

## Remotely tuned metamaterials with emergent elastic properties

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We introduce a framework for remotely tunable dynamical metamaterials based on interacting nodes embedded in an elastic matrix, effectively forming meta-atoms governed by a programmable nonlinear landscape. Using secondary acoustic radiation forces between microbubble arrays, we predict and experimentally realize an acoustically actuated linear unit cell exhibiting geometric reconfiguration, discontinuous switching, and memory. Building on these results, we introduce a general numerical model for the system that predicts diverse behaviors in larger lattices, including a remote stiffness softening up to 60% and auxeticity up to  $\nu = -0.22$ . The theory applies to other modes of actuation (e.g., optical or magnetic) and reveals universal behavior in these systems arising purely from the interplay of local  $r^{-n}$  forces and elastic interactions in the lattice.

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## I. INTRODUCTION

Metamaterials have garnered widespread interest across domains because of their general ability to exhibit otherwise unrealizable material properties through mesoscale geometric patterning [1]. In elastic systems, this principle has produced materials with negative Poisson's ratios, programmable stiffness, and multistability, opening opportunities in soft robotics, adaptive materials, and mechanical computation [2–9]. Recent efforts have sought to render such systems dynamic, enabling reversible and programmable reconfiguration under thermal, magnetic, electric, or mechanical stimuli [1,2,10,11]. Most existing approaches rely on modification of global material properties [12] or triggering of predesigned mechanical instabilities [10]. An alternative approach relies on externally imposed fields to control a metamaterial's response, such as magnetic fields that impact a crystal's response to flow [13] and sound fields that arrange particles through the primary radiation force into a regular crystal with tunable acoustic band gaps [14]. Such remotely actuated metamaterials could lead to new approaches in material computation and actuation that are not only programmable, but also responsive to local fields

and even time dynamic, supporting novel non-Hermitian and topological behaviors [1,15].

However, existing field-tunable metamaterials are limited by their reliance on globally defined compositional changes that are specific to a single system at large scales. For example, acoustically reprogrammable metamaterials track a globally determined resonance mode shape in a fixed chamber, and buckling elastic metamaterials follow preprogrammed mechanical pathways under load. While such demonstrations have illustrated key new concepts in dynamic metamaterials, the results do not provide mechanistic pathways to realize microstructured materials that could be implemented more broadly. If, rather than using external fields to move metamaterials through predefined global pathways, one could instead externally tune the interactions between meta-atoms, it would establish a new paradigm for designing more varied tunable metamaterial responses.

Here, we introduce a general framework for remotely actuated elastic metamaterials, defined as a matrix of meta-atoms connected by elastic springs. In an external field, induced interactions between each pair of nodes give rise to a central force of the form  $F \sim 1/r^n$ , where  $r$  is the distance between nodes. In practice, such induced forces could be dipole-dipole forces in a magnetic field ( $1/r^4$ ), optical binding forces between nanoparticles ( $1/r^4$ ) [16], or secondary radiation forces of external fields ( $1/r^2$ ) [17]. The induced force is countered by spring forces between connected nodes, allowing the external fields to controllably alter the effective interaction potential that governs the dynamics and equilibrium of the lattice.

As the building block of a metamaterial, we consider underwater nodes containing air bubbles, which are actuated by Bjerknes forces. Bjerknes forces are secondary acoustic radiation forces that arise between air bubbles in a sound field [17,18]. When a bubble is insonified, it oscillates radially,

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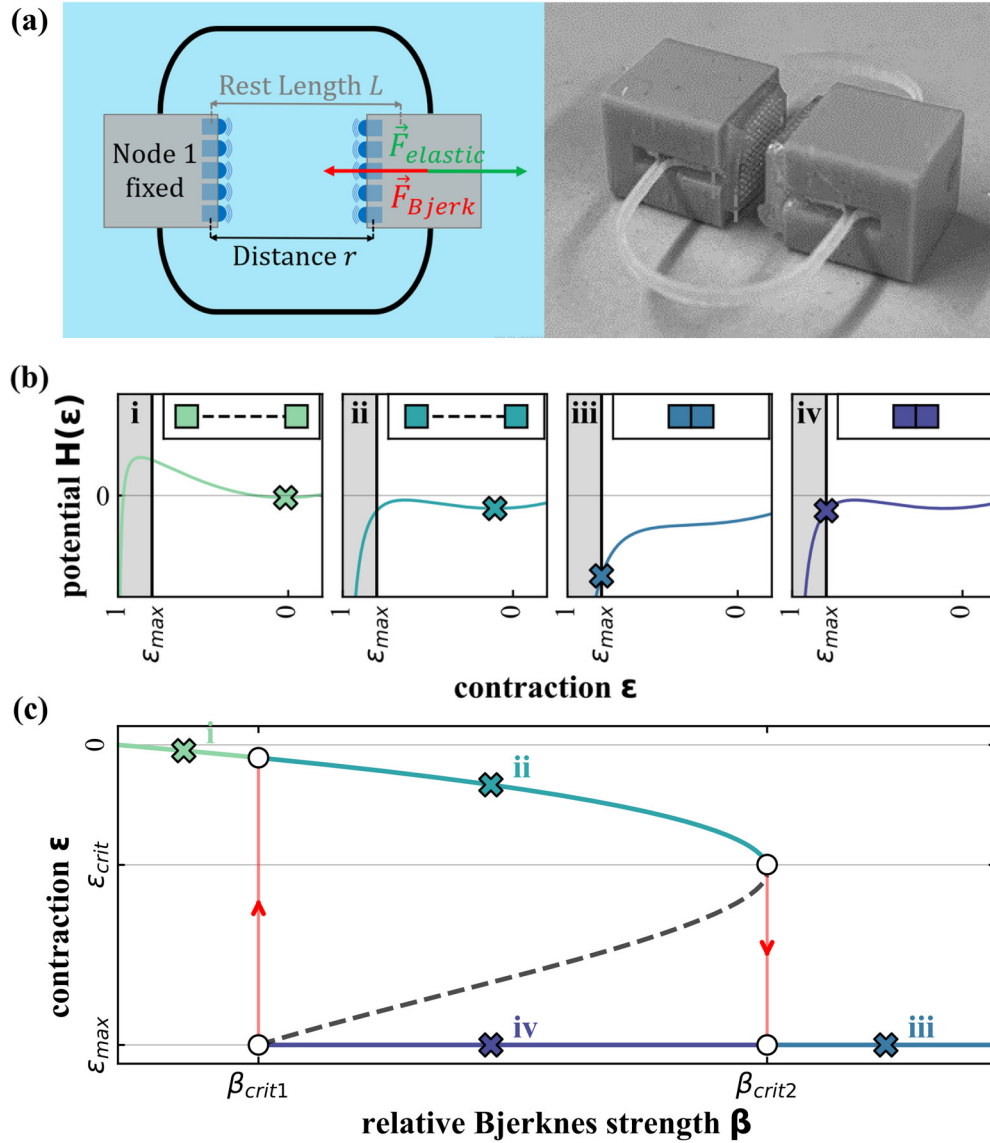


FIG. 1. (a) Schematic and image of the two-node actuator unit. Interacting bubble arrays are connected by a linear spring and the free node experiences competing Bjerknes and elastic forces. (b) Potential energy landscape for different acoustic driving regimes  $\beta$ , plotted as a function of contraction  $\epsilon = 1 - r/L \in [0, 1]$ . States (ii) and (iv) correspond to different local minima at the same  $\beta$ . (c) Hysteretic actuation profile of the actuator unit with acoustic driving. Branches (i) and (ii) support continuous actuation, while the system remains contracted after a discontinuous jump in panels (iii) and (iv).

producing a scattered pressure field. If, instead, two bubbles oscillate near each other in an acoustic field, the interaction of each bubble's own oscillation with the scattered field of the other leads to a small, symmetric force between the bubbles, known as the Bjerknes force. Such forces can be attractive or repulsive, but we focus on the attractive regime, where all bubbles oscillate in phase. The Bjerknes force follows an inverse-square law  $F_B = \alpha/r^2$  [18], where  $\alpha$  is referred to as the Bjerknes parameter and  $r$  represents the distance between bubbles.

## II. METHODS

### A. Experimental methods

The unit-cell actuators for the metamaterial were constructed from two-bubble arrays coupled by a Polydimethyl-

siloxane (PDMS) flexural spring. The spring and bubble arrays were mounted on three-dimensional (3D)-printed cubes such that the spring was centered between the cubes and the bubble arrays faced each other [Fig. 1(a)]. Each bubble array consisted of an  $11 \times 11$  square lattice of air cavities molded in PDMS (Sylgard 184, 10:1 base-to-curing-agent ratio). The cavities stably trapped air when submerged in water. Individual cavities had a diameter of  $300 \mu\text{m}$  and a height of  $250 \mu\text{m}$ , giving roughly an order of magnitude separation between the bubble diameter and the node-to-node spacing ( $L \sim 1 \text{ cm}$ ); this separation of length scales is used below to justify the pairwise interaction model.

