

Quantum-limited imaging of a nanomechanical resonator with a spatial mode sorter

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(Received 19 December 2024; accepted 14 July 2025; published 8 August 2025; corrected 6 November 2025)

We explore the use of a spatial mode sorter to image a nanomechanical resonator, with the goal of studying the quantum limits of active imaging and extending the toolbox for optomechanical force sensing. In our experiment, we reflect a Gaussian laser beam from a vibrating nanoribbon and pass the reflected beam through a commercial spatial mode demultiplexer (Cailabs Proteus). The intensity in each demultiplexed channel depends on the mechanical modes and encodes information about their displacement amplitudes. As a concrete demonstration, we monitor the angular displacement of the ribbon's fundamental torsion mode by illuminating in the fundamental Hermite-Gauss mode (HG₀₀) and reading out in the HG₁₀ mode. We show that this technique permits readout of the ribbon's torsional vibration with a precision near the quantum limit. Our results highlight new opportunities at the interface of quantum imaging and quantum optomechanics.

DOI: 10.1103/6msj-wh2f

Spatial mode demultiplexing (SPADE) has emerged as a powerful tool for extreme imaging applications ranging from exoplanet detection [1] to superresolution microscopy [2]. Motivations include the advance of low-cross-talk commercial technologies based on multiplane light conversion (MPLC) [3] and the identification of protocols for traditional low-light imaging applications (e.g., ground-based telescopy), in which SPADE arises as an optimally efficient receiver [4,5].

A canonical application of SPADE is tracking small distortions (such as displacement) of a laser beam [6]. As opposed to traditional cameras, which sample an image in the position basis, SPADE receivers project in a natural paraxial basis, such as the Laguerre- or Hermite-Gauss (HG) modes [7]. The reduced dimensionality (typically ~ 10) of this approach comes with the advantage of higher bandwidth and access to quantum-limited detection of the sorted light [8], including at the single photon level [9]. These trade-offs form the basis for quantum-enhanced [10,11] or inspired [12,13] SPADE protocols using classical [12,13] and nonclassical [10,11] light.

Here we consider SPADE for dynamic imaging of a nanomechanical resonator, a measurement task rooted in practical applications [14,15] and fundamental experiments [16], whose implementation has close parallels to laser beam tracking. To our knowledge, MPLC-based tracking of a nanomechanical resonator has not been reported in the literature; however, a recent study applied photonic-integrated SPADE to dynamic imaging of a levitated nanoparticle [17]. Both studies are timely, given the emergence of ultracohesent vacuum-levitated [18] and tethered nanomechanical resonators [19] for which the use of “cavity-free” multimode

imaging techniques is approaching quantum limits [20,21]. In these experiments, the resonator couples different spatial modes of a laser field, and the readout task becomes similar to spatial mode sorting.

The basic experimental concept is illustrated in Fig. 1. A laser with transverse modeshape u_{in} is reflected from a vibrating membrane into a spatial mode sorter (u_{det}). The membrane modeshape ϕ and displacement z_0 are encoded into two orthogonal modes comprising the reflected field profile [22]

$$u_{\text{ref}} = u_{\text{in}} e^{2ikz_0\phi} \approx u_{\text{in}} + 2ikz_0(\beta_{\parallel}u_{\text{in}} + \beta_{\perp}u_{\perp}), \quad (1)$$

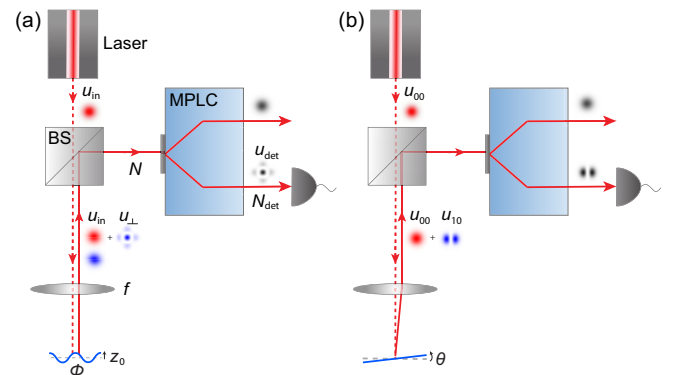


FIG. 1. Concept for readout of a mechanical oscillator with a spatial mode sorter. (a) A laser with transverse modeshape u_{in} reflects off a vibrating membrane with modeshape ϕ and amplitude z_0 . A fraction (blue) of the reflected light is inelastically scattered into spatially orthogonal mode $u_{\perp}$. A mode sorter (MPLC, modeshape u_{det}) distills this light and a small amount of elastically scattered (red) u_{in} light, serving as a local oscillator for self-homodyne detection. (b) Case of an optical lever, in which angular displacement θ is encoded into fundamental and first-order Hermite-Gauss modes u_{mn} .

