

Observation and control of nonmonotonic recoils in a viscoelastic fluid

Salil S. Vaidya¹, Lokesh Muruga¹, Juliana Caspers², Matthias Krüger², and Clemens Bechinger¹

¹*Fachbereich Physik, Universität Konstanz, 78457 Konstanz, Germany*

²*Institute for Theoretical Physics, Georg-August Universität Göttingen, 37073 Göttingen, Germany*



(Received 29 April 2025; accepted 26 June 2025; published 23 July 2025)

We experimentally study the relaxation dynamics of a colloidal particle in a micellar viscoelastic fluid following different driving protocols. When the particle is driven at constant velocity for a finite duration, its recovery to equilibrium is always monotonic. In contrast, altering the driving velocity during the protocol induces nonmonotonic relaxation. Our results are in quantitative agreement with the analytical solution of a minimal micromechanical model exhibiting only two distinct eigenmodes independent of the specific protocol. Notably, the model enables selective suppression of one or both modes—an effect confirmed experimentally. Because the model is broadly applicable to diverse viscoelastic fluids, our findings offer a general framework for tailoring relaxation dynamics in complex environments.

DOI: [10.1103/74nx-3t1h](https://doi.org/10.1103/74nx-3t1h)

I. INTRODUCTION

The relaxation dynamics of materials in response to external perturbations provides not only key insights into their structural and dynamical properties but is also important for technical applications. Remarkably, diverse systems including tectonic plates, superconductors, spin and colloidal glasses, granular materials, and crumpled sheets exhibit similar non-exponential relaxation behaviors [1–8]. Typically, material properties monotonically recover to equilibrium after a single-step excitation, e.g., after a sudden application of stress. However, in more complex excitation protocols, relaxation processes characterized by a nonmonotonic departure from equilibrium have been observed. In general, such behavior is a signature of memory effects that can be encoded in materials when subjected to certain excitation protocols. The first such observation was reported by Kovacs, who discovered a non-monotonic relaxation of the isobaric volume of a polymeric glass after a two-step temperature protocol where the sample was first rapidly cooled down, allowed to relax for some time, and then heated up [9]. Since then, similar behaviors have been observed in various materials, e.g., foams, rock-salt, metallic sheets, glass beads, granular systems, and active matter, for other control parameters, e.g., stress [10–15]. To model this effect, typically broad distributions of relaxation times have been proposed as a key factor [10,16,17]. The presence of large numbers of relaxation times, however, hinders effective control of the recovery process.

Here, we experimentally investigate the nonmonotonic stress relaxation of a micrometer-sized colloidal particle in a viscoelastic solution that exhibits only two eigenmodes. For one-step protocols, where mechanical stress is applied to

the fluid by driving the particle with constant velocity over a finite time interval, we always find a monotonic particle relaxation (recoil) back to equilibrium. In the case of two-step protocols, however, where the direction and magnitude of the particle's driving force is changed during the protocol, a complex relaxation behavior is observed: Our experiments show that either one of the eigenmodes can be fully suppressed, turning the recoil into a single exponential function that can point along the direction of the latest driving, or opposite to it. Using a three-step protocol, the recoil can even be suppressed, i.e., the particle completely stands still after removing the drive. These features, which have to our knowledge not been demonstrated before, allow for a high degree of controllability of the relaxation dynamics in such systems. Our findings agree remarkably well with analytical solutions of a minimal micromechanical model, which is also applicable to other complex fluids.

II. EXPERIMENTAL METHODS

A. Solution and sample preparation

Experiments were performed using 2.73- μm silica particles (microParticles GmbH[®]) suspended in an 8-mM equimolar solution of cetylpyridinium chloride monohydrate (CPyCl) and sodium salicylate (NaSal). The solution is composed of 286.4 mg of CPyCl (99%), 128.1 mg of NaSal (99.5%), and 104.4 g of deionized water. After mixing, it is slowly stirred (100 rpm) for 24 h at 45 °C for a complete mixing. Afterwards the temperature is cooled to 25 °C for sample storage while keeping the same stirring speed. Under such conditions, the fluid forms an entangled network of giant wormlike micelles exhibiting pronounced viscoelasticity [18] and exponential relaxation behavior [19–21] (see Appendix A for microrheological characterization of the fluid). The sample is prepared by first adding 0.5 ml of the aqueous CPyCl-NaSal solution into an Eppendorf[®] tube followed by $\sim 0.5\ \mu\text{l}$ of a concentrated solution of 2.73- μm silica particles. A 100- μm -thick

Published by the American Physical Society under the terms of the Creative Commons Attribution 4.0 International license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

capillary (Rectangle Boro Tubing, VitroTubes®) is dipped in the solution, which immediately fills with the fluid due to capillary forces. Finally, the capillary is closed with beeswax (Hampton Research®) and reinforced with epoxy glue to provide chemical inertness and mechanical stability. It is then affixed on a glass slide with a hole for better handling. To achieve complete equilibration of the system, samples are kept 48 h on a temperature-controlled sample holder before conducting experiments.