The interconnecting flexural spring was laser cut from PDMS with a rectangular cross section of  $1 \text{ mm} \times 0.3 \text{ mm}$  and a measured spring constant of  $0.25 \text{ N m}^{-1}$ . The actuator was operated under acoustic driving at frequencies in the

vicinity of the bubble resonance (4.1 kHz, measured independently; see Appendix C).

Experiments were performed in a cylindrical water tank of 70 mm diameter. One actuator node was fixed to the tank bottom, while the second node was free to move. Acoustic excitation was provided by a 27-mm-diameter piezoelectric buzzer (RS PRO) bonded to the tank base. The buzzer was driven by a signal generator (Tektronix AFG1022) followed by a 50× voltage amplifier (Falco Systems WMA-300).

The motion of the free node was recorded from above using a digital camera with a macro lens. The driving signal and video acquisition were synchronized. Actuator displacement was extracted by image analysis implemented in Python. Detailed experimental descriptions, including fabrication protocols and resonance-frequency characterization, can be found in Appendix C.

### B. Numerical model

To predict the behavior of larger assemblies of nodes, we developed a numerical Hamiltonian framework that generalizes the two-bubble interaction to arbitrary lattice geometries. The total potential energy is built from pairwise Bjercknes attractions and local elastic restoring forces between connected nodes. In addition to minimizing this Hamiltonian, our numerical framework allows physical and geometric constraints to be imposed, such as incompressible connections between nodes, finite-element sizes, and nonoverlap conditions. The bulk lattice response to an acoustic excitation is then solved as a constrained energy minimization, allowing both physical and geometric constraints to be satisfied simultaneously. This is achieved by introducing a Lagrangian functional that combines both the physical Hamiltonian and all geometric constraints. The model takes as input the designed lattice geometry, as well as the experimentally measured elastic constants  $k_{ij}$  and acoustic interaction strengths  $\alpha_{ij}$ . The lattice deformations under acoustic driving are then calculated by finding the extrema of the functional using a hybrid solver that combines Newton's method [19] with sequential quadratic programming (SQP) [20]. Full details of the solver, together with a justification of the pairwise-interaction approximation underlying it, are given in Appendix E. The solver, along with the calculations performed in this article and the necessary data, are available on Zenodo [21].

## III. RESULTS

### A. Theoretical behavior of the two-node actuator

To describe the behavior of such a metamaterial, we first consider its fundamental building block: a two-node actuator connected by a spring. This unit actuator is shown with the relevant forces in Fig. 1(a), alongside an image of the experimental realization. Experimentally, we use bubble arrays rather than single bubbles at each node because of the larger forces generated [22]. The arrays can be analytically approximated as single bubbles interacting under a modified Bjercknes parameter (see Appendix D).

The competition between acoustic attraction and the elastic restoring force defines the equilibrium and dynamic behavior

of the actuator unit. The corresponding potential can be written as  $\mathcal{H}(r) = -\frac{\alpha}{r} + \frac{k}{2}(r - L)^2$ .

The force competition generates a nonmonotonic potential whose local minima determine stable configurations of the actuator for a given acoustic driving. In Fig. 1(b), we plot the potential in terms of the contraction  $\varepsilon = 1 - r/L$ , which varies from the rest state ( $\varepsilon = 0$ ) to a maximally contracted state where the bubbles are just barely touching ( $\varepsilon_{\max}$  at  $r = 2a$ , bubble radius  $a$ ).

The structure of the potential is governed by the dimensionless ratio  $\beta = \frac{\alpha}{kL^3}$ , which reflects the relative strength of acoustic and elastic interactions and is referred to as the relative Bjercknes strength. As shown in Fig. 1(b), as  $\beta$  is increased from 0, the divergence in the potential deepens until a second minimum occurs in the accessible contraction regime. A further increase of  $\beta$  deepens the singularity until the local minimum vanishes and the only stable configuration is the fully contracted state.

Due to its implicit dependence on the acoustic driving, the role of  $\beta$  reveals the possibility of acoustic control over the contraction of the actuator. If  $\beta$  is slowly varied from rest [Fig. 1(b)(i)] such that the actuator tracks the local minimum of the potential landscape adiabatically, a characteristic contraction curve can be constructed from  $\mathcal{H}(r)$  [Fig. 1(c)]. As the acoustic radiation force (and thus  $\beta$ ) is increased, the equilibrium follows the upper continuous branch [branches (i)/(ii)] until a critical threshold  $\beta_{\text{crit}2}$  is reached, where the local minimum vanishes and the system transitions discontinuously to the collapsed state at maximum contraction  $\varepsilon_{\max}$  [branch (iii)]. When the acoustic excitation is then decreased, the system remains trapped in this contracted configuration [branch (iv)] until a second threshold  $\beta_{\text{crit}1} < \beta_{\text{crit}2}$  is crossed, resulting in a pronounced hysteresis in the contraction-Bjercknes strength response.

This bistability endows the actuator with mechanical memory, encoding its excitation history through its configuration, and enables remotely reconfigurable mechanical states actuated purely by sound. Notably, the critical contraction limit  $\varepsilon_{\text{crit}} = 1/3$  and corresponding  $\beta_{\text{crit}2} = 4/27$  emerge as universal properties of the system, introducing a distinct discontinuity associated with bistable and hysteretic behavior from the interaction of Bjercknes and elastic forces alone (see Appendix A).

### B. Experimental validation of the two-node actuator

To validate this predicted behavior, we constructed a macroscopic actuator unit [Fig. 1(a); see Sec. II A and Appendix C for details] and measured the contraction as a function of the driving frequency  $f$  and acoustic amplitude  $V$ , which both implicitly affect  $\beta$ . For a fixed frequency, increasing the amplitude produced a continuous contraction up to a critical strain  $\varepsilon_{\text{crit}} = 1/3$ , followed by a sudden, discontinuous collapse to the fully closed configuration, consistent with the disappearance of a local minimum at  $\beta_{\max}$ . Similarly, frequency sweeps at constant amplitude revealed a pronounced and sudden total contraction around the resonant frequency of the bubble arrays. As the driving amplitude was increased, the frequency range corresponding to the collapsed state broadened, as shown in Fig. 2(a). In a small number of cases, the

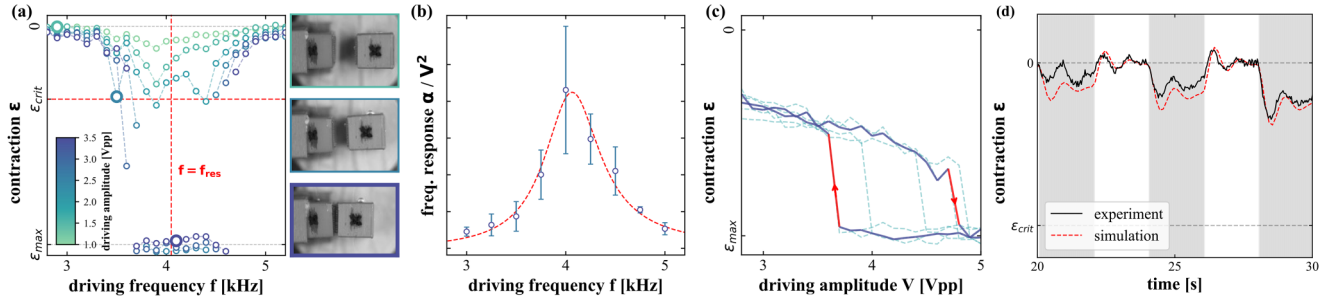


FIG. 2. Experimental results: (a) Frequency-dependent actuation for multiple acoustic strengths reveals a clear contraction peak around resonance. The actuator exhibits the predicted jump between states at  $\varepsilon_{\text{crit}} = 1/3$  for sufficiently strong actuation. Experimental snapshots demonstrate the predicted actuation branches. (b) The actuator response can be described by a master equation that captures all acoustic dependencies. (c) Slowly increasing then decreasing the acoustic driving reveals the hysteretic behavior of three individual actuators driven at resonance. The light dashed lines correspond to individual measurements and the bold line is the median. (d) Experimental and numerically predicted dynamics of an actuator unit driven by sound at sequentially increasing frequencies. Comparison of experimental data from a frequency sweep (black) and the solution of the dynamic pairwise model [Eq. (B2)].  $\beta(t) = \alpha(f(t), V)/kL^3$  was evaluated using the calibrated parameters  $\psi$  and  $\gamma$  from the previous panels; the only free fitting parameter was the damping ratio  $\zeta = 0.015 \text{ kg s}^{-1}$ .

actuator contraction exceeded  $\varepsilon_{\text{crit}}$  without fully collapsing, which we attribute to the single-bubble approximation and external forces in the experiment (see Appendix D). Besides these few isolated points, the results from 150 independent experiments strongly support the existence of the universal collapse strain  $\varepsilon_{\text{crit}} = 1/3$  predicted by our model, experimentally demonstrating the discontinuous switching and memory behavior of the unit cell.