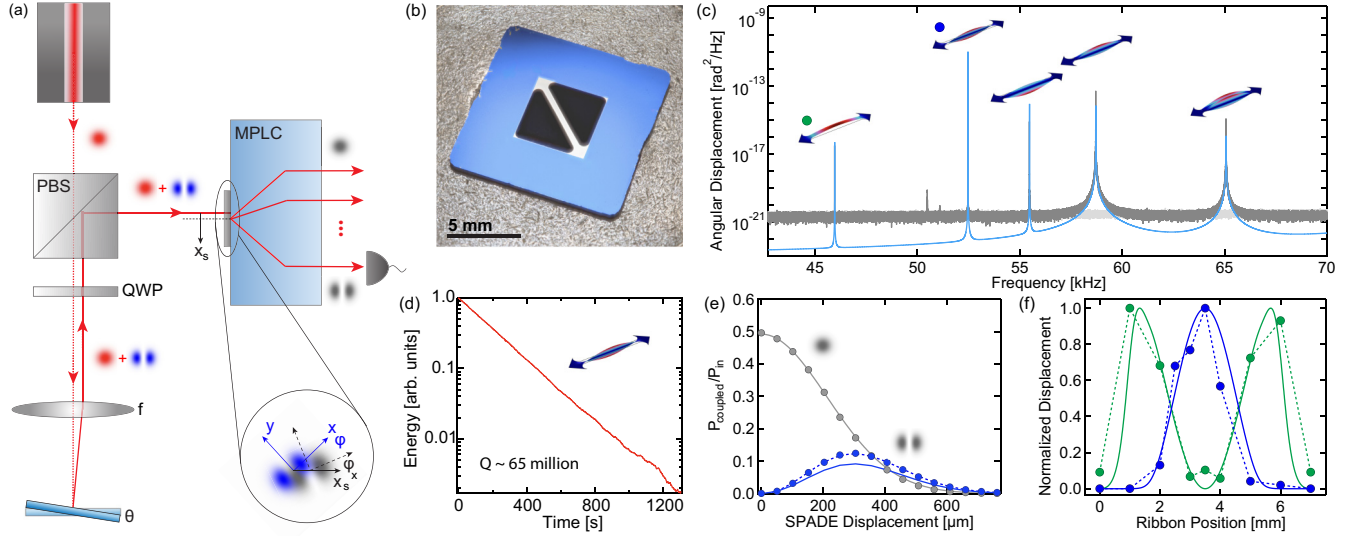


FIG. 2. SPADE-based optical lever readout of a nanomechanical torsion oscillator. (a) Sketch of the experiment: A nanoribbon is illuminated by an HG_{00} laser beam. Angular displacement θ of the ribbon is encoded in the HG_{10} component of reflected field, which is sorted using an HG-mode MPLC. The reference frame of the torsion oscillator (red) is rotated with respect to the MPLCs (black). Inset: Reference frame of MPLC (black dashed axes and gray HG_{10} intensity profile), inelastically scattered field (blue axes and blue HG_{10} intensity profile), and laboratory (black axis). (b) Photograph of the Si_3N_4 nanoribbon: 380- μm wide, 7-mm long, and 75-nm thick. (c) Broadband power spectral density of the HG_{10} photocurrent calibrated in angular displacement units. Blue curve is a multimode thermal noise model. Simulated modeshapes are assigned to each noise peak. (d) Ringdown of the fundamental torsion mode, revealing a quality factor of $Q = 65$ million. (e) Power coupling efficiency of the reflected field into HG_{00} and HG_{10} ports of the MPLC vs transverse misalignment x . (f) Area beneath the fundamental flexural and torsion mode peaks in (c) vs beam position on the ribbon (see main text for details).