B. Optical tweezers setup

The optical trap is generated using a 532-nm diode pumped laser (Coherent® Verdi-V2) whose intensity is modulated through an acoustic-optical modulator (AA Opto-Electronic® MT80-A1.5-VIS). The beam is directed to the back aperture of a 100× oil-immersion microscope objective (Zeiss® EC Plan-Neofluar M27, NA=1.3) that focuses it into the sample cell to create an optical trapping potential. To avoid any interactions with walls, the focusing plane is about in the midplane of the sample cell. The sample is positioned onto a piezo-actuated translational stage (Piezoconcept® LT3.300) enabling precise control of the stage's motion relative to the trap with nanometer resolution. To ensure a constant temperature of the sample cell, it is mounted onto an aluminum sample holder whose temperature is kept at 25 °C by a water thermostat. Furthermore, the oil-immersion objective, which is in direct contact with the sample cell, is kept at the same temperature using an objective heater (Okolab®).

C. Single-step and multistep protocols

Single- and multistep protocols for forward and backward driving of a colloidal particle through a viscoelastic fluid are realized by controlling the motion of the optical tweezers, which holds the particle relative to the fluid. In the single-step protocol, the colloid is translated during t_f at velocity v_f by steadily moving the stage relative to the trap. At $t = 0$, the laser intensity is turned off, and particle relaxation is recorded via digital video microscopy with a frame rate of 100 Hz and ± 6 nm spatial resolution. Afterwards, the protocol is applied in the reverse direction. Each protocol is typically repeated at least 20 times to yield sufficient statistics and to calculate noise-free mean trajectories. In the two-step protocol as shown in Fig. 1(a), the particle is first driven forward along the x axis for time t_f and with velocity v_f and then for t_b with v_b in the backward direction ($-x$ axis). At $t = 0$, the optical trap is turned off and the recoil trajectory is measured. After 25 s, the trap is turned back on, and the process is mirrored in the opposite direction. Similarly, a three-step protocol has been implemented by adding another reversal in the particle driving during $t_{f,2}$ at a velocity $v_{f,2}$. Note that the driving time at the beginning of each protocol ($t_f = 60$ s) is considerably longer than the system's relaxation times (on the order of seconds). This guarantees that each protocol is independent when repeating such measurements.

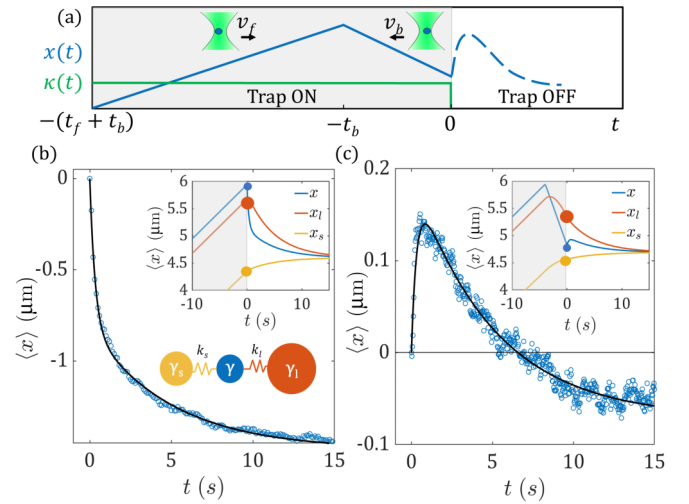


FIG. 1. (a) Sketch of a two-step driving protocol (solid line). First the optical trap moves in the x direction during time t_f and velocity v_f and then during t_b and v_b in the opposite direction ($-x$). At $t = 0$ the optical trap stiffness κ_{OT} is set to zero, which marks the beginning of the particle relaxation (dashed line). (b) Measured (symbols) relaxation after a one-step protocol with $t_f = 60$ s, $v_f = 0.1$ μm/s. The solid line corresponds to the solution of a micromechanical model illustrated in the inset. The upper inset shows the calculated positions of all particles within the model during and after the protocol. (c) Corresponding data for a two-step protocol with parameters $t_f = 60$ s, $v_f = 0.1$ μm/s, $t_b = 4$ s, and $v_b = 0.3$ μm/s. Note the different relative positions of the two bath particles relative to the probe particle.

III. RESULTS AND DISCUSSION

Shear protocols are applied by dragging a colloidal particle at constant velocity through the fluid using an optical trap formed by a tightly focused laser beam ($\lambda = 532$ nm, 100×, NA=1.3) yielding a trap stiffness of $\kappa_{OT} = 20 \pm 2$ μN/m.

