

Neural-network-assisted Boltzmann approach for dilute microswimmer suspensions

Haruki Hayano¹* and Akira Furukawa¹†*Institute of Industrial Science, The University of Tokyo, Meguro-ku, Tokyo 153-8505, Japan*Kang Kim²‡*Division of Chemical Engineering, Department of Materials Engineering Science, Graduate School of Engineering Science, The University of Osaka, Toyonaka, Osaka 560-8531, Japan*

(Received 28 October 2025; accepted 31 July 2026; published 21 August 2026)

We introduce a neural-network-assisted Boltzmann framework that learns the binary-collision map of microswimmers from two-body scattering data and uses it as microscopic input to evaluate Boltzmann collision integrals. For a representative swimmer model, the learned map provides quantitative predictions for rotational and translational diffusivities, in close agreement with direct many-body hydrodynamic simulations, and it also enables a linear-stability analysis of the isotropic state against polar ordering within the same dilute, binary-collision-dominated regime. The key advance is not merely that a neural network approximates collision outcomes, but that it provides a compact and continuous representation of the full collision-geometry manifold, thereby making the Boltzmann collision integrals directly accessible and revealing which collisions dominate the overall transport and alignment/disalignment. Because the framework relies only on two-body collision statistics, it provides a transferable, data-driven route from microscopic scattering data to transport properties and ordering tendencies in dilute active suspensions.

DOI: [10.1103/7565-51x2](https://doi.org/10.1103/7565-51x2)

Suspensions of microswimmers, including bacteria, algae, and synthetic active colloids, exhibit dynamical behaviors absent in passive colloids, such as anomalous rheology, mesoscale turbulence, and motility-induced transport [1–16]. These phenomena arise from the interplay of self-propulsion, hydrodynamic interactions (HIs), steric and chemical interactions, fluctuations, and boundary effects or external fields.

Interparticle/intercellular interactions govern both local dynamics and emergent collective states. In mesoscopic descriptions such as Smoluchowski kinetics and continuum hydrodynamics, their effects are often incorporated into effective collision kernels and transport coefficients [16–20]. Obtaining reliable values for these quantities requires resolving the elementary collisional processes, particularly those controlled by near-field HIs and steric contact. In dilute suspensions, binary collisions dominate, and a Boltzmann-type kinetic equation that treats them explicitly provides a natural coarse-graining route [21–31]. However, near-field coupling makes the two-body scattering map highly nonlinear and sensitive to swimmer geometry and propulsion type (pusher, puller, neutral) [32,33]. While detailed two-body solutions exist for specific model swimmers such as squirmers [34–36],

extending those results to arbitrary swimmer designs remains challenging.

In this Letter, we introduce a physics-structured neural-network-assisted Boltzmann framework for dilute active suspensions. Rather than using a neural network (NN) as a black-box predictor, we learn the full binary-collision map of microswimmers from direct hydrodynamic simulations and use it as microscopic input to evaluate the Boltzmann collision integrals. For a representative microswimmer model, this yields quantitative predictions for the rotational and translational diffusivities and for the linear stability of the isotropic state against polar ordering. Notably, because the learned map resolves the collision geometry continuously, it makes visible which binary encounters dominate transport and polar ordering. We validate these predictions through many-body hydrodynamic simulations in the dilute regime ($\phi \lesssim 0.04$): Once binary-collision data are available, either numerically or experimentally, the same workflow can be transferred to other swimmer models in the dilute, binary-collision-dominated regime.

Model swimmers and simulation method. For the preparation of the dataset, we conduct direct hydrodynamic simulations of a binary collision of two model microswimmers. Our model swimmer used in this study is exactly the same as that used in our previous shear flow study [37], which is a simplified model of a rodlike swimmer with a flagellum such as *Escherichia coli*. The model microswimmer, schematically shown in Fig. 1(a), is composed of body and flagellum parts. The body part is treated as a rigid body, while the flagellum part is regarded as a massless “phantom” particle simply following the body’s motions. A self-propulsion force $F_A \hat{n}$ is exerted on the body part, the flagellum part exerts

