic Floquet models exhibit analogous phenomenology, local Hamiltonian dynamics leads to a parametrically slower resource growth [98].

Most of these works, however, focus on *global* resource measures, such as the stabilizer entropy [21] or participation entropy [99–108] of the full many-body state. Global metrics quantify how much resource the system carries overall, but do not track its distribution and propagation. Fault-tolerant architectures require logical operations that spread errors only locally, e.g., via geometrically local gates [109–111]. Local resource spreading can also underlie the quantum Mpemba

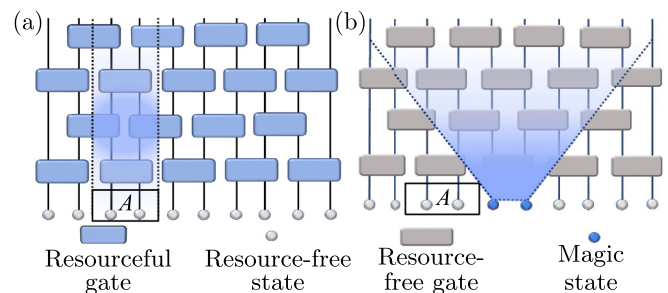


FIG. 1. Our setup. (a) *Local resource dynamics*: We track the resource content of a subsystem A under brickwall circuit dynamics with resource-generating gates starting from a resource-free initial state, uncovering a universal rise-peak-fall profile. (b) *Resource spreading*: Starting from an initial state where the resource is confined to a small region and evolving with circuits whose gates do not generate the resource, we scan the spatiotemporal position of A and expose ballistic spreading of the resource.

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effect [112–115]. Hence, for understanding information scrambling, thermalization, and experimentally relevant local probes, the *local* structure of quantum resource growth is crucial.

In this work, we study the *local dynamics* of quantum resources in one-dimensional random brickwall circuits, focusing on three paradigmatic resources: nonstabilizerness, coherence, and fermionic non-Gaussianity. We consider a subsystem A and analyze two complementary setups (see Fig. 1). In the first setup, circuits are composed of random resource-generating two-qubit gates acting on an initial computational-basis product state. This allows us to characterize *local resource dynamics*, from the initial buildup induced by the gates to the subsequent decay of the resource content of A as it approaches a maximally mixed state. In the second setup, we consider circuits whose gates do not generate the resource, acting on an initial state where the resource is localized in a finite region. By shifting the position of A , we reveal ballistic spreading of the initially localized resourceful cluster.

II. QUANTUM RESOURCES

Resource theories provide a general framework for quantifying nonclassical features of quantum states by distinguishing a set of *free states*, which can be prepared without cost, and *free operations*, which do not generate the resource from free states. Any state outside the free set is deemed resourceful. This naturally leads to the definition of monotones $M(\rho)$, i.e., functions of state ρ of L qubits that satisfy two basic requirements: (1) *faithfulness*— $M(\rho) = 0$ if and only if ρ is a free state, and (2) *monotonicity*— $M(\Lambda[\rho]) \leq M(\rho)$ for all free operations Λ .

Stabilizer states, denoted $|\sigma\rangle$, are pure states obtained from $|0\rangle^{\otimes L}$ by action of Clifford unitaries, i.e., operators that map a Pauli string (an L -fold tensor product of Pauli operators I, X, Y, Z) to another Pauli string. In the resource theory of nonstabilizerness, the free states are those in the convex hull $\text{STAB} \equiv \{\sum_i p_i |\sigma_i\rangle\langle\sigma_i| : p_i > 0, \sum_i p_i = 1\}$ of pure stabilizer states. The free operations are stabilizer protocols [116], which include, in particular, the action of Clifford unitaries. Stabilizer entropy [117] is an experimentally relevant [118–121] measure of nonstabilizerness, but it is monotone only for pure states [117] and is not a faithful measure of nonstabilizerness for mixed states [74, 116]. Since we aim to quantify the nonstabilizerness of a subsystem A consisting of L_A qubits, which is in general in a mixed state ρ_A , we instead employ the log-robustness of magic (LRoM) [24, 122, 123] defined as

$$\mathcal{L}(\rho_A) = \log_2 \min_{x_1, \dots, x_k} \left\{ \sum_{i=1}^k |x_i| : \rho_A = \sum_{i=1}^k x_i |\sigma_i\rangle\langle\sigma_i| \right\}, \quad (1)$$

where k is an integer. The LRoM quantifies the distance of ρ_A from STAB and is a faithful monotone in the resource theory of nonstabilizerness.

Coherence is at the heart of interference phenomena. In the theory of quantum coherence, one distinguishes a specific basis, chosen here as the computational basis $\mathcal{B} = \{|i\rangle\}_{i=1, \dots, 2^L}$, and defines the set of free states as the convex hull $\text{INCOH} = \{\sum_i p_i |i\rangle\langle i| : p_i > 0, \sum_i p_i = 1\}$, referred to as the

incoherent states. Free operations in the theory of coherence [25, 124] are not able to create coherence from the incoherent states. The relative entropy of coherence [25–27, 124],

$$C_d(\rho_A) = S(\rho_A^D) - S(\rho_A), \quad (2)$$

where $S(\rho) = -\text{tr}(\rho \log_2 \rho)$ is the von Neumann entropy and $\rho_A^D = \sum_z \langle z| \rho_A |z\rangle |z\rangle\langle z|$ is a faithful monotone quantifying the coherence of the subsystem A .

Fermionic Gaussian states are of the form $\rho_G = c \exp(\frac{i}{2} \sum_{mn} g_{mn} \gamma_m \gamma_n)$ [125, 126], where $\gamma_{2k-1} = (\prod_{m=1}^{k-1} Z_m) X_k$, $\gamma_{2k} = (\prod_{m=1}^{k-1} Z_m) Y_k$ are Majorana operators, the coefficients g_{mn} form an antisymmetric $2L \times 2L$ matrix, and c is a constant. In the resource theory of fermionic non-Gaussianity $\text{GAUSS} = \{\rho_G\}$ is the set of free states, while free operations include fermionic Gaussian unitaries, i.e., operators of the form $U_G = \exp(\frac{i}{2} \sum_{mn} h_{mn} \gamma_m \gamma_n)$. A faithful monotone of fermionic non-Gaussianity is the relative entropy of non-Gaussianity [31, 32]

$$\mathcal{NG}(\rho_A) = S(\rho_A^F) - S(\rho_A), \quad (3)$$

where ρ_A^F is the fermionic Gaussian state with the covariance matrix $\Gamma_{ab} = -\frac{i}{2} \text{tr}[\rho_A (\gamma_a \gamma_b - \gamma_b \gamma_a)]$. The von Neumann entropy of ρ_A^F is evaluated using free-fermion techniques [127] as $S(\rho_A^F) = \sum_{j=1}^{L_A} H(\frac{1+\lambda_j}{2})$, where λ_j are the Williamson eigenvalues [128, 129] of Γ_{ab} and $H(x) = -x \log_2 x - (1-x) \log_2 (1-x)$.

