

Disorder-to-order transition in one-dimensional nonreciprocal Cahn-Hilliard model

Navdeep Rana^{1,*} and Ramin Golestanian^{1,2,†}¹Max Planck Institute for Dynamics and Self-Organization (MPI-DS), D-37077 Göttingen, Germany²Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford OX1 3PU, United Kingdom

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We present the phenomenology of the one-dimensional nonreciprocal Cahn-Hilliard model for varying nonreciprocity (α) and different boundary conditions. At small α , a perturbed uniform state evolves to a defect-laden configuration that lacks global polar order. Defects are the sources and sinks of traveling waves. For a given α , defects with a unique wave number that increases monotonically with α are selected. A critical threshold α_c marks the onset of a transition to states with finite global polar order. For periodic boundary conditions, above α_c , the system shows traveling waves that are completely ordered. In contrast, traveling waves are incompatible with the Neumann and Dirichlet boundary conditions. Instead, for $\alpha \gtrsim \alpha_c$, we find fluctuating domains that show intermittent polar order, and at large α , the system partitions into two domains with opposite polar order.

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I. INTRODUCTION

The nonreciprocal Cahn-Hilliard (NRCH) model describes phase separation of multicomponent mixtures in the presence of nonreciprocal couplings [1–8]. Systems with nonreciprocity are intrinsically out of equilibrium and show rich phenomenology [9–16]. In the NRCH model, the spontaneous breaking of parity and time-reversal symmetries leads to the formation of traveling density bands [1,2], suppressed coarsening dynamics [1,17], localized states [18], chaotic steady states [8], true long-range polar order in two dimensions [7], and enhanced stability in multispecies mixtures [5]. Introduced phenomenologically, the NRCH model emerges as a universal amplitude equation for a conserved-Hopf instability in systems with two conservation laws [19] and can also be derived from a systematic coarse graining of a microscopic model of phoretically active Janus colloids [20,21]. Nonreciprocal interactions emerge in a variety of nonequilibrium systems with hydrodynamic interactions [22] and vision-cone interactions [23–25], active mixtures [12,14,15,26–29], mixtures of active and passive components [30,31], and some mass-conserved reaction-diffusion systems [32,33].

Studies of the NRCH model in two dimensions (2D) have revealed a rich phenomenology due to the presence of defects, namely, spirals with unit magnitude topological charge and topologically neutral targets [3,4]. These defects are the sources of the traveling waves, and thus the precursors of global polar order. For a given strength of nonreciprocal coupling α , defects with a unique asymptotic wave number,

$k_\infty \propto \sqrt{\alpha}$, are selected [3]. At a critical threshold of nonreciprocity α_c , we find a disorder-to-order transition. For $\alpha < \alpha_c$, random disordered states evolve to quasistationary defect networks with no global polar order. On the other hand, for $\alpha > \alpha_c$ the system admits traveling waves without any defects. Studying the interactions between pairs of defects reveals that the stability of targets in this model fundamentally alters the nature of dynamics of the defects [4], as compared with previously studied nonequilibrium systems with nonconserved order parameter [34–37]. Thus, the NRCH model provides a fertile ground to investigate the defect dynamics in nonreciprocal systems with a conserved order parameter.

In this paper, we present an extensive study on the defects of the NRCH model in one dimension (1D). As compared to our previous investigations in 2D [3,4], which have showcased the complexity of defect dynamics in the NRCH model, the 1D configuration provides the opportunity to investigate the role of the defects in a simpler setting, for a number of reasons: (1) Topologically charged defects, which would be the analog of spirals in 2D, are ruled out. (2) Sources of traveling waves in 1D are the analog of targets in 2D. Similarly, sinks are the 1D analog of domain walls. (3) Waves emanated from the sources end at the sinks; thus, sources and sinks are arranged in an alternating manner on the 1D domain. Furthermore, the defect motion is also limited to the 1D line. (4) Since 1D allows only for sources and sinks, defect interactions are also simplified. At small separations, a source-and-sink pair annihilates. At larger distances, they form stable configurations. These reasons afford us a deeper and more pedagogical representation of the interplay between defect dynamics and the emergent properties of the NRCH model.

In addition to these effects, the 1D geometry allows us to explore the role of boundary conditions in the dynamics of the NRCH model. While the disorder-to-order transition at higher α leads to traveling waves for periodic boundary conditions, pure unidirectional traveling waves are incompatible with Neumann and Dirichlet boundary conditions, which

*Contact author: navdeep.rana@ds.mpg.de†Contact author: ramin.golestanian@ds.mpg.de

are more relevant to experimental systems. How the transition proceeds in such cases and what kind of phases emerge from these transitions remain unknown. We investigate the defect dynamics and the effect of the boundary conditions on the disorder-order transition in the 1D NRCH model under different boundary conditions. Our main results are as follows: (1) We find a transition from disordered, defect-laden states to polar ordered states. (2) Below the transition threshold ($\alpha < \alpha_c$), sources and sinks with a particular wave number are selected. (3) For periodic boundary conditions with $\alpha > \alpha_c$, we find traveling waves with perfect global polar order. (4) For Neumann and Dirichlet boundary conditions, the phenomenology above α_c is different. For α slightly above α_c , the system shows intermittent behavior, with fluctuating domains of polar order. For larger α , the system splits into subdomains with opposing polar order. (5) The transition point $\alpha_c \sim 0.60$ coincides with the crossover value $\alpha_\times \sim 0.62$ predicted by the Eckhaus instability of the plane waves. This is in a stark contrast with 2D, where the transition point $\alpha_c \sim 0.28$ is much smaller than the Eckhaus threshold $\alpha_\times \sim 0.58$.

The rest of the paper is organized as follows. In Sec. II, we present our model and methods. In Sec. III, we present our results for periodic boundary conditions. We highlight the features of the source and sink solutions, wave number selection, defect density, polar order, and the disorder-to-order transition. In Sec. IV, we consider Dirichlet and Neumann boundary conditions and discuss how the boundary conditions alter the phenomenology. We conclude the paper with a discussion and future perspectives.