*Contact author: hayano@iis.u-tokyo.ac.jp

†Contact author: furu@iis.u-tokyo.ac.jp

‡Contact author: kk@cheng.es.osaka-u.ac.jp

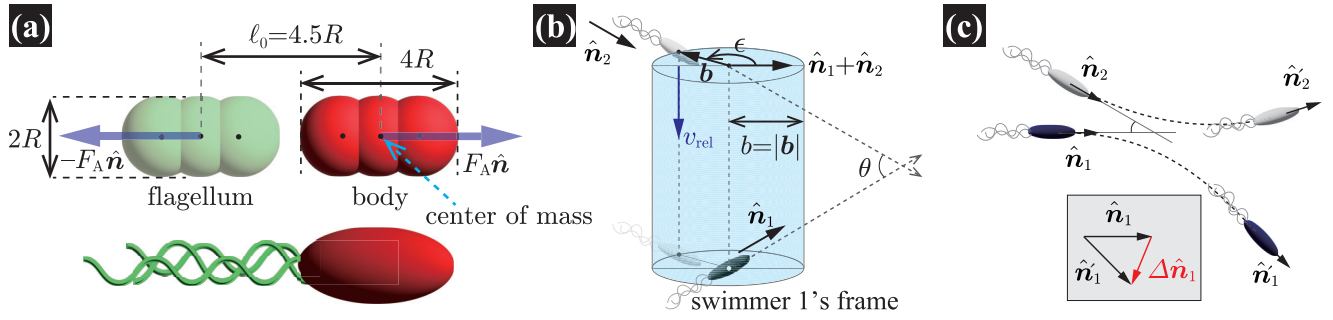


FIG. 1. (a) Schematic of our model microswimmer resembling *E. coli*. The body is treated as rigid, whereas the flagellum is modeled as a massless “phantom” particle. Both the body and the flagellum are represented by three overlapping spheres of radius R . Further details of the swimmer model and its construction are provided in Ref. [37]. (b) Reference precollisional geometry of two microswimmers specified by the relative angle θ , impact parameter b , and azimuthal angle ϵ . These parameters are defined from hypothetical noninteracting straight trajectories at the swimming speed v_s ; thus, b labels the incoming geometry rather than a quantity held fixed during the actual collision. The vector $\mathbf{b}$ is perpendicular to the relative velocity: $\mathbf{b} \perp \mathbf{v}_{\text{rel}}$. See Sec. I A of the Supplemental Material [41] for details of the initial configuration. (c) Schematic of an actual interacting binary collision, during which the swimmers can be reoriented and their trajectories can be deflected by hydrodynamic and steric interactions. The precollisional orientations $\hat{n}_1$ and $\hat{n}_2$ change to the postcollisional orientations $\hat{n}'_1$ and $\hat{n}'_2$ after the collision. We denote the orientation changes as $\Delta \hat{n}_1 = \hat{n}'_1 - \hat{n}_1$ and $\Delta \hat{n}_2 = \hat{n}'_2 - \hat{n}_2$.

the opposite force $-F_A \hat{n}$ directly on the solvent fluid. Here, $\hat{n}$ is the unit vector of the orientation of the swimmer. Full details of the model swimmer can be found in our previous work [37]. The hydrodynamic interactions between swimmers are incorporated using the smoothed profile method [38–40], which is one of the mesoscopic simulation techniques that can treat HIs in a computationally efficient manner.

Binary collision of microswimmers. A precollisional configuration of two swimmers with orientations $\hat{n}_1$ and $\hat{n}_2$ is parametrized by three variables (θ, b, ϵ) , as illustrated in Fig. 1(b). These variables are defined from hypothetical noninteracting straight trajectories of isolated swimmers with speed v_s , and therefore serve only as labels of the incoming geometry. During an actual hydrodynamic encounter, neither the trajectories nor the closest separation are constrained to remain fixed. The three parameters are defined as follows: (1) The relative angle is defined as $\theta \equiv \arccos(\hat{n}_1 \cdot \hat{n}_2)$ ($0 < \theta \leq \pi$). (2) Let $\mathbf{b}$ be the vector connecting the centers of the two swimmers at the point of closest approach of the corresponding noninteracting reference trajectories, with $\mathbf{b} \perp \mathbf{v}_{\text{rel}}$, where $\mathbf{v}_{\text{rel}} = v_s(\hat{n}_1 - \hat{n}_2)$ and v_s is the swimming speed of an isolated swimmer. The impact parameter is defined as $b \equiv |\mathbf{b}|$ ($0 \leq b < \infty$). (3) The azimuthal angle ϵ ($-\pi < \epsilon \leq \pi$) is the angle between $\mathbf{b}$ and $\hat{n}_1 + \hat{n}_2$. For identical axisymmetric swimmers, particle-exchange and axial symmetries allow the range to be limited to $0 \leq \epsilon \leq \pi$.

Based on (θ, b, ϵ) , we generated a dataset of 2048 simulated collisions storing the postcollisional orientations $(\hat{n}'_1, \hat{n}'_2)$, schematically illustrated in Fig. 1(c). The parameter ranges were chosen as follows: $\pi/16 \leq \theta \leq \pi$, because for small relative angles the orientational change is sufficiently small to neglect and introducing a lower cutoff in θ does not affect the predictions; $0 \leq b \leq 2\ell_0$, since the orientational change is negligible for $b > 2\ell_0$ in our simulations (refer to Sec. I B of the Supplemental Material [41]), where ℓ_0 is the swimmer length, and $0 \leq \epsilon \leq \pi$.