III. RANDOM CIRCUITS

We now study how these quantum resources evolve under random circuit dynamics. Specifically, we consider a one-dimensional chain of L qubits (see Fig. 1) evolving under a brickwall circuit. The evolution operator reads $U_t = \prod_{r=1}^t U^{(r)}$, where t is the circuit depth, referred to as time. The circuit layers $U^{(r)}$ are given by

$$U^{(2m)} = \prod_{i=1}^{L/2-1} u_{2i, 2i+1}, \quad U^{(2m+1)} = \prod_{i=1}^{L/2} u_{2i-1, 2i}, \quad (4)$$

where each two-qubit gate $u_{i,j}$ is drawn with probability ϵ from an ensemble (depending on the setup and resource) and is the identity $\mathbb{1}_{i,j}$ with probability $1 - \epsilon$. The dilution parameter ϵ controls the rate of the circuit dynamics.

In the first setup, we fix the subsystem A and, starting from the computational basis state $|\psi_1\rangle = |0 \dots 0\rangle$, we monitor the resource monotones [Eqs. (1)–(3)] in the reduced state $\rho_A(t) = \text{tr}_{\bar{A}}[|\psi(t)\rangle\langle\psi(t)|]$, where $|\psi(t)\rangle = U_t |\psi_1\rangle$ is the time-evolved state and $\bar{A}$ denotes the complement of A . For suitable gate ensembles, this setup enables us to track how the circuits generate the resources starting from $|\psi_1\rangle$, which is a free state in the theories of nonstabilizerness, coherence, and non-Gaussianity.

In the second setup, the circuit comprises gates that are free operations, while the resource is localized in a finite “cluster” of the initial state $|\psi_2\rangle$, taken of the form $|\psi_2\rangle = |0 \dots 0\rangle \otimes |m\rangle \otimes |0 \dots 0\rangle$, where $|m\rangle$ denotes a resourceful state supported on L_M central sites of the chain. By varying the position of the subsystem A , this setup allows us to track how resources spread throughout the chain under the circuit dynamics.

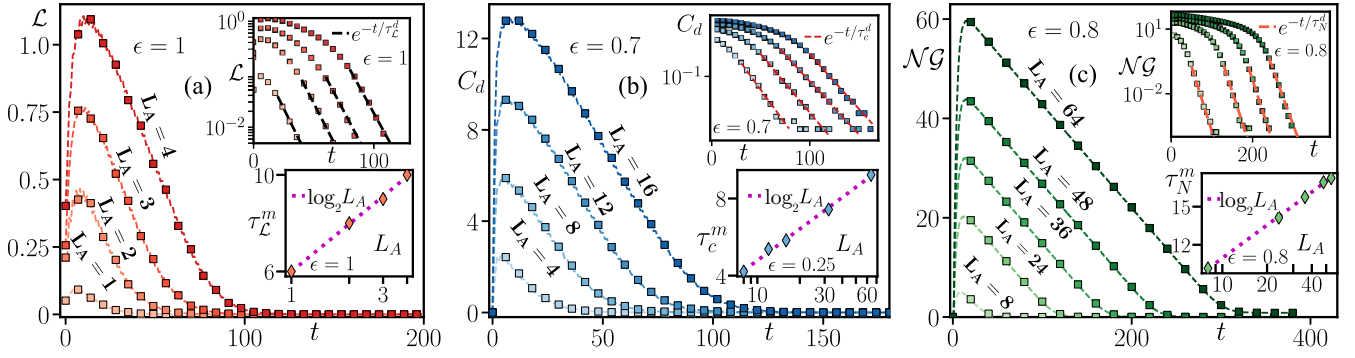


FIG. 2. Local dynamics of (a) LRoM $\mathcal{L}(\rho_A)$ ($L = 24$), (b) relative entropy of coherence $C_d(\rho_A)$ ($L = 128$), and (c) relative entropy of non-Gaussianity $\mathcal{NG}(\rho_A)$ ($L = 128$) for subsystems of size L_A under resource-generating circuits initialized in the free state $|\psi_1\rangle$. Results are averaged over at least 10^4 circuit realizations; the dilution parameter ϵ in each panel is chosen for clarity. Main panels: the rise-peak-fall structure of resource content of $\rho_A(t)$. Top insets: exponential relaxation of the monotones at large t . Bottom insets: peak times scale logarithmically with subsystem size, $\tau^m \sim \log_2 L_A$.

IV. LOCAL RESOURCE GROWTH AND DECAY

We now study the first setup, fixing the initial state to $|\psi_1\rangle$ and monitoring the resource content of the subsystem state ρ_A . For all three resource theories, this protocol reveals the same qualitative behavior: The local resource content first grows, reaches a maximum at a time scaling logarithmically with the subsystem size, and eventually decays as ρ_A approaches the maximally mixed state.

A. Nonstabilizerness

To track the dynamics of LRoM, [Eq. (1)], we choose the gates $u_{i,j}$ in Eq. (4) as Haar-random unitaries from the unitary group $\mathcal{U}(4)$. Fixing the system size to $L = 24$ and considering $L_A \leq 4$, we perform full state-vector simulations to compute $\rho_A(t)$ and then evaluate $\mathcal{L}(\rho_A(t))$ [123]. The results are shown in Fig. 2(a). The gates generate nonstabilizerness and $\mathcal{L}(\rho_A)$ grows, reaching a maximum at time τ_L^m . This maximum occurs at a time $\tau_L^m \sim \log_2 L_A$, as shown in the bottom inset of Fig. 2(a). The logarithmic scaling of τ_L^m is reminiscent of the logarithmic-in-system-size saturation of global nonstabilizerness measures under random circuit dynamics [97]. Nevertheless, the maximum reported here is a qualitatively different result concerning the circuit depth at which a *local subsystem* of size L_A reaches the maximal value. At later times, the dynamics strongly entangles subsystem A with its complement $\bar{A}$, and ρ_A gradually approaches the maximally mixed state, $\rho_A \xrightarrow{t \gg 1} \mathbb{1}_A/2^{L_A}$, where $\mathbb{1}_A$ is the identity operator on A . The maximally mixed state belongs to **STAB**, and LRoM decays exponentially to 0, $\mathcal{L}(\rho_A(t)) \propto e^{-t/\tau_L^d}$, with a decay time τ_L^d independent of the subsystem size; $\mathcal{L}(\rho_A(t))$ reaches a fixed threshold $\theta \ll 1$ on a timescale proportional to L_A (see Appendix C for further analysis).