The observed variations of  $\varepsilon$  with  $f$  and  $V$  can be quantitatively described by the dependence of the Bjerknes parameter  $\alpha$  on these parameters. The Bjerknes parameter for singular spherical bubbles scales with the square of both the bubble oscillation amplitude and the driving frequency [18,23]:  $\alpha \sim \delta^2 f^2$ . Similarly, the bubble scattering amplitude typically follows a Lorentzian profile around the resonance and is proportional to the acoustic pressure [24], which is itself proportional to the driving voltage. These trends suggest the functional form

$$\alpha(f, V; f_{\text{res}}, \gamma, \psi) = \frac{\psi f^2}{(2ff_{\text{res}}\gamma)^2 + (f_{\text{res}}^2 - f^2)^2} V^2, \quad (1)$$

where  $f_{\text{res}}$  is the resonant frequency of the bubble arrays,  $\gamma$  is a damping coefficient for the resonance, and  $\psi$  is a driving efficiency, reflecting how effectively the driving voltage is converted into a linear bubble oscillation amplitude via the acoustic pressure. Combining Eq. (1) and the solution of the Hamiltonian  $\mathcal{H}$ , a master actuator equation can be constructed that defines actuator contraction as a function of the acoustic driving parameters:

$$\frac{\alpha(f, V; f_{\text{res}}, \gamma, \psi)}{kL^3} = \varepsilon(1 - \varepsilon)^2. \quad (2)$$

Note that this actuator equation is only valid in the upper continuous branch of the contraction curve [branches (i) and (ii)] and has no solution for the maximally contracted state [branches (iii) and (iv)]. To evaluate the accuracy of this model, we plot the calibrated Bjerknes parameter  $\alpha/V^2$  as a function of driving frequency for 108 independent experiments in Fig. 2. The data demonstrate excellent agreement with the quadratically modulated Lorentzian line shape around the resonance  $f_{\text{res}} = 4.1 \text{ kHz}$ , which was measured

independently (see Appendix C). The damping coefficient  $\gamma = 0.08$  and driving efficiency  $\psi = 0.0019 \text{ N m}^2 \text{ V}^{-2} \text{ s}^{-2}$  were empirically determined as fitting coefficients. This result not only supports the validity of the master actuator equation (2) in predicting contraction behavior, but also provides a framework for empirical calibration of the response parameters  $\gamma$  and  $\psi$ , which is critical for predicting the behavior of larger lattices.

In addition to exhibiting the universal collapse strain and resonant behavior, the actuator unit also demonstrated clear hysteretic behavior [Fig. 2(c)]. As the driving voltage was adiabatically increased, the actuator slowly contracted until the critical strain, where it collapsed into the lower stable branch of the contraction curve [branch (iii)] and stayed fully contracted as the voltage was further increased. As the voltage was then decreased, the actuator remained in the contracted state beyond the critical cutoff point, following the predicted lower stable branch [branch (iv)] of the contraction curve. Quantitatively, the hysteresis in the experiments was less pronounced than in the point-bubble model, since the single-bubble approximation breaks down in the near field (see Appendix D). Nonetheless, the pronounced hysteresis of the actuator's response confirms the theoretical predictions and highlights the potential of the system as acoustically addressable memory.

Beyond the equilibrium (adiabatic) description above, the actuator's transient response is also well captured by the pairwise model. Figure 2(d) compares the experimentally measured contraction during a sequential frequency sweep to the numerical solution of the corresponding equation of motion (see Appendix B), using only the damping ratio as a free fitting parameter. The model reproduces the inertial overshoot and damped oscillatory relaxation observed experimentally, both when the acoustic field is switched on and when it is switched off.

### C. Predicted behavior of larger lattices

The actuator unit provides the base response from which more complex metamaterial lattices can be built. Building on the experimentally validated unit-cell model, we use our

numerical framework to predict how larger assemblies of nodes behave. In addition to wireless actuation and memory effects demonstrated above, our simulations of larger assemblies predict qualitatively new collective behaviors including dynamically tunable stiffness and auxetic mechanical responses, as shown below. These behaviors are expected to emerge in extended lattices because each bubble simultaneously interacts acoustically with multiple neighbors while being constrained by the surrounding elastic network. This interplay gives rise to a highly coupled and nonlinear energy landscape, from which new mechanical responses and actuation paradigms can emerge.

As the number of interacting elements increases, predicting the energy landscape and associated lattice response becomes intractable with analytical approaches, particularly in the face of geometric constraints such as steric node interactions or rigid connections. Our numerical Hamiltonian framework allows for the prediction and design of actuation behaviors that are not possible with the two-node actuator [Fig. 3(a)]. For example, the model predicts that a cross-shaped rigid skeleton at the center of a square lattice deforms the structure into an octagon with acoustically responsive angles [panel (i)]. Replacing square nodes with hexagonal ones is predicted to enable the densest possible contracted configuration [panel (ii)]. Finally, sparse interaction matrices (e.g., nodes without bubbles on all sides) are predicted to give rise to shear deformations [panel (iii)]. These lattice-scale predictions have not yet been experimentally validated at the multinode level; the pathways toward such experimental realization are discussed below.

Beyond controlling the macroscopic shape, our model reveals how the interplay between field-mediated forces and elastic forces can enable active modulation of the lattice's effective stiffness. When the lattice is subjected to an external load  $F_{\text{ext}}$ , acoustic excitation is predicted to alter the curvature of the potential energy landscape that governs node separation, changing the lattice stiffness. The predicted stiffness [Fig. 3(b)] of the actuator unit exhibits a pronounced asymmetry arising from the nonlinear Bjerknes interaction that also gives rise to the bistability. Under compressive loading, the actuator is rapidly contracted beyond the energy barrier into the collapsed state, resulting in a highly nonlinear spring. For tensile loading, however, the response remains approximately linear. The stiffness in the linear regime is predicted to be tunable wirelessly via the acoustic driving strength  $\beta$ . The effective stiffness ( $k_{\text{eff}} = F_{\text{ext}}/\Delta r$ ) is predicted to decrease with increasing  $\beta$  [Fig. 3(c)], with a maximal reduction in the original stiffness of nearly 62%. Tunable softening is predicted to be possible in more complex lattices as well: The effective stiffness of an  $N$ -node actuator chain decreases even more rapidly with increasing  $\beta$  for longer chains [Fig. 3(c)]. However, because of the stronger multibubble interactions, longer chains are pushed into the contracted state at lower  $\beta$ , and shorter chains can access a wider range of effective stiffness.

In addition to tunable elasticity, the model predicts that two-dimensional metalattices can exhibit an acoustically adjustable auxetic response. The magnitude of this predicted auxetic effect grows with increasing  $\beta$ , as stronger acoustic attraction deepens the contraction and amplifies the

directional asymmetry of the force balance. Consequently, the Poisson's ratio is predicted to decrease as the system approaches the edge of the continuous contraction regime [branches (i)/(ii)]. Once the system enters the fully contracted state, the hysteresis associated with the collapse eliminates the auxetic response. This imposes a hard bound on the achievable Poisson's ratios within this architecture: The strongest negative Poisson's ratios are predicted to arise only when the material is driven precisely at the boundary between continuous contraction and collapse. Since the auxetic response stems from this nonlinear interaction landscape, it is not a homogeneous property with a single fixed Poisson's ratio for all external forces. The simulations were performed on two-dimensional square lattices of varying size, geometrically modeled on the experimentally characterized actuator unit, under a loading protocol with external forces of  $F = 0.5kL$ , chosen to be comparable to the intrinsic lattice stiffness. The strongest auxetic response found in all simulations had a Poisson's ratio of  $\nu = -0.22$ , which is comparable to previously reported elastic metamaterials [25,26]. We further verified numerically that this predicted auxetic behavior is robust to fabrication-scale disorder: Introducing random variations of up to 10% in individual spring rest lengths and stiffnesses left the auxetic response essentially unchanged, confirming that it is a consequence of the nonlinear force balance rather than a symmetry-dependent artifact of an idealized lattice (Appendix G).

While other previously reported elastic metamaterials have been able to achieve auxetic responses up to  $\nu = -0.9$  [27], such behavior is programmed into the design and cannot be changed on the fly. Other approaches leveraging temperature-dependent materials have exhibited auxetic responses that can be actuated to  $\nu = -0.5$  [12], but thermal actuation typically involves long switching times between states.

In contrast, we predict that a Bjerknes metamaterial would be the first to provide an auxetic response that is continuously and wirelessly tuned with switching times on subsecond timescales, based on the switching speeds measured for the unit cell. The auxeticity emerges from the nonlinear interaction of Bjerknes and restoring forces that we show to be valid experimentally. Further numerical modeling indicates the auxetic behavior would further be robust to imperfections in the fabrication of the lattice (see Appendix G). Therefore, in practical realization we anticipate the main factor determining auxetic performance of large lattices will be to generate a strong and uniform sound field across the entire lattice, which everywhere generates forces on the order of the spring stiffness.

