

Direct estimation of arbitrary observables of an oscillator

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Quantum harmonic oscillators serve as fundamental building blocks for quantum information processing, particularly in the context of the bosonic circuit quantum electrodynamics (cQED) platform. Conventional methods for extracting oscillator properties rely on predefined analytical gate sequences to access a restricted set of observables or resource-intensive tomography processes. Here, we introduce the Optimized Routine for Estimation of any Observable (OREO), a numerically optimized protocol that maps the expectation value of arbitrary oscillator observables onto that of an ancillary qubit. We demonstrate OREO in a bosonic cQED system as a means to efficiently measure phase-space quadratures and their higher moments, directly obtain faithful non-Gaussianity ranks, and effectively achieve state preparation independent of initial conditions in the oscillator. These results position OREO as a valuable tool for direct and efficient information extraction from bosonic quantum states, unlocking possibilities for measurement, control, and state preparation in continuous-variable quantum information processing.

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I. INTRODUCTION

Quantum harmonic oscillators offer a powerful platform for generating and manipulating photons, which are fundamental resources for a wide range of quantum information processing applications [1]. Among various physical implementations, the bosonic circuit quantum electrodynamics (cQED) platform, which leverages superconducting microwave cavities coupled to nonlinear ancillary transmon qubit, stands out for its excellent quantum coherence and control versatility [2–4]. Thanks to these features, bosonic cQED systems have enabled many remarkable achievements, from the creation of robust logical qubits [5–7] to the demonstrations of effective quantum metrology [8,9].

In typical bosonic cQED devices, information about the harmonic oscillator is extracted via a controlled unitary operation that maps a target property of the oscillator to a distinct transmon state. Three main analytical gate sequences have been developed to enable direct experimental access to the expectation values of the excitation number [10], parity [11], and displacement [12] operators. Each of these affords a path toward full tomography [13–16] by incorporating many displacement points in phase space, which then

allows experimentalists to extract any target information from the reconstructed oscillator state via postprocessing. However, such tomographic approaches are typically resource intensive, and often excessive when the desired information is associated with a specific observable. Thus, it is desirable to explore a more versatile experimental procedure capable of directly estimating arbitrary observables.

In this work, we present a one-fits-all technique to estimate the value of any observable associated with a harmonic oscillator via standard qubit measurements. This method, dubbed the Optimized Routine for Estimation of any Observable (OREO), employs numerically optimized pulses to encode the expectation value of a target observable into the state of an ancillary transmon, allowing oscillator properties to be extracted through standard qubit readout (Fig. 1). OREO is implementable on any oscillator-qubit system that allows universal control on the oscillator space [17], such as trapped ions/atoms [18,19], cavity QED [20], and quantum acoustics [21]. Compared to the standard Hadamard test, OREO offers a notable reduction in the resources required to obtain the expectation value of an arbitrary Hermitian operator [17]. Here, we experimentally execute OREO in a bosonic cQED architecture, enabling efficient measurement of several useful observables that are typically inaccessible directly. In the first experiment, we probe the phase-space quadratures and their higher-order moments, bridging a common gap in bosonic cQED systems, where direct quadrature measurement is typically hindered by the stationary nature of the oscillator mode. In the second, we apply OREO to quantify degrees of non-Gaussianity, providing an efficient means to assess the quality of quantum resources. The last experiment expands on our technique by using the qubit to herald the projection of the oscillator onto an arbitrary state, which affords a strategy to

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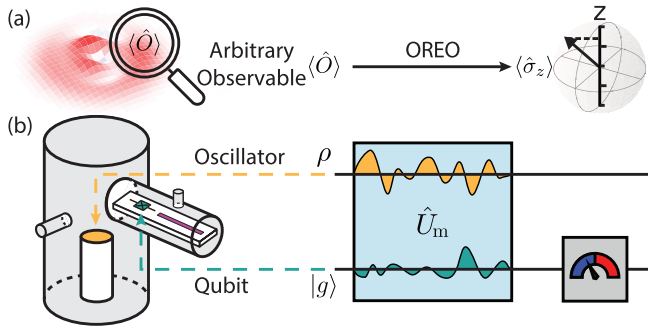


FIG. 1. Observable mapping. (a) The OREO maps the expected value of any target observable $\langle \hat{O} \rangle$ of a harmonic oscillator state onto the z axis of a qubit's Bloch vector $\langle \hat{\sigma}_z \rangle$. (b) The cQED setup and protocol, which consists of a numerically optimized pulse $\hat{U}_m$ applied to the oscillator-qubit system, followed by a qubit readout. A step-by-step implementation guide is included in the Supplemental Material [17].

perform robust state preparation independent of the initial condition of the oscillator.