B. Coherence and non-Gaussianity

The characteristic rise-peak-fall structure of the local resource content under local random circuit dynamics is also found for the measures of coherence [Eq. (2)] and fermionic non-Gaussianity [Eq. (3)], as we demonstrate below. To obtain

a clearer picture of the local resource dynamics, we restrict the gates $u_{i,j}$ in Eq. (4) to be Clifford gates which are, in general, *not* free operations in the resource theories of coherence and non-Gaussianity. The use of Clifford gates enables numerical simulations of systems comprising hundreds of qubits via the tableaux formalism [130], far beyond the reach of full state-vector simulations.

To study the relative entropy of coherence of ρ_A , we choose the gates $u_{i,j}$ from the subset $C_2 \setminus C_2^{\text{inc}}$ of the 11 520 two-qubit Clifford gates C_2 , where C_2^{inc} denotes the set of 768 *incoherent* Clifford unitaries that do not generate coherence (superpositions of two or more computational-basis states) when acting on computational-basis states. Using numerical simulations [131] of a chain of $L = 128$ qubits and computing $C_d(\rho_A)$ within the tableaux formalism using the formula derived in Ref. [114], we obtain the results shown in Fig. 2(b). As in the case of nonstabilizerness, we observe a rise-peak-fall structure of $C_d(\rho_A(t))$, with a peak at times $\tau_c^m \propto \log_2 L_A$, reminiscent of the log-depth anticoncentration [3,91], now visible over a considerably wider range of subsystem sizes $L_A \leq 64$. For $t \gg 1$, $C_d(\rho_A(t))$ decays exponentially $C_d(\rho_A(t)) \propto e^{-t/\tau_c^d}$, and the relative entropy of coherence saturates down to a fixed threshold $\theta \ll 1$ on timescales scaling linearly with L_A (see also Appendix C).

Clifford gates C_2 contain a subset of 192 Clifford matchgates C_2^{Gauss} , which are Clifford unitaries that also fulfill the matchgate condition [132–134], i.e., they are fermionic Gaussian unitaries. To study the evolution of $\mathcal{NG}(\rho_A(t))$, we choose the gates $u_{i,j}$ in Eq. (4) randomly from the set $C_2 \setminus C_2^{\text{Gauss}}$ and simulate a chain of $L = 128$ qubits within the tableaux formalism [131]. The results shown in Fig. 2(c) again exhibit a rise-peak-fall structure, with the peak time $\tau_N^m \propto \log_2 L_A$, consistently with the similarity of fermionic magic resources growth [34] to dynamics of coherence and nonstabilizerness. For $t \gg 1$, we observe an exponential decay $\mathcal{NG}(\rho_A(t)) \propto e^{-t/\tau_N^d}$.

V. RESOURCE SPREADING

We now turn to the second setup, where the system is initialized in a locally resourceful state $|\psi_2\rangle$ and the gates $u_{i,j}$

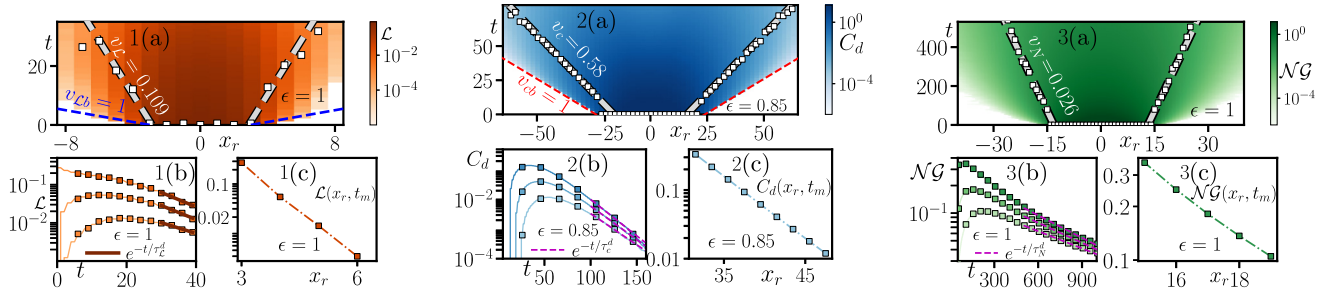


FIG. 3. Spreading of (1) nonstabilizerness ($L = 24$), (2) coherence ($L = 256$), and (3) non-Gaussianity ($L = 128$) from a localized resource cluster under circuits comprising resource-free gates. The results are averaged over 10^4 circuit realizations. Panel (a) displays the emergence of ballistic light cone structure of the spreading; panel (b) shows the evolution of the resource content of ρ_A at fixed x_r ($-x_r = (5, 4, 3), (47.5, 41.5, 35.5)$, and $(19, 17, 15)$ for $R = \mathcal{L}, c, N$, respectively); and panel (c) highlights the exponential attenuation of the peak local resource with distance along the light cone $x = v_R t + \text{const}$ (with $R = \mathcal{L}, c, N$).

are chosen as free operations of the corresponding resource theory. By scanning the position of the subsystem A along the chain, we resolve the spatiotemporal structure of the resource monotoness and reveal the ballistic spreading of the initially localized resource.

A. Nonstabilizerness

To study the spreading of nonstabilizerness, we draw each gate $u_{i,j}$ uniformly from the set C_2 of two-qubit Clifford gates and prepare the initial state $|\psi_2\rangle = |0 \cdots 0\rangle \otimes |m\rangle \otimes |0 \cdots 0\rangle$, with $|m\rangle = |T\rangle^{\otimes L_M}$ a product of single-qubit magic states $|T\rangle = \cos(\pi/8)|0\rangle + \sin(\pi/8)|1\rangle$ [135]. We fix $L_M = 4$, perform state-vector simulations for $L = 24$ qubits to obtain the time-evolved state $|\psi(t)\rangle = U_t |\psi_2\rangle$, and compute $\mathcal{L}(\rho_A(t))$. We introduce a variable $x_r \in [-L/2, L/2]$ denoting the relative position of the center of mass of subsystem A respective to the center of the magic region. The resulting spatiotemporal profile of $\mathcal{L}(\rho_A)$ is shown in Fig. 3(1a).

The spacetime profile of nonstabilizerness exhibits a clear light-cone structure, signaling ballistic spreading of the resource. The earliest detectable nonstabilizerness time grows linearly with the distance x_r of subsystem A from the initial magic cluster, defining an outer light cone expanding with velocity $v_{cb} = 1$ set by gate locality. Along this front, $\mathcal{L}(\rho_A)$ decays exponentially with distance and, at larger distances (e.g. $x_r = 8$ and $t = 8$), it falls below the threshold 10^{-6} . After the first nonzero signal at fixed x_r , the nonstabilizerness of ρ_A increases [see Fig. 3(1b)] reaching a maximum at times t_m satisfying $x_r = v_{\mathcal{L}} t_m + \text{const}$, which reflects ballistic spreading of nonstabilizerness. The peak LRoM decays exponentially with x_r [Fig. 3(1c)]. At late times, $\mathcal{L}(\rho_A)$ decays exponentially back to zero [Fig. 3(1b)] as ρ_A approaches the maximally mixed state. This return of every small subsystem to a free state at long times shows that nonstabilizerness is not a conserved local density under free two-body dynamics: The initially localized resource is transferred into highly nonlocal correlations, so that local magic resources quantified by $\mathcal{L}(\rho_A)$ vanish even though global nonstabilizerness measures of $|\psi(t)\rangle$ remain constant at all t , since U_t consists of stabilizer operations.