Figure 1(b) shows the mean particle response $\langle x(t) \rangle$ after applying a single-step protocol characterized by $t_f = 60$ s and $v_f = 0.1$ μm/s. Consistent with previous relaxation studies on various viscoelastic solutions—including micellar systems, semidilute polymers, and λ -phage DNA—the observed recovery to such single-step protocols follows a biexponential decay, as indicated by the solid lines [22–25]:

$$\langle x(t) \rangle = A_{\text{tot}} - A_s e^{-t/\tau_s} - A_l e^{-t/\tau_l}, \quad (1)$$

with amplitudes A_s , A_l , and $A_{\text{tot}} = A_s + A_l$ and timescales τ_s and τ_l corresponding to the eigenmodes of the specific system. Fitting this expression to our data, we obtain $A_s = 0.81$ μm, $A_l = 0.67$ μm, $\tau_s = 0.33$ s, and $\tau_l = 4.96$ s, respectively [solid line in Fig. 1(b)].

In Fig. 1(c) we plot the response of the same colloidal particle to a two-step protocol with parameters $t_f = 60$ s, $v_f = 0.1$ μm/s, $t_b = 4$ s, and $v_b = 0.3$ μm/s. Here, we find a strongly nonmonotonic relaxation with a pronounced maximum around 1 s. To investigate the influence of protocol parameters on the relaxation behavior, we varied t_b between 0 and 10 s [Fig. 2(a)]. With increasing t_b , the direction of relaxation gradually reverses, indicating the decreasing influence of the protocol for $t < -t_b$. For t_b between 3 and 6 s the

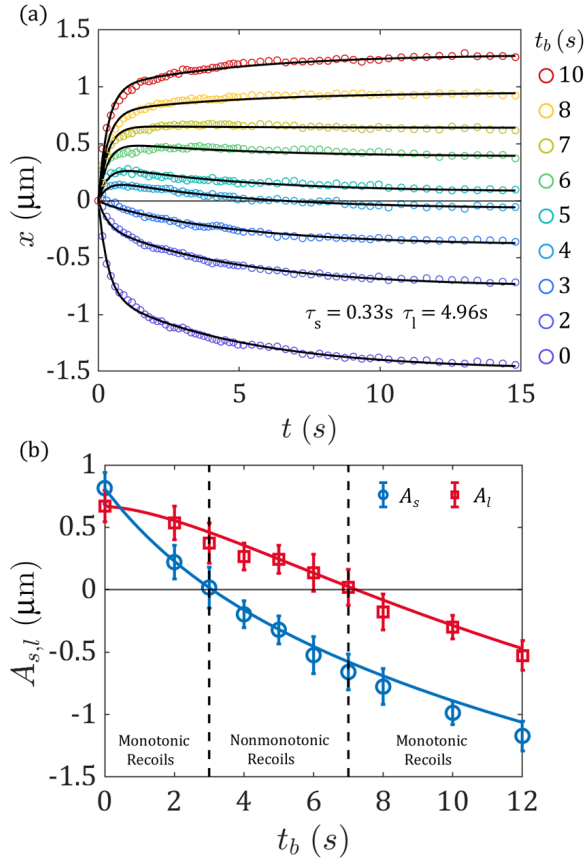


FIG. 2. (a) Measured (symbols) particle relaxation after the two-step protocol for the same parameters as Fig. 1 and with t_b varied between 0 and 10 s. The fits (solid lines) correspond to a biexponential fit with the amplitudes $A_{s,l}$ used as free fit parameters and the same two relaxation times for all curves. (b) Amplitudes $A_{s,l}$ obtained from fitting in (a) (symbols) and theoretical prediction according to Eq. (8) (solid line).

curves exhibit a maximum demonstrating the robustness of the nonmonotonic relaxation observed in Fig. 1(c). Notably, all curves are in excellent agreement with Eq. (1) but with positive and negative amplitudes. The solid lines in Fig. 2(a) are fits with amplitudes A_s and A_l as free fitting parameters but using the same values $\tau_s = 0.33$ s and $\tau_l = 4.96$ s as before. The dependence of $A_{s,l}$ on t_b is plotted as symbols in Fig. 2(b). When the signs of the amplitudes are opposite (equal), this results in a nonmonotonic (monotonic) recoil. Because each amplitude becomes zero for a special t_b value, this demonstrates that single eigenmodes of the system can be selectively suppressed during relaxation. The recoil can therefore be controlled to relax slowly with τ_l ($t_b \approx 3$ s), or quickly with τ_s ($t_b \approx 7$ s).