II. METHODS

Our implementation of OREO employs a standard bosonic cQED device [22], shown in Fig. 1, in which a three-dimensional superconducting cavity couples to a transmon qubit in the dispersive regime [17]. A planar resonator patterned on the same transmon chip allows direct single-shot measurement of the transmon state. In such a system, universal control of the oscillator state has been demonstrated using numerically optimized drives on both the cavity and transmon [23,24]. Here, we expand the capabilities of this approach to realize direct mapping of the expectation value of any observable of the bosonic mode to the transmon state.

Conceptually, any observable associated with a quantum harmonic oscillator can be written as $\hat{O} = \sum_k \Lambda_k |\psi_k\rangle \langle \psi_k|$, where $\Lambda_k \in \mathbb{R}$ is any real number and $\{|\psi_k\rangle\}$ is any basis. Since the qubit measurement restricts the outcomes to be $\langle \hat{\sigma}_z \rangle \in [-1, 1]$, we define a scaled version of the observable, $\hat{O} = \hat{O}/f = \sum_k \lambda_k |\psi_k\rangle \langle \psi_k|$, with $f = \max_k |\Lambda_k|$ such that $\lambda_k \in [-1, 1]$. Next, we numerically optimize a pulse sequence to enact a unitary operation, $\hat{U}_m(\hat{O})$, which is applied to the initial state $|g\rangle \langle g| \otimes \rho$, where $|g\rangle$ is the ground state of the qubit and ρ is an arbitrary state of the oscillator, to implement the mapping process. More specifically, our framework optimizes the drives to minimize the elementwise mean square error between $\hat{O}$ and $\hat{O}_{\text{eff}}$, where $\hat{O}_{\text{eff}} \equiv 2\langle g|\hat{U}_m^\dagger|g\rangle\langle g|\hat{U}_m|g\rangle - \mathbb{I}_c$ denotes an effective observable with $\mathbb{I}_c$ the identity operator within a chosen truncation dimension D where the oscillator state is fully contained [17]. Finally, a standard readout is performed on the qubit after $\hat{U}_m$ to obtain

$$\langle \hat{\sigma}_z \rangle = 2p_g - 1 = \text{tr}_c(\hat{O}_{\text{eff}} \rho), \quad (1)$$

where $\hat{\sigma}_z$ denotes the Pauli- z operator, p_g the probability of the qubit in its ground state, and tr_c the trace with respect to the oscillator. This completes the full OREO process and leads to a direct reading of an arbitrary $\langle \hat{O} \rangle$ [17]. OREO enables

a significantly more efficient estimation of $\langle \hat{O} \rangle$ compared to calculating it from a reconstructed density matrix obtained via full-state tomography.

III. RESULTS

To demonstrate the efficacy of OREO in a practical use case, we apply the procedure to directly obtain the expectation values of the phase-space quadratures of the oscillator states residing in the superconducting cavity, as well as their higher moments. Phase-space quadratures are directly accessed in optical systems through homodyne detection and are frequently featured in continuous-variable information processing protocols, such as quantum teleportation, cryptography, and error correction [1]. In contrast, the oscillator states encoded in high- Q cavities are strongly isolated from the environment, a feature that affords excellent lifetimes but hinders direct access to its quadrature information via homodyne detection. OREO bridges this gap, providing an efficient and practical method to extract the values of quadrature moments of arbitrary order without special hardware requirements. Access to such information provides a valuable primitive for tasks such as the construction of entanglement witnesses [25], implementations of nonlinear phase states [26,27], and universal quantum computation via nonlinear squeezing [28–30].