B. Coherence and non-Gaussianity

We restrict the gates $u_{i,j}$ to Clifford unitaries [130,131] to study the spreading of coherence and fermionic

non-Gaussianity in chains comprising hundreds of qubits.

For coherence, we choose each $u_{i,j}$ to be the swap gate with probability p , and with probability $1 - p$ a randomly chosen gate from the set C_2^{inc} . The initial state is prepared as $|\psi_2\rangle = |0 \cdots 0\rangle \otimes |m\rangle \otimes |0 \cdots 0\rangle$, with $|m\rangle = [(|0\rangle + |1\rangle)/\sqrt{2}]^{\otimes L_c}$ acting as a source of coherence in the system. Fixing $L = 256$, $L_c = 32$, $L_A = 32$, and $p = 0.8$, we compute the relative entropy of coherence $C_d(\rho_A)$ as a function of time t and the position x_r of subsystem A . Our results, shown in Figs. 3(2a)–3(2c), uncover a spatiotemporal pattern of $C_d(\rho_A)$ analogous to that in the nonstabilizerness case, with light cones at velocity $v_{cb} = 1$ delimiting the region of nonzero coherence and a maximum of $C_d(\rho_A)$ propagating along a light cone with velocity $v_c < 1$.

For non-Gaussianity, we choose the gates $u_{i,j}$ in Eq. (4) randomly from the Clifford matchgate set C_2^{Gauss} , while the initial state is $|\psi_2\rangle = |0 \cdots 0\rangle \otimes |m\rangle \otimes |0 \cdots 0\rangle$, with $|m\rangle = [\frac{1}{4}(|0000\rangle + |1100\rangle + |0011\rangle - |1111\rangle)]^{\otimes L_N}$ a product of four-qubit fermionic magic states [136]. Setting $L = 128$, $L_N = 4$, and $L_A = 16$, we obtain the results for $\mathcal{NG}(\rho_A)$ shown in Figs. 3(3a)–3(3c), which reveal a spatiotemporal structure of non-Gaussianity spreading closely analogous to the nonstabilizerness and coherence cases; the ballistic propagation of the outer front of the non-Gaussianity spreading is further analyzed in Appendix B.

C. Local coherence spreading using sparse-vector simulation

The coherence-spreading results presented above were obtained for circuits of Clifford gates. To corroborate them in a generic, beyond-Clifford setting, we introduce a method of sparse-vector simulation, which allows us to access system sizes far beyond the reach of full state-vector simulations. The key structural property we exploit is that coherence-preserving local two-qubit gates are of the form $U_{\pi, \vec{\phi}} = \sum_{b \in \{00, 01, 10, 11\}} e^{i\phi_b} |\pi(b)\rangle \langle b|$, where π is one of the 24 permutations of the two-qubit computational-basis states and $\vec{\phi} \equiv (\phi_b)$ denotes four arbitrary phases, so that the gate merely permutes computational-basis states and decorates them with phases. Consider a one-dimensional qubit chain of length L prepared in the state $|\psi_2\rangle = |0 \cdots 0\rangle \otimes |m\rangle \otimes |0 \cdots 0\rangle$, where $|m\rangle$ is a superposition of basis states on L_c qubits. The state

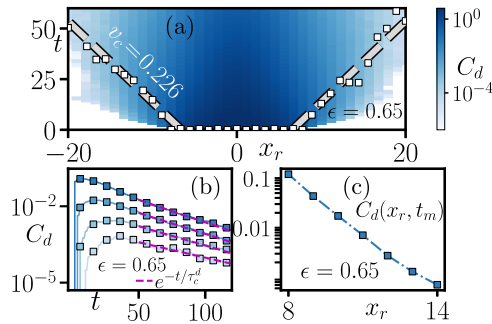


FIG. 4. Coherence spreading for a chain of $L = 64$ qubits from initially localized resource cluster, evolved under random coherence preserving gates and computed via sparse-vector simulations, with $L_c = 8$ and $L_A = 8$. Panel (a) reveals a clear two-front structure in the dynamics, with a sharply defined inner light cone. Panels (b), (c) illustrate the late-time relaxation of the local resource content back to a resource-free steady state and exponential attenuation of the peak local resource with distance from the initial cluster. All the results are averaged over at least 10^4 circuit realizations and overall dilution ϵ is chosen to be 0.65.

$|\psi_2\rangle$ is supported on exactly $N_+(0) = 2^{L_c}$ computational-basis configurations out of the full Hilbert space of dimension 2^L . Because the global time-evolution operator is built as a product of two-qubit gates that only permute local basis states and add phases, the number of nonzero amplitudes is conserved at all times, $N_+(t) = N_+(0) = 2^{L_c}$ for every circuit layer t . This conservation of sparsity of the state vector is what renders sparse-vector simulation efficient even for large L : Although the underlying Hilbert space is exponentially large, the dynamics is confined to a smaller subspace of size $2^{L_c} \ll 2^L$, and it suffices to keep track of only 2^{L_c} nonzero amplitudes to compute the state of the system at any circuit depth t . This yields a simulation method with computational cost scaling exponentially with L_c but only polynomially with system size L , similarly to the approach of Refs. [137,138] for Clifford circuits doped with non-Clifford gates.

In our numerical simulations, we investigate a chain of total size $L = 64$ with an initial magic cluster of size $L_c = 8$ prepared in $|+\rangle$ states and embedded in a background of resource-free states $|0\rangle$, such that $N_+(t) = 2^{L_c} = 256$ for all t . The dynamics is generated by the coherence-preserving two-qubit gates specified above, the relative entropy of coherence $C_d(\rho_A)$ is evaluated for a subsystem of size $L_A = 8$, the overall gate dilution is fixed at $\epsilon = 0.65$, and all observables are averaged over 10^4 random circuit realizations. The results shown in Fig. 4 provide clear evidence for the ballistic spreading of the local resource content $C_d(\rho_A)$ and an exponential suppression of $C_d(\rho_A)$ at larger times and distances, supporting our findings in a beyond-Clifford setup.

VI. STATISTICAL MECHANICS MODEL FOR THE COHERENCE DYNAMIC

To rationalize the universal phenomenology observed in Secs. IV and V, we now develop the analytical framework underlying the dynamics of the relative entropy of coherence $C_d(\rho_A)$, the monotone most amenable to an exact treatment.