To understand our observations, we perform analytical calculations based on a minimal micromechanical model that has been experimentally confirmed in different viscoelastic fluids (including the micellar system under study) [22–25]. In this model, the interaction between the viscoelastic fluid and a colloid is described by introducing two so-called *bath* particles labeled with indices s, l and connected with harmonic springs $\kappa_{s,l}$ to the colloidal (*tracer*) particle (the latter having

no index). All particles are characterized by their friction coefficients γ , γ_s , and γ_l , respectively [see inset Fig. 1(b)]. The dynamics of the colloid and the bath particles is described by the following set of overdamped Langevin equations ($i \in \{s, l\}$):

$$\gamma \dot{x}(t) = -\kappa(t)[x(t) - x_0(t)] - \sum_i \kappa_i[x(t) - x_i(t)] + \xi(t), \quad (2)$$

$$\gamma_i \dot{x}_i(t) = -\kappa_i[x_i(t) - x(t)] + \xi_i(t) \quad (3)$$

with Gaussian white noises $[(\xi_i, \xi_j) \in \{\xi, \xi_s, \xi_l\}]$,

$$\langle \xi_i(t) \rangle = 0, \quad \langle \xi_i(t) \xi_j(t') \rangle = \delta_{ij} 2k_B T \gamma_i \delta(t - t'). \quad (4)$$

The recoil protocol is contained in the first term on the right-hand side of Eq. (2), which describes the time-dependent trapping force acting on the probe particle. $x_0(t)$ denotes the position of the harmonic optical potential that follows the specific driving protocol. In the case of a two-step protocol it follows

$$x_0(t) = \begin{cases} -v_f t_f + v_b t_b, & t < -(t_b + t_f) \\ v_f(t + t_b) + v_b t_b, & (t_b + t_f) \leq t < -t_b \\ -v_b t, & -t_b \leq t < 0 \end{cases} \quad (5)$$

and

$$\kappa(t) = \begin{cases} \kappa_{OT}, & t < 0 \\ 0, & t > 0. \end{cases} \quad (6)$$

We take the means of Eqs. (2) and (3), and consider, for simplicity, the limit of large κ_{OT} , for which $x(t) = x_0(t)$ for $t < 0$, and find that, due to models linearity, independent of the number of steps and the parameters of the protocol, the averaged particle relaxation always follows the double-exponential function of Eq. (1) (for details, see Appendix B [26]). The timescales read

$$\tau_{s,l}^{-1} = \frac{1}{2\gamma} [\zeta_s + \zeta_l \pm \sqrt{(\zeta_s + \zeta_l)^2 + 4(\kappa_s \kappa_l - \zeta_s \zeta_l)}] \quad (7)$$

with $\zeta_i = (\gamma + \gamma_i)\kappa_i/\gamma_i$. The positive (negative) sign in Eq. (7) corresponds to the shorter (longer) timescale τ_s (τ_l).

Application of the above two-step protocol results in the following expressions for the recoil amplitudes (for details, see Appendix B):

$$A_s = \frac{\gamma_s \tilde{\tau}_s [v_b(1 - e^{-t_b/\tilde{\tau}_s}) - v_f e^{-t_b/\tilde{\tau}_s} (1 - e^{-t_f/\tilde{\tau}_s})]}{2(\gamma + \gamma_s + \gamma_l)} \times \left[1 + \frac{\tilde{\tau}_l [\zeta_l(\tilde{\tau}_l - \tilde{\tau}_s) + \gamma_s + \gamma_l]}{(\gamma + \gamma_s + \gamma_l)(\tau_l - \tau_s)} \right] + \frac{\gamma_l \tilde{\tau}_l [v_b(1 - e^{-t_b/\tilde{\tau}_l}) - v_f e^{-t_b/\tilde{\tau}_l} (1 - e^{-t_f/\tilde{\tau}_l})]}{2(\gamma + \gamma_s + \gamma_l)} \times \left[1 - \frac{\tilde{\tau}_s [\zeta_s(\tilde{\tau}_s - \tilde{\tau}_l) + \gamma_s + \gamma_l]}{(\gamma + \gamma_s + \gamma_l)(\tau_s - \tau_l)} \right] \quad (8)$$

with $\tilde{\tau}_i = \gamma_i/\kappa_i$; A_l follows from Eq. (8) by changing indices $s \leftrightarrow l$ [27].

The solid lines in Fig. 2(b) have been obtained from Eqs. (7) and (8) using the following model parameters: $\gamma_s/\gamma = 3.8$, $\gamma_l/\gamma = 23.1$, $\kappa_s/\gamma = 1.5$ s⁻¹, and $\kappa_l/\gamma = 1.3$ s⁻¹. Because all parameters are expressed relative to