Experimentally, we obtain $\langle \hat{\sigma}_z \rangle$ from standard single-shot readout after applying drives obtained numerically to map the quadratures and their higher moments onto the qubit. We then perform a simple correction to account for readout errors [17]. The results for the observables $\langle \hat{x} \rangle$, $\langle \hat{x}^2 \rangle$, $\langle \hat{p} + \hat{x}^2 \rangle$, and $\langle (\hat{p} + \hat{x}^2)^2 \rangle$ for a coherent state $|\alpha\rangle$ displaced to various locations in phase space are shown in Fig. 2. The data show a strong quantitative agreement with the theoretical values, which shows that OREO is capable of reliably measuring arbitrary quadrature functions. The minor deviations observed for larger displacements, for example, $|\alpha| \approx \sqrt{2}$, is due to the leakage of the Fock-state population outside the truncation dimension of $D = 6$ used in this experiment and can be improved by increasing the truncation dimension [17].

Next, we apply OREO to benchmark the quality of bosonic quantum states by probing an observable directly for the first time to infer their non-Gaussianity rank [31–34], beyond simply Wigner negativity [35–40]. For Fock states and their superpositions, an accurate observable is introduced in Ref. [34]:

$$\exists \lambda : \langle \hat{\Pi}_n + \lambda \hat{\Pi}_{n+1} \rangle > F_n(\lambda), \quad (2)$$

where $F_n(\lambda)$ is a calculated threshold [17], $\hat{\Pi}_n$ is the projector onto the n th Fock state, and λ is a free parameter. A state satisfying Eq. (2) means that it could not have been prepared by applying any Gaussian operation on Fock-state superpositions involving up to $|n-1\rangle$ [32]. This imposes a hierarchy, certifying the quality of Fock states with higher n , which are crucial resources, e.g., for quantum metrology [41,42].

We investigate this experimentally for two specific non-Gaussian resource states. In the first case, a Fock state $|3\rangle$ was prepared using a standard numerically optimized state-transfer process [24]. We also performed a standard Wigner

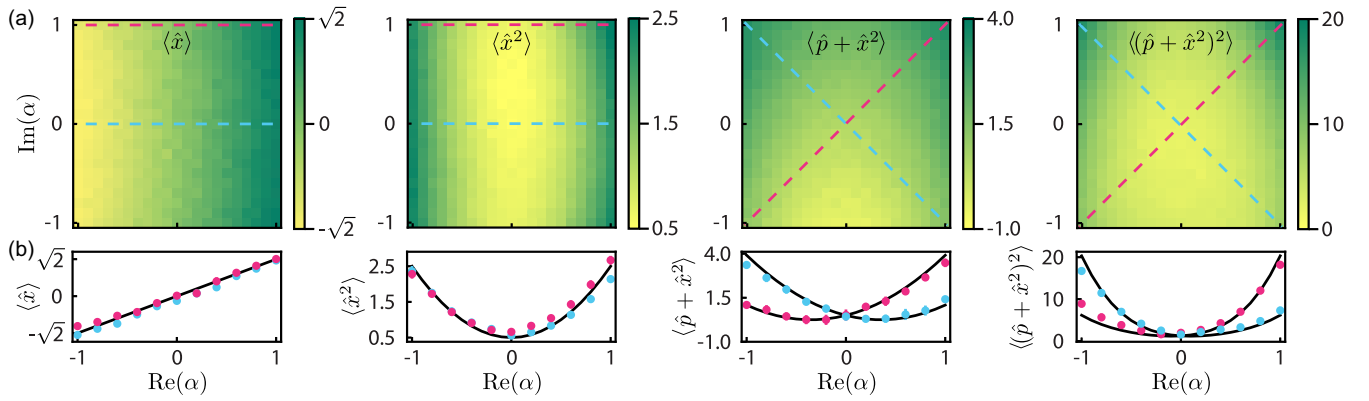


FIG. 2. Direct estimation of polynomials of phase-space quadratures. (a) Application of OREO to directly obtain the values of $\langle \hat{x} \rangle$, $\langle \hat{x}^2 \rangle$, $\langle \hat{p} + \hat{x}^2 \rangle$, and $\langle (\hat{p} + \hat{x}^2)^2 \rangle$ of a variable coherent state $|\alpha\rangle$. The dashed lines indicate one-dimensional cuts in the data plotted in panel (b), which quantitatively agree with the theoretical expected values (solid curves).