II. MODEL AND METHODS

We consider a minimal model of two conserved scalar fields $\phi_1(x, t)$ and $\phi_2(x, t)$ with nonreciprocal interactions on a one-dimensional domain. The conserved dynamics of the complex scalar order parameter $\phi = \phi_1 + i\phi_2$ is governed by the continuity equation $\partial_t \phi = \partial_x^2 \mu$, where $\mu = \delta\mathcal{F}/\delta\phi^* + i\alpha\phi$ is the nonequilibrium chemical potential. The equilibrium contribution to μ promotes phase separation and is derived from the free energy functional $\mathcal{F} = \int dx (-|\phi|^2/2 + |\phi|^4/4 + |\nabla\phi|^2/2)$. The nonequilibrium part encodes the nonreciprocal interactions between the two species, quantified by the parameter α . For $\alpha = 0$, the model describes equilibrium phase separation of two interacting species, and an initial disordered state coarsens toward a bulk-phase-separated state. Any $\alpha \neq 0$ sets the system out of equilibrium and it describes nonreciprocal phase separation. A positive α implies that ϕ_1 chases ϕ_2 , and the model is invariant under the simultaneous transformation $\alpha \rightarrow -\alpha$ and $\phi_{1,2} \rightarrow \phi_{2,1}$. Combining these terms, we obtain the following nondimensional equation for the evolution of $\phi(x, t)$ [1–4]:

$$\partial_t \phi = \partial_x^2 [(-1 + i\alpha)\phi + |\phi|^2 \phi - \partial_x^2 \phi]. \quad (1)$$

The homogeneous state $\phi = 0$ is unstable to small perturbations and a linear stability yields that perturbations of the form $\delta\phi(k, t) \exp(ikx)$ evolve as

$$\partial_t \delta\phi(k, t) = k^2(1 - i\alpha - k^2)\delta\phi(k, t), \quad (2)$$

which reveals that $\forall |k| < 1$ the perturbations grow in an oscillatory manner. The behavior of the system depends on α ,

domain length L , and the choice of boundary conditions. We systematically study the phase behavior of Eq. (1) with varying strength of nonreciprocity (α) for various domain sizes (L) and boundary conditions. We consider three different kinds of boundary conditions that are relevant to the phenomenon of phase separation [38]:

- (1) Periodic boundary conditions (P-BCs), where

$$\phi(x + L) = \phi(x) \quad \text{and} \quad \mu(x + L) = \mu(x). \quad (3)$$

- (2) Neumann boundary conditions (N-BCs), where

$$\partial_x \phi|_0 = \partial_x \phi|_L = 0 \quad \text{and} \quad \partial_x \mu|_0 = \partial_x \mu|_L = 0. \quad (4)$$

- (3) Dirichlet boundary conditions (D-BCs), where we fix ϕ on boundaries and μ is flux-free as

$$\phi(0) = \phi(L) = 0 \quad \text{and} \quad \partial_x \mu|_0 = \partial_x \mu|_L = 0. \quad (5)$$

One can also consider fixed boundary conditions for μ , but it is not compatible with the conservation of mass; thus, we do not consider this case. To study the phenomenology, we numerically integrate Eq. (1) on a 1D domain of length L that is discretized using N points. For P-BCs, we use a pseudospectral algorithm with a second-order exponential time-differencing scheme [39] for time marching. For N-BCs and D-BCs, we perform the simulations using a Chebyshev-polynomial-based spectral Tau method implemented by the Dedalus framework [40]. Additionally, we use Dedalus to cross-check our results for the periodic domain. The source code to simulate the 1D NRCH model for all the boundary conditions considered here using Dedalus is available at [41]

We start all our simulations with a small perturbation to the homogeneous state, i.e., $\phi(x, 0) = \delta\phi(x)$, where at every point x the initial perturbation $\delta\phi(x)$ is drawn from a uniform distribution. Further, we impose $\int_0^L \phi(x, 0) dx = 0$, which remains constant throughout the entire time evolution due to the conservation of mass. Unless reported otherwise, we average over eight or more independent realizations of initial conditions for each data point reported in the paper.

III. PERIODIC BOUNDARY CONDITIONS

For P-BCs, the NRCH model (1) admits traveling wave solutions of the form

$$\phi(x, t) = R e^{i(kx - \omega t)}, \quad (6)$$

where $k < 1$, $R^2 = 1 - k^2$, and $\omega = \alpha k^2$. For small k , traveling states are stable to small perturbations; however, an Eckhaus instability kicks in at large k and restricts the allowed wave number range to $0 \leq k^2 < 1/3$ [3,8,35,42]. In our previous studies [3,4], we have shown that in 2D, at low α , disordered states evolve to defect configurations composed of spirals and targets. In 1D, we find similar phenomenology. A random state evolves into multidefect configurations that persist up to the end of simulation time (see Figs. 1 and 2). We now discuss the properties of these states in detail.

A. Sources and sinks

The 1D NRCH model [Eq. (1)] admits solutions of the form

$$\phi(x, t) = R(x) e^{i(Z(x) - \omega t)}, \quad (7)$$

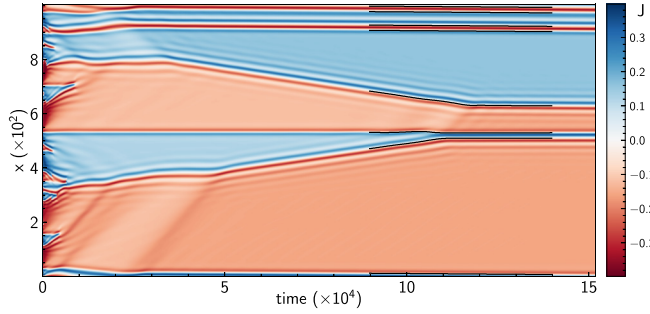


FIG. 1. Kymograph plot of the polar order parameter $J(x, t)$ highlighting the evolution of disordered states. Defects are the zeros of $J(x, t)$, which appear white in the kymograph. We also mark the exact defect positions (small black points) at late times. Initially, numerous source-sink pairs are formed, which merge, and the system settles into a stable configuration, which persists till the end of the simulations.

where x is measured from defect location, $R(x)$ is the amplitude, $Z(x)$ is the phase, and $k(x) \equiv dZ/dx$. They are the sources and sinks of traveling waves and we shall collectively call them defects. In Fig. 2(a), we plot a representative multi-defect configuration obtained from our P-BC simulations. The sources and sinks are arranged in an alternating manner on the 1D domain and the number of sources is always equal to the number of sinks. It is easy to identify the defect locations from the polar order parameter

$$J(x, t) = \text{Im}(\bar{\phi} \partial_x \phi) = \frac{1}{2i}(\bar{\phi} \partial_x \phi - \phi \partial_x \bar{\phi}). \quad (8)$$

As shown in Fig. 2(b), at a source (sink), J vanishes with a positive (negative) slope; thus, at a given time t , the number of defects is equal to the number of zeros of $J(x, t)$. Quantities similar to $J(x, t)$ have been used earlier to analyze the defect dynamics in systems with complex order parameter [43]. Substituting the defect ansatz in the definition of $J(x, t)$, we further find that at the defect core $\partial_x(R^2) = \partial_x(\phi \bar{\phi}) = 2\bar{\phi} \partial_x \phi = 2\phi \partial_x \bar{\phi}$. Thus, at the defect positions,

$$\partial_x R(x)|_{x=0} = 0 \quad \text{and} \quad k(0) = 0. \quad (9)$$

In Figs. 2(c) and 2(d), we plot the profiles of $R(x)$ and $k(x)$ for sources and sinks for different α . For a source, $k(x) > 0$ (< 0) when $x > 0$ (< 0), and vice versa for a sink. It is evident that $R(-x) = R(x)$ and $k(-x) = -k(x)$, which implies that for small x , $R(x) \sim a_0 - a_2 x^2$ and $k(x) \sim b_1 x - b_3 x^3$. Far away from the defect center, the wave front approaches that of a traveling wave, meaning that for $x \gg 1$, $k(x) \rightarrow \pm k_\infty$ for a source and $k(x) \rightarrow \mp k_\infty$ for a sink [see Fig. 2(a)]. The amplitude $R(x) \rightarrow \sqrt{1 - k_\infty^2}$ also becomes constant, and to ensure proper oscillations, we require $\omega = \alpha k_\infty^2$. Note that the sources are the direct 1D analog of targets [3,4] and topologically charged defects do not exist in 1D.