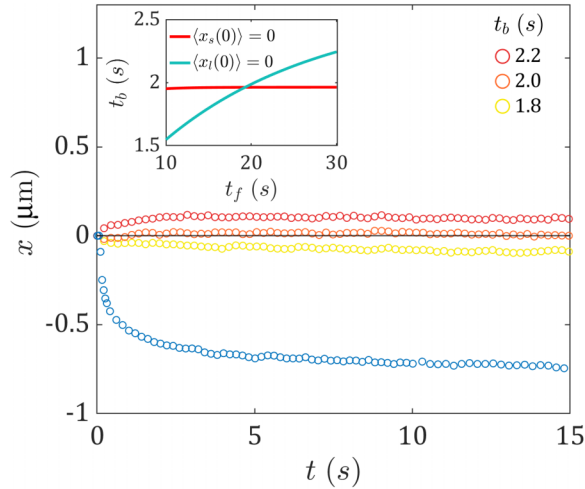


FIG. 3. Inset: Calculated values of t_b and t_f yielding $\langle x_{s,l}(0) \rangle = 0$ for a three-step protocol with fixed parameters $v_f = 0.1 \mu\text{m/s}$, $v_b = 1 \mu\text{m/s}$, $v_{f,2} = 0.2 \mu\text{m/s}$, and $t_{f,2} = 3.5$ s. The crossing point at $t_b = 1.96$ s and $t_f = 19.3$ s defines the conditions for zero recoil. Main graph: Measured particle relaxation after a three-step protocol with the above parameters and $t_b = 1.8, 2.0, 2.2$ s. To demonstrate the almost perfect suppression of the relaxation, as a reference we plot the measured recoil for $t_f = 19.3$ s.

the friction coefficient of the colloid $\gamma = 0.44 \mu\text{Ns/m}$, independently determined via flow curve experiments [24], this renders our results independent of the fluid's viscosity. The good agreement with the measured amplitudes confirms the accuracy of Eq. (8) for our data.

The observed nonmonotonic recoil can be understood by considering the relative positions of the colloidal and the bath particles. In a single-step protocol, both bath particles are on the same side of the colloid at the moment of release, i.e., $t = 0$ [see inset of Fig. 1(b)]. In contrast, with a two-step protocol, they can also be on opposite sides of the colloid at $t = 0$, depending on the protocol parameters [see inset of Fig. 1(c)]. Due to their different friction coefficients $\gamma_{s,l}$ and coupling stiffnesses $\kappa_{s,l}$, this asymmetry explains the differences in the recovery of the system depending on the driving protocol.

We now turn to three-step protocols. Such protocols are implemented by adding another forward driving period $t_{f,2}$, $v_{f,2}$ after t_b . Within the two-bath particle model, the particle relaxation again follows Eq. (1) with the same relaxation times τ_s and τ_l as before. It is insightful to regard the mean positions of the two bath particles at the moment of their release, which is given by (see Appendix B)

$$\begin{aligned} \langle x_i(0) \rangle = & -v_f \tilde{\tau}_i e^{-(t_b+t_{f,2})/\tilde{\tau}_i} (1 - e^{-t_f/\tilde{\tau}_i}) \\ & + v_b \tilde{\tau}_i e^{-t_{f,2}/\tilde{\tau}_i} (1 - e^{-t_b/\tilde{\tau}_i}) \\ & - v_{f,2} \tilde{\tau}_i (1 - e^{-t_{f,2}/\tilde{\tau}_i}). \end{aligned} \quad (9)$$

Notably, if $\langle x_i(0) \rangle = 0$ for $i = s, l$, all three particles, i.e., tracer and bath particles, are at the origin at $t = 0$. In this case, the recoil is thus *completely erased* in our model. As an example we have calculated the $\langle x_i(0) \rangle = 0$ curves of both bath particles as a function of t_f , t_b (inset of Fig. 3). Recoil erasure occurs at the crossing of the curves.

Is such erasure possible in our experiments? The main graph of Fig. 3 shows the experimental results for a three-step protocol where t_b is stepwise varied between 2.2 and 1.8 s, which is around the expected erasure point (see inset of Fig. 3). Compared to a one-step recoil (shown as blue symbols), the recoil almost fully vanishes in accordance with our model. Our numerical simulations show that it is possible to suppress the recoil for a wide combination of parameters (see Appendix B). Of course, for $t_b \rightarrow \infty$ any previous strain imposed to the fluid has fully decayed and the situation effectively corresponds to a two-step protocol where a zero recoil no longer can be achieved.

The above experiments demonstrate that both eigenmodes of the system can be suppressed by an appropriate choice of protocol parameters that may also be relevant in situations where a fast recovery after an external perturbation is required. We note that, depending on the specific situation, there might be a trade off between using a simple protocol without reversal and realizing the case of a designed recoil.

IV. CONCLUSION

In summary, we have demonstrated a variety of relaxation behaviors of colloidal particles driven by one-, two-, and three-step driving protocols through a micellar viscoelastic fluid. Our results are in excellent agreement with a micromechanical model exhibiting two eigenmodes of the system. Depending on the specific driving conditions, each mode can be selectively excited, with amplitudes of equal or opposite signs, which allows one to fully control and even suppress the relaxation behavior in such systems. As a final remark we expect that the presence of n eigenmodes requires, in general, a protocol of $n + 1$ steps for a full control, making the small number of modes in the system at hand specifically handy.