tomography and obtained the reconstructed density matrix via Bayesian inference [13], from which we confirm that a state $|3\rangle$ was prepared with a fidelity of $F = 0.92(3)$ [Fig. 3(a)]. We then perform OREO to directly extract $\langle \hat{\Pi}_3 + \lambda \hat{\Pi}_4 \rangle$, instead of constructing it by separately measuring $\langle \hat{\Pi}_3 \rangle$ and $\langle \hat{\Pi}_4 \rangle$, from single-shot qubit measurements for $\lambda = -1, -0.5, 0, 0.5, 1$. The measurement outcomes, shown as

blue circles in Fig. 3(b), are considerably above the threshold $F_3(\lambda)$ (lower boundary of the blue shaded region), indicating that the state has non-Gaussianity rank of 3. The results, obtained from a single measurement at each value of λ , are consistent with those derived from the reconstruction of the full density matrix via Wigner tomography (dotted line), while reducing the raw measurement time by nearly three orders of magnitude [17].

To further illustrate the reliability of OREO in identifying the non-Gaussianity rank, we execute the same experiment with a variable wait time between state preparation and rank inference, allowing the oscillator state to decay under single photon loss. For a $200\ \mu\text{s}$ wait time, which is approximately 20% of single-photon lifetime $T_{1,c}$ of the cavity [17], the measurement outcomes, shown as green triangles, show that $\langle \hat{\Pi}_3 + \lambda \hat{\Pi}_4 \rangle$ is consistently under the threshold and indicate that we can no longer infer rank 3 non-Gaussianity. For shorter wait times, e.g., 50 and $100\ \mu\text{s}$, there are still values of λ that satisfy the condition in Eq. (2), signifying that the state still has rank 3. Thus, the flexibility afforded by OREO to extract information on the more complex quantity $\langle \hat{\Pi}_3 + \lambda \hat{\Pi}_4 \rangle$ provides an accurate and efficient mechanism to determine the non-Gaussianity rank and its depth respective to dominant cavity damping in the system [43].

This ability to reliably extract the non-Gaussianity rank can also reveal subtle imperfections in the resource state that are otherwise not obvious. For instance, we investigate a target state of $(|0\rangle + |3\rangle)/\sqrt{2}$ in the oscillator and use an observable $\langle \Xi_3 + \lambda \Pi_4 \rangle$ with $\Xi_3 \equiv |3\rangle\langle 0| + |0\rangle\langle 3|$, which captures the coherence of the quantum superposition, to infer its non-Gaussian coherence rank [44–46]. Experimentally, we prepare the target state in the oscillator using a numerically optimized state-transfer pulse [24] and then induce a dephasing mechanism on the cavity via a strong pulse on the readout resonator, corresponding to the oscillator's coherence time of $T_{\phi,c}^{\text{strong}} \approx 42\ \mu\text{s}$ [17]. The reconstructed Wigner function of state under strong dephasing drive is shown in Fig. 3(c) with a fidelity of $0.79(2)$ to the ideal target state. It shows three well-defined negative regions that may lead to a naive conclusion that the state has rank 3. In contrast, when we use OREO to directly probe $\langle \Xi_3 + \lambda \Pi_4 \rangle$, the outcome, shown