## IV. DISCUSSION AND CONCLUSION

### A. Validity of the pairwise interaction model

The lattice-scale predictions above rest on approximating the many-body acoustic interaction between nodes as a sum of pairwise Bjerknes forces. Based on the close alignment between our pairwise-force simulations and our measurements, we believe this approximation to be well justified for underwater bubbles. Unlike acoustically levitated rigid-particle clusters [28,29], where thermoviscous boundary-layer

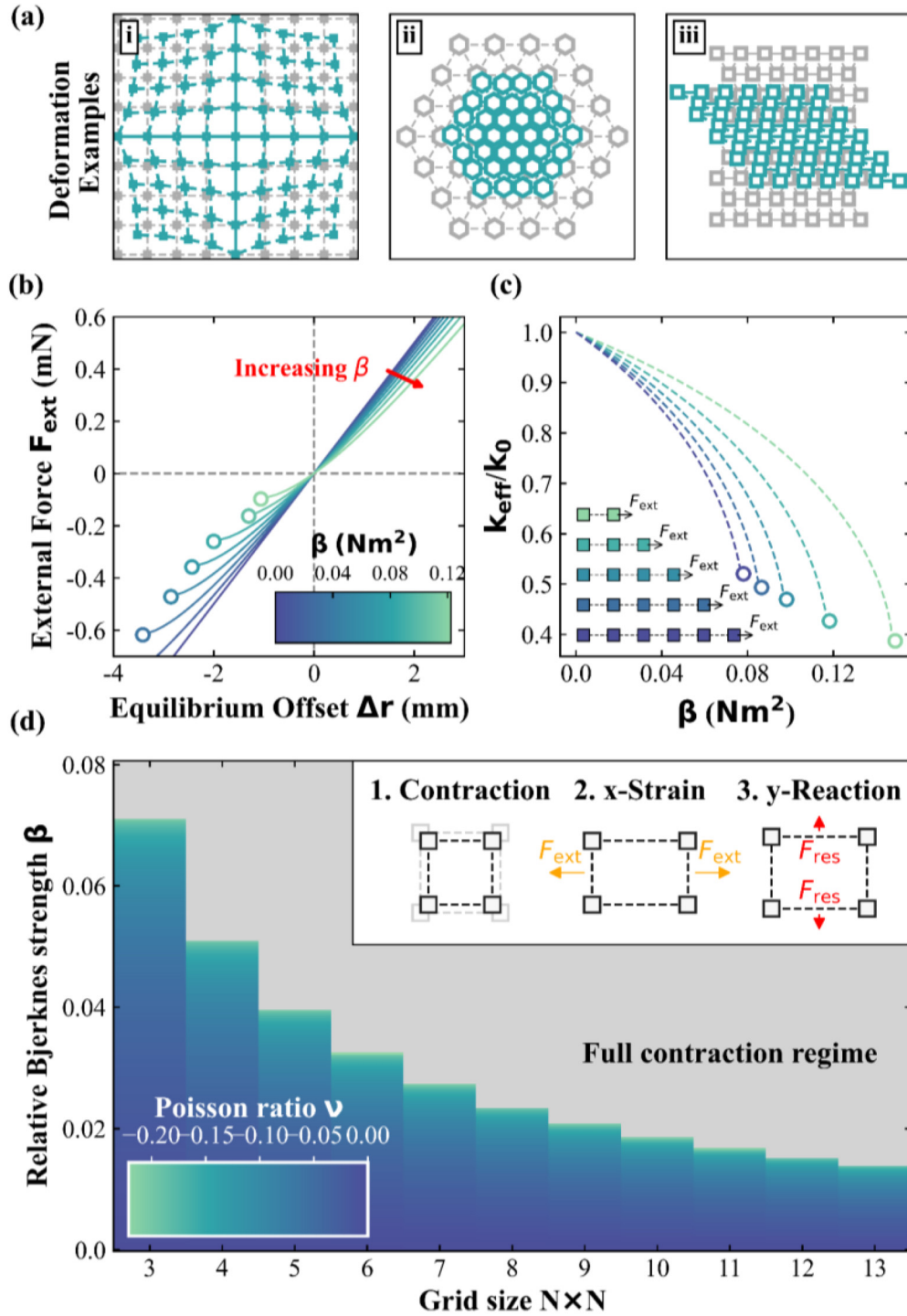


FIG. 3. Numerical predictions of complex deformation patterns and emergent behavior in larger lattices. (a) Lattice geometry and constraints provide a rich landscape of predicted lattice deformations: (i) constraint-guided shape change, (ii) lattice-dependent volumetric contractions, and (iii) asymmetry-enabled shearing deformations. (b) Predicted acoustically tunable elasticity emerges from nonlinearity in the Bjerknes force. (c) Predicted acoustically driven softening effect extends to  $N$ -node actuator chains: Longer chains soften more rapidly with  $\beta$ . (d) Predicted emergent auxetic response in two-dimensional square lattices of different sizes: The Poisson's ratio  $\nu$  decreases with  $\beta$  until the lattice fully contracts, with minimal  $\nu = -0.22$ . Inset: Auxetic behavior arises from spatial symmetry breaking during loading.

coupling, multibody, and nonconservative interactions can become significant, our system operates in a regime where these effects are expected to be small. First, our nodes are bubble arrays rather than single rigid particles, introducing an additional order of magnitude between the individual

bubble diameter ( $\sim 500 \mu\text{m}$ ) and the node-to-node spacing ( $L \sim 1 \text{ cm}$ ). For our 4 kHz driving in water, the viscous and thermal boundary-layer depths are approximately  $\delta_v = 9 \mu\text{m}$  and  $\delta_\alpha = 3 \mu\text{m}$ , respectively, so that both  $\delta_v, \delta_\alpha \ll R \ll L$ . In this regime, boundary-layer-driven coupling between distant

nodes is negligible, and we do not observe experimentally the kinds of thermoviscous coupling effects that have been reported for closely spaced rigid particles. In the fully collapsed actuator configuration, hydrodynamic coupling could in principle become more relevant; however, this was not observed experimentally for millimeter-scale bubble separations in Ref. [22], where a model relying solely on pairwise radiation-force interactions accurately predicted actuator dynamics. The agreement between the pairwise model and both the equilibrium and transient dynamics of our actuator unit [Fig. 2(d)] is consistent with this picture. We note that thermal and viscous streaming interactions could become more relevant for microbubble lattices with tighter spacing; for example, frequency-multiplexed bidirectional actuation exploiting the interplay of radiation and streaming forces has been demonstrated in two-bubble microscale systems [30].

We also expect nonlinear and multibody bubble effects to play a negligible role in our experiments. Our bubble arrays are driven in the linear regime, corresponding to linear monopolar oscillations [22]; high-speed microscopy of the driven arrays did not reveal surface oscillations, subharmonic oscillations, or higher-harmonic responses characteristic of nonlinear bubble dynamics. With the secondary radiation force as the dominant interaction pathway, the main route for higher-order multibody effects would be multiple acoustic scattering between bubbles. Prior treatments of multiple-scattering effects in bubble lattices indicate that their primary consequences are (1) lattice-driven attenuation within an acoustic band gap and (2) a shift of the collective resonance away from the single-bubble resonance frequency [31]. In Ref. [22], such resonance shifts were observed for bubble-array spacings below  $\sim 2$  mm, but were negligible at the centimeter-scale spacings used here. We therefore do not expect multiple-scattering effects to be significant for the node spacings and array sizes considered in this work, though this should be reevaluated for denser or more tightly packed lattices.

Similarly, this model needs to be reevaluated when the lattices are scaled down to the microscale and designed with singular microbubbles instead of arrays at the nodes. For this regime, no experiments were performed in this work and thus no conclusive statements about the validity can be drawn.

### B. Toward experimental realization of larger lattices

The lattice-scale behaviors discussed above—shape reconfiguration, tunable stiffness, and auxeticity—are, at this stage, numerical predictions built on an experimentally validated unit-cell model; they have not yet been demonstrated in a fabricated multinode lattice. Realizing such a lattice experimentally raises practical considerations beyond those of the single actuator, including inter-unit-cell variability, fabrication tolerances in bubble geometry and spring stiffness, and the need for an acoustic field that is sufficiently strong and spatially uniform across the entire lattice to generate comparable forces at every node. Our numerical model already indicates that the auxetic response is robust to random disorder of up to 10% in individual spring properties (Appendix G), suggesting that moderate fabrication tolerances are unlikely to be limiting. We anticipate that the dominant practical chal-

lenge for larger lattices will instead be generating an acoustic field that is uniform enough, across the full lattice footprint, to drive every node into a comparable regime of  $\beta$ . Our goal in this work is to establish the physical framework and the numerical tools that were validated against a well-characterized unit cell needed to design and predict such lattices; a direct multinode (e.g.,  $3 \times 3$  or larger) experimental demonstration is a natural and important next step, which we leave to future work.