Training and evaluation of neural network. We train an NN to predict the postcollisional state of a binary collision from its precollisional configuration. The NN is a fully connected

feedforward model with three hidden layers, each consisting of 384 neurons with rectified linear unit (ReLU) activation. The input layer contains four variables representing the precollisional state: $\cos \theta$, $\sin \theta$, $b \cos \epsilon$, and $b \sin \epsilon$. When (θ, b, ϵ) were used directly as the network inputs, the training loss did not decrease satisfactorily. One possible reason is that this representation does not reflect the natural Cartesian structure of the quantities relevant to the network outputs. For example, the relative orientation is more naturally represented by $\sin \theta$ and $\cos \theta$, whereas the transverse relative position is represented by $b \sin \epsilon$ and $b \cos \epsilon$. In addition, the direct use of angular variables introduces artificial discontinuities due to their periodicity. The output layer contains six variables corresponding to the postcollisional orientation vectors of the two swimmers, $\hat{n}'_1 = (\hat{n}'_{1,x}, \hat{n}'_{1,y}, \hat{n}'_{1,z})$ and $\hat{n}'_2 = (\hat{n}'_{2,x}, \hat{n}'_{2,y}, \hat{n}'_{2,z})$.

The network is trained using the Adam optimizer with a learning rate of 8×10^{-4} and a batch size of 16. The loss function is defined as the mean squared error between the predicted and simulated postcollisional orientation vectors. Training is performed for 100 epochs with a 4:1 split between training and validation data, and cross-validation is used to suppress overfitting. The relative error and mean angular error of the trained NN are evaluated on the test dataset (10% of the total dataset and not included in the training data) and found to be 4.7% and 0.048 rad ($= 2.7^\circ$), respectively. In addition, with the present convention for the reduced parameter domain, swimmer exchange corresponds to $\epsilon \rightarrow \pi - \epsilon$. We numerically checked this symmetry by evaluating

$$\left| \frac{[\hat{n}'_1 \cdot \hat{n}_1](\theta, b, \epsilon) - [\hat{n}'_2 \cdot \hat{n}_2](\theta, b, \pi - \epsilon)}{[\hat{n}'_1 \cdot \hat{n}_1](\theta, b, \epsilon)} \right|$$

and found that the mean relative error is 1.4%, indicating that the trained NN respects the exchange symmetry to good accuracy. All computations are implemented in Python using TENSORFLOW with the Keras API [42].

Figure 2 shows the predicted orientational change of swimmer 1, $|\Delta \hat{n}_1| = |\hat{n}'_1 - \hat{n}_1|$, due to a collision with precollisional parameters (θ, b, ϵ) . The results indicate a strong

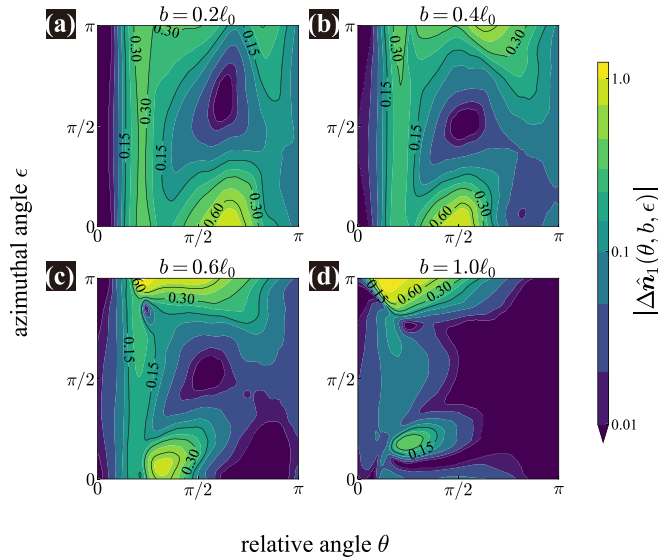


FIG. 2. Predicted orientational change of the swimmer 1, $|\Delta\hat{\mathbf{n}}_1| = |\hat{\mathbf{n}}'_1 - \hat{\mathbf{n}}_1|$, as a function of the relative angle θ , azimuthal angle ϵ , and impact parameter b . The color indicates the magnitude of the orientational change.

dependence on the impact parameter b . For smaller b , swimmer 1 undergoes large orientational changes over a wide range of initial conditions (θ, ϵ) . When the two swimmers pass each other at close proximity, near-field HIs are strong enough to substantially modulate their trajectories during the encounter. In particular, for initial values $\theta \simeq \pi/4$ and $(\theta, \epsilon) = (\pi/2, 0)$, the extensile flow generated in their front regions effectively acts as a repulsive interaction, producing pronounced reorientation. In contrast, for larger b , the trajectories are only weakly perturbed over the passing time, and significant reorientation occurs only within a narrow range of (θ, ϵ) that corresponds to direct head-to-tail collisions—specifically, between the head of swimmer 2 and the tail of swimmer 1. In our model, the flagellum (tail) is treated as a massless phantom particle, while direct repulsive (excluded-volume) interactions between swimmers are retained. As a result, when a swimmer collides with another's flagellum—located far from the latter's center of mass—the large lever-arm effect produces a significant torque, which in turn causes a larger reorientation of the swimming direction. Note that, for most b values, the configuration $(\theta, \epsilon) = (\pi/2, \pi/2)$ produces minimal reorientation because this parameter corresponds to a side-by-side configuration where the swimmers weakly interact hydrodynamically.

