

Temperature-induced superconductivity enhancement under large exchange fields

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(Received 25 November 2025; accepted 1 October 2026; published 20 October 2026)

Through a comprehensive free energy analysis, we demonstrate that finite temperature can simultaneously weaken superconductivity and mitigate Zeeman-effect-induced depairing, leading to potential nonmonotonic temperature-dependent behaviors in superconductors subjected to large exchange fields. Remarkably, superconductivity can be counterintuitively enhanced by temperature when the Zeeman energy exceeds the superconducting order parameter, owing to the competition between thermal and magnetic effects. We propose that multiband effect offers one possible microscopic route for this temperature-induced enhancement and demonstrate it explicitly within a two-band superconducting model. A detailed parameter analysis identifies the conditions under which this phenomenon emerges, suggesting that temperature-enhanced superconductivity may be observable in materials such as MgB₂ and FeSe through transport and tunneling measurements.

DOI: [10.1103/6q41-1cxp](https://doi.org/10.1103/6q41-1cxp)

I. INTRODUCTION

The behavior of superconductors under large exchange fields has been a central topic in condensed matter physics for decades. The zero-temperature critical exchange field H_c was first determined independently by Clogston and Chandrasekhar [1,2], now known as the Pauli limit. The finite temperature extension of this theory was later developed by Sarma through a free energy analysis [3–5], which revealed that the transition from the homogeneous superconducting state to the normal state is first order at low temperatures and second order at higher temperatures. Subsequent studies demonstrated that an inhomogeneous superconducting state with finite-momentum pairing, the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state, can become energetically favorable slightly beyond the Pauli limit [6,7]. However, realization of the FFLO state requires exceptionally clean systems to sustain the spatial modulation of the order parameter. These theoretical predictions have been supported by experimental observations in superconducting thin films subjected to in-plane magnetic fields [8–15].

We focus on the temperature-induced nonmonotonic behavior of superconductivity under strong exchange fields. In conventional understanding, increasing temperature suppresses superconductivity, causing the order parameter to decrease monotonically. However, superconductors containing magnetic impurities or exhibiting spontaneous ferromagnetism challenge this expectation. Sarma and de Gennes theoretically demonstrated that, in superconductors with

an appropriate concentration of magnetic impurities, the temperature-dependent polarization of the impurities can lead to nonmonotonic behavior: H_c can be enhanced by thermal effects at low temperatures [16,17]. This prediction was supported by experiments on superconducting dilute rare-earth alloys [18–20]. Furthermore, several independent studies have reported temperature-induced nonmonotonic or reentrant superconductivity in various ferromagnetic superconductors [21–27], which were explained in our recent work by incorporating temperature-dependent ferromagnetism [28].

In this work, we investigate finite temperature one-band and two-band superconducting models with spin-singlet pairing and general momentum-dependent gap functions. By analyzing the free energy, we show that temperature has two competing effects: It weakens superconductivity through thermal quasiparticle excitations, but it also reduces the exchange-field-induced depairing effect in superconductors without magnetic impurities or magnetic order. The competition between these two effects can give rise to a nonmonotonic temperature dependence of the superconducting order parameter when the Zeeman energy exceeds the relevant superconducting gap scale. Furthermore, we explicitly examine temperature-induced superconductivity enhancement within a two-band framework and identify the parameter regimes in which this phenomenon emerges.

II. MODEL AND THEORETICAL FRAMEWORK

For a two-band superconducting film subjected to an external in-plane magnetic field H , which acts purely as an exchange field, the Hamiltonian is expressed as

$$\begin{aligned} \hat{H} = & \sum_{i=1,2} \sum_{\mathbf{k}, \sigma} (\xi_i(\mathbf{k}) + \sigma H) \hat{c}_{i\sigma}^\dagger(\mathbf{k}) \hat{c}_{i\sigma}(\mathbf{k}) \\ & + \sum_{i,j=1,2} \sum_{\mathbf{k}, \mathbf{k}'} U_{ij}(\mathbf{k}, \mathbf{k}') \hat{c}_{i\uparrow}^\dagger(\mathbf{k}) \hat{c}_{i\downarrow}^\dagger(-\mathbf{k}) \hat{c}_{j\downarrow}(-\mathbf{k}') \hat{c}_{j\uparrow}(\mathbf{k}'), \end{aligned} \quad (1)$$