B. Wave number selection

As we show in Fig. 3, for a given α , sources and sinks with a particular value of the asymptotic wave number k_∞ are selected. k_∞ increases monotonically with α , and from a least-squares fit, we obtain $k_\infty \sim \alpha^{0.6}$. Furthermore, k_∞ is comparable to the wave number selected for targets in 2D. In

Fig. 3 (inset), we verify that k_∞ for a single source-sink pair configuration is the same as the k_∞ obtained from multidefect configurations that evolved from disordered initial conditions. A monotonically increasing k_∞ predicts a crossover value of nonreciprocity $\alpha_\times$, such that when $\alpha \rightarrow \alpha_\times$, $k_\infty \rightarrow 1$ and $R_\infty \rightarrow 0$. This implies that the defect solutions will cease to exist for $\alpha \geq \alpha_\times$. Using a least-squares fit, we find $\alpha_\times \sim 1.5$. However, this prediction of $\alpha_\times$ is not entirely correct as it does not account for the Eckhaus instability. Since the plane waves generated by the defects are unstable for $k^2 \geq 1/3$ [3,8,35,42], we expect that the defect solutions will vanish for $k_\infty^2 \geq 1/3$, which yields $\alpha_\times \sim 0.62$. Thus, wave number selection and the Eckhaus instability restrict the defect solutions to the range $0 < \alpha < 0.62$. It is interesting to note that this threshold for the stability of the defects is directly connected to the onset of the emergence of global polar order (see below).

C. Defect density and the global polar order

We now focus on the statistical properties of the multi-defect configurations. Using the fact that $J = 0$ at a source or a sink, we define the 1D defect density ρ_D simply as the number of zeros of J per unit length. In Fig. 4(a), we plot ρ_D in the steady state for different values of α . Since $\alpha = 0$ is the equilibrium limit of the model, ρ_D vanishes as $\alpha \rightarrow 0$. Furthermore, ρ_D vanishes above $\alpha_c \sim 0.6$ and instead we find traveling waves. This marks the onset of disorder-to-order transition that was also observed in 2D [3]. To better analyze the transition, in Fig. 4(b), we plot the average polar order $\bar{J}$ versus α , in the steady state. $\bar{J}$ is defined as

$$\bar{J} = \frac{1}{(q_p - q_p^3)} \langle |J(x, t)| \rangle, \quad (10)$$

where $\langle \dots \rangle$ implies averaging over space and time in the steady state, and we use the dominant wave number of the patterns q_p for normalization. For traveling waves, $J = R^2 k = k - k^3$, and $q_p = k$, which sets $\bar{J} = 1$ for all traveling waves, irrespective of their wave number. For defects, $q_p = k_\infty$ is a suitable choice. Since defects are sources or sinks of traveling waves in both the directions, we expect $\bar{J}$ to vanish in the presence of defects. For $\alpha < \alpha_c$, $\bar{J}$ is close to zero, and for $\alpha > \alpha_c$, consistent with the traveling states, $\bar{J} \sim 1$.

For very small α , or for $\alpha \lesssim \alpha_c$, ρ_D is small, which implies that there are only a handful of defects in the domain. This can lead to finite-size effects and large fluctuations that are reflected in the plot of $\bar{J}$. However, these effects become smaller as the domain size increases.

Another consequence of the finite domains is that in the vicinity of α_c (gray shaded region), the steady state varies across the independent realizations of the initial conditions. Some of the simulations transition to traveling waves, while others show defects. Thus, to find a better estimate of α_c , we compute the fraction of simulations (F_D) that show defect states [see Fig. 4(b), inset]. For $\alpha \lesssim 0.4$, $F_D = 1$, which implies that the entire ensemble shows defect states. For $\alpha > 0.6$, all the simulations for the largest domains end up in traveling wave states. From the plots of ρ_D and $\bar{J}$, we conclude that the disorder-to-order transition in 1D occurs at $\alpha_c \sim 0.6$, which is in agreement with the crossover threshold