To that end, we map the Haar-averaged circuit dynamics onto a two-dimensional classical statistical mechanics model whose boundary conditions encode the two entropies entering the relative entropy of coherence [Eq. (2)].

We consider brickwall circuits of Haar-random two-qubit gates on the free initial state $|\psi_1\rangle = |0 \cdots 0\rangle$, and work with the annealed proxy for $C_d(\rho_A)$ defined as

$$\tilde{C}_d(\rho_A) = \tilde{S}_2(\rho_A^D) - \tilde{S}_2(\rho_A), \quad \tilde{S}_2(\rho) \equiv -\log_2 \overline{\text{tr}(\rho^2)}, \quad (5)$$

which retains the quantitative temporal behavior of C_d provided that fluctuations of $\text{tr}(\rho^2)$ are limited, as confirmed *a posteriori* by comparison with the Clifford simulations of Sec. IV.

A. Replica tensor network mapping

Both terms in Eq. (5) can be written as traces of $k = 2$ copies of $|\psi(t)\rangle\langle\psi(t)|$ with an appropriate operator; cf. Ref. [139] for a pedagogical discussion. Averaging each gate comprising the circuit (4) over $\mathcal{U}(4)$ via the Weingarten formula projects the d^{2k} -dimensional replicated space at each site onto the two-dimensional S_2 subspace [83,84], written in the Choi representation [140,141] as

$$|e\rangle\rangle_x = \sum_{i,j=0,1} |ijj\rangle_x, \quad |s\rangle\rangle_x = \sum_{i,j=0,1} |iji\rangle_x, \quad (6)$$

associated with the identity e and swap s of the permutation group of two elements S_2 . The Haar-averaged circuit becomes a classical partition function with spins $\sigma_x \in \{e, s\}$, governed by the elementary Boltzmann weight $2K_d = 2d/(d^2 + 1) = 4/5$ for qubits [3,83,84,91].

The bulk of the statistical model is fixed by the arrangement of the gates in the circuit, while the bottom boundary is fixed by the initial state $|\psi_1\rangle$, for both terms in Eq. (5) [see Figs. 5(a) and 5(b)]. The two terms differ only in the top boundary at time t . On the complement $\bar{A}$, both are contracted against $\langle\langle e|$. On A , the purity $\text{tr}(\rho_A^2)$ uses the swap boundary $\langle\langle s|$ (partition function Z_P), while the diagonal purity $\text{tr}((\rho_A^D)^2)$ uses the *book boundary condition* [4,102,142] $\langle\langle D|_x = \sum_{i=0,1} \langle iii|_x$, leading to partition function Z_Q . The relative entropy of coherence thus reads

$$\tilde{C}_d(\rho_A) = \log_2 Z_P - \log_2 Z_Q, \quad (7)$$

a free-energy difference between two boundary configurations of the same bulk model. Similarly to Refs. [97,142], we numerically compute $\tilde{C}_d(\rho_A)$ by implementing Z_P and Z_Q as tensor network contractions involving a system of L on-site spins $\sigma_x \in \{e, s\}$.

B. Domain-wall dynamics

The purity Z_P admits the standard domain-wall picture [83,84]: The boundaries of A propagate backward as domain walls whose two end points perform independent random walks, each step suppressed by $2K_d < 1$ [Fig. 5(a)]. This yields the linear growth of the entanglement entropy $\tilde{S}_2(\rho_A) \propto vt$ [Fig. 5(c)], followed by saturation once the typical domain wall size reaches the system size L .

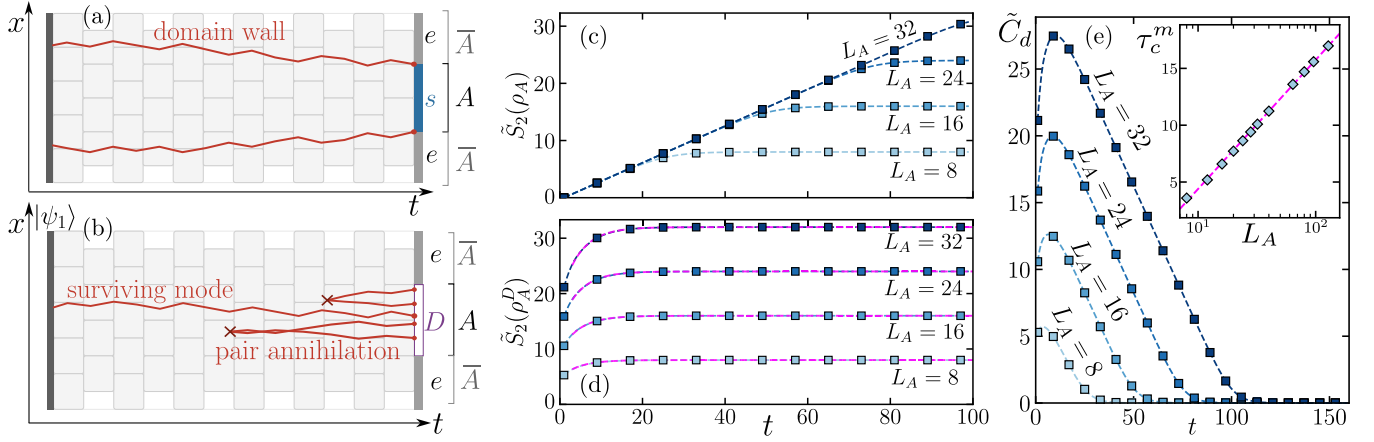


FIG. 5. Replica theory calculation of the relative entropy of coherence $\tilde{C}_d(\rho_A)$. (a), (b) Sketch of Z_P and Z_Q partition functions: time runs left to right, space vertically. The left boundary is fixed by the initial state $|\psi_1\rangle$; the right boundary carries the swap $\langle s|$ [panel (a)] or book $\langle D|$ [panel (b)] operator on subsystem A , and $\langle e|$ on the complement $\bar{A}$. (c), (d) $\tilde{S}_2(\rho_A)$ and $\tilde{S}_2(\rho_A^D)$ vs t from the replica tensor network contraction ($L = 512$); magenta dashed lines: exponential saturation fits. (e) $\tilde{C}_d(\rho_A)$ vs t . Inset: Logarithmic scaling of the peak time τ_c^m as a function of the subsystem size L_A .

The partition function Z_Q is markedly richer [3,91]: The projection of $|D\rangle$ generates a superposition of multiple defect configurations. Most defects pair annihilate during the backward propagation; the leading contribution at longer times is provided by a single surviving mode [Fig. 5(b)]. This mode controls the exponential relaxation $Z_Q(t) - Z_Q^H \sim a_Q L_A (2K_d)^t$ toward the long-time saturation value, where a_Q is a constant and the proportionality factor L_A appears since the surviving mode may originate anywhere in the subsystem L_A . This behavior is shown in Fig. 5(d), where magenta curves show the exponential saturation fits.