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where $\hat{c}_{i\sigma}^\dagger$ ($\hat{c}_{i\sigma}$) represents the creation (annihilation) operator for an electron with spin σ in the i th band and U_{ij} denotes the coupling strength between the i th and j th bands. Within the mean-field and weak-coupling treatment of multiband superconductivity, Barzykin and Gor'kov showed that the two gaps can be described by an approximately temperature- and field-independent gap ratio Δ_2/Δ_1 [29,30], which substantially simplifies the calculation. Within the present free energy formulation, this constant-gap-ratio reduction additionally requires sufficiently strong interband locking, as discussed and tested in Sec. II of the Supplemental Material [31]. Under these conditions, the free energy $F(H, T)$ can be derived in closed form (see Sec. I of the Supplemental Material [31] for details):

$$F = \sum_i \int_0^{2\pi} d\phi \left[\chi^2(\phi) N_i \Delta_i^2 \left(\ln \frac{\Delta_i}{\Delta_{i0}} - \frac{1}{2} \right) - \frac{2N_i}{\beta} \int_0^\infty d\xi \ln \left(1 + 2 \cosh(\beta H) e^{-\beta \sqrt{\xi^2 + \Delta_i^2 \chi^2(\phi)}} + e^{-2\beta \sqrt{\xi^2 + \Delta_i^2 \chi^2(\phi)}} \right) \right], \quad (2)$$

where N_i is the density of states near Fermi level for the i th band, Δ_i the corresponding superconducting order parameter with $\Delta_{i0} = \Delta_i(H=0, T=0)$, and $\beta = 1/k_B T$. The function $\chi(\phi)$ characterizes the order parameter anisotropy [$\Delta_i(\phi) = \Delta_i \chi(\phi)$], with $\chi(\phi) = 1$ for isotropic s -wave pairing and $\chi(\phi) = \cos 2\phi$ for anisotropic d -wave pairing. For simplicity, only the s -wave case is discussed in the main text, while results for d wave and other forms of $\chi(\phi)$ are provided in Secs. V, VII, and VIII of the Supplemental Material [31]. Minimizing the free energy $F(H, T)$ yields the self-consistent $\Delta(H, T)$, enabling quantitative analysis of the temperature-induced superconductivity enhancement.

For computational convenience, Eq. (2) is expressed in dimensionless form as $f(x) = F/(N_1 \Delta_{10}^2)$, with $x = \Delta_i/\Delta_{i0}$. The other dimensionless parameters are defined as

$$\alpha = \frac{\Delta_{20}}{\Delta_{10}}, \quad \gamma = \frac{N_1}{N_2}, \quad h = \frac{H}{\Delta_{10}}, \quad t = \frac{k_B T}{\Delta_{10}}. \quad (3)$$

The explicit form of the dimensionless two-band free energy is given in Sec. I of the Supplemental Material [31]. Because Δ_1 and Δ_2 are interchangeable, we take Δ_1 to be the smaller order parameter in the following discussion without loss of generality, implying $\alpha \geq 1$. The two-band case naturally reduces to the one-band limit when $\alpha = 1$, which will be discussed first as an illustrative example. To analyze the free energy difference between the normal state ($x=0$) and the superconducting state ($x \neq 0$), we define

$$f_{s0}(x, h, t) = f(x, h, t) - f(0, h, t). \quad (4)$$

Since $f(0, h, t)$ is independent of the order parameter, minimizing $f_{s0}(x, h, t)$ yields the same self-consistent solution for x . The difference is that $f_{s0}(x, h, t)$ defines the free energy of the normal state $f_{s0}(0, h, t)$ as a constant zero reference, independent of h and t . Figure 1(a) shows the evolution of f_{s0} at zero temperature for various h in a one-band s -wave superconductor, where the minima of the curves correspond to the equilibrium order parameters. The order parameter exhibits an