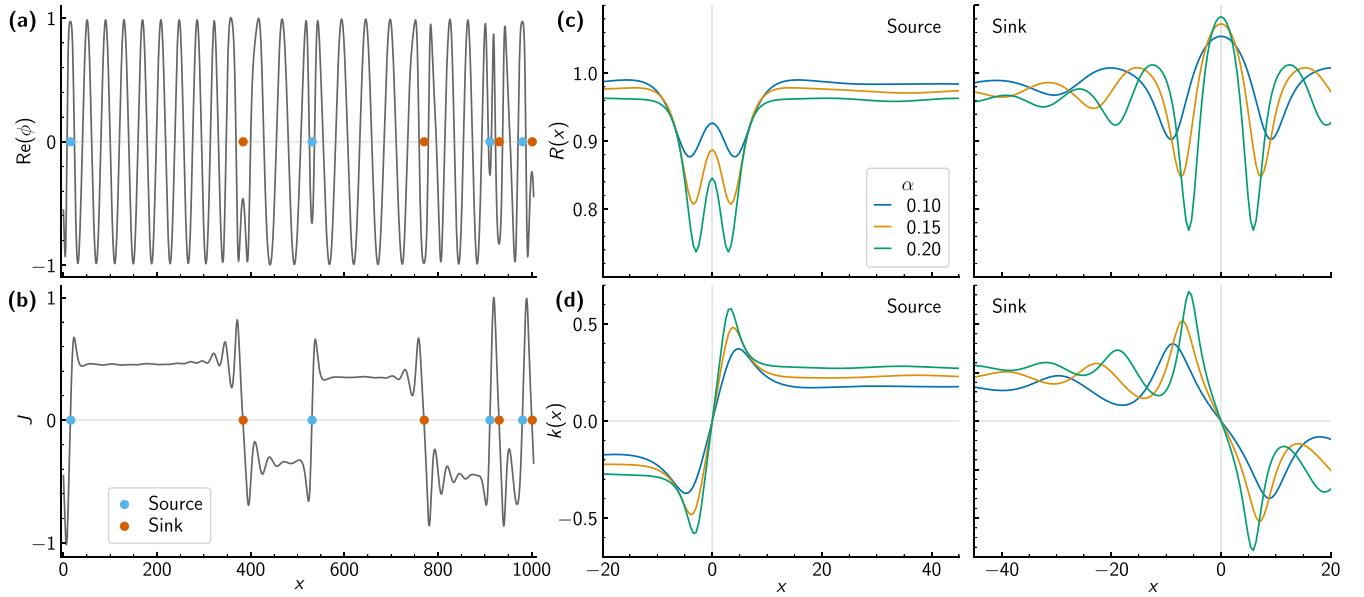


FIG. 2. Sources and sinks in the 1D NRCH model. (a), (b) Representative plot of $\text{Re}(\phi)$ and the polar order parameter J (normalized by its maximum value), respectively. Sources (blue circles) and sinks (orange circles) are arranged in an alternating manner on the 1D domain. J vanishes at defect centers and is constant far from defects, where the wave front is that of a plane wave. (c), (d) Plot of $R(x)$ and $k(x)$ for sources and sinks for different values of α . For these plots, x is measured from the center of the defects. At defect centers, $\partial_x R$ and $k(x)$ vanish. Far from the defects, $R(x)$ and $k(x)$ approach their plane wave forms.

$\alpha_\times \sim 0.62$ predicted by the wave number selection and the Eckhaus instability.

To understand this result, we note that the traveling wave solutions of NRCH are linearly stable at all values of α [8], and therefore, the only reason they might not be adopted at relatively smaller values of α is the emergence of relatively stable defects that form an arrested defect network [3,4]. Once these defects are destabilized (beyond $\alpha_\times \sim 0.62$) due to Eckhaus instability, the only stable solutions that remain will be the traveling waves. While the above argument, which supports $\alpha_c \simeq \alpha_\times$, provides a simple and comprehensive

explanation for the 1D phase diagram, we note that this scenario does not hold for the disorder-to-order transition in 2D, where $\alpha_c \sim 0.28$ is much smaller than $\alpha_\times \sim 0.58$ [3]. This suggests that in 2D there must be other effects (not present in 1D) that can destabilize the defects before the Eckhaus instability occurs. These effects could originate from the role of geometry of the domains, the existence of different types of defects, etc. and highlight the role of dimensionality in the phenomenology of the NRCH model.

D. Resonances

A careful analysis of the ρ_D versus α plot reveals nontrivial features of the defect configurations. We find that ρ_D shows sharp minima at certain values of α , which we call “resonances” [see Fig. 4(a)]. A visual inspection of the evolution of the defect configurations reveals that at resonances, the defects keep merging even at longer times, resulting in smaller defect density. For further analysis, we compute the time it takes for the defect configurations to reach a steady state (t_{st}), where t_{st} is defined as the time after which no defect merger event takes place and the defect density becomes constant. We find that for the majority of α space, defects attain a steady state relatively quickly; however, in the resonance regions, t_{st} can be order of magnitude higher [see Fig. 5(a)]. Consistent with these observations, we also observe sharp rise in the nearest-neighbor separation d_{min} for the resonances [see Fig. 5(b)]. Similar phenomenon, where d_{min} rises sharply and decreases gradually, was also observed in the study of pairwise defect interactions in 2D [4].

IV. EFFECT OF BOUNDARY CONDITIONS

We now consider the effects of varying boundary conditions on the phenomenology of the NRCH model. As

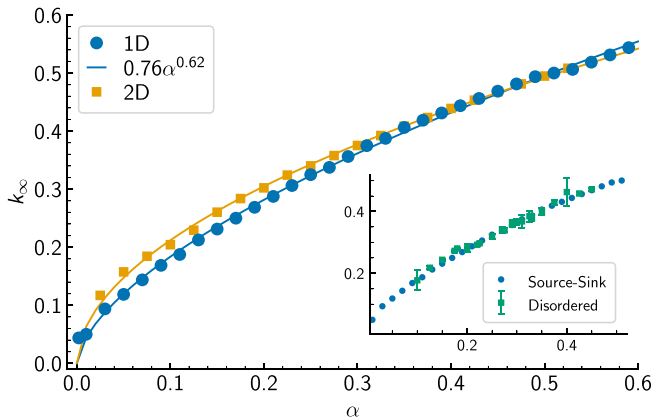


FIG. 3. Wave number selection in 1D. For a given α , sources and sinks with a particular wave number are selected. Using a least-squares fit to power-law forms, we obtain $k_\infty \sim \alpha^{0.62}$ (solid black line). The selected wave number is comparable to the k_∞ obtained for the targets in 2D (orange circles) [3]. Inset: k_∞ for a single source-sink pair is identical to the k_∞ for a multidefect disordered configuration.