where k is the laser wave number. To estimate z_0 , a photodetector monitors the sorted photon flux $N_{\text{det}} = |\langle u_{\text{det}} | u_{\text{ref}} \rangle|^2 N$, where N is the reflected flux and $\langle \cdot, \cdot \rangle$ is the transverse spatial overlap [23] at the mode sorter. In the limit that $\beta_{\parallel} = 0$ and $|\langle u_{\text{det}} | u_{\perp} \rangle| \gg |\langle u_{\text{det}} | u_{\text{in}} \rangle| > 0$, displacement is fully encoded into the “interacting” mode $u_{\perp}$ and interference between the elastically scattered u_{in} field (acting as a local oscillator), and inelastically scattered $u_{\perp}$ field yields a quantum-limited displacement imprecision, in spectral density units [24], of

$$S_{z_0}^{\text{imp}} = \left(\frac{dN_{\text{det}}}{dz_0} \right)^{-2} S_{N_{\text{det}}} \quad (2a)$$

$$\geq \frac{1}{8Nk^2\beta_{\perp}^2} \frac{1}{|\langle u_{\text{det}} | u_{\perp} \rangle|^2} \frac{1}{\sin^2 \phi_{\text{in},\perp}} \geq \frac{\hbar^2}{S_F^{\text{BA}}}, \quad (2b)$$

where $\phi_{\text{in},\perp}$ is the relative phase of u_{in} and $u_{\perp}$ at the mode sorter, $S_{N_{\text{det}}} = 2N_{\text{det}}$ is the photon shot noise at the detector, and $S_F^{\text{BA}} = 8\hbar^2k^2N(\beta_{\parallel}^2 + \beta_{\perp}^2)$ is the backaction force due to spatiotemporal photon shot noise at the membrane [22].

To explore Eq. (2), we specialize to an HG_{00} laser reflected from a torsion mode, $\phi \propto x$. In this case, our setup embodies an optical lever (OL), which encodes angular displacement $\theta = z_0\phi'(x)$ into a superposition of HG_{00} and HG_{10} [21,25],

$$u_{\text{ref}} \approx u_{00} + 2i(\theta/\theta_D)u_{10}, \quad (3)$$

where θ_D is the diffraction angle of the reflected beam. When aligned into an HG mode sorter and monitored through the

HG_{10} port ($u_{\text{det}} = u_{10}$), Eq. (2) predicts a lower bound of [26]

$$S_{\theta}^{\text{imp}} = \frac{\theta_D^2}{8N}, \quad (4)$$

which is the Heisenberg limit for an OL measurement subject to a quantum torque backaction of $S_{\tau} = 8\hbar^2N/\theta_D^2$ [21,22,27].

Our experimental setup is similar to that in Ref. [27], where we used an OL to monitor the thermally driven torsion mode of a Si_3N_4 nanoribbon, using a split photodetector (SPD) as a receiver. For this study, we use an identical nanoribbon with a torsional resonance frequency of $\omega_m = 2\pi \times 52.5$ kHz and a quality factor of $Q_m = 65 \times 10^6$. The SPD is replaced with a commercial ten-channel HG-mode MPLC (Cailabs Proteus-C), which has a nominal throughput and cross-talk of ~ 1 and ~ 20 dB, respectively, at a wavelength of $\lambda = 1550$ nm. As illustrated in Fig. 2(a), the ribbon is housed in a high vacuum chamber ($\sim 10^{-8}$ mbar) beneath the OL setup. The optical breadboard, collimation lens, and MPLC are mounted on individual micropositioning stages for fine alignment [26].

In order to achieve the optimal sensitivity given by Eq. (4), it is necessary to perform an ideal measurement of the phase quadrature of the scattered HG_{10} field. Rather than interfering the demultiplexed field with an independent local oscillator (LO), we take advantage of the fact that HG_{00} and HG_{10} fields acquire a relative Gouy phase of $\phi_{00,10} \approx \pi/2$ in the far field and disalign the mode sorter (by translating its optical axis) so that a fraction of the HG_{00} field is coupled into the detection port ($|\langle u_{\text{det}} | u_{00} \rangle| > 0$). In this self-homodyne scheme [28], the HG_{00} field serves as an LO, at the expense of a small loss of information due to misalignment ($|\langle u_{\text{det}} | u_{10} \rangle| < 1$).