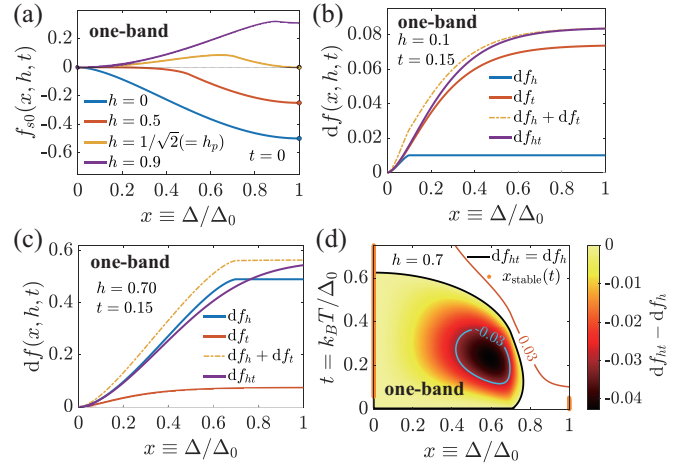


FIG. 1. Free energy analysis for one-band s -wave superconductors. (a) Landscapes of $f_{s0}(x)$ at $t=0$ for $h=0.0, 0.5, 1/\sqrt{2}$, and 0.9 . Corresponding profiles of $df_h(x)$, $df_t(x)$, $df_h(x)+df_t(x)$, and $df_{ht}(x)$ for (b) $h=0.10, t=0.15$ and (c) $h=0.70, t=0.15$. (d) Temperature evolution map of $df_{ht}(x) - df_h(x)$ at $h=0.70$. The solid black line marks the condition $df_{ht}(x) = df_h(x)$, orange points x_{stable} denote the equilibrium order parameter, and the color scale represents the amplitude of $df_{ht}(x) - df_h(x)$, corresponding to the negative of the energy saved by temperature effects. Contour lines of $df_{ht}(x) - df_h(x)$ illustrate the distribution of the energy saving. Regions where $df_{ht}(x) - df_h(x) \geq 0$ are shown as transparent but remain traceable through their positive contour levels.

abrupt drop from $x=1$ to $x=0$ once h exceeds the Pauli limit $h_p = 1/\sqrt{2}$, consistent with previous studies [1–3]. Notably, f_{s0} increases with h and displays pronounced x dependence.

III. THERMODYNAMIC ORIGIN OF TEMPERATURE-INDUCED ENHANCEMENT

To quantify the variation of f_{s0} induced by the exchange field and temperature, we offset $f_{s0}(x, h, t)$ by its zero-field and zero-temperature reference value $f_{s0}(x, h=0, t=0)$, as illustrated by the blue curve in Fig. 1(a), leading to the definition

$$df(x, h, t) = f_{s0}(x, h, t) - f_{s0}(x, h=0, t=0). \quad (5)$$

Based on Eq. (5) and the discussion above, $df(x, h, t) \equiv df_{ht}$, $df(x, h, t=0) \equiv df_h$, and $df(x, h=0, t) \equiv df_t$ represent the lifted free energies associated with the combined effects of finite h and t , the pure Zeeman effect, and the pure thermal effects, respectively. These quantities, calculated for finite h and t and shown in Figs. 1(b) and 1(c), illustrate the competition between the temperature-induced reduction of Zeeman-type depairing and the thermal suppression of superconductivity.