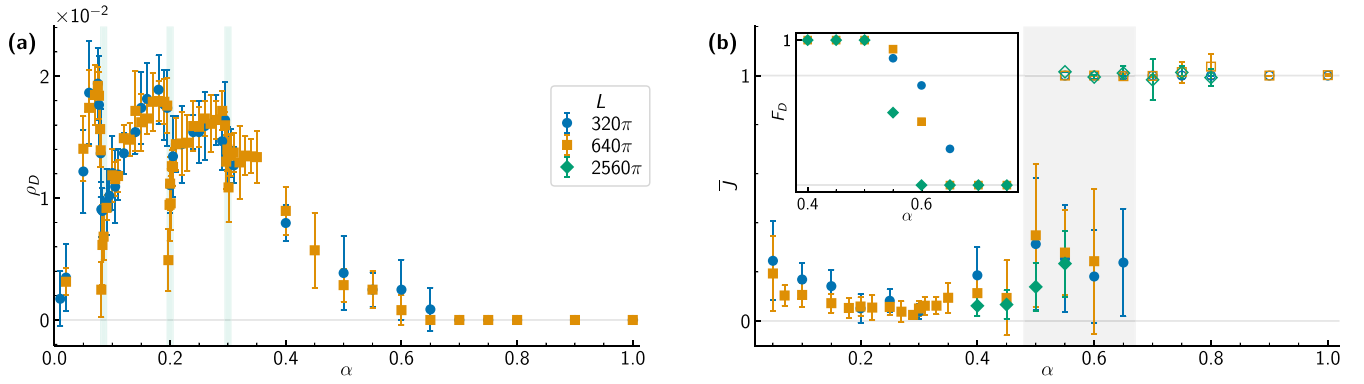


FIG. 4. (a) Defect density ρ_D vs α for P-BCs. Overall ρ_D peaks around $\alpha \sim 0.2$ and vanishes for $\alpha \rightarrow 0$ and $\alpha \rightarrow \alpha_c$. Above $\alpha_c \sim 0.6$, $\rho_D = 0$, which implies the traveling wave phase. At certain values of $\alpha \simeq 0.08, 0.2, 0.3$, which we call “resonances” (shaded green), ρ_D significantly dips. (b) Average polar order $\bar{J}$ vs α . For small α , $\bar{J} \sim 0$, and at large α , $\bar{J} \sim 1$. For $\alpha \sim 0$ or $\alpha \sim \alpha_c$ (gray shaded region), $\bar{J}$ shows large fluctuations due to finite-size effects. We compute $\bar{J}$ separately for defect-riddled states (solid markers) and traveling states (hollow markers). In the shaded region, some realizations show defects and some show traveling waves. Inset: The fraction of ensemble (F_D) that show defect states for α close to α_c . Below α_c , $F_D = 1$ and above α_c , all simulations transition to traveling waves; thus, $F_D = 0$. Larger domains show sharper transitions. From these plots, we infer $\alpha_c \sim 0.6$.

discussed in Sec. II, in addition to the periodic boundary conditions, we consider Neumann and Dirichlet boundary conditions. Traveling waves of the form (6) are incompatible with both N-BCs and D-BCs; thus, we do not expect states with complete polar order to emerge beyond α_c . Further, for both N-BC and D-BC, polar order vanishes at the boundaries, i.e.,

$$J(x=0, t) = J(x=L, t) = 0 \quad \forall t, \quad (11)$$

as either ϕ or $\partial_x \phi$ vanish at the boundaries. Thus, the boundaries act as additional sources or sinks, and in our analysis, we ignore the defects on the boundaries and only consider defects that are of the form described in Fig. 2 and are located inside the domain within $x \in (x_{\text{pad}}, L - x_{\text{pad}})$. For both N-BCs and D-BCs, we heuristically choose $x_{\text{pad}} = 50$, which we find filters the boundary defects very well.

A. Defects below α_c

Below α_c , we once again find defect configurations. As shown in Fig. 6, similar to the P-BCs, for both N-BCs and

D-BCs, defects with a particular wave number are selected. However, wave number selection is affected by the presence of boundaries; for a given α , the selected wave number is higher than that for the P-BCs, and a least-squares fit yields $k_\infty = \alpha^{0.52}$. Thus, one expects that defect solutions will vanish around $\alpha \sim 1$. It is tempting to use the Eckhaus instability to arrive at a crossover threshold of nonreciprocity; $\alpha_\times \sim 0.33$ for N-BCs and D-BCs. However, we note that the instability analysis assumes traveling wave states, which are not permitted solutions for these boundary conditions. Our numerical analysis suggests that the defect states persist up to $\alpha_c \sim 0.6$.

The phenomenology below α_c is qualitatively similar to the P-BCs. The defect density ρ_D agrees with the results from the P-BCs [see Figs. 7(a) and 7(d)]. Further, the average polar order vanishes for defect states.

B. Fluctuating polar order beyond α_c

Since traveling waves are only permitted for P-BCs, we expect the phenomenology for $\alpha > \alpha_c$ to vary significantly

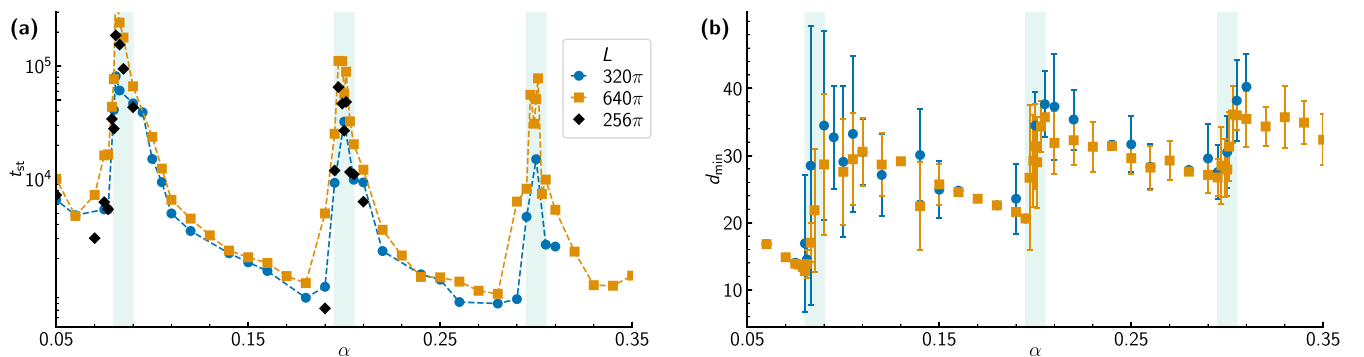


FIG. 5. Analysis of resonances. (a) Time (t_{st}) after which the system reaches the steady-state value of ρ_D . At resonances, defect merger events continue for long times; thus, t_{st} is orders of magnitude larger. To verify that the resonances are not the effect of domain size, we also plot results for $L = 256\pi$ (black points), which shows similar behavior. (b) Minimum interdefect separation d_{min} vs α . As defects keep merging for long times, we find sharp jumps in d_{min} in the resonance regions.

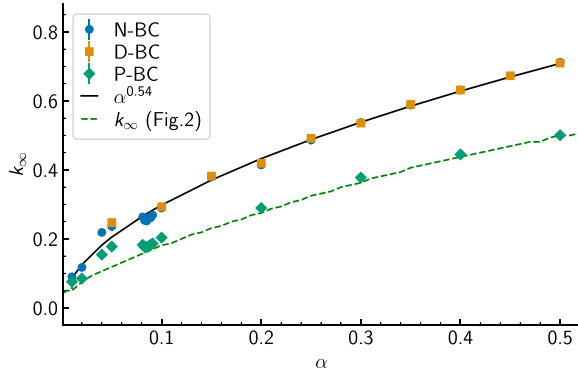


FIG. 6. Comparison of k_∞ for different α for $\alpha < \alpha_c$ for different boundary conditions. For a given α , k_∞ is higher for both N-BC and D-BC when compared to the k_∞ for the P-BCs. As a check, we also plot the k_∞ obtained from Dedalus for the P-BCs, which matches with our own simulations (dashed black line).

for N-BCs and D-BCs. Indeed, beyond α_c , the system does not attain a single traveling wave phase. For $\alpha \gtrsim \alpha_c$, both N-BCs and D-BCs show intermittent behavior. The emergence of global polar order is thwarted by the incompatibility of traveling bands with the boundary conditions. Thus, the system does not settle down, but shows transient, fluctuating states with patchy polar order that sustain over long times (see Fig. 8).