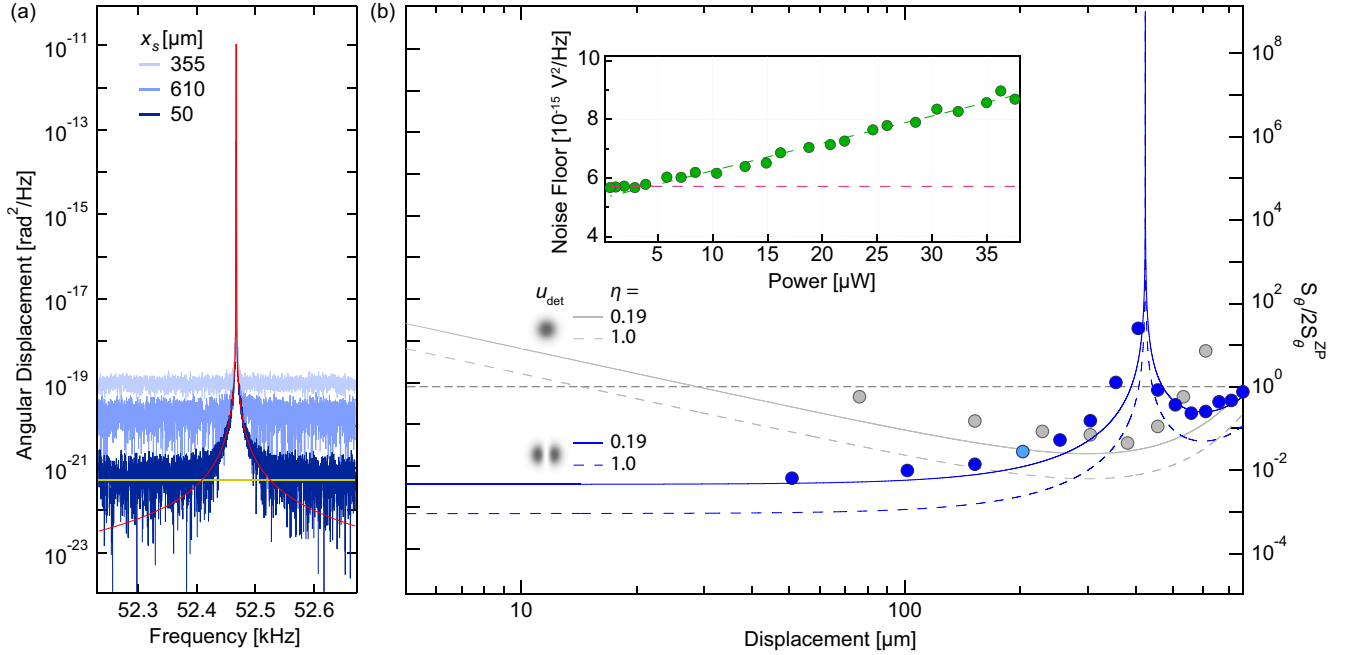


FIG. 3. Quantum-limited readout of a nanomechanical torsion oscillator with a spatial mode sorter. (a) HG_{10} photocurrent spectrum near the torsion mode resonance frequency for three different MPLC positions x_s (blue data), calibrated in angular displacement units by bootstrapping to a thermal noise model (red curve). The inferred angular displacement imprecision (noise floor) for $x_s = 50 \mu\text{m}$ is highlighted with a yellow line. (b) Displacement imprecision vs x_s (round markers) compared to the model in Eq. (5) (lines). The light blue marker corresponds to the dataset in Fig. 2(c). The solid blue line is a fit to data. The dashed blue line assumes a perfect quantum efficiency $\eta = 1$. Gray markers and curves are corresponding measurements and models for the HG_{00} photosignal (see Supplemental Material [26]). Inset: Raw HG_{10} photocurrent noise vs power on the HG_{10} photodetector. The dashed green and red lines are models for shot noise and detector noise, respectively.