Evaluation of diffusivity by kinetic theory with a trained NN. We evaluate self-diffusion in dilute microswimmer suspensions by isolating the binary-collision contribution via a kinetic-theory calculation in which the postcollision orientation change is supplied by a trained neural network, and we validate the prediction against direct many-body hydrodynamic simulations.

The translational diffusion coefficient D_t is defined by

$$D_t = \lim_{t \rightarrow \infty} \frac{\langle |\Delta \mathbf{R}(t)|^2 \rangle}{6t}, \quad (1)$$

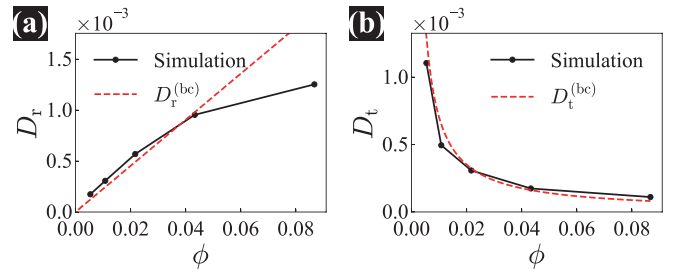


FIG. 3. (a) Rotational diffusion coefficient D_r from many-body simulations (solid line) and the binary-collision prediction $D_r^{(bc)}$ from Eq. (4) (dashed line) vs swimmer volume fraction ϕ . (b) Translational diffusion coefficient D_t from simulations (solid line) and $D_t^{(bc)}$ from Eq. (5) (dashed line) vs ϕ .

where $\langle \dots \rangle$ denotes an average over swimmers and time origins, and $\Delta \mathbf{R}(t)$ is the displacement of the swimmer's center of mass [see Eq. (S15) of the Supplemental Material [41] for its explicit definition]. Analogously, the rotational diffusion coefficient D_r is

$$D_r = \lim_{t \rightarrow \infty} \frac{\langle |\Delta \Theta(t)|^2 \rangle}{4t}, \quad (2)$$

with $\Delta \Theta(t)$ the unbounded angular displacement [see Eq. (S16) of the Supplemental Material [41] for its explicit definition].

To focus on the effect of binary collisions, we evaluate the Boltzmann collision integral with the integrand (single-collision orientation change) provided by the NN. The binary-collision contribution to rotational diffusion is

$$D_r^{(bc)} = \lim_{t \rightarrow 0} \frac{\langle |\Delta \hat{\mathbf{n}}(t)|^2 \rangle}{4t} = \frac{1}{4} \int d\Gamma(\theta, b, \epsilon) |\Delta \hat{\mathbf{n}}(\theta, b, \epsilon)|^2, \quad (3)$$

where $d\Gamma = \rho |\mathbf{v}_{\text{rel}}| P(\theta) d\theta d\sigma$ is the differential collision rate. Here, ρ is the number density, $|\mathbf{v}_{\text{rel}}| = 2v_s \sin(\theta/2)$ the relative speed, $P(\theta) = (1/2) \sin \theta$ the isotropic distribution of relative angles, and $d\sigma = b db d\epsilon$ the differential cross section [43]. Substituting $|\mathbf{v}_{\text{rel}}| = 2v_s \sin(\theta/2)$, $P(\theta) = (1/2) \sin \theta$, and $d\sigma = b db d\epsilon$ into Eq. (3), the product $|\mathbf{v}_{\text{rel}}| P(\theta)$ becomes $v_s \sin \theta \sin(\theta/2)$. We thus obtain

$$D_r^{(bc)} = \frac{\rho v_s}{4} \int_0^{b_{\text{max}}} db \int_0^\pi d\theta \int_{-\pi}^\pi d\epsilon \times b \sin \theta \sin\left(\frac{\theta}{2}\right) |\Delta \hat{\mathbf{n}}(\theta, b, \epsilon)|^2. \quad (4)$$

Similar expressions are found in the literature [12–15].