In the case of P-BCs, a single wave number dominates the steady states; for defect configurations below α_c , it is the selected wave number k_∞ and for the traveling states above α_c , it is the wave number of the traveling wave. For N-BCs and D-BCs, we find that the intermittent patchy ordered states exhibit fluctuations at multiple wave numbers. To verify this, we compute various relevant scales using the structure factor $S(k, t)$:

(1) The dominant mode k_d defined as the wave number at which $S(k)$ peaks, i.e., $k_d = \text{argmax}_k S(k)$.

(2) The coarsening mode $k_c = \sum_k k S(k) / \sum_k S(k)$.

(3) The integral mode $k_i = \sum_k S(k) / (\sum_k S(k)/k)$.

Here, $S(k, t) = |\psi(k, t)|^2$ is the structure factor, where $\psi(k, t) = \sum_x \psi(x, t) \exp(-ikx)$ is the Fourier transform of the windowed field $\psi(x, t) = w(x)\phi(x, t)$. Note that we apply the Hanning window $w(x) = \sin^2(\pi x/L)$ on the data for the N-BCs and D-BCs to avoid spurious amplitude errors arising from the aperiodicity.

As shown in Figs. 7(c) and 7(f), k_d , k_c , and k_i are in excellent agreement with each other and match with k_∞ for $\alpha < \alpha_c$, emphasizing that k_∞ governs the dynamics for defect configurations. Slightly above α_c , different modes disagree with each other, and exhibit large fluctuations, signifying that the transient states show fluctuations at multiple length scales. Consequently, it is not possible to choose a suitable normalization for the average polar order; i.e., a suitable choice for q_p in Eq. (10) is not available for these states, which is also reflected in large fluctuations of $\bar{J}$ [see Figs. 7(b) and 7(d)] where we have used $q_p = k_c$.

For $\alpha > \alpha_c$, the system solves the incompatibility with the traveling waves by creating two domains with waves traveling in the opposite direction (see Fig. 8). These domains are

separated by a partition where $|\phi|$ vanishes, and in addition to the domain boundaries, it acts as a source or sink for the traveling waves. The partition can form anywhere inside the domain and fluctuates slowly with time. The distribution function of $J(x, t)$ is primarily bimodal, where the strength of the two peaks depends on the location of partition in the domain. As a consequence, we find large nonzero polar order when the partition is closer to one of the domains, or small polar order if it is close to the middle of the simulation domain. Finally, various scales, k_d , k_c , and k_i all converge to similar values.

V. CONCLUSIONS

We have presented a systematic study of the disorder-to-order transition for the 1D NRCH model. In 1D, defect solutions are the sources and sinks of traveling waves. Sources are analogous to the two-dimensional topologically neutral targets, whereas sinks are similar to the disclination lines where the waves emanated from sources meet. For a given α , defects select a unique wave number k_∞ that increases monotonically with α . Wave number selection predicts a crossover threshold $\alpha_\times \sim 0.62$ above which the defect solutions cannot exist.

At small α , disordered initial states evolve into multidefect configurations, where sources and sinks are arranged in an alternating manner. With increasing α , overall the defect density increases, peaks at $\alpha \sim 0.2$, and then vanishes at $\alpha_c \sim 0.6$, which marks the onset of disorder-to-order transition. In accordance with this observation, we find that there is no significant average polar order for $\alpha < \alpha_c$, but traveling states for $\alpha > \alpha_c$ show perfect order in the long-time steady state. In a direct contrast to 2D, where the transition occurs at $\alpha_c \sim 0.28 \ll \alpha_\times \sim 0.58$, the transition point in 1D, $\alpha_c \sim 0.6$ is in close agreement with the crossover threshold $\alpha_\times \sim 0.62$ predicted by wave number selection. A closer inspection of the defect dynamics reveals that at certain resonance values of nonreciprocity below α_c , ρ_D shows sharp minima, which is a consequence of the fact that for these values of α , defect merger events continue to occur even at very long times. Additionally, the average interdefect separation rises sharply for these α values, which was also observed in the study of pairwise defect interactions in 2D [4].

Our numerical simulations with N-BCs and D-BCs, which are inconsistent with the traveling waves, show a different flavor of the disorder-order transition. The defect dynamics at $\alpha < \alpha_c$ is similar to the P-BC, but the boundary conditions significantly affect the phenomenology above the transition point. Slightly above the threshold, we find intermittent patches of polar order. For larger values of α , the system partitions into two subdomains that sustain over time.

To conclude, our study shows that for the P-BCs the NRCH model shows similar phenomenological behavior in both 1D and 2D, with a crucial difference that the disorder-to-order transition occurs at the crossover value predicted by wave number selection in 1D. We further show that boundary conditions can significantly affect the phenomenology. These findings call for future studies to further investigate the defect dynamics and defect interactions for conserved systems with