C. Rise-peak-fall profile

The contrasting dynamics of Z_P and Z_Q directly explain the profile of $\tilde{C}_d$ [Fig. 5(e)]. Using Eq. (7) and gathering the information about Z_P and Z_Q , we find the leading-order scaling (at $t \lesssim L$)

$$\tilde{C}_d(t) = S_2^H - a_Q L_A (2K_d)^t - vt, \quad (8)$$

where $S_2^H = -\log_2 Z_Q^H$. At early times, the fast exponential saturation of the $-\log_2 Z_Q(t)$ contribution leads to an increase of $\tilde{C}_d(t)$. At longer times, the entanglement contribution vt amounts to a linear decay of $\tilde{C}_d(t)$ that persists up to $t \approx L$, followed by an exponential decay of $\tilde{C}_d(t)$ when the entanglement contribution saturates to its long-time value. Differentiating Eq. (8) with respect to time, and taking into account the prefactor $a_Q L_A \propto L_A$, we find the logarithmic scaling $\tau_c^m \sim \log_2(L_A)$ of the maximum of local resource content $\tilde{C}_d(t)$.

D. Coherence spreading

The features of the above replica mapping of $\tilde{C}_d(t)$ also provide insight into the coherence spreading of Sec. V (Fig. 3). In that scenario, the bulk of the statistical mechanics model is *markedly distinct* from the case analyzed above: It comprises gates that do not generate coherence, i.e., leave Z_Q invariant. Nevertheless, the entanglement entropy—

$\log_2 Z_P$ grows under the action of the gates. These two factors, together with the causal structure fixed by the locality of gates, lead to the light-cone spatiotemporal profile of the coherence spreading from an initially localized resourceful region.

E. Generalization

The replica approach above exploits the fact that $\tilde{C}_d$ [Eq. (5)] is built from Rényi-2 entropies—quadratic functions of the density matrix—which are naturally computed within a $k = 2$ replica framework. Other resource monotones studied in this work do not admit such a representation: The relative entropy of non-Gaussianity $\mathcal{NG}(\rho_A)$ [Eq. (3)] involves the entropy of a fermionic Gaussian state, which through the dependence on the full covariance matrix becomes a function of ρ_A of arbitrary order; the log-robustness of magic $\mathcal{L}(\rho_A)$ [Eq. (1)] is a highly nonlinear problem of a constrained optimization over the stabilizer polytope; and the mana $\mathcal{M}(\rho_A)$ (see Appendix A) contains absolute values of Wigner-function entries that break any polynomial dependence on ρ_A . None of these can be cast as a free-energy difference of a replica partition function.

Notably, the numerical results of Secs. IV and V demonstrate that the rise-peak-fall profile and the spatiotemporal spreading patterns are *universal*: They appear for every resource monotone we examine, well beyond the reach of the replica description. This universality points to a general mechanism governing subsystem resource dynamics in random circuits: the competition between fast-growing “bulk” contributions—associated with local scrambling within A , which saturate exponentially with an $O(1)$ rate—and slower “boundary” contributions—controlled by entanglement generation across the $A/\bar{A}$ cut, which grow on a timescale proportional to L_A . The mismatch of these two timescales generically produces the rise-peak-fall profile, while the causal structure imposed by the locality of gates determines the spatial spreading front, independently of the specific monotone employed.

VII. DISCUSSION AND CONCLUSIONS

In this work, we studied the spreading of quantum resources that underlie the classical complexity of quantum states—nonstabilizerness, coherence, and fermionic non-Gaussianity—in local random circuit dynamics. For resource-generating gates, we uncovered a rise-peak-decay behavior of the local resource content, with peak times scaling logarithmically with the subsystem size, $\tau^m \sim \log_2 L_A$. This signals a rapid onset of local resourcefulness and the buildup of nonclassical features in local subsystems on a timescale parametrically shorter than the linear timescale required to generate correlations across the subsystem. In a complementary setup, where the gates are free operations that do not generate the resource and the chain is initialized with a localized resourceful cluster, we found ballistic spreading of the local resource content. Taken together, our results demonstrate a unified phenomenology for the spatiotemporal dynamics of nonstabilizerness, coherence, and fermionic non-Gaussianity, pointing to a form of universality in resource spreading governed primarily by locality and unitarity, rather than microscopic details of the dynamics or of the specific resource theory. We therefore conjecture that the phenomenology uncovered here applies broadly to local ergodic quantum many-body systems.

For the coherence, the rise-peak-fall of the local resource content (Fig. 2) is explained quantitatively by the replica statistical mechanics model of Sec. VI, which traces this profile to the mismatch between the fast, bulklike saturation of the contribution— $\log_2 Z_Q$ —governed by local resource injection within A —and the slow, linear-in-time growth of the entanglement contribution— $\log_2 Z_P$ across the $A/\bar{A}$ cut, yielding peak times $\tau_m \sim \log_2 L_A$. While the replica approach applies directly only to $\tilde{C}_d$ —a two-replica quantity—our numerical simulations indicate that *the same phenomenology*, including the logarithmic peak-time scaling and the exponential late-time decay, holds for *all resource monotones* studied here. This demonstrates a *robust universality* in the dynamics of local quantum resources. A similar behavior was recently observed [143] for entanglement flatness [144], which bounds nonlocal nonstabilizerness [145], and for asymmetry [146]. The ballistic spreading of resources (Fig. 3) is rooted in a similar mechanism: The causal structure of local gates propagates the resource front ballistically, while the exponential attenuation behind the front—driven by the “boundary term,” i.e., the entanglement buildup—sharply distinguishes resource spreading from the transport of conserved local densities, as further confirmed beyond the Clifford setting by the sparse-vector simulations of Sec. VC. We further corroborate these findings in Appendix A, where we show identical dynamics for mana [19] in qutrit chains evolved under local circuits.

Our results open several conceptual avenues for exploration. A central challenge is to extend the analytical framework for the phenomenology uncovered here, for instance by relating nonstabilizerness and other resources to the slow hydrodynamic modes [89,147–150] of many-body dynamics or utilizing the replica framework to perform calculations, a first step in that direction in the limit of infinite local Hilbert space dimension was already done in Ref. [151]. It would

also be compelling to move beyond random circuits to fully ergodic Floquet drives [98,152] and genuinely chaotic Hamiltonian evolutions [153], where similar universal structures may emerge in a less coarse-grained setting. Moreover, incorporating symmetries [98,150,154] and probing the role of integrability [155] and ergodicity breaking [55,56,156–158] could shed further light on more exotic regimes of quantum resource spreading. We leave these questions for future work.