Figure 3(a) compares D_r from many-body simulations with the kinetic-theory prediction [Eq. (4)] using the trained NN varying the volume fraction of the body parts ϕ . The procedures for extracting both D_r and D_t from the many-body simulation data are described in Sec. II of the Supplemental Material [41]. In evaluating Eq. (4), we set $b_{\text{max}} = 2\ell_0$ and use uniform quadrature with 100 points per variable. As shown in Fig. 3(a), D_r and $D_r^{(bc)}$ agree for $\phi \lesssim 0.04$, indicating that binary collisions dominate rotational diffusion in the dilute regime. Assuming a constant swimming speed v_s , the translational diffusion coefficient follows from a Green-Kubo

relation as

$$D_t^{(bc)} = \frac{1}{3} \int_0^\infty \langle \mathbf{v}_s(t) \cdot \mathbf{v}_s(0) \rangle dt = \frac{v_s^2}{6D_r^{(bc)}}. \quad (5)$$

The constant-speed approximation is used only at the kinetic-theory level, namely, in the collision rate and in Eq. (5); it is not imposed in the direct hydrodynamic simulations used to generate the binary-collision dataset. Note that this expression assumes that the swimming direction decorrelates solely via binary collisions instantaneously and is essentially equivalent to that of a run-and-tumble particle [18]. Figure 3(b) shows that $D_t^{(bc)}$ is almost consistent with D_t over the range examined [44].

Linear stability of the isotropic state. We assess the linear stability of the spatially homogeneous isotropic state within a Boltzmann description of binary collisions [21,22,25,29]. We neglect spatial dependence and consider only the orientational distribution $f(\hat{\mathbf{n}}, t)$, normalized by $\int d\hat{\mathbf{n}} f = 1$. Its evolution obeys

$$\partial_t f(\hat{\mathbf{n}}, t) = D_r^{(th)} \nabla_{\hat{\mathbf{n}}}^2 f(\hat{\mathbf{n}}, t) + C[f], \quad (6)$$

where $\nabla_{\hat{\mathbf{n}}}$ is the gradient operator on the unit sphere, $D_r^{(th)}$ the thermal rotational diffusivity (absent in the present simulations), and $C[f]$ the collision integral [see Eq. (S18) of the Supplemental Material [41] for its explicit definition].

The isotropic steady state is $f_0 = 1/(4\pi)$. Considering small perturbations, we write $f = f_0 + \delta f$ with

$$\delta f(\hat{\mathbf{n}}, t) = \frac{3}{4\pi} \mathbf{m}(t) \cdot \hat{\mathbf{n}}, \quad \mathbf{m}(t) \equiv \int d\hat{\mathbf{n}} \hat{\mathbf{n}} \delta f(\hat{\mathbf{n}}, t).$$

Multiplying Eq. (6) by $\hat{\mathbf{n}}$ and integrating over orientations yields

$$\frac{d\mathbf{m}(t)}{dt} = [-2D_r^{(th)} + \nu^{(bc)}] \mathbf{m}(t) \quad (7)$$

and

$$\nu^{(bc)} = \int d\Gamma(\theta, b, \epsilon) (\Delta\hat{\mathbf{n}}_1 + \Delta\hat{\mathbf{n}}_2) \cdot (\hat{\mathbf{n}}_1 + \hat{\mathbf{n}}_2). \quad (8)$$

Here, $d\Gamma = \rho |\mathbf{v}_{rel}| P(\theta) d\theta d\sigma$, with $|\mathbf{v}_{rel}| = 2v_s \sin(\theta/2)$, $P(\theta) = (1/2) \sin \theta$, and $d\sigma = b db d\epsilon$. The single-collision orientation increments $\Delta\hat{\mathbf{n}}_i(\theta, b, \epsilon)$ ($i = 1, 2$) are provided by the trained NN. Note that Eq. (8) is the same as that derived in Ref. [29] for two-dimensional (2D) systems and see Sec. III of the Supplemental Material [41] for a detailed derivation of Eq. (8).

The growth rate (eigenvalue) is $\nu \equiv -2D_r^{(th)} + \nu^{(bc)}$. The isotropic state is linearly stable for $\nu < 0$ and unstable for $\nu > 0$. Using the trained NN, we obtain $\nu^{(bc)}/\Gamma_0 \simeq -0.13$, so the isotropic state remains stable even for $D_r^{(th)} = 0$, where the per-particle total collision rate is $\Gamma_0 = \int d\Gamma(\theta, b, \epsilon)$.

To further understand the collision-induced alignment, we examine the integrand in Eq. (8). We define the alignment metric

$$\Delta A(\theta, b, \epsilon) \equiv (\Delta\hat{\mathbf{n}}_1 + \Delta\hat{\mathbf{n}}_2) \cdot (\hat{\mathbf{n}}_1 + \hat{\mathbf{n}}_2). \quad (9)$$

The alignment metric ΔA quantifies the collision-induced change in the polar alignment of the two swimmers. Specifically, $\Delta A > 0$ indicates enhanced polar alignment, whereas $\Delta A < 0$ indicates reduced polar alignment. Since $\hat{\mathbf{n}}_1 + \hat{\mathbf{n}}_2$ is