For a small $h=0.1$ with relatively large $t=0.15$, Fig. 1(b) shows that $df_h + df_t \geq df_{ht} \geq df_h$ holds for all x , indicating that temperature reduces the superconducting free energy via coupling with the exchange field but cannot offset the increase arising from superconductivity suppression. In contrast, for $h=0.7$ near h_p and $t=0.15$, $df_h \geq df_{ht}$ appears at small and intermediate x , as shown in Fig. 1(c). To examine this temperature-induced energy saving in detail, the evolution of

$df_h - df_{ht}$ with temperature is mapped in Fig. 1(d). The solid black line in Fig. 1(d) is consistent with the results in Fig. 1(c), showing that free energy reduction occurs only when the Zeeman energy generally exceeds the order parameter. Therefore, at sufficiently large exchange fields, a finite temperature can diminish Zeeman-type depairing more effectively than it suppresses superconductivity, resulting in a net energy gain relative to the zero-temperature case (see Sec. IV of the Supplemental Material [31] for details). As temperature rises, this saved energy initially increases and then rapidly decreases after reaching a maximum at a moderate temperature.

For a one-band s -wave superconductor, the order parameter remains constant ($x = 1$) as h increases and abruptly drops to $x = 0$ once $h \geq h_p$. Consequently, thermal effects cannot reduce the free energy, and no temperature-induced enhancement of superconductivity appears. This motivates the search for scenarios in which the order parameter can be partially suppressed before superconductivity vanishes, allowing the system to accommodate a large Zeeman energy relative to its order parameter. One such possibility is a highly anisotropic order parameter, briefly discussed in Sec. V of the Supplemental Material [31], while another arises in two-band superconductors [29,30,32].

IV. TEMPERATURE-INDUCED ENHANCEMENT IN TWO-BAND SYSTEMS

The superconducting phase diagrams of one-band and two-band s -wave systems are presented for comparison. For the one-band case, the h - t phase diagram in Fig. 2(a) shows that the critical field h_c reaches its maximum at zero temperature and gradually decreases with increasing temperature. The contour lines of the order parameter extend monotonically from the upper-left to the lower-right corner, indicating that superconductivity is not enhanced by thermal effects. At a fixed in-plane magnetic field, the order parameter x decreases steadily with temperature [Fig. 2(c)], further confirming the absence of temperature-induced superconductivity enhancement.

In contrast, temperature-induced superconductivity enhancement appears in the two-band system under specific parameter conditions. As highlighted by the dashed white line in Fig. 2(b), the maximum of $h_c(t)$ occurs at a finite temperature rather than at zero temperature, confirming the enhancement of superconductivity at finite temperature. We note that a recent work on two-band superconductivity also reported a nonmonotonic field-temperature boundary by solving the gap equation in the limit $\Delta \rightarrow 0$ [33]. However, this criterion identifies the physical critical field only for a continuous, second-order transition. In the presence of a first-order transition, the $\Delta \rightarrow 0$ gap-equation criterion gives a metastability limit, or supercooling field, which is not necessarily energetically favored and therefore does not generally correspond to the physical thermodynamic critical field [34,35]. This distinction is discussed in detail in Sec. III of the Supplemental Material [31].

As shown by the dashed gray lines in Figs. 2(a) and 2(b), a nonmonotonic temperature dependence of the supercooling field can emerge even in the one-band case, although the physical critical field remains monotonic. Therefore, a