Accounting for additional loss due to mismatch of the beam waist and transverse axis orientation [see inset in Fig. 2(a)], we consider the following model for the spatial overlap [26]:

$$|\langle u_{\text{det}} | u_{10} \rangle| \approx e^{-x_s^2/2w^2} \cos \varphi \left(1 - \frac{x_s^2}{2w^2} \left(1 + \frac{\cos(\varphi - 2\varphi_x)}{\cos \varphi} \right) \right) \quad (5)$$

and angular displacement imprecision

$$S_{\theta}^{\text{imp}} = \frac{\theta_D^2}{8N} \frac{1}{\eta_m \eta_d} \frac{1}{|\langle u_{\text{det}} | u_{10} \rangle|^2} \frac{1}{\sin^2 \phi_{00,10}} \equiv \frac{1}{\eta} \frac{\theta_D^2}{8N}, \quad (6)$$

where x_s is the displacement between the MPLC and reflected field optical axes; w is the beam waist on the MPLC; φ and φ_x are the angular misalignment of the MPLC from the torsion and x_s axis, respectively; and η_m , η_d , and η are the loss due to MPLC internal loss and mode mismatch, detector quantum efficiency, and total quantum efficiency, respectively.

Measurements characterizing the alignment of our MPLC are shown in Fig. 2(e), implying access to a quantum efficiency of $\eta \gtrsim 0.5$. Specifically, we record the fraction of the photon flux N_{det}^{ij} coupled into the HG_{00} (u_{det}^{00}) and HG_{10} (u_{det}^{10}) ports and compare to the idealized (waist-matched) model [29],

$$N_{\text{det}}^{00}/N = \eta_m^{00} |\langle u_{\text{det}}^{00} | u_{00} \rangle|^2 \approx \eta_m^{00} e^{-x_s^2/w^2}, \quad (7a)$$

$$N_{\text{det}}^{10}/N = \eta_m^{10} |\langle u_{\text{det}}^{10} | u_{00} \rangle|^2 \approx \eta_m^{10} (x_s^2/w^2) e^{-x_s^2/w^2} \cos^2 \varphi_x, \quad (7b)$$

where η_m^{ij} is the mode match efficiency for channel ij . Results agree qualitatively well with Eq. (7) using $w = 300 \mu\text{m}$, $\varphi_x = 45^\circ$, $\eta_m^{00} = 0.50$, and $\eta_m^{10} = 0.67$ (see Ref. [26] for details).

Armed with these expectations, we carried out a series of experiments to explore the quantum efficiency of our SPADE-based OL, using the torsion mode of the nanoribbon in Fig. 2(b) as a test mass. For these experiments, the incident beam was focused to a spot size of $w_0 \approx 150 \mu\text{m}$ [$\theta_D = \lambda/(\pi w_0) = 3.3 \text{ mrad}$] on the ribbon, the reflected power was fixed at $P = 2.5 \text{ mW}$ ($N = P\lambda/hc \approx 2.0 \times 10^{16}$), and the MPLC was displaced by $x_s \gtrsim 50 \mu\text{m}$ (sufficient to increase photon shot noise above detector noise). The power spectrum of the HG_{10} photocurrent was then recorded over a broad range of frequencies, using a low noise digitizer [30]. A representative measurement with $x \approx 200 \mu\text{m}$ is shown in Fig. 2(c).

We first remark that—as is typical in readout of high- Q nanomechanical resonators—multiple thermal noise peaks appear in the broadband HG_{10} photocurrent spectrum in Fig. 2(c), including the fundamental torsion mode at 52.5 kHz, with modeshape $\phi = (x/w_r) \sin(\pi y/L)$, and fundamental flexure mode at 47 kHz, with $\phi = \sin(\pi y/L)$, where $w_r = 380 \mu\text{m}$ and $L \approx 7 \text{ mm}$ are the ribbon width and length, respectively [27]. The flexure mode should, in principle, not couple to HG_{10} , implying a small MPLC misorientation, $\varphi > 0$. To test this theory, we translated the beam position y_0 along the ribbon axis $x = 0$ while monitoring the area $\langle \theta^2 \rangle$ beneath the torsion and flexure mode peaks, yielding qualitative agreement with the toy model $\langle \theta^2 \rangle(y_0) \propto \beta_{10}^2(y_0) \cos^2 \varphi +$

$\beta_{01}^2(y_0) \sin^2 \varphi \propto (\phi'_x(y_0))^2 \cos^2 \varphi + (\phi'_y(y_0))^2 \sin^2 \varphi$. We note that this model is only valid for sufficiently small spot size, $w_0 \ll w_r, L_r$ (see Ref. [26]), appropriate for our experiment.