### C. Generality to other field-mediated interactions

While our experiments and detailed model center on acoustic Bjerknes interactions, the qualitative conclusions of the theory extend to metamaterials built from other field-mediated interactions, such as magnetic or optical forces, that follow a similar power-law dependence on separation. The critical strain for a general attractive potential of the form  $-\alpha/r^N$  is  $\varepsilon_{\text{crit}} = 1 - \frac{N+1}{N+2}$ , so that a nonlinear dropoff in the interaction, and the associated bistability, hysteresis, stiffness reduction, and auxeticity, arises generically for any power-law interaction, including  $r^{-3}$  magnetic dipole-dipole interactions [32] or optical binding forces [16]. Moreover, because these phenomena occur within the continuously contracted regime, extending this regime enhances the magnitude of the elastic response.

This generality should not be mistaken for straightforward realizability, however. Each field-actuation mechanism introduces its own challenges: Our acoustic implementation is intrinsically an underwater one, since it relies on radiating bubbles in a liquid, and magnetic or optical implementations would introduce their own considerations, such as interaction anisotropy, phase-dependent coupling, intrinsic many-body effects, and nonconservative force components, which are not present in the idealized isotropic pairwise picture used here. The characteristic scale of magnetic and optical interactions also means that lattices built from these interactions would necessarily operate at micro- and nanoscales, where a central design challenge will be developing elastic elements (springs or other restoring interactions) of comparable stiffness to counteract the field-induced attraction at these scales. The identification of a common underlying mechanism across these platforms, rather than a demonstrated universal implementation, is the main generalizable contribution of this work.

### D. Conclusion

Our results demonstrate that the coupling of acoustic and elastic forces can generate rich, nonlinear behaviors in metamaterials. Using an experimentally validated two-node actuator, we have shown that acoustic radiation forces alone are sufficient to produce discontinuous switching and mechanical memory purely through sound. By exploiting these effects, we predict that the intrinsic mechanical properties of a metalattice can be reversibly tuned in a fully wireless manner, enabling nearly instantaneous transitions between soft and stiff states as well as controllable auxetic responses. The potential of such a metamaterial to change geometry, stiffness, and Poisson's ratio quickly and wirelessly could offer benefits for applications in soft robotics and fluid handling,

particularly when interfacing with soft media such as biological cultures and tissues. The architecture also lends itself to be extended in multiple directions by scaling (see Appendix F) or diversifying the physical interactions incorporated into the metamaterial. For instance, by tuning bubble geometry and the interaction forces, both attractive and repulsive interactions can occur [30,33,34], expanding the range of accessible deformation patterns and bulk elastic properties. Scaling the system to smaller bubbles and springs can also introduce additional thermal and viscous effects that can enable further rich control opportunities [30] alongside the Bjerknes forces (see Appendix F). Similar behaviors may be realizable using other switchable local interactions (magnetic, optical), which present alternate possibilities for realizing our metamaterial architecture at micro- and nanoscales, subject to the practical caveats discussed above. Our results are supported by robust and extensible analytical and numerical models that allow the behavior of complex lattices to be predicted and therefore designed. The system introduced here provides a foundation for designing complex, functional metalattices actuated by remote fields, opening new opportunities for microscale actuation and physical computing architectures, with direct multinode experimental realization as a clear next step.

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A.G.A. and P.F. conceived of the work; L.S., P.F., and A.G.A. designed experiments; L.S. developed analytical and numerical models; L.S. performed experiments; L.S. and A.G.A. analyzed the data; L.S. wrote the initial manuscript draft; L.S., P.F., and A.G.A. edited the manuscript; and A.G.A. and P.F. supervised the work.

#### DATA AVAILABILITY

The data that support the findings of this article are openly available [21].

#### APPENDIX A: ANALYTICAL SOLUTION TO THE ACTUATOR UNIT

Here, we derive the equilibrium configuration of the actuator unit within a Hamiltonian framework. Since we are interested in static equilibria, kinetic terms are for now neglected and the problem reduces to minimizing a potential energy. The actuator unit consists of two acoustically interacting bubbles connected by an elastic spring of stiffness  $k$  and rest length  $L$ . The interaction between the bubbles is governed by the secondary Bjerknes force, characterized by

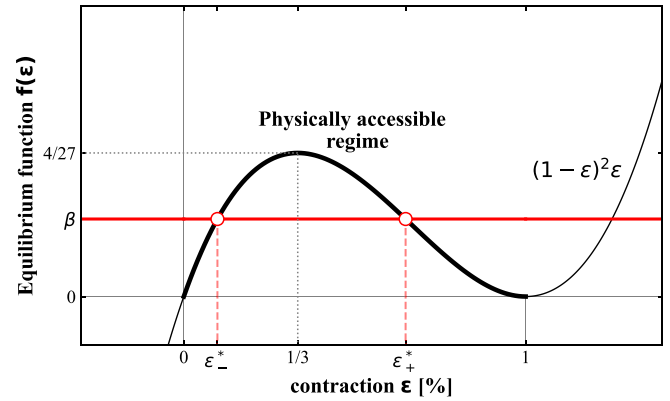


FIG. 4. Equilibrium function  $f(\varepsilon)$  whose intersections with a constant  $\beta$  define equilibrium positions of an actuator unit. For attractive acoustic interactions, the equilibrium is constrained to the range  $0 \leq \varepsilon \leq 1$ . When acoustic forces exceed elastic restoring forces by more than a factor of  $\beta_{\text{crit}2} = 4/27$ , then the only solution is beyond  $\varepsilon > 1$  and the actuator is fully contracted.

the parameter  $\alpha$ , where  $\alpha > 0$  corresponds to attraction and  $\alpha < 0$  to repulsion. Restricting the dynamics to one dimension along the actuator axis, the Hamiltonian reads

$$\mathcal{H}(r) = -\frac{\alpha}{r} + \frac{k}{2}(r - L)^2, \quad (\text{A1})$$

where  $r$  denotes the separation between the bubble center positions. Mechanical equilibrium at a separation  $r^*$  is obtained by setting the gradient of  $\mathcal{H}$  to zero. The force balance yields

$$\partial_r \mathcal{H}|_{r^*} = \frac{\alpha}{r^{*2}} + k(r^* - L) = 0. \quad (\text{A2})$$

By introducing the dimensionless relative contraction  $\varepsilon = 1 - r/L$  and the relative Bjerknes strength  $\beta = \alpha/kL^3$ , the equilibrium condition can be written compactly in terms of the equilibrium contraction  $\varepsilon^*$ :

$$\beta = f(\varepsilon^*) = (1 - \varepsilon^*)^2 \varepsilon^*. \quad (\text{A3})$$

Equation (A3) defines the extrema of the Hamiltonian as the intersections of  $\beta$  with the equilibrium function  $f(\varepsilon)$ , as shown in Fig. 4. While a fixed acoustic driving profile defines an equilibrium  $\varepsilon^*$ , as acoustic properties such as driving amplitude are varied,  $\alpha$  and thus  $\beta$  can change, allowing the system to be acoustically actuated.

For attractive interactions ( $\beta > 0$ ), physically accessible contractions satisfy  $0 \leq \varepsilon \leq 1$ , while repulsive interactions correspond to  $\varepsilon < 0$ .

Equilibrium configurations are generally mechanically stable only if they correspond to local minima of the potential energy. The function  $f(\varepsilon)$  possesses a single maximum at  $\varepsilon_{\text{crit}} = \frac{1}{3}$ , with a maximum value  $\beta_{\text{crit}2} = f(\frac{1}{3}) = \frac{4}{27}$ . As a consequence, stable equilibria exist only for  $\beta \leq 4/27$ . For  $0 < \beta < 4/27$ , two extrema appear in the potential: a stable minimum at larger separation (smaller contraction,  $\varepsilon^*_-$ ) and an unstable maximum at smaller separation (larger contraction,  $\varepsilon^*_+$ ). At  $\beta = 4/27$ , these extrema merge in a saddle-point bifurcation, and for  $\beta > 4/27$  no equilibrium exists. In this regime, the acoustic attraction overwhelms the elastic restoring force and the actuator collapses into the fully contracted