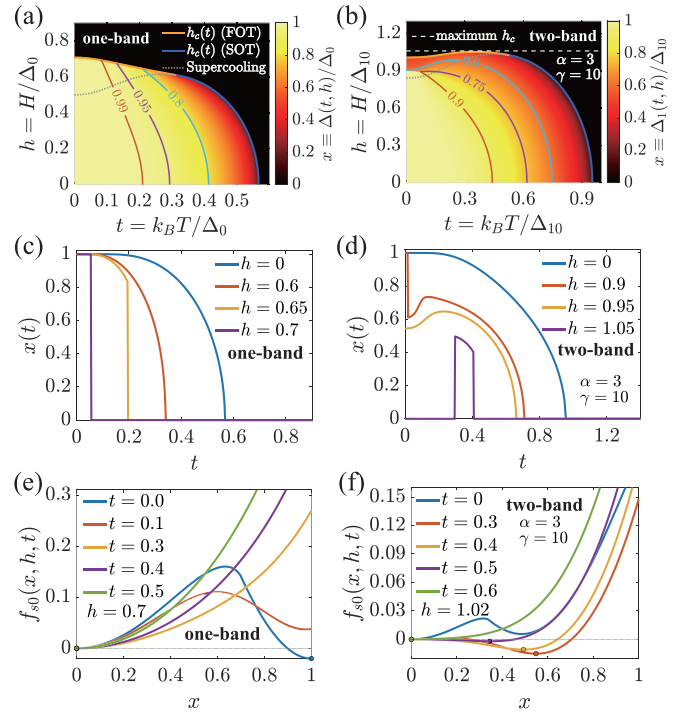


FIG. 2. Superconductivity under large exchange fields in one-band and two-band s -wave systems. h - t phase diagrams for (a) the one-band system and (b) the two-band system with specific parameters. The physical critical fields $h_c(t)$ associated with first-order transitions (FOTs) and second-order transitions (SOTs) are denoted by orange and blue solid lines, respectively. The temperature-dependent supercooling field is indicated by the gray dashed line. Color scales indicate the magnitude of the order parameter x , and contour lines highlight the monotonic and nonmonotonic superconducting features. The same color scales and line conventions are used in the following figures. Temperature dependence of the order parameter $x(t)$ at various exchange fields for (c) the one-band system and (d) the two-band system. Temperature-dependent landscapes of $f_{s0}(x)$ for (e) the one-band system at $h = 0.70$ and (f) the two-band system at $h = 1.02$.

nonmonotonic supercooling field alone should not be regarded as evidence for temperature-induced superconductivity enhancement. In the following discussion, we focus on the physical critical field determined from the global minimum of the free energy.

The physical temperature-induced nonmonotonic behavior is further illustrated by the order-parameter contours. As shown in Fig. 2(b), the contour line corresponding to a small order-parameter value, for example, $x = 0.6$, exhibits a nonmonotonic temperature dependence in the high-field regime. Moreover, Fig. 2(d) shows that the temperature dependence of x varies with the applied field h . For relatively small h , the order parameter remains $x = 1$ at zero temperature and decreases monotonically with increasing temperature, consistent with the one-band case. When h is sufficiently large to drive an FOT and satisfies $h > x$, a temperature-induced nonmonotonic behavior of the order parameter emerges.

For a relatively large h that triggers an FOT at finite temperature rather than at zero temperature, the order parameter

exhibits a sequence of behaviors: an abrupt drop, a gradual recovery, and then suppression with further temperature increase as illustrated by the $h = 0.9$ case in Fig. 2(d). With increasing h , when the FOT already occurs at zero temperature but the field remains below h_c , the order parameter first increases and then decreases with temperature, as seen for $h = 0.95$ in Fig. 2(d). Most interestingly, when h slightly exceeds h_c at zero temperature, a reentrant superconducting behavior emerges, where the order parameter becomes nonzero only within an intermediate temperature range.

This phenomenon can be better understood from the landscapes of $f_{s0}(x)$ shown in Fig. 2(f). At large h , the free energy associated with an intermediate order parameter, $x \approx 0.5$, is significantly lowered with increasing temperature in both one-band and two-band systems [Figs. 2(e) and 2(f)], consistent with the behavior shown in Figs. 1(c) and 1(d). However, only the two-band system can sustain superconductivity in the large-field regime $h > x$, thereby allowing temperature-induced reentrant superconductivity to appear. In practice, additional nonmonotonic temperature dependences of the order parameter can appear for certain parameter sets. The corresponding results are provided in Sec. VIII of the Supplemental Material [31].