ACKNOWLEDGMENTS

This project was funded by the Deutsche Forschungsgemeinschaft (DFG), Grant No. SFB 1432–Project ID 425217212. L.M. acknowledges support by the Alexander von Humboldt foundation. C.B. acknowledges funding by the ERC through the Advanced Grant BRONEB (Grant No. 101141477).

S.V. and L.M. contributed equally to this work.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. However, all data will be made available upon reasonable request.

APPENDIX A: MICRORHEOLOGICAL SAMPLE CHARACTERIZATION

In this appendix, we study the microrheological behavior of the micellar solution. To investigate this, the viscosity of the fluid was measured using a traditional Stokes experiment, in which a particle is dragged using an optical trap with a

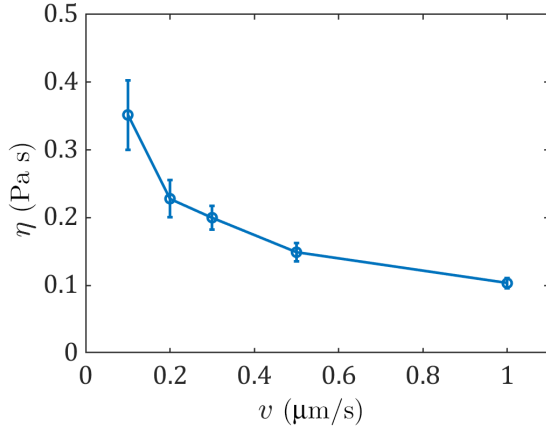


FIG. 4. Flow curve showing the viscosity as a function of velocity for a 2.73- μm silica particle suspended in an 8-mM CPyCl–NaSal solution at 25 $^{\circ}\text{C}$. We observe a steady shear thinning of the fluid.

known stiffness (κ_{OT}) at a constant velocity. In practice, this was performed by moving the sample with a piezoelectric stage while keeping the optical trap stationary. The viscosity was then calculated using Stokes' law, $\eta = \frac{\kappa_{\text{OT}} x_0}{6\pi r v}$, based on the average displacement of the particle from the trap center, denoted as x_0 . The resulting flow curve can be observed in Fig. 4.

APPENDIX B: NONMONOTONIC STRESS RELAXATION IN THE TWO-BATH PARTICLE MODEL

Previous work [24,25] has shown that the biexponential relaxation of a probe particle during recoil is well described by a linear two-bath particle model. In this model, the probe particle at position $x(t)$ at time t is linearly coupled to two bath particles at positions $x_s(t)$ and $x_l(t)$, with coupling strengths κ_s and κ_l . The particles are characterized by friction coefficients γ , γ_s , and γ_l , respectively. Their dynamics is described by the following set of overdamped Langevin equations ($i \in \{s, l\}$):

$$\gamma \dot{x}(t) = -\kappa(t)[x(t) - x_0(t)] - \sum_i \kappa_i[x(t) - x_i(t)] + \xi(t), \quad (\text{B1})$$

$$\gamma_i \dot{x}_i(t) = -\kappa_i[x_i(t) - x(t)] + \xi_i(t) \quad (\text{B2})$$

with Gaussian white noises $[\langle \xi_i, \xi_j \rangle \in \{\xi, \xi_s, \xi_l\}]$,

$$\langle \xi_i(t) \rangle = 0, \quad \langle \xi_i(t) \xi_j(t') \rangle = \delta_{ij} 2k_B T \gamma_i \delta(t - t'). \quad (\text{B3})$$

Equation (B1) is the equation of motion of the probe particle. Equation (B2) is the equation of motion of the i th bath particle.

The driving protocol is contained in the first term on the right-hand side of Eq. (B1), the trapping force acting on the probe particle. $x_0(t)$ denotes the position of the harmonic optical potential. For a two-step protocol, it follows

$$x_0(t) = \begin{cases} -v_f t_f + v_b t_b, & t < -(t_b + t_f) \\ v_f(t + t_b) + v_b t_b, & -(t_b + t_f) \leq t < -t_b \\ -v_b t, & -t_b \leq t < 0. \end{cases} \quad (\text{B4})$$

Thus, it is driven with constant velocity v_f from time $t = -(t_b + t_f)$ to time $t = -t_b$ and with v_b from time $t = -t_b$ to

time $t = 0$. At time $t = 0$ the probe particle is released and the recoil is observed. This is described by the time-dependent stiffness of the optical trap, $\kappa(t)$. It is equal to κ_{OT} for $t < 0$ and zero during recoil (for $t \geq 0$).