Note added. Recently, we became aware of two independent studies by Maity and Hamazaki [159] and Bejan *et al.* [160] on the spreading of local magic under Clifford dynamics. While approach of our study is markedly different, the results agree in the overlapping regime.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [161].

APPENDIX A: LOCAL MANA SPREADING IN QUTRITS

We further substantiate the picture of nonstabilizerness spreading established for qubits in Secs. IV and V by examining local resource dynamics in a one-dimensional chain of L qutrits (on-site Hilbert space dimension $d = 3$). As a probe of the resource content in a subsystem A , we employ the mana [19], a Wigner-function-based quantifier of nonstabilizerness that constitutes a faithful nonstabilizerness monotone for mixed states. Let $\omega = e^{2\pi i/d}$ and introduce the generalized Pauli operators

$$Z = \sum_{m=0}^{d-1} \omega^m |m\rangle\langle m|, \quad X = \sum_{m=0}^{d-1} |m+1 \bmod d\rangle\langle m|. \quad (\text{A1})$$

On an L -site chain, we denote by X_ℓ and Z_ℓ the corresponding operators acting on site ℓ . For each phase-space point $r = (r_1^x, r_1^z, \dots, r_L^x, r_L^z) \in \mathbb{Z}_d^{2L}$, we define the Pauli string $P_r = \prod_{\ell=1}^L \omega^{\frac{(d+1)}{2} r_\ell^z r_\ell^x} X_\ell^{r_\ell^x} Z_\ell^{r_\ell^z}$. These operators generate the

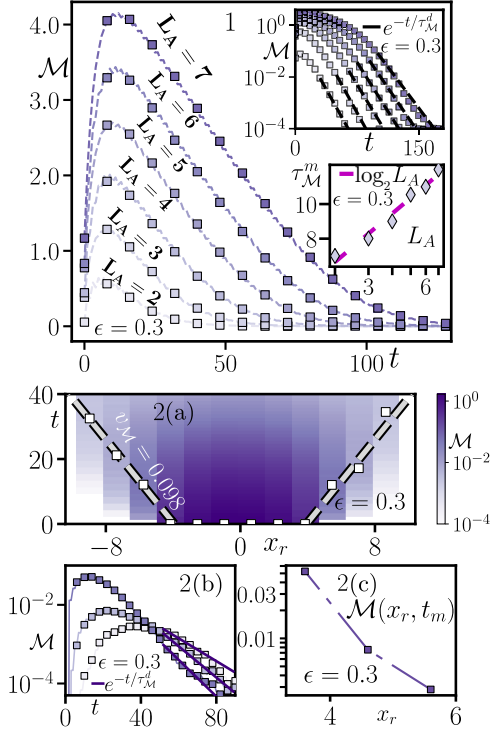


FIG. 6. Dynamics of mana $\mathcal{M}(\rho_A)$ in qutrit chains. Panel (1): Evolution from a resource-free initial state under resourceful two-qutrit Haar gates for $L = 16$ and various L_A , showing a universal rise-peak-fall profile (main), late-time exponential decay (top inset), and peak times scaling logarithmically with L_A (bottom inset). Panel (2): $\mathcal{M}(\rho_A)$ for $L = 14$ with an initial magical cluster of size $L_M = 4$ and $L_A = 3$ under resource-free gates, exhibiting a light-cone spreading pattern and late-time exponential decay to a free state. Results averaged over at least 10^4 circuit realizations and overall dilution ϵ in each panel are chosen for clarity.

phase-space point operators [162]

$$A_0 = \frac{1}{d^L} \sum_{r \in \mathbb{Z}_d^{2L}} P_r, \quad A_r = P_r A_0 P_r^\dagger, \quad (\text{A2})$$

which in turn define the discrete Wigner function

$$W_r(\rho) = \text{tr}(A_r \rho) / d^L. \quad (\text{A3})$$

The nonstabilizerness of a state ρ is then quantified by the mana [19,97,163,164]

$$\mathcal{M}(\rho) = \log_2 \left(\sum_{r \in \mathbb{Z}_d^{2L}} |W_r(\rho)| \right), \quad (\text{A4})$$

which vanishes for stabilizer states and increases monotonically with the negativity of the Wigner quasiprobability distribution.

With these qutrit degrees of freedom in place, we perform an analysis fully analogous to the qubit case, computing the time-evolved state as a state vector and calculating mana with methods of Ref. [165]. In the first setup, starting from the resource-free product state $|0 \cdots 0\rangle$ and evolving under local two-qutrit Haar-random gates, we find that the local nonstabilizerness in A displays the same universal rise-peak-fall profile

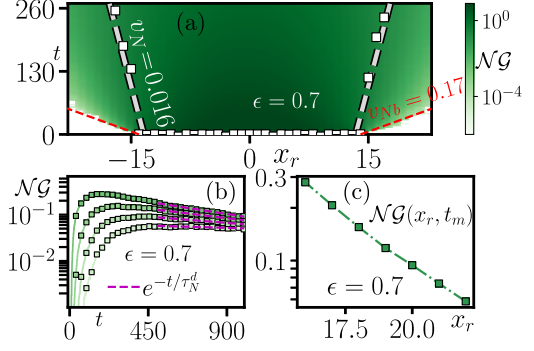


FIG. 7. Spatiotemporal spreading of a localized non-Gaussian magic cluster in a 128-qubit chain under resource-free two-body Gaussian (matchgate) dynamics, selecting $u_{i,j}$ gates from ten Clifford matchgates propagating the initial non-Gaussian cluster in left- and right-moving ballistic modes and ten generic Clifford matchgates. We fix $(L_N, L_A) = (16, 16)$, set $\epsilon = 0.7$, and average over 10^4 circuit realizations. The outer front in panel (a) is markedly more visible than in the case of Fig. 3(3a), highlighting the role of the left/right propagating gates; all other features in panels (a)–(c) remain qualitatively the same as in Fig. 3.

as observed for qubits, illustrated in Fig. 6(1a) for $L = 16$ and $L_A = 2-7$. In particular, the timescale τ_M^m at which the reduced mana in A attains its maximum again grows only logarithmically with the subsystem size, $\tau_M^m \propto \log_2 L_A$ [bottom inset of Fig. 6(1a)], reinforcing the fast-scrambling phenomenology of local magic spreading across different on-site Hilbert space dimensions. Furthermore, Figs. 6(2a)–6(2c) show the spacetime pattern of local mana in a chain of $L = 14$ qutrits, initialized in a state featuring a magic cluster of size $L_M = 4$, where each single-qutrit magic state is $|m\rangle = e^{-i\frac{2\pi}{9}(X_i + X_i^\dagger)}|0\rangle$, and evolved under a brickwall circuit composed of uniformly sampled two-qutrit Clifford gates $u \in C_2(d)$ [166]. The resulting mana fronts for a subsystem of size $L_A = 3$ closely parallel the nonstabilizerness spreading observed in the qubit case [Figs. 3(1a)–3(1c)], indicating that the qualitative features of local magic spreading are robust to the underlying on-site dimensionality.