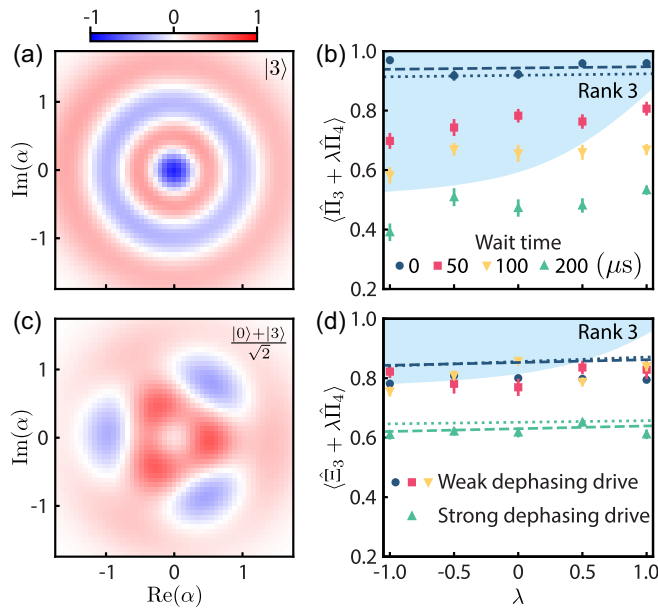


FIG. 3. Non-Gaussianity rank. (a) Reconstructed Wigner of Fock state $|3\rangle$. (b) Measurement of $\langle \hat{\Pi}_3 + \lambda \hat{\Pi}_4 \rangle$ ($\hat{\Pi}_n \equiv |n\rangle\langle n|$) with OREO is used to infer the rank under photon loss for a variable wait time and parameter λ . The theoretical threshold is shown as the lower boundary of the blue shaded region, above which one infers rank 3 non-Gaussianity. (c) Reconstructed Wigner for $(|0\rangle + |3\rangle)/\sqrt{2}$, prepared with strong induced dephasing drive. (d) Measurement of $\langle \hat{\Xi}_3 + \lambda \hat{\Pi}_4 \rangle$ ($\hat{\Xi}_n \equiv |n\rangle\langle 0| + |0\rangle\langle n|$) with OREO under both strong and weak dephasing drives. Both in panels (b) and (d), the outcomes obtained via OREO agree with values derived from the full density matrix reconstruction (dotted lines) as well as full simulations with decoherence (dashed lines).

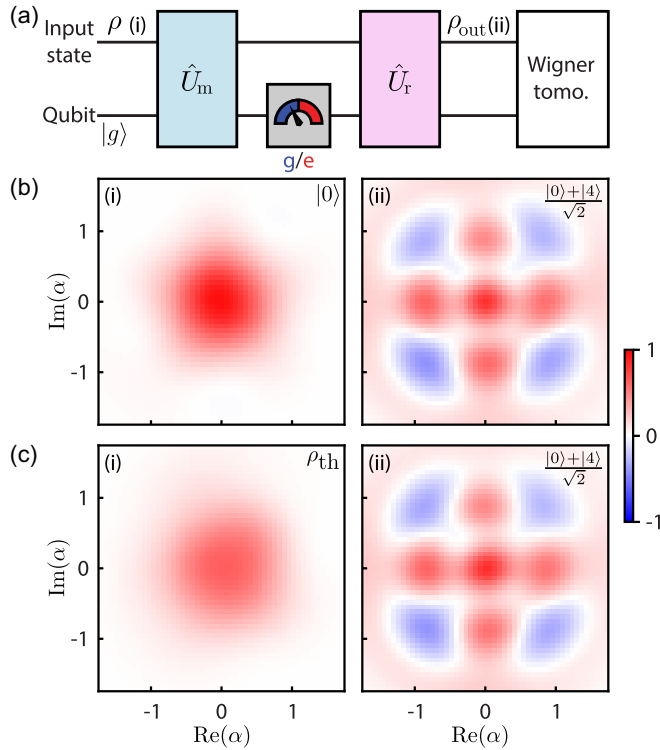


FIG. 4. Projection to a target oscillator state. (a) The protocol that extends OREO, which involves a second numerically optimized pulse $\hat{U}_r$ that reverts the mapping done by $\hat{U}_m$. The resulting oscillator states after successful projections are characterized using Wigner tomography. The reconstructed Wigner of the initial and final states of the cavity after the projection into the state $|\psi\rangle = (|0\rangle + |4\rangle)/\sqrt{2}$ with the cavity prepared in $|0\rangle$ [panel (b)] and in a thermal state with $n_{\text{th}} = 0.24$ [panel (c)]. The projected oscillator states in panels (b) and (c) have similar fidelities of 0.84(3) and 0.85(2), respectively, to the ideal binomial state and non-Gaussianity coherence rank 4 [17] despite the differences in initial conditions.

in Fig. 3(d), indicates that the state with strong dephasing drive is consistently below the threshold. This is consistent with the knowledge that strong dephasing has been induced on the cavity state, which should reduce the corresponding non-Gaussianity coherence rank. Furthermore, as we reduce the strength of the dephasing drive, which results in an improved coherence time of $T_{\phi,c}^{\text{weak}} \approx 263 \mu\text{s}$ [17], the values of $\langle \Xi_3 + \lambda \Pi_4 \rangle$ obtained via OREO increase in accordance and approach the threshold for rank 3. We include data from different iterations of this measurement to verify the consistency and accuracy of the process. Although several of these points marginally cross the calculated threshold, we do not have statistically significant evidence to conclude its coherence rank with certainty. Nonetheless, this example highlights the ability of OREO as a sensitive diagnosis tool to assess the coherence of non-Gaussian states.