Within the weak-coupling approximation, the temperature- and field-independent gap ratio α encodes the effect of the interband coupling. According to Eq. (2), the properties of the two-band system are then determined by α and the ratio of densities of states γ . For two-band systems with large α and small γ , superconductivity is dominated by the larger order parameter Δ_2 , resembling a one-band system characterized by Δ_2 . Conversely, for small α ($\alpha \approx 1$) and large γ , the behavior is governed by the smaller order parameter Δ_1 , also effectively mimicking a one-band system. As shown in Figs. 3(a) and 3(b), the h - t phase diagrams of these two limiting cases closely resemble that of the one-band system in Fig. 2(a). However, the Δ_2 -dominated two-band system can exhibit weak nonmonotonic temperature dependence near an SOT at $h \approx x$, as indicated by the $x = 0.95$ contour line in Fig. 3(a), while the Δ_1 -dominated system does not, since it always satisfies $h < x$. Moreover, both types of one order parameter dominated systems show no temperature-induced enhancement near the critical field, as evidenced by the maximum h_c occurring at zero temperature, confirming that superconductivity enhancement is sensitive to the parameters α and γ .

The enhancement of superconductivity is characterized by two key features: the temperature-induced nonmonotonic behavior of the order parameter and h_c enhancement. Because the nonmonotonic evolution of the order parameter is difficult to quantify directly, we focus on the parameter dependence of the h_c enhancement. We define the critical exchange field at zero temperature as h_{c0} and the maximum critical field as $h_{c,\max}$, which occurs at temperature $t_{\max}$. Accordingly, temperature-induced superconductivity enhancement is identified when $h_{c,\max}/h_{c0} > 1$. The maps of $h_{c,\max}/h_{c0}$ and $t_{\max}$ in the α - γ parameter space were calculated to reveal this dependence. As shown in Fig. 3(c), the enhancement of h_c appears only within an appropriate region of the parameter space, where α and γ are roughly comparable. In other words,

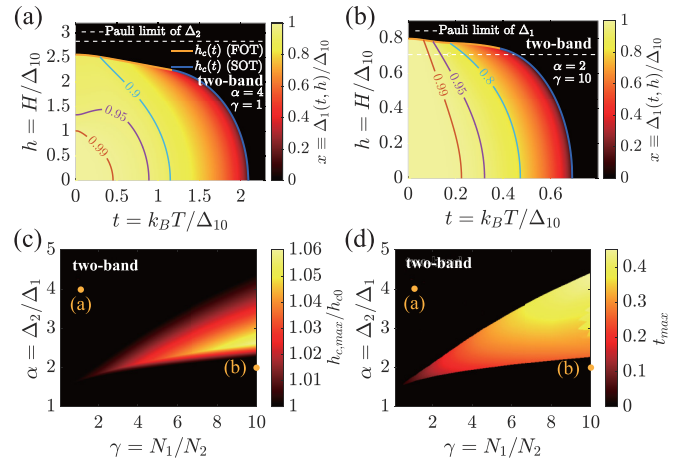


FIG. 3. α - γ parameter dependence of superconductivity enhancement. h - t phase diagrams for two-band systems with (a) large α and small γ , and (b) small α and large γ . The dashed white lines in panels (a) and (b) indicate the Pauli limit associated with the dominant gap. Maps of (c) $h_{c,\max}/h_{c0}$ and (d) $t_{\max}$ in the α - γ parameter space. The color scales in panels (c) and (d) represent the amplitudes of $h_{c,\max}/h_{c0}$ and $t_{\max}$, respectively. The parameter points corresponding to panels (a) and (b) are indicated by orange markers.

the parameters should avoid approaching the one-band limits, thereby retaining the intrinsic two-band characteristics. The corresponding $t_{\max}$ map in Fig. 3(d) further indicates that the maximal superconductivity enhancement always occurs at a relatively small but nonzero temperature.