The mean recoil is obtained analytically for the model and protocol described above. We take the means of Eqs. (B1) and (B2), and solve for $\langle x(t) \rangle$ for $t > 0$. The resulting set of ordinary differential equations is solved with initial conditions $\langle x(0) \rangle$ and $\langle x_i(0) \rangle$ that depend on the protocol of driving. These initial positions are found via

$$\langle x_i(0) \rangle = \langle x(0) \rangle - \int_{-\infty}^0 dt \langle \dot{x}(t) \rangle e^{t/\tilde{\tau}_i}. \quad (\text{B5})$$

$\tilde{\tau}_i = \gamma_i / \kappa_i$ ($i \in \{s, l\}$) denotes the individual relaxation time of the i th bath particle, denoted the nonreciprocal timescale in Ref. [24]. In the following, we take the limit $\kappa_{\text{OT}} \rightarrow \infty$ so that $x(t) = x_0(t)$ for $t < 0$. The motion of the probe particle thus follows the protocol, $x(t) = x_0(t)$. Inserting Eq. (B4) into Eq. (B5) then results in

$$\begin{aligned} \langle x_i(0) \rangle - \langle x(0) \rangle = & -v_f \tilde{\tau}_i e^{-t_b/\tilde{\tau}_i} (1 - e^{-t_f/\tilde{\tau}_i}) \dots \\ & \dots + v_b \tilde{\tau}_i (1 - e^{-t_b/\tilde{\tau}_i}). \end{aligned} \quad (\text{B6})$$

Finally, we solve Eqs. (B1) and (B2) for the mean recoil $\langle x(t) \rangle$ for $t > 0$ with the initial conditions determined by Eq. (B6) using the *Mathematica* differential equation solver. The mean recoil is given by a double-exponential relaxation,

$$\langle x(t) \rangle = A_{\text{tot}} - A_s e^{-t/\tau_s} - A_l e^{-t/\tau_l} \quad (\text{B7})$$

with amplitudes A_s , A_l , and $A_{\text{tot}} = A_s + A_l$ and timescales τ_s and τ_l . The timescales and amplitudes read as Eqs. (7) and (8) in the main text.

From Eq. (8), one may explore the protocol parameters that make either A_s or A_l vanish [see Fig. 2(b) in the main text]. Notably, $A_i = 0$, i.e., vanishing eigenmode i does not correspond to $\langle x_i(0) \rangle = 0$ [24].

For a three-step protocol, we solve the equations analogously. For this case, it is possible to achieve situations where

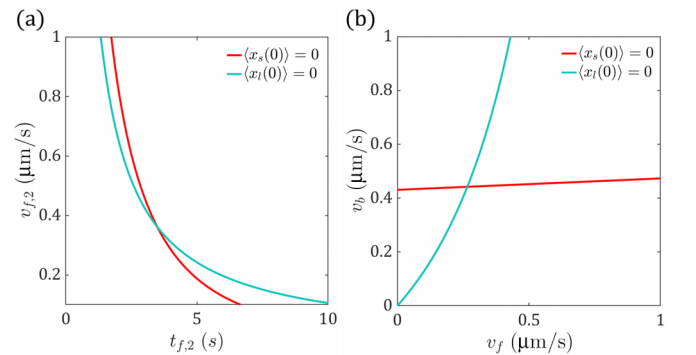


FIG. 5. Additional predictions for the case of zero recoils: (a) For $t_f = 60$ s, $v_f = 0.1$ $\mu\text{m/s}$, $t_b = 1.6$ s, and $v_b = 2$ $\mu\text{m/s}$, cancellation of recoils is predicted at $t_{f,2} = 3.48$ s with $v_{f,2} = 0.36$ $\mu\text{m/s}$. (b) Similarly, for $t_f = 20$ s, $t_b = 10$ s, and $t_{f,2} = 3.5$ s, recoils are canceled at $v_f = 0.27$ $\mu\text{m/s}$, $v_b = 0.44$ $\mu\text{m/s}$, with $v_{f,2} = 0.2$ $\mu\text{m/s}$.

the positions of two bath particles, at the moment of release, coincide with the position of the tracer. In this case, no recoil is observed. We provide the analytical solution for the initial tracer positions in the main text, and graphically give an

example for the condition of zero recoil in the inset of Fig. 3 in the main text (see Fig. 5 for more examples). For brevity, we do not provide the lengthy expressions for the amplitudes for a three-step protocol.