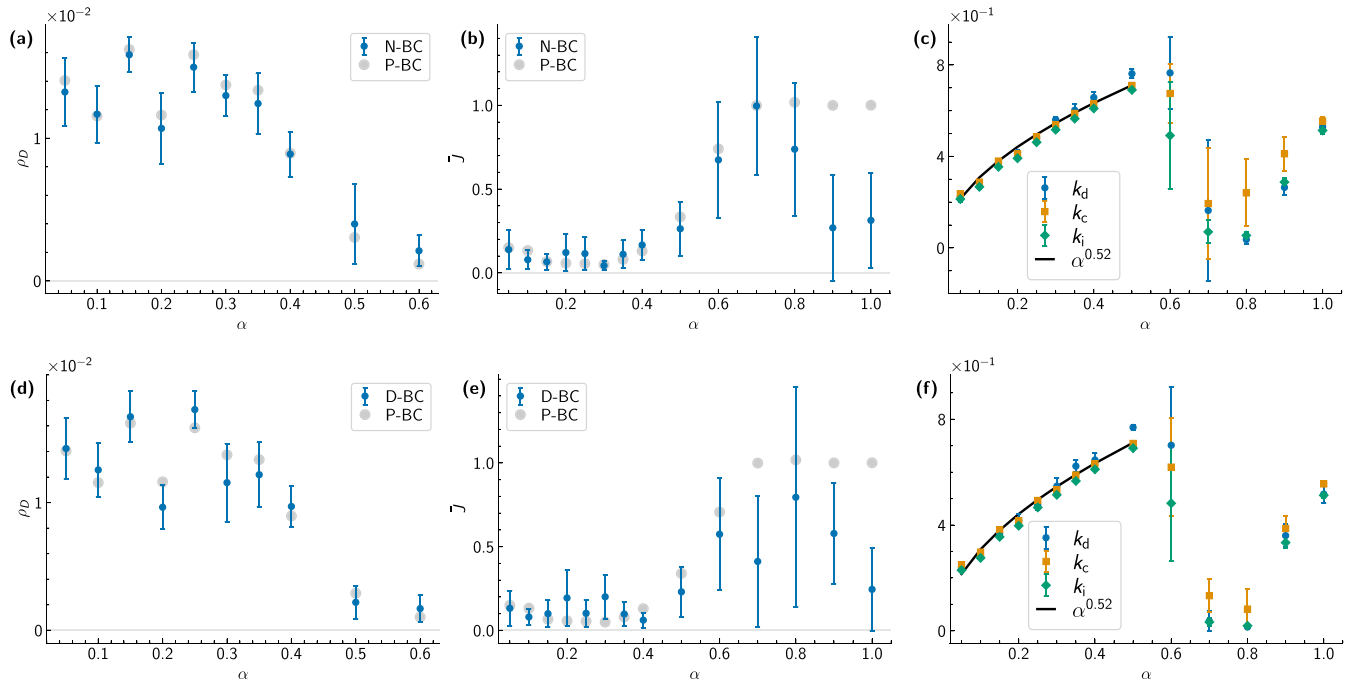


FIG. 7. Phenomenology for the N-BCs [panels (a)–(c)] and D-BCs [panels (d)–(f)]. For $\alpha < \alpha_c$, the phenomenology is similar to the P-BCs. (a), (d) Plot of average defect density ρ_D vs α , which is consistent with the statistics for P-BCs within the error bars. (b), (e) Plot of the average polar order. For $\alpha < \alpha_c$, we find $\bar{J} \sim 0$. For $\alpha > \alpha_c$, the system shows partial order. Near the transition point, we observe large intermittent fluctuations, which are reflected in the large fluctuations in $\bar{J}$ as well. (c), (f) Comparison of different length scales. For $\alpha < \alpha_c$, k_∞ is the dominant length scale and all other scales converge to it. Near the transition, we find large disagreements as well as large fluctuations and different length scales, which once again converge at larger $\alpha \sim 0.9$.

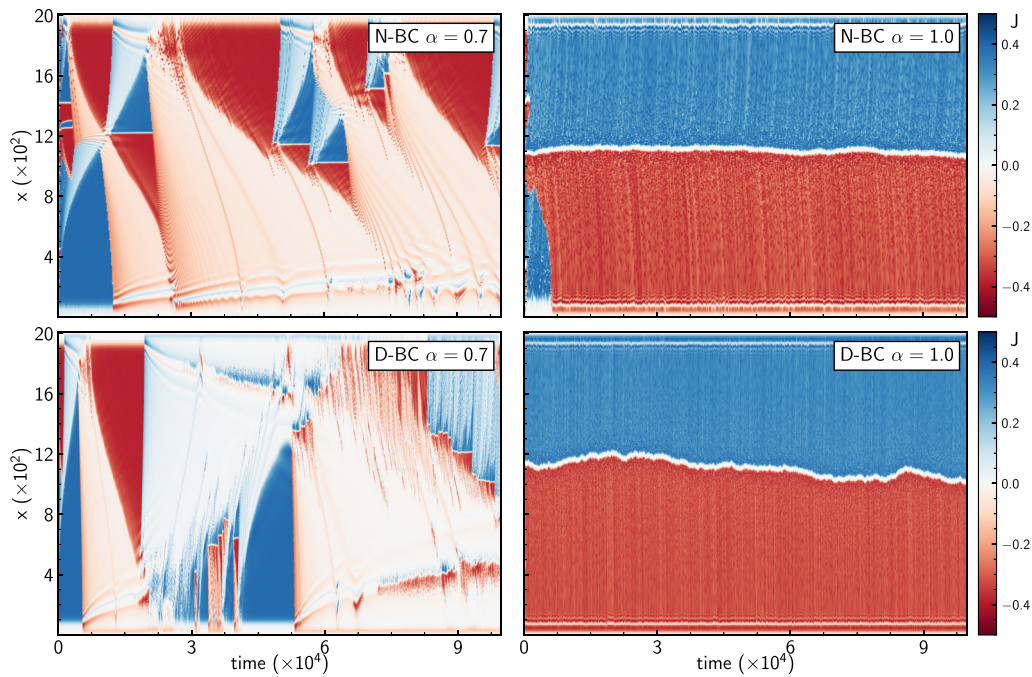


FIG. 8. Kymographs of $J(x, t)$ for N-BCs and D-BCs for $\alpha = 0.70$ (slightly above α_c) and for $\alpha = 1.0$. Closer to the transition point, we observe large transient and intermittent patches of polar order in the domain that keep emerging and vanishing. For large α , both N-BCs and D-BCs show that the system develops two partitions with polar order in opposite directions. The position of the partition fluctuates with time.

nonreciprocal interactions. In particular, it will be interesting to explore what determines the value of α_c in 1D and 2D and to find out the mechanism that drives the resonances for certain values of α .