APPENDIX B: BALLISTIC PROPAGATION OF THE OUTER FRONT IN THE NON-GAUSSIANITY SPREADING

In Fig. 3, we observed that the spatiotemporal regions of nonzero nonstabilizerness and coherence are delimited by light cones with velocities $v_{\mathcal{L}b} = 1 = v_{cb}$. In contrast, the light cone bounding the region of nonvanishing $\mathcal{NG}(\rho_A)$ corresponds to a smaller velocity. To elucidate this behavior, we uncover a directional structure within the set of Clifford matchgates: There exist two distinguished classes of Clifford matchgates, one transporting non-Gaussianity ballistically to the left and the other to the right, while no single matchgate generates simultaneous left- and right-moving ballistic propagation (as the swap gate does in the coherence case). Moreover, the associated ballistic velocity is comparatively smaller, so non-Gaussianity still spreads ballistically but more slowly than coherence and nonstabilizerness. In addition, the resource content in subsystems away from the non-Gaussian

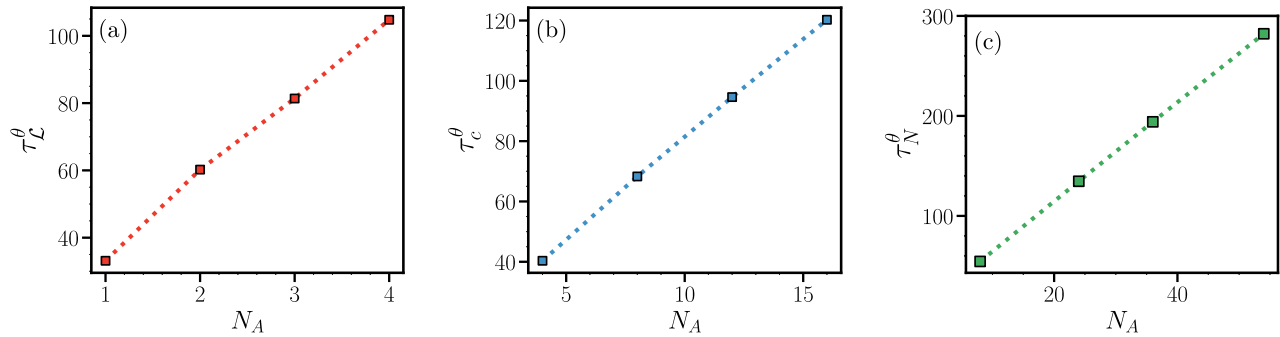


FIG. 8. Subsystem-size dependence of the threshold time τ_θ for three representative resource measures: (a) nonstabilizerness ($L = 24$ qubits), (b) coherence ($L = 128$ qubits), and (c) non-Gaussianity ($L = 128$ qubits). Starting from a resourceless initial state, the system evolves under resourceful dynamics generated by two-qubit operations for the setup shown in Fig. 2. We define τ_θ implicitly by the threshold condition $M(\tau_\theta) = \theta$, with θ fixed to 0.01 for nonstabilizerness and to 0.1 for coherence and non-Gaussianity. Across all three measures, τ_θ grows linearly with the subsystem size L_A .

center decays exponentially with distance, further suppressing long-range non-Gaussian correlations. Taken together, these effects explain why a sharp ballistic outer front is not readily visible for non-Gaussianity spreading.

To sharpen this picture, we perform numerical simulations in which we explicitly include both the left- and right-moving Clifford matchgates, supplemented by an additional set of ten generic matchgates, for a chain of length $L = 128$, averaging the results over at least 10^4 circuit realizations. In this tailored setting, the ballistic propagation of the region of nonzero $\mathcal{NG}(\rho_A)$ becomes markedly more pronounced than in the fully generic Clifford matchgate ensemble, as shown in panel (a) of Fig. 7. Other features of the spatiotemporal fermionic non-Gaussianity propagation remain qualitatively the same as those in Figs. 3(3a)–3(3c).

APPENDIX C: SUBSYSTEM-SIZE DEPENDENCE OF RESOURCE-MONOTONE DECAY TIMES

As discussed in Sec. IV, the resource monotones, under resource-generating dynamics, at late times relax exponentially to zero as the subsystem approaches the maximally mixed state. Here, we quantify this decay using two complementary approaches. First, we fit the late-time profiles of resource monotones to exponentially decaying functions to extract the corresponding decay times and study their dependence on the subsystem size L_A . Second, we employ a threshold-based characterization of the decay time by defining a threshold time τ_θ such that for $t > \tau_\theta$ the resource measures

fulfill $M(\rho_A(t)) < \theta \ll 1$, and we examine how τ_θ scales with L_A .

Restricting to the first setup (Sec. IV), i.e., resourceful dynamics initialized in an initially resourceless state, we examine the late-time decay of the local resource content for three resource quantifiers: nonstabilizerness, coherence, and non-Gaussianity. Fitting the long time decays of monotones, respectively, with $\mathcal{L}(\rho_A(t)) \propto e^{-t/\tau_\mathcal{L}^d}$, $C_d(\rho_A(t)) \propto e^{-t/\tau_c^d}$, and $\mathcal{NG}(\rho_A(t)) \propto e^{-t/\tau_N^d}$, we extract the decay constants τ_R^d (with $R = \mathcal{L}, c, N$). We obtain $\tau_\mathcal{L}^d = (9.42, 9.30, 9.50)$ for $L_A = (2, 3, 4)$ (nonstabilizerness), $\tau_c^d = (9.90, 10.01, 9.65)$ for $L_A = (8, 12, 16)$ (coherence), and $\tau_N^d = (7.20, 7.33, 7.48, 7.42, 7.46)$ for $L_A = (8, 16, 24, 32, 36)$ (non-Gaussianity). Across all three diagnostics, the extracted τ_R^d values fluctuate by at most $\sim 5\%$ over the explored range of L_A , indicating an essentially subsystem-size-independent late-time decay.

In Fig. 8, we turn to the threshold timescale τ_θ for the same setup and the same three resources. Here, the behavior is markedly different: τ_θ increases linearly with L_A , indicating $\tau_\theta \propto L_A$. Taken together, these observations point to a clear separation of dynamical regimes. The scaling $\tau_\theta \propto L_A$ is consistent with ballistic propagation of correlations, where the onset of resource depletion is set by a light-cone traversal across a subsystem of size L_A . In contrast, the subsequent late-time relaxation is governed by local dynamics, leading to an exponential decay of the resource monotones with an L_A -independent timescale τ_d . In this sense, the ballistic spreading controls the *onset* of resource depletion, while the *rate* of the asymptotic decay is determined by a local equilibration process.

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