As discussed in Fig. 1, energy saving occurs only when the Zeeman energy exceeds the superconducting order parameter, which is always accompanied by a partially suppressed order parameter. For a two-band superconductor at zero temperature, the critical field h_{c0} induces an FOT, at which the homogeneous order parameter abruptly drops to zero from a finite value. To evaluate the order parameter immediately before the transition, denoted as $x(h_{c0}^-)$, we introduce a small field increment $dh = 1 \times 10^{-5}$ and define

$$x(h_{c0}^-) = x(h_{c0} - dh). \quad (6)$$

The α and γ parameter dependence of $x(h_{c0}^-)$ is shown in Fig. 4(a). The contour lines display nonmonotonic variation primarily along the α direction, indicating that the minimum of $x(h_{c0}^-)$ occurs at large γ and intermediate α , located near the middle-right region of the map. This behavior can be further understood from the order parameter profiles in Fig. 4(b). For small α and large γ systems, where the superconductivity is dominated by Δ_1 , only a single FOT appears, similar to the one-band case. For intermediate α with large γ , where the stronger order parameter partially protects the weaker one, an additional FOT emerges at relatively low h , leading to significant suppression of the order parameters. For large α with relatively small γ , where the stronger order parameter effectively protects the weaker one, only an SOT occurs at low h , resulting in minimal suppression. Additional details are provided in Sec. VI of the Supplemental Material [31].

The contour line distributions of $x(h_{c0}^-)$ are broadly consistent with the h_c enhancement map in Fig. 3(c), indicating that superconductivity enhancement is always associated with

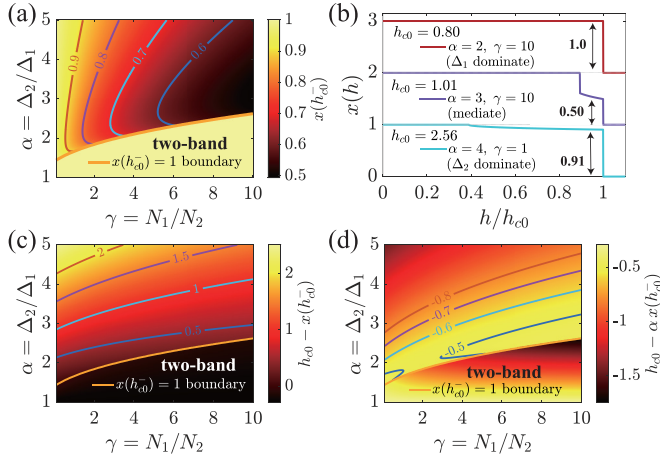


FIG. 4. α - γ parameter dependence of the relation between $x(h_{c0})$ and h_{c0} at zero temperature. (a) Map of $x(h_{c0})$. (b) Order parameter profiles $x(h)$ for different α and γ values at zero temperature. Each curve is vertically offset by 1 for clarity, and the drop near h_{c0} is marked by double-sided arrows. Maps of (c) $h_{c0} - x(h_{c0})$ and (d) $h_{c0} - \alpha x(h_{c0})$. The color scales in panels (a), (c), and (d) represent the amplitudes of $x(h_{c0})$, $h_{c0} - x(h_{c0})$, and $h_{c0} - \alpha x(h_{c0})$, respectively. Contour lines of $h_{c0} - \alpha x(h_{c0})$ below the boundary defined by $x(h_{c0}) = 1$ are omitted for clarity.

partial suppression of the order parameters. Minor discrepancies arise from the dominance of Δ_2 at high fields in systems with large α . To further compare the Zeeman energy and order parameters at the critical field, we calculated the maps of $h_{c0} - x(h_{c0})$ and $h_{c0} - \alpha x(h_{c0})$, as shown in Figs. 4(c) and 4(d). In Fig. 4(c), increasing α and decreasing γ make h_{c0} significantly larger than the weaker order parameter $x = \Delta_1/\Delta_{10}$, enabling nonmonotonic order parameter behavior near $h = x$, as seen in Fig. 3(a). In contrast, Fig. 4(d) shows that the Zeeman energy h_{c0} remains smaller than the stronger component of order parameter αx , and the system behaves effectively as a one-band superconductor dominated by Δ_2 when α is too large and γ too small, suppressing any h_c enhancement.