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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- [1] S. Saha, J. Agudo-Canalejo, and R. Golestanian, Scalar active mixtures: The nonreciprocal Cahn-Hilliard model, *Phys. Rev. X* **10**, 041009 (2020).
 - [2] Z. You, A. Baskaran, and M. C. Marchetti, Nonreciprocity as a generic route to traveling states, *Proc. Natl. Acad. Sci. USA* **117**, 19767 (2020).
 - [3] N. Rana and R. Golestanian, Defect solutions of the nonreciprocal Cahn-Hilliard model: Spirals and targets, *Phys. Rev. Lett.* **133**, 078301 (2024).
 - [4] N. Rana and R. Golestanian, Defect interactions in the nonreciprocal Cahn-Hilliard model, *New J. Phys.* **26**, 123008 (2024).
 - [5] L. Parkavousi, N. Rana, R. Golestanian, and S. Saha, Enhanced stability and chaotic condensates in multispecies nonreciprocal mixtures, *Phys. Rev. Lett.* **134**, 148301 (2025).
 - [6] G. Pisegna, S. Saha, and R. Golestanian, Emergent polar order in nonpolar mixtures with nonreciprocal interactions, *Proc. Natl. Acad. Sci. USA* **121**, e2407705121 (2024).
 - [7] G. Pisegna, N. Rana, R. Golestanian, and S. Saha, Nonreciprocal mixtures in suspension: The role of hydrodynamic interactions, *Phys. Rev. Lett.* **135**, 108301 (2025).
 - [8] S. Saha and R. Golestanian, Effervescence in a binary mixture with nonlinear non-reciprocal interactions, *Nat. Commun.* **16**, 7310 (2025).
 - [9] S. A. M. Loos and S. H. L. Klapp, Irreversibility, heat and information flows induced by non-reciprocal interactions, *New J. Phys.* **22**, 123051 (2020).
 - [10] S. Osat and R. Golestanian, Non-reciprocal multifarious self-organization, *Nat. Nanotechnol.* **18**, 79 (2023).
 - [11] S. Osat, J. Metson, M. Kardar, and R. Golestanian, Escaping kinetic traps using nonreciprocal interactions, *Phys. Rev. Lett.* **133**, 028301 (2024).
 - [12] R. Soto and R. Golestanian, Self-assembly of catalytically active colloidal molecules: Tailoring activity through surface chemistry, *Phys. Rev. Lett.* **112**, 068301 (2014).
 - [13] A. V. Ivlev, J. Bartnick, M. Heinen, C.-R. Du, V. Nosenko, and H. Löwen, Statistical mechanics where Newton's third law is broken, *Phys. Rev. X* **5**, 011035 (2015).
 - [14] J. Agudo-Canalejo and R. Golestanian, Active phase separation in mixtures of chemically interacting particles, *Phys. Rev. Lett.* **123**, 018101 (2019).
 - [15] M. Fruchart, R. Hanai, P. B. Littlewood, and V. Vitelli, Non-reciprocal phase transitions, *Nature (London)* **592**, 363 (2021).
 - [16] R. Golestanian, Non-reciprocal active-matter: A tale of "loving hate, brawling love" across the scales, *Europhys. News* **55**, 12 (2024).
 - [17] T. Frohoff-Hülsmann and U. Thiele, Localized states in coupled Cahn-Hilliard equations, *IMA J. Appl. Math.* **86**, 924 (2021).
 - [18] T. Frohoff-Hülsmann, J. Wrembel, and U. Thiele, Suppression of coarsening and emergence of oscillatory behavior in a Cahn-Hilliard model with nonvariational coupling, *Phys. Rev. E* **103**, 042602 (2021).
 - [19] T. Frohoff-Hülsmann and U. Thiele, Nonreciprocal Cahn-Hilliard model emerges as a universal amplitude equation, *Phys. Rev. Lett.* **131**, 107201 (2023).
 - [20] R. Golestanian, Phoretic active matter, in *Active Matter and Nonequilibrium Statistical Physics: Lecture Notes of the Les Houches Summer School: September 2018*, edited by J. Tailleur, G. Gompper, M. C. Marchetti, J. M. Yeomans, and C. Salomon (Oxford University Press, Oxford, UK, 2022), Vol. 112, pp. 230–293.
 - [21] G. Tucci, R. Golestanian, and S. Saha, Nonreciprocal collective dynamics in a mixture of phoretic Janus colloids, *New J. Phys.* **26**, 073006 (2024).
 - [22] N. Uchida and R. Golestanian, Synchronization and collective dynamics in a carpet of microfluidic rotors, *Phys. Rev. Lett.* **104**, 178103 (2010).
 - [23] L. F. A., W. Hugo, B. Tobias, and B. Clemens, Group formation and cohesion of active particles with visual perception-dependent motility, *Science* **364**, 70 (2019).
 - [24] J. A. Cohen and R. Golestanian, Emergent cometlike swarming of optically driven thermally active colloids, *Phys. Rev. Lett.* **112**, 068302 (2014).
 - [25] U. Khadka, V. Holubec, H. Yang, and F. Cichos, Active particles bound by information flows, *Nat. Commun.* **9**, 3864 (2018).
 - [26] R. Soto and R. Golestanian, Self-assembly of active colloidal molecules with dynamic function, *Phys. Rev. E* **91**, 052304 (2015).
 - [27] S. Saha, S. Ramaswamy, and R. Golestanian, Pairing, waltzing and scattering of chemotactic active colloids, *New J. Phys.* **21**, 063006 (2019).
 - [28] L. P. Dadhichi, J. Kethapelli, R. Chajwa, S. Ramaswamy, and A. Maitra, Nonmutual torques and the unimportance of motility for long-range order in two-dimensional flocks, *Phys. Rev. E* **101**, 052601 (2020).
 - [29] V. Ouazan-Reboul, R. Golestanian, and J. Agudo-Canalejo, Network effects lead to self-organization in metabolic cycles of self-repelling catalysts, *Phys. Rev. Lett.* **131**, 128301 (2023).
 - [30] R. Wittkowski, J. Stenhammar, and M. E. Cates, Nonequilibrium dynamics of mixtures of active and passive colloidal particles, *New J. Phys.* **19**, 105003 (2017).
 - [31] A. M. Tayar, F. Caballero, T. Anderberg, O. A. Saleh, M. Cristina Marchetti, and Z. Dogic, Controlling liquid-liquid phase behaviour with an active fluid, *Nat. Mater.* **22**, 1401 (2023).
 - [32] F. Brauns, G. Pawlik, J. Halatek, J. Kerssemakers, E. Frey, and C. Dekker, Bulk-surface coupling identifies the mechanistic connection between min-protein patterns *in vivo* and *in vitro*, *Nat. Commun.* **12**, 3312 (2021).
 - [33] L. Würthner, F. Brauns, G. Pawlik, J. Halatek, J. Kerssemakers, C. Dekker, and E. Frey, Bridging scales in a multiscale

- pattern-forming system, *Proc. Natl. Acad. Sci. USA* **119**, e2206888119 (2022).
- [34] I. S. Aranson, L. Kramer, and A. Weber, Theory of interaction and bound states of spiral waves in oscillatory media, *Phys. Rev. E* **47**, 3231 (1993).
- [35] I. S. Aranson and L. Kramer, The world of the complex Ginzburg-Landau equation, *Rev. Mod. Phys.* **74**, 99 (2002).
- [36] P. S. Hagan, Target patterns in reaction-diffusion systems, *Adv. Appl. Math.* **2**, 400 (1981).
- [37] P. S. Hagan, Spiral waves in reaction-diffusion equations, *SIAM J. Appl. Math.* **42**, 762 (1982).
- [38] *Mathematical Models for Phase Change Problems: Proceedings of the European Workshop Held at Óbidos, Portugal, October 1–3, 1988*, International Series of Numerical Mathematics Vol. 88, edited by J.-F. Rodrigues (Birkhäuser Verlag, Basel, 1989).
- [39] S. Cox and P. Matthews, Exponential time differencing for stiff systems, *J. Comput. Phys.* **176**, 430 (2002).
- [40] K. J. Burns, G. M. Vasil, J. S. Oishi, D. Lecoanet, and B. P. Brown, Dedalus: A flexible framework for numerical simulations with spectral methods, *Phys. Rev. Res.* **2**, 023068 (2020).
- [41] <https://github.com/navdeeprana/DedalusScripts>.
- [42] W. Zimmermann, Stability of traveling waves for a conserved field, *Physica A* **237**, 405 (1997).
- [43] L. Angheluta, P. Jeraldo, and N. Goldenfeld, Anisotropic velocity statistics of topological defects under shear flow, *Phys. Rev. E* **85**, 011153 (2012).