Finally, we show that OREO can be readily extended to perform a projective measurement and collapse the oscillator state ρ into an arbitrary target $|\psi\rangle$ [Fig. 4(a)]. This goes beyond the Fock-state projection, the only currently available technique in cQED in the strong dispersive regime [10]. It builds on the original OREO procedure with a

unitary $\hat{U}_m$, optimized for estimating the observable $\langle |\psi\rangle \langle \psi| \rangle$, and an additional unitary $\hat{U}_r$ that reverts the mapping of $\hat{U}_m$, optimized such that $\langle g | \hat{U}_r | g \rangle = \langle g | \hat{U}_m^\dagger | g \rangle$ [17]. Here, the intermediate qubit measurement extracts the overlap observable $p = \langle \psi | \rho | \psi \rangle$ and heralds the correct final state in the oscillator [17].

As an illustration, we optimize $\hat{U}_m$ and $\hat{U}_r$ for the observable $\hat{\Pi}_\psi = |\psi\rangle \langle \psi|$, where $|\psi\rangle$ is the binomial state $(|0\rangle + |4\rangle)/\sqrt{2}$. Measurement of the qubit ground state signals a successful projection to the target state, whereas the oscillator state associated with the qubit in $|e\rangle$ is discarded. We execute this procedure with the oscillator initialized in the vacuum state [Fig. 4(b)(i)] and Wigner tomography performed in the end to obtain the reconstructed density matrix of the output state ρ_{out} . The reconstructed Wigner [Fig. 4(b)(ii)] shows close agreement with the ideal binomial state, with a success rate of $p = 0.49$ and fidelity of 0.84(3), limited by qubit Ramsey time and measurement imperfections [17].

Next, we prepare the oscillator in an thermal state [Fig. 4(c)(i)], with mean photon number $n_{\text{th}} \approx 0.24$ [17] with a fidelity of 0.97(2) to the ideal state, and perform the same projective operation as described in Fig. 4(a). Conceptually, a projective operation brings the system to the target state regardless of the initial condition. Here, we observe that the reconstructed Wigner of the output state [Fig. 4(c)(ii)] shows a fidelity of 0.85(2) to the ideal binomial state, a clear indication that the quality of the projection is not affected by the oscillator's initial condition. However, it comes at the cost of a minor reduction in the success rate ($p = 0.44$), which scales with the overlap between the initial and target states. This simple demonstration shows that OREO can be easily extended to implement a projective operation, which serves as an effective oscillator state initialization tool. Although it is a heralded nondeterministic process, its robustness against the initial condition of the oscillator is a valuable feature that is beyond the reach of the standard state-transfer protocols. This is particularly useful for platforms where the oscillator may not have a well-defined initial state due to the presence of residual thermal excitations such as bosonic cQED [47], optomechanical [48], and quantum acoustic [21] devices as well as state generation protocols requiring projective measurements [49–53].

IV. CONCLUSION

In conclusion, this work demonstrates the robust capabilities of the OREO in bridging critical measurement gaps in bosonic cQED systems. By harnessing numerically optimized pulses to encode the expectation value of target observables into an auxiliary transmon qubit, OREO enables single-shot detection of a diverse range of oscillator properties, from phase-space quadratures and their higher-order moments to non-Gaussianity metrics. The performance of OREO is mainly limited by qubit decoherence during the optimized drives, which can potentially be alleviated by known methods to extend the qubit coherence properties [54,55] or reduce the drive durations [26]. Even with our modest system parameters [17], experimental results presented in this work

offer compelling validation to OREO's value and practical efficacy. Collectively, these findings underscore OREO as a versatile and effective tool for advancing continuous-variable quantum information processing.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [60].

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- Correction:* The name of the seventh author was misspelled and has been corrected.