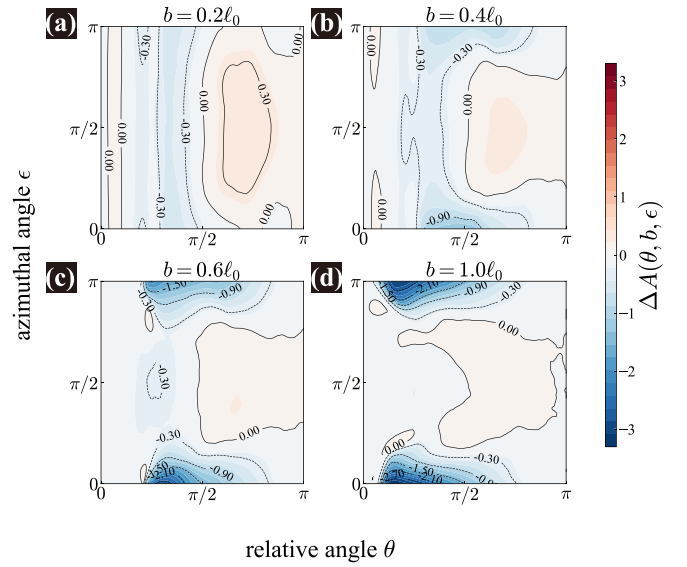


FIG. 4. Alignment metric $\Delta A \equiv (\Delta\hat{\mathbf{n}}_1 + \Delta\hat{\mathbf{n}}_2) \cdot (\hat{\mathbf{n}}_1 + \hat{\mathbf{n}}_2)$ as a function of relative angle θ , azimuth ϵ , and impact parameter b . Colors indicate the magnitude of the alignment (negative values: disalignment).

the total precollision polarization, ΔA measures the component of its collision-induced change along the precollision polarization direction. Figure 4 shows ΔA versus the collision parameters (θ, b, ϵ) . The alignment effect is predominantly negative (disaligning), with particularly strong disalignment near $\theta \simeq \pi/4$ at small b , and at large b for head-to-tail configurations $\epsilon \simeq 0, \pi$. For collisions with $\theta \lesssim \pi/2$, when the swimmers are initially aligned, they tend to disalign after the collision, while for $\theta \gtrsim \pi/2$, when they are initially antialigned, they tend to align. By axial symmetry ΔA is even about $\epsilon = \pi/2$, a property almost captured by the NN. The average over the variables other than b (see Sec. IV of the Supplemental Material [41]) shows that collisions promote alignment only for very small b ($\lesssim 0.2\ell_0$), whereas they lead to disalignment otherwise. As a complementary test, it would be interesting to examine whether the present framework can also capture relaxation from initially aligned states. A systematic comparison with direct simulations would require averaging over many independently prepared initial conditions and is therefore left for future work.

Concluding remarks. We have introduced a neural-network-assisted Boltzmann framework that learns the binary-collision law directly from data and evaluates collision integrals efficiently. For a representative microswimmer, the learned law yields quantitative predictions for rotational and translational self-diffusion in the dilute regime and matches many-body simulations for $\phi \lesssim 0.04$. A linear analysis further shows that collisions are predominantly disaligning, stabilizing the isotropic state. Because the surrogate model encodes only the single-encounter input-output map, the present framework is largely independent of how the collision data are obtained and can, in principle, be applied to other classes of self-propelled particles once their binary-collision statistics are available, whether from experiments or direct simulations. As a minimal test of such transferability across swimmer

classes, we show in Sec. V of the Supplemental Material [41] that the same workflow, without special tuning, provides $\nu^{(bc)} > 0$ for a neutral squirmer model, indicating that binary collisions between neutral squirmers promote polar alignment and thereby favor the emergence of polar order, consistent with previous studies [30,45,46].

A natural next step is to extend the present framework to suspensions under shear flow. In the quiescent system studied here, global rotational invariance reduces the precollisional geometry to (θ, b, ϵ) , whereas under shear this symmetry is broken and the collision map must be generalized. Such a shear-dependent collision operator would enable kinetic predictions of the orientational distribution, active stress, and effective viscosity in dilute suspensions, and would naturally

extend Ref. [37] beyond its analysis of transitions among a limited set of discrete swimming states.

Acknowledgments. This work was supported by the Japan Society for the Promotion of Science (JSPS) KAKENHI (Grants No. JP20H05619, No. JP25K00968, No. JP25K00967, and No. JP26H02008), the JSPS Core-to-Core Program “Advanced core-to-core network for the physics of self-organizing active matter” (Grant No. JPJSCCA20230002), and the Japan Science and Technology Agency (JST) SPRING (Grant No. JPMJSP2108).

Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] S. Ramaswamy, The mechanics and statistics of active matter, *Annu. Rev. Condens. Matter Phys.* **1**, 323 (2010).
- [2] M. C. Marchetti, J.-F. Joanny, S. Ramaswamy, T. B. Liverpool, J. Prost, M. Rao, and R. A. Simha, Hydrodynamics of soft active matter, *Rev. Mod. Phys.* **85**, 1143 (2013).
- [3] E. Lauga, Bacterial hydrodynamics, *Annu. Rev. Fluid Mech.* **48**, 105 (2016).
- [4] V. A. Martinez, E. Clément, J. Arlt, C. Douarche, A. Dawson, J. Schwarz-Linek, A. K. Creppy, V. Škultéty, A. N. Morozov, H. Auradou, and W. C. K. Poon, A combined rheometry and imaging study of viscosity reduction in bacterial suspensions, *Proc. Natl. Acad. Sci. USA* **117**, 2326 (2020).
- [5] J. Gachelin, A. Rousselet, A. Lindner, and E. Clement, Collective motion in an active suspension of *Escherichia coli* bacteria, *New J. Phys.* **16**, 025003 (2014).
- [6] A. Sokolov and I. S. Aranson, Physical properties of collective motion in suspensions of bacteria, *Phys. Rev. Lett.* **109**, 248109 (2012).
- [7] R. Alert, J. Casademunt, and J.-F. Joanny, Active turbulence, *Annu. Rev. Condens. Matter Phys.* **13**, 143 (2022).
- [8] P. Baconnier, O. Dauchot, V. Démery, G. Düring, S. Henkes, C. Huepe, and A. Shee, Self-aligning polar active matter, *Rev. Mod. Phys.* **97**, 015007 (2025).
- [9] A. Sokolov and I. S. Aranson, Reduction of viscosity in suspension of swimming bacteria, *Phys. Rev. Lett.* **103**, 148101 (2009).
- [10] H. M. López, J. Gachelin, C. Douarche, H. Auradou, and E. Clément, Turning bacteria suspensions into superfluids, *Phys. Rev. Lett.* **115**, 028301 (2015).
- [11] X.-L. Wu and A. Libchaber, Particle diffusion in a quasi-two-dimensional bacterial bath, *Phys. Rev. Lett.* **84**, 3017 (2000).
- [12] G. Mino, T. E. Mallouk, T. Darnige, M. Hoyos, J. Dauchet, J. Dunstan, R. Soto, Y. Wang, A. Rousselet, and E. Clement, Enhanced diffusion due to active swimmers at a solid surface, *Phys. Rev. Lett.* **106**, 048102 (2011).
- [13] Z. Lin, J.-L. Thiffeault, and S. Childress, Stirring by squirmers, *J. Fluid Mech.* **669**, 167 (2011).
- [14] J.-L. Thiffeault and S. Childress, Stirring by swimming bodies, *Phys. Lett. A* **374**, 3487 (2010).
- [15] A. Morozov and D. Marenduzzo, Enhanced diffusion of tracer particles in dilute bacterial suspensions, *Soft Matter* **10**, 2748 (2014).
- [16] H. H. Wensink, J. Dunkel, S. Heidenreich, K. Drescher, R. E. Goldstein, H. Löwen, and J. M. Yeomans, Meso-scale turbulence in living fluids, *Proc. Natl. Acad. Sci. USA* **109**, 14308 (2012).
- [17] D. Saintillan and M. J. Shelley, Instabilities and pattern formation in active particle suspensions: Kinetic theory and continuum simulations, *Phys. Rev. Lett.* **100**, 178103 (2008).
- [18] D. Saintillan, Rheology of active fluids, *Annu. Rev. Fluid Mech.* **50**, 563 (2018).
- [19] A. Baskaran and M. C. Marchetti, Enhanced diffusion and ordering of self-propelled rods, *Phys. Rev. Lett.* **101**, 268101 (2008).
- [20] A. Baskaran and M. C. Marchetti, Statistical mechanics and hydrodynamics of bacterial suspensions, *Proc. Natl. Acad. Sci. USA* **106**, 15567 (2009).
- [21] I. S. Aranson and L. S. Tsimring, Pattern formation of microtubules and motors: Inelastic interaction of polar rods, *Phys. Rev. E* **71**, 050901 (2005).
- [22] E. Bertin, M. Droz, and G. Grégoire, Boltzmann and hydrodynamic description for self-propelled particles, *Phys. Rev. E* **74**, 022101 (2006).
- [23] E. Bertin, M. Droz, and G. Grégoire, Hydrodynamic equations for self-propelled particles: Microscopic derivation and stability analysis, *J. Phys. A: Math. Theor.* **42**, 445001 (2009).
- [24] A. Peshkov, E. Bertin, F. Ginelli, H. Chaté, and M. Leoni, Nonlinear field equations for aligning self-propelled rods, *Phys. Rev. Lett.* **109**, 268701 (2012).
- [25] T. Hanke, C. A. Weber, and E. Frey, Understanding collective dynamics of soft active colloids by binary scattering, *Phys. Rev. E* **88**, 052309 (2013).
- [26] C. A. Weber, F. Thüroff, and E. Frey, Role of particle conservation in self-propelled particle systems, *New J. Phys.* **15**, 045014 (2013).
- [27] F. Thüroff, C. A. Weber, and E. Frey, Critical assessment of the Boltzmann approach to active systems, *Phys. Rev. Lett.* **111**, 190601 (2013).
- [28] A. Peshkov, E. Bertin, F. Ginelli, and H. Chaté, Boltzmann-Ginzburg-Landau approach for continuous descriptions of generic Vicsek-like models, *Eur. Phys. J. Spec. Top.* **223**, 1315 (2014).
- [29] R. Suzuki, C. A. Weber, E. Frey, and A. R. Bausch, Polar pattern formation in driven filament systems requires non-binary particle collisions, *Nat. Phys.* **11**, 839 (2015).