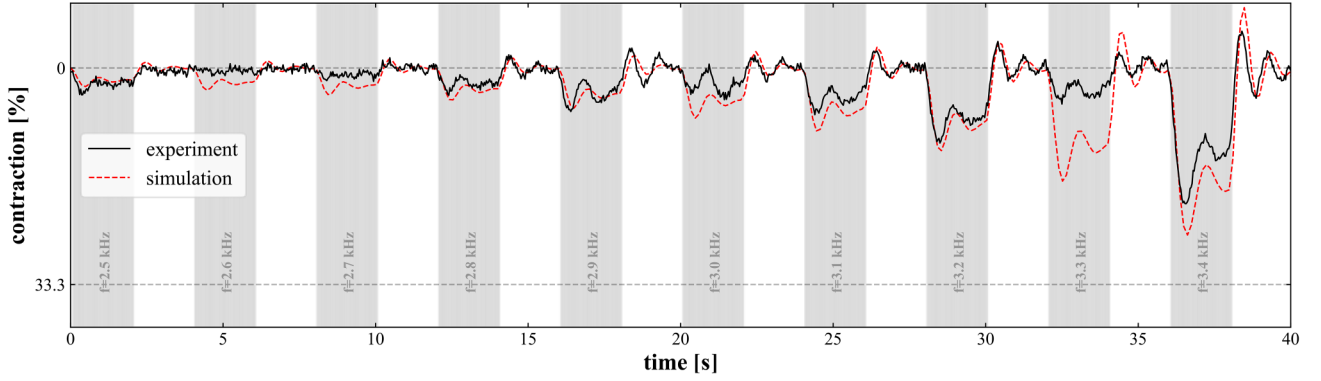


FIG. 5. Experimental and numerically predicted dynamics of an actuator unit being sequentially driven by sound at increasing frequencies. Comparison of experimental data from a frequency sweep (black) and the solution of Eq. (B2). The actuator unit in the experiment had a rest length of  $L = 6.5$  mm and a spring constant of  $k = 0.25$  N m $^{-1}$ . The nodes of the actuator unit had a mass of  $m = 5$  g.  $\beta(t) = \alpha(f(t), V)/kL^3$  was simulated through the calibrated parameters  $\psi$  and  $\gamma$ . The only fitting parameter for this simulation was the damping ratio  $\zeta = 0.015$  kg s $^{-1}$ . The good match between the predicted and qualitative dynamics supports that our theoretical analysis suffices to capture both the dynamics and the equilibrium of the actuator unit.

state  $\varepsilon_{\max} = 1 - a/L$ , with  $a$  being the diameter of a bubble. Following the unstable branch of the hysteresis curve reveals the second critical point at  $\beta_{\text{crit1}} = f(\varepsilon_{\max})$ .

Starting at a zero-driving equilibrium ( $\beta = 0$ ), this analysis establishes a universal contraction limit for the actuator unit:

$$0 \leq \varepsilon^* \leq \varepsilon_{\text{crit}} = \frac{1}{3}. \quad (\text{A4})$$

Up to this limit, the system can be continuously actuated. If the acoustic driving is further increased, the actuator discontinuously collapses to the closed  $\varepsilon = \varepsilon_{\max}$  state. To actuate along the other branch  $1/3 \leq \varepsilon^* \leq \varepsilon_{\max}$  of the equilibrium curve, the system must first be put into the collapsed state, and the acoustic driving then slowly reduced, as discussed in the main text.

If the actuator unit or the lattice in general is perturbed by an external force, this corresponds mathematically to a perturbed Hamiltonian:

$$\mathcal{H}'(r) = \mathcal{H}(r) - F_{\text{ext}}r. \quad (\text{A5})$$

Generally, this was solved numerically but for the actuator unit perturbed by an external force, an analytical equilibrium formula can also be derived. Again, we start by setting the

gradient of the perturbed Hamiltonian to zero

$$\partial_r \mathcal{H} = \frac{\alpha}{r^2} + k(r - L) - F_{\text{ext}} = 0. \quad (\text{A6})$$

Introducing  $\varepsilon$ ,  $\beta$ , and  $\gamma = F_{\text{ext}}/kL$  gives rise to a perturbed equilibrium condition

$$\beta = f'(\varepsilon', \gamma) = (1 - \varepsilon')^2 \varepsilon' + \gamma(1 - \varepsilon')^2. \quad (\text{A7})$$

We know that the unperturbed equilibrium condition is maximized at  $f(\varepsilon) = 4/27$ . Setting  $\beta$  to this maximum and calculating  $\varepsilon' - \varepsilon$  for different values of  $\gamma$  gives rise to the force-displacement curve with the lowest effective spring constant possible. This effective constant is a 61.97% reduction of the original value of  $k$ .

## APPENDIX B: DYNAMICS IN THE ACTUATOR UNIT

Adding kinetic and damping terms to the Hamiltonian leads to the following equation of motion for the actuator unit:

$$m\ddot{r} + \zeta\dot{r} + k(r - L) + \frac{\alpha}{r^2} = 0. \quad (\text{B1})$$

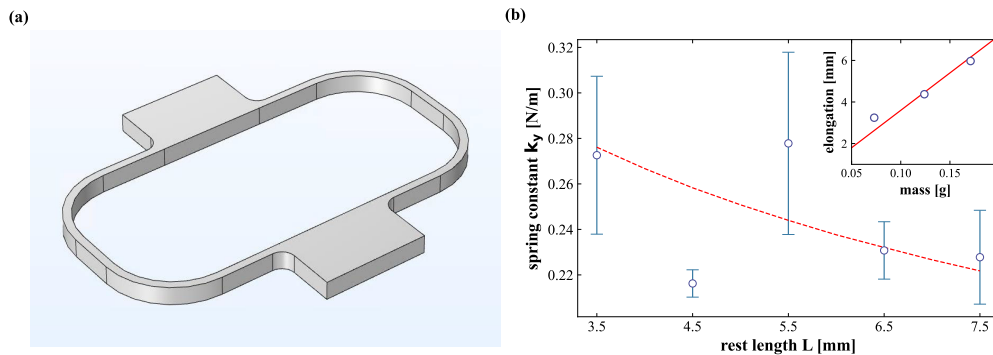


FIG. 6. (a) 3D model of the flexural springs that were used for the actuator unit. The thickness in  $z$  direction was difficult to control and therefore left constant. The spring constant was controlled via the rest length and the thickness in the  $xy$  plane. (b) Experimental measurements of the spring constant of springs with varying rest lengths and the finite-element simulation agree well. The inset shows a force-displacement curve for a spring with  $L = 3.5$  mm.

As before, we introduce the relative contraction  $\varepsilon$  via  $r = (1 - \varepsilon)L$ , with corresponding time derivatives  $\dot{r} = -L\dot{\varepsilon}$  and  $\ddot{r} = -L\ddot{\varepsilon}$ . The time evolution of the relative contraction  $\varepsilon(t)$  during actuation is then governed by

$$\ddot{\varepsilon} = -\omega_0^2 \left( \xi \dot{\varepsilon} + \varepsilon + \frac{\beta}{(1 - \varepsilon)^2} \right), \quad (\text{B2})$$

where  $\omega_0 = \sqrt{k/m}$  is the natural frequency associated with the elastic spring and the effective mass of the moving node,  $\xi = \zeta/k$  is the relative damping coefficient, and  $\beta$  is the relative Bjerknes strength during the frequency and amplitude sweeps each experiment therefore consisted of a single pulse of acoustic excitation followed by complete relaxation to the rest state. Repeating this procedure for different driving frequencies or amplitudes yielded reproducible contraction measurements that are independent of actuation history.

Starting from rest-state initial conditions ( $\varepsilon=0$  and  $\dot{\varepsilon}=0$ ), numerical solutions of Eq. (B2) reproduce the characteristic temporal response observed experimentally (Fig. 5). When the acoustic field is switched on, the attractive Bjerknes force drives contraction until the elastic restoring force balances the acoustic interaction at the equilibrium position. Due to inertia, the moving element overshoots this equilibrium, resulting in damped oscillations that decay toward the stable minimum.

When the acoustic excitation is switched off, the Bjerknes interaction vanishes and the system reduces to a damped harmonic oscillator displaced from its rest length. The actuator then relaxes smoothly back to  $\varepsilon = 0$ .

The qualitative agreement between experimental measurements and numerical solutions captures all salient features of the observed dynamics, including overshoot, oscillatory relaxation, and return to the undeformed state. This agreement indicates that the nonlinear  $1/r^2$  dependence of the secondary Bjerknes force, combined with linear elastic restoring forces and damping, is sufficient to explain the observed actuation dynamics.

## APPENDIX C: EXPERIMENTAL PROTOCOL TO BUILD AND USE THE ACTUATOR UNIT

### 1. Design and assembly

We fabricate the macroscopic actuator unit from the following components: two PDMS bubble arrays, a compliant PDMS spring, and two 3D-printed cubes for assembly. The spring connects the two cubes to provide a restoring force while the bubble arrays are mounted facing each other on the cube face, as shown in Fig. 1(a).