V. SUMMARY AND DISCUSSION

In summary, we have studied finite temperature one-band and two-band superconducting models with spin-singlet pairing and general momentum-dependent gap functions. Through a systematic free energy analysis, we showed that finite temperature has two competing effects in superconductors under exchange fields: It suppresses superconductivity through thermal quasiparticle excitations, but it can also reduce the exchange-field-induced depairing and thereby promote superconductivity. When the Zeeman energy exceeds the relevant superconducting gap scale, the latter effect can dominate, leading to temperature-induced superconductivity enhancement. Possible microscopic routes to this regime include spin-orbit-parity coupling and two-band effects, where previous works [30,36] reported slight temperature-induced h_c enhancement in h - t phase diagrams, although this phenomenon was not explicitly identified or emphasized.

In the present study, we analyzed two-band s -wave superconductors in detail and demonstrated that both the nonmonotonic temperature dependence of the order parameter and the enhancement of $h_c(t)$ serve as signatures of temperature-enhanced superconductivity. We found that this enhancement is sensitive to the parameters α and γ : A pronounced h_c enhancement appears only for large γ and intermediate α , whereas nonmonotonic order parameter behavior can arise over a broader parameter range. Throughout most of the analysis, we employ the weak-coupling, constant-gap-ratio approximation introduced by Gor'kov and Barzykin, in which Δ_2/Δ_1 is taken to be independent of temperature and exchange field. A more detailed free energy analysis shows that, within the present formulation, this reduction additionally requires strong interband locking [31]. We therefore also examine regimes with weaker interband locking. These calculations show that the temperature-induced enhancement of h_c persists beyond the strongly locked regime, although its quantitative magnitude decreases as the interband locking is weakened. Moreover, the nonmonotonic temperature dependence of the superconducting gaps remains present even for very weak interband locking. Thus, the constant-gap-ratio approximation captures the essential physical mechanism, while its quantitative applicability depends on the microscopic coupling matrix.

Although the present theory is formulated at the mean-field level, thermal fluctuations are expected to be important mainly within the narrow critical region near T_c . Since the temperature-induced enhancement identified here occurs in a relatively low-temperature regime away from T_c , such fluctuations may quantitatively broaden transitions or shift phase boundaries, but are not expected to qualitatively eliminate the thermodynamic mechanism proposed in this work.

Our theoretical predictions, the temperature-induced h_c enhancement and the nonmonotonic temperature dependence of the order parameter, can be directly tested by transport and tunneling measurements on multiband superconducting thin films under large in-plane magnetic fields. Two-band s -wave systems such as MgB_2 [37–40] and NbSe_2 [41–44], as well as anisotropic (possibly d -wave) two-band superconductors including CeCoIn_5 [45–48] and FeSe [49–52], provide ideal platforms for such verification.

In MgB_2 , tunneling measurements [40] reveal spatial variations in the ratio of densities of states γ , offering a natural setting to test our s -wave predictions. Moreover, previous studies have shown that both the densities of states ratio γ and the order parameter ratio α can be tuned by substrate-induced hole doping [53,54], making FeSe a particularly promising candidate for experimentally validating our theoretical framework. Using representative parameters extracted from tunneling spectra [39,40,54,55], we estimate that both MgB_2 and FeSe may exhibit a temperature-induced critical field enhancement of approximately 5% in the strongly interband-locked limit [31]. For MgB_2 , where the relevant paramagnetic field scale is on the order of several tens of tesla, this corresponds to a field increase on the order of 1–2 T. For FeSe , with critical fields exceeding several tesla, the same relative enhancement corresponds to several tenths of a tesla. These

estimates suggest that the predicted enhancement should be experimentally resolvable.

ACKNOWLEDGMENTS

X.W. acknowledges X.-Y. Zhou, Z.-C. Zhang, L.-C. Ji, Y.-Q. Yan, J.-B. Liu, and M. Shu for stimulating discussions. This work was financially supported by Quantum Science and Technology-National Science and Technology

Major Project (Grant No. 2023ZD0300500) and the National Natural Science Foundation of China (Grants No. 11427903, No. 52388201, and No. 12375136).

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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