-
- [1] L. Gulia and S. Wiemer, Real-time discrimination of earthquake foreshocks and aftershocks, *Nature (London)* **574**, 193 (2019).
 - [2] Y. Kim, C. Hempstead, and A. Strnad, Critical persistent currents in hard superconductors, *Phys. Rev. Lett.* **9**, 306 (1962).
 - [3] L. F. Cugliandolo and J. Kurchan, Analytical solution of the off-equilibrium dynamics of a long-range spin-glass model, *Phys. Rev. Lett.* **71**, 173 (1993).
 - [4] A. Gurevich and H. K pfer, Time scales of the flux creep in superconductors, *Phys. Rev. B* **48**, 6477 (1993).
 - [5] A. Vaknin, Z. Ovadyahu, and M. Pollak, Aging effects in an anderson insulator, *Phys. Rev. Lett.* **84**, 3402 (2000).
 - [6] K. Matan, R. B. Williams, T. A. Witten, and S. R. Nagel, Crumpling a thin sheet, *Phys. Rev. Lett.* **88**, 076101 (2002).
 - [7] O. Ben-David, S. M. Rubinstein, and J. Fineberg, Slip-stick and the evolution of frictional strength, *Nature (London)* **463**, 76 (2010).
 - [8] D. M. Kaz, R. McGorty, M. Mani, M. P. Brenner, and V. N. Manoharan, Physical ageing of the contact line on colloidal particles at liquid interfaces, *Nat. Mater.* **11**, 138 (2012).
 - [9] A. Kovacs, Glass transition in amorphous polymers: A phenomenological study, *Adv. Polym. Sci.* **3**, 394 (1963).
 - [10] Y. Lahini, O. Gottesman, A. Amir, and S. M. Rubinstein, Nonmonotonic aging and memory retention in disordered mechanical systems, *Phys. Rev. Lett.* **118**, 085501 (2017).
 - [11] R. K rsten, V. Sushkov, and T. Ihle, Giant Kovacs-like memory effect for active particles, *Phys. Rev. Lett.* **119**, 188001 (2017).
 - [12] S. Dillavou and S. M. Rubinstein, Nonmonotonic aging and memory in a frictional interface, *Phys. Rev. Lett.* **120**, 224101 (2018).
 - [13] Y. He, D. Jiang, J. Chen, R. Liu, J. Fan, and X. Jiang, Non-monotonic relaxation and memory effect of rock salt, *Rock Mech. Rock Eng.* **52**, 2471 (2019).
 - [14] K. A. Murphy, J. W. Kruppe, and H. M. Jaeger, Memory in nonmonotonic stress relaxation of a granular system, *Phys. Rev. Lett.* **124**, 168002 (2020).
 - [15] Y. Tong, L. Song, Y. Gao, L. Fan, F. Li, Y. Yang, G. Mo, Y. Liu, X. Shui, Y. Zhang *et al.*, Strain-driven Kovacs-like memory effect in glasses, *Nat. Commun.* **14**, 8407 (2023).
 - [16] A. Amir, Y. Oreg, and Y. Imry, On relaxations and aging of various glasses, *Proc. Natl. Acad. Sci. USA* **109**, 1850 (2012).
 - [17] R. Mandal, D. Tapias, and P. Sollich, Memory in non-monotonic stress response of an athermal disordered solid, *Phys. Rev. Res.* **3**, 043153 (2021).
 - [18] M. Baiesi, S. Iubini, and E. Orlandini, The rise and fall of branching: A slowing down mechanism in relaxing wormlike micellar networks, *J. Chem. Phys.* **155**, 214905 (2021).
 - [19] M. E. Cates and S. J. Candau, Statics and dynamics of wormlike surfactant micelles, *J. Phys.: Condens. Matter* **2**, 6869 (1990).
 - [20] N. A. Spenley, M. E. Cates, and T. C. B. McLeish, Nonlinear rheology of wormlike micelles, *Phys. Rev. Lett.* **71**, 939 (1993).
 - [21] H. Rehage and H. Hoffmann, Rheological properties of viscoelastic surfactant systems, *J. Phys. Chem.* **92**, 4712 (1988).
 - [22] J. R. Gomez-Solano and C. Bechinger, Transient dynamics of a colloidal particle driven through a viscoelastic fluid, *New J. Phys.* **17**, 103032 (2015).
 - [23] M. Khan, K. Regan, and R. M. Robertson-Anderson, Optical tweezers microrheology maps the dynamics of strain-induced local inhomogeneities in entangled polymers, *Phys. Rev. Lett.* **123**, 038001 (2019).
 - [24] F. Ginot, J. Caspers, L. F. Reinalter, K. K. Kumar, M. Kr ger, and C. Bechinger, Recoil experiments determine the eigenmodes of viscoelastic fluids, *New J. Phys.* **24**, 123013 (2022).
 - [25] J. Caspers, N. Ditz, K. Krishna Kumar, F. Ginot, C. Bechinger, M. Fuchs, and M. Kr ger, How are mobility and friction related in viscoelastic fluids? *J. Chem. Phys.* **158**, 024901 (2023).
 - [26] This observation may be understood from the superposition principle applicable in this linear system (see Ref. [17]).
 - [27] The sign convention here is opposite to that used in [24,25]. Previous results are retrieved for $t_b = 0$ and $v_f = -v$.