The bubble arrays were fabricated using soft lithography. A reusable silicon wafer mold was produced via photolithography using SU8-100 photoresist. The resist thickness defined the cavity height and was patterned using a custom bitmap mask printed at high resolution (Epson ET 1810) and exposed in a mask aligner (Kloe UV-KUB 2). After development and postbaking, the wafer contained arrays of cylindrical pillars serving as negative templates. PDMS (Sylgard 184, 10:1 base-to-curing-agent ratio) was degassed for 30 min, poured onto the wafer to a height of approximately 1 mm, degassed again, and cured at 65°C for at least 12 h. After curing, the PDMS

slab was peeled off, producing arrays of cylindrical cavities that trap air bubbles.

The resonance frequency of the bubble arrays was measured experimentally using high-speed imaging. Due to optical lensing effects, oscillating bubbles produce intensity fluctuations proportional to their radial oscillation amplitude [22]. Video frames were masked to isolate individual bubbles, and the mean pixel intensity was tracked. A rolling window of 15 frames was used to fit sinusoidal functions, extracting local oscillation amplitudes and frequencies. The resonance frequency was identified as the frequency at which the oscillation amplitude reached its maximum. A linear frequency chirp from 1 to 12 kHz over 1 s was applied while recording bubble oscillations at 50 000 fps using a Phantom TMX 6410 camera mounted on a custom 10× microscope. The measured resonance frequencies was  $f_{\text{res}} = 4.1$  kHz.

The PDMS springs were laser cut from uniformly cast PDMS sheets with thicknesses in the 1 mm range (Sylgard 184, 10:1 base-to-curing-agent ratio). The spring geometry, shown in Fig. 6(a), was designed with the help of finite-element simulations, using a Young's modulus for PDMS that was measured independently in a tensile tester ( $E = 2.66 \pm 0.07$  MPa). The simulations revealed strong sensitivity to beam thickness and weaker dependence on rest length. The springs were found to behave linearly, with diagonal stiffness tensor components. The desired spring geometry was expanded before cutting to account for kerf of the laser cutting beam. The cut springs were inspected optically and the spring constants were experimentally measured by measuring gravitational elongation of the springs under the weight of one of the mounting slabs. The measured spring constants showed good agreement with simulation results [Fig. 6(b)].

The mechanical connection was provided by cubic mounts 3D printed on a Prusa MK4S from polylactic acid. The cubes had external dimensions of 1 cm × 1 cm × 1 cm. Each cube contains a narrow slot for inserting the PDMS spring and a square recess for mounting the bubble array, ensuring precise and reproducible assembly of the actuator.

### 2. Actuator characterization

The actuator was submerged in a 70-mm-diameter water tank, with one cube rigidly fixed and the second cube free to move. Acoustic excitation was provided by a piezoelectric buzzer (RS PRO, 27 mm diameter) glued to the container bottom. The transducer was driven by a signal generator (Tektronix AFG1022) and amplified using a 50× amplifier (Falco Systems WMA-300). The container consisted of a Petri dish combined with a 3D-printed polylactic acid wall extension. Motion was recorded using a digital camera with a 2× macro lens. The cubes were marked with a high-contrast cross, which was tracked using the following image processing pipeline in Python: Frames were converted to grayscale, thresholded to isolate the crosses, and centroid positions were extracted for each cube independently. Image data were synchronized with the acoustic driving by analyzing the audio track embedded in the video. Frames where the sound levels exceeded 70 dB indicated the start of acoustic excitation. Pixel distances were converted into physical units using known geometric

TABLE I. Geometric and calibration parameters of the bubble array.

|                  |                                                         |
|------------------|---------------------------------------------------------|
| Height           | 300 $\mu\text{m}$                                       |
| Diameter         | 300 $\mu\text{m}$                                       |
| Spacing          | 500 $\mu\text{m}$                                       |
| $N$              | 11                                                      |
| $\psi$           | 0.0019 N m <sup>2</sup> V <sup>-2</sup> s <sup>-2</sup> |
| $\gamma$         | 0.08                                                    |
| $f_{\text{res}}$ | 4.1 kHz                                                 |

references from the actuator design, allowing computation of contraction as a function of time.

The geometric and calibrated data for the bubble array are collected in Table I.

#### APPENDIX D: SINGLE-BUBBLE APPROXIMATION FOR BUBBLE ARRAYS

Both in the analytical and numerical solutions, bubble arrays are modeled as effective singular bubbles with an amplified Bjerknes interaction strength. Here, we justify this approximation and quantify its limitations. For two oscillating point bubbles separated by a distance  $r$ , the attractive secondary Bjerknes force scales as

$$\mathbf{F} = \frac{\alpha_0}{r^2} \mathbf{e}_r, \quad (\text{D1})$$

where  $\alpha_0$  is the microscopic Bjerknes parameter. In the experiments, each actuator element consists of an  $N \times N$  array of bubbles, and the total force between two arrays is obtained by summing over all pairwise interactions,

$$\mathbf{F}_{\text{tot}} = \alpha_0 \sum_{i \in \mathbf{R}_1} \sum_{j \in \mathbf{R}_2} \frac{\mathbf{x}_i - \mathbf{x}_j}{|\mathbf{x}_i - \mathbf{x}_j|^3}, \quad (\text{D2})$$

where  $\mathbf{R}_1$  and  $\mathbf{R}_2$  denote the bubble positions in the two arrays. For two identical, planar  $N \times N$  arrays facing each other along the  $x$  axis and separated by a distance  $r$ , the symmetry of the configuration allows the total force magnitude to be written as

$$F_{\text{tot}} = \frac{\alpha_{\text{macro}}(r)}{r^2}, \quad (\text{D3})$$

with a distance-dependent effective Bjerknes parameter

$$\alpha_{\text{macro}}^{N \times N}(r) = \alpha_0 \left[ N^2 + \sum_{x=0}^{N-1} \sum_{y=1}^{N-1} \frac{4(N-x)(N-y)}{1 + (x^2 + y^2)\xi^2} \right], \quad (\text{D4})$$

where  $\xi = \Delta/r$  is the ratio of the intra-array spacing  $\Delta$  to the interarray separation  $r$ . This expression interpolates between two limiting cases: (1)  $\alpha_{\text{macro}} \rightarrow \alpha_0 N^2$  for  $\xi \rightarrow \infty$ , where only directly opposing bubbles interact, and (2)  $\alpha_{\text{macro}} \rightarrow \alpha_0 N^4$  for  $\xi \rightarrow 0$ , corresponding to a point-like cluster. For analytical tractability, we approximated the array interaction by a radially constant amplification,

$$\alpha_{\text{macro}}^{N \times N}(r) \approx \alpha_0 M, \quad M \in [N^2, N^4]. \quad (\text{D5})$$

This single-bubble approximation [Eq. (D5)] does not capture the exact spatial dependence of Eq. (D4), but its validity

only needs to hold in the continuously contracting regime of the actuator unit. Numerical evaluation using the experimental values for  $\alpha$  shows that, within this regime, the force-contraction curves obtained from the exact array interaction and from the single-bubble approximation agree closely (Fig. 7). Deviations become significant only near full collapse, where the system transitions discontinuously and the equilibrium description breaks down.

The distance dependence of  $\alpha_{\text{macro}}(r)$  introduces a weak dependence of the relative Bjerknes strength  $\beta$  on the contraction  $\varepsilon$ . As a result, the equilibrium condition  $\beta = f(\varepsilon)$  is modified quantitatively. For the experimentally relevant array geometry, this shifts the maximum attainable contraction in the continuous regime from the idealized value  $\varepsilon_{\text{crit}} = 1/3$  to  $\varepsilon_{\text{crit}} \approx 0.40$ . This correction does not alter the qualitative behavior of the actuator but explains the slightly extended continuous contraction observed in experiments. Since the single-bubble approximation breaks down in the near field, no quantitative agreement between theory and the experimental results can be found for the lower branch of the hysteresis curve. The analytical value for  $\beta_{\text{crit1}} = f(a)$  that was derived above will not hold for bubble arrays.

In the actuator unit, the two bubble arrays are aligned parallel to each other, and the interaction is effectively one dimensional. For higher-dimensional lattices, however, bubble arrays interact at arbitrary angles, and the angular dependence of the amplified force becomes relevant. While no closed-form expression exists for  $\alpha_{\text{macro}}(\mathbf{r})$  in this general case, direct numerical summation of Eq. (D2) shows that near-field deviations from radial symmetry are confined to distances comparable to the array size. In the far field, the force field generated by a cube carrying bubble arrays on multiple faces converges to that of an effective monopole with amplified strength. Consequently, bubble arrays can be accurately modeled as effective singular bubbles in the continuous actuation regime and at distances exceeding the array dimensions. This approximation underlies the analytical treatment and the numerical lattice simulations presented in the main text.