- [30] N. Oyama, J. J. Molina, and R. Yamamoto, Do hydrodynamically assisted binary collisions lead to orientational ordering of microswimmers? *Eur. Phys. J. E* **40**, 95 (2017).
- [31] R. Großmann, I. S. Aranson, and F. Peruani, A particle-field approach bridges phase separation and collective motion in active matter, *Nat. Commun.* **11**, 5365 (2020).
- [32] G. P. Alexander, C. Pooley, and J. M. Yeomans, Scattering of low-Reynolds-number swimmers, *Phys. Rev. E* **78**, 045302 (2008).
- [33] K. Drescher, J. Dunkel, L. H. Cisneros, S. Ganguly, and R. E. Goldstein, Fluid dynamics and noise in bacterial cell–cell and cell–surface scattering, *Proc. Natl. Acad. Sci. USA* **108**, 10940 (2011).
- [34] T. Ishikawa, M. Simmonds, and T. J. Pedley, Hydrodynamic interaction of two swimming model micro-organisms, *J. Fluid Mech.* **568**, 119 (2006).
- [35] D. Papavassiliou and G. P. Alexander, Exact solutions for hydrodynamic interactions of two squirmer spheres, *J. Fluid Mech.* **813**, 618 (2017).
- [36] I. O. Götze and G. Gompper, Mesoscale simulations of hydrodynamic squirmer interactions, *Phys. Rev. E* **82**, 041921 (2010).
- [37] H. Hayano and A. Furukawa, Hydrodynamic interactions in anomalous rheology of active suspensions, *Phys. Rev. Res.* **4**, 043091 (2022).
- [38] Y. Nakayama and R. Yamamoto, Simulation method to resolve hydrodynamic interactions in colloidal dispersions, *Phys. Rev. E* **71**, 036707 (2005).
- [39] J. J. Molina, K. Otomura, H. Shiba, H. Kobayashi, M. Sano, and R. Yamamoto, Rheological evaluation of colloidal dispersions using the smoothed profile method: Formulation and applications, *J. Fluid Mech.* **792**, 590 (2016).
- [40] R. Yamamoto, J. J. Molina, and Y. Nakayama, Smoothed profile method for direct numerical simulations of hydrodynamically interacting particles, *Soft Matter* **17**, 4226 (2021).
- [41] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/7565-51x2> for (1) the initial configuration and parameter cutoffs used in the hydrodynamic simulations, (2) the evaluation of self-diffusivity, (3) the derivation of the linearized equation for polar order, (4) the dependence of the alignment metric on collision parameters, (5) an application of the same framework to a neutral squirmer model as a minimal test of transferability, (6) the computational cost and practical advantage of the learned collision map, and (7) an assessment of dataset-size sufficiency based on the training and validation mean absolute errors for different training-set sizes.
- [42] M. Abadi, A. Agarwal, P. Barham, E. Brevdo, Z. Chen, C. Citro, G. S. Corrado, A. Davis, J. Dean, M. Devin, *et al.*, TensorFlow: Large-scale machine learning on heterogeneous systems (2015), <https://www.tensorflow.org/extras/tensorflow-whitepaper2015.pdf>.
- [43] Equation (3) gives the short-time slope of the mean-square orientation change produced by a single (uncorrelated) binary collision. If the swimming direction decorrelates sufficiently between collisions, as in dilute suspensions, this short-time slope coincides with the long-time rotational diffusivity defined in Eq. (2).
- [44] The agreement for $\phi \gtrsim 0.04$ may be coincidental. Although a significant deviation appears in D_r for $\phi \gtrsim 0.04$, this discrepancy is not reflected in D_t . The deviation in D_r is attributable to crowding: Increasing ϕ slows rotational dynamics. At the same time, crowding also reduces the actual v_s , whereas the evaluation of $D_t^{(bc)}$ assumes a constant dilute-limit speed. In practice, both D_r and v_s decrease with ϕ ; these two effects combine to make D_t appear consistent with $D_t^{(bc)}$ even for $\phi \gtrsim 0.04$, where the deviation in D_r is already evident.
- [45] A. A. Evans, T. Ishikawa, T. Yamaguchi, and E. Lauga, Orientational order in concentrated suspensions of spherical microswimmers, *Phys. Fluids* **23**, 111702 (2011).
- [46] F. Alarcón and I. Pagonabarraga, Spontaneous aggregation and global polar ordering in squirmer suspensions, *J. Mol. Liq.* **185**, 56 (2013).