#### APPENDIX E: IMPLEMENTATION OF THE NUMERICAL EQUILIBRIUM SOLVER

To determine the equilibrium configurations of arbitrary acoustic metamaterial lattices, we developed a numerical solver that minimizes a many-body Hamiltonian subject to physical and geometric constraints. The system is described using a Hamiltonian framework in which the total potential energy consists of contributions from secondary Bjerknes interactions and elastic spring forces. For a lattice of  $N$  interacting bubbles (or bubble arrays), the Hamiltonian is written as

$$\mathcal{H}(\mathbf{R}) = \sum_{\substack{i=1 \\ j>i}}^N \left( -\frac{\alpha_{ij}}{|\mathbf{r}_i - \mathbf{r}_j|} + \frac{1}{2} (\mathbf{r}_i - \mathbf{r}_j - \mathbf{L}_{ij})^T \mathbf{K}_{ij} (\mathbf{r}_i - \mathbf{r}_j - \mathbf{L}_{ij}) \right), \quad (\text{E1})$$

where  $\mathbf{r}_i$  denotes the position of bubble  $i$ ,  $\alpha_{ij}$  is the Bjerknes interaction strength,  $\mathbf{L}_{ij}$  is the spring rest-length vector, and

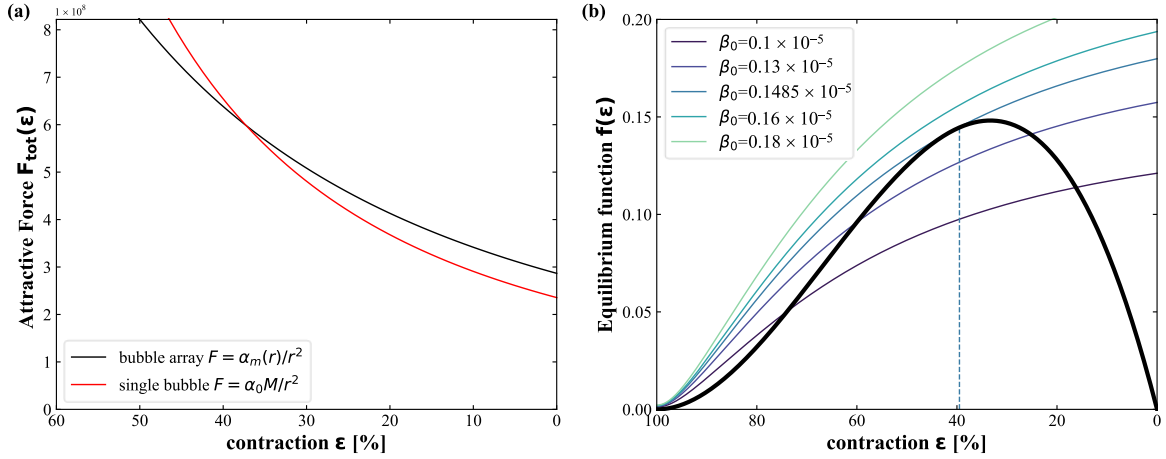


FIG. 7. (a) Comparison of the attractive Bjerknes force in the single-bubble approximation and for a bubble array type A in an actuator unit with  $L = 6.5$  mm. In this example,  $M = 9951.7$ . (b) For the single-bubble actuator unit, the equilibrium condition is  $\beta = f(\epsilon)$ . This is changed for bubble arrays to  $\beta(\epsilon) = f(\epsilon)$ . The plot shows  $\beta(\epsilon)$  for different  $\beta_0 = \alpha_0/kL^3$  and the equilibrium function in black. Because of the curvature of  $\beta(\epsilon)$ , the nonlinear cutoff for the continuous contraction regime moves to  $\epsilon_{\text{crit}} = 0.4$ .

$\mathbf{K}_{ij}$  is the anisotropic spring stiffness tensor expressed in the global coordinate frame. The equilibrium configuration corresponds to a minimum of  $\mathcal{H}$ .

Physical constraints—such as rigid connections, fixed nodes, and nonoverlap conditions—are incorporated using Lagrange multipliers. The constrained optimization problem is formulated via the Lagrange function

$$\mathcal{L}(\mathbf{R}, \lambda, \psi) = \mathcal{H}(\mathbf{R}) + \sum_i \lambda_i g_i(\mathbf{R}) + \sum_j \psi_j h_j(\mathbf{R}), \quad (\text{E2})$$

where  $g_i(\mathbf{R}) = 0$  represent equality constraints (e.g., fixed distances or rigid connections) and  $h_j(\mathbf{R}) \leq 0$  represent inequality constraints (e.g., nonoverlap conditions). This formulation ensures that equilibrium configurations satisfy both mechanical force balance and imposed geometric restrictions. Equilibrium states are obtained by solving the stationarity

conditions

$$\nabla_{\mathbf{R}, \lambda, \psi} \mathcal{L} = 0, \quad (\text{E3})$$

which generally correspond to saddle points of the Lagrange function rather than strict minima. Consequently, conventional gradient-descent methods are insufficient. Instead, we employ a hybrid strategy combining Newton's method and SQP. Newton's method provides rapid convergence by incorporating second-derivative information through the Hessian of  $\mathcal{L}$ . However, the Hessian can become singular or ill-conditioned due to constraints. To overcome this limitation, we implement SQP, which replaces the full Hessian inversion with a sequence of quadratic subproblems that remain well posed and computationally efficient. The optimization is implemented using the SciPy routine `scipy.optimize.minimize` with the sequential least squares quadratic programming backend [35]. Tight convergence tolerances are used to accurately resolve the shallow energy landscapes encountered at low driving amplitudes.

The computational framework scales efficiently to lattices containing tens to hundreds of nodes and enables systematic exploration of deformation modes, multistability, stiffness tuning, and auxetic behavior under acoustic actuation.

## APPENDIX F: SCALING LAWS

While the macroscopic actuators presented here provide an effective test bed for exploring the fundamental properties of acoustically actuated elastic lattices, their practical relevance is inherently limited. Their primary advantages lie in rapid prototyping, iterative design, and direct visualization of deformation dynamics. In contrast, the most promising real-world applications emerge at the microscale.

Importantly, the physical principles governing actuator dynamics, namely, the balance between secondary Bjerknes forces and elastic restoring forces, exhibit scale invariance. Both force contributions scale identically with geometric dimensions, ensuring that their interplay remains effective

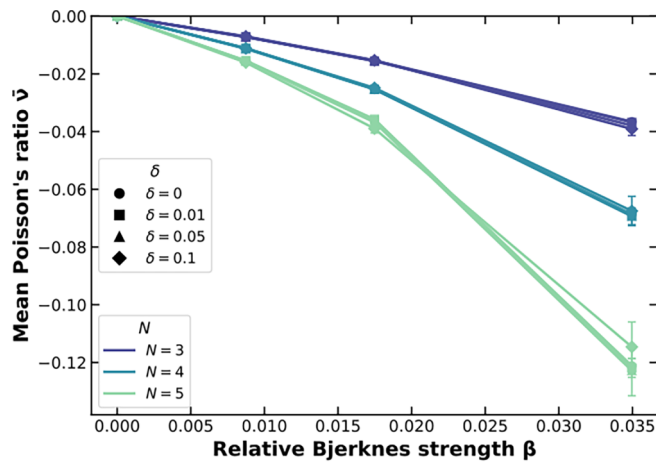


FIG. 8. For varying lattice sizes, the Poisson's ratio is computed as a function of the relative acoustic excitation. This analysis was performed for increasing levels of imperfections until 10%. No significant differences could be quantified.

across length scales. Consequently, the conceptual framework developed at the macroscale naturally translates to microscale systems.

This scaling behavior can be illustrated by two back-of-the-envelope estimates,

$$F_{\text{Bjerk}} = \frac{\alpha}{r^2} \rightarrow \frac{[\alpha]}{d^2} = \frac{d^6[\omega^2]}{d^2} = d^2, \\ F_{\text{spring}} = k(r - L) \rightarrow [k]d = d^2, \quad (\text{F1})$$

where  $[\cdot]$  denotes the scaling component of the corresponding quantity and  $d$  is the characteristic system size. Distance variables scale linearly with  $d$ . The Bjerknes coupling parameter  $\alpha$  contains the bubble radius to the sixth power and the driving frequency squared. Assuming resonance driving, the resonance frequency scales inversely with system size, such that the Bjerknes force scales quadratically with  $d$ .

For the elastic restoring force, the scaling of the spring constant  $k$  must be determined. The analytical expression for

the stiffness of a helical spring [36], supported by finite-element simulations in COSMOL, indicates a linear scaling law,  $k \sim d$ .

## APPENDIX G: AUXETICITY UNDER NOISE

To validate that the auxetic behavior of the lattice stems from the nonlinear interaction between the individual nodes and not due to symmetry effects and is therefore not affected by irregularities in the fabrication, the Poisson ratio was calculated for irregular  $N \times N$  lattices with rest lengths of  $L = (1 + \epsilon)L_0$  and  $k = (1 + \epsilon)k_0$ , where  $\epsilon$  was randomly sampled from  $[-\delta, \delta]$ .

The results of this analysis, shown in Fig. 8, clearly show that imperfections in the lattice even until a variation of 10% do not significantly change the auxetic behavior, and that the auxeticity is not a symmetry-induced effect.

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