Without access to backaction, Eq. (3), estimating the quantum efficiency of a displacement measurement requires direct calibration and comparison to a model. To this end, following a standard approach [31], we calibrate the HG₁₀ photocurrent spectrum in angular displacement units by bootstrapping the wings of the thermal noise peak at $\omega_m = 2\pi \times 52.5$ kHz to a narrowband model $S_\theta^{\text{th}}(\omega) \approx S_\theta^{\text{th}}/(1 + 4Q_m^2(\omega - \omega_m)^2/\omega_m^2)$, where $S_\theta^{\text{th}} = 4k_B T_0 Q_m / I \omega_m$ is the resonant thermal displacement, $I = 2.8 \times 10^{-18}$ kg m² is the effective moment of inertia of the fundamental torsion mode [26,30], and $T_0 = 293$ K is the mode temperature assuming negligible photothermal heating [32]. We also carefully subtract the detector noise (measured by blocking the laser) and confirm that the remainder is shot noise by recording the raw voltage noise S_V versus P , yielding the characteristic scaling $S_V \propto P$ [Fig. 3(b), inset].

Figure 3 summarizes our main result: We recorded calibrated thermal noise spectra versus MPLC displacement x_s , and inferred an angular displacement imprecision as low as 5×10^{-22} rad²/Hz at $x_s = 50$ μ m, corresponding to a quantum efficiency of $\eta \approx 0.14$ relative to the ideal value $\theta_D^2/(8N) = 7 \times 10^{-23}$ rad²/Hz. The solid (dashed) line in Fig. 3(b) is an overlay of the model in Eq. (6) with $\varphi_x = 45^\circ$, $\varphi = 0^\circ$, $w = 300$ μ m, and $\eta = 0.19$ (1.0), and features a prominent peak $x_s \approx \sqrt{2}w$, consistent with $dN_{\text{det}}/d\theta \propto |\langle u_{10}|u_{\text{det}}\rangle| \rightarrow 0$ [26]. Also shown are corresponding measurements and models for detection through the HG₀₀ port of the MPLC [26]. In conjunction with the estimated relative Gouy phase for our optical setup $\phi_{00,10} \approx 0.66$ [26], the inferred quantum efficiency agrees well with the model value $\eta(x_s \rightarrow 0) = \eta_m \eta_d \sin^2(\phi_{00,10}) = 0.25 \eta_d$ given by Eq. (6). A summary is given in Table 1 of the Supplemental Material [26].

The observed high quantum efficiency and absolute sensitivity of our SPADE-based OL are independent of the small mass and high Q of the nanoribbon resonator (as long as the spot size of the beam is smaller than the ribbon [33]). Combining these features gives access to spatiotemporal radiation pressure backaction [Eq. (2)] and new opportunities at the interface of quantum imaging and quantum optomechanics [22].

To highlight this potential, a key figure of merit is the ratio of imprecision to the zero-point displacement spectral density of the oscillator, corresponding to the added measurement noise at the standard quantum limit [34]. For the $x = 50$ μ m measurement in Fig. 3, the imprecision is 23 dB below the zero-point motion of the torsion mode $S_\theta^{\text{ZP}} = \hbar Q_m / (2I \omega_m^2) = 9 \times 10^{-20}$ rad²/√Hz, corresponding to a phonon-equivalent imprecision and a quantum backaction of $n_{\text{imp}} \equiv S_\theta^{\text{imp}} / 2S_\theta^{\text{ZP}} = 0.003$ and $n_{\text{BA}} = (16n_{\text{imp}}\eta)^{-1} \approx 150$, respectively [33,35].

A canonical application of quantum-limited measurement in optomechanics is radiation pressure feedback cooling [35,36]. This might be implemented in our setup, for example, by imprinting the HG₁₀ photocurrent onto the HG₁₀ amplitude of the incident field using a pair of SPADEs, as shown in Fig. 4(c). For ideal derivative feedback, a final occupation of $n_m = 2\sqrt{n_{\text{imp}}(n_{\text{BA}} + n_{\text{th}})} - 0.5 \geq 0.5(1/\sqrt{\eta} - 1)$ [35,36] is achievable, where $n_{\text{th}} = k_B T_0 / \hbar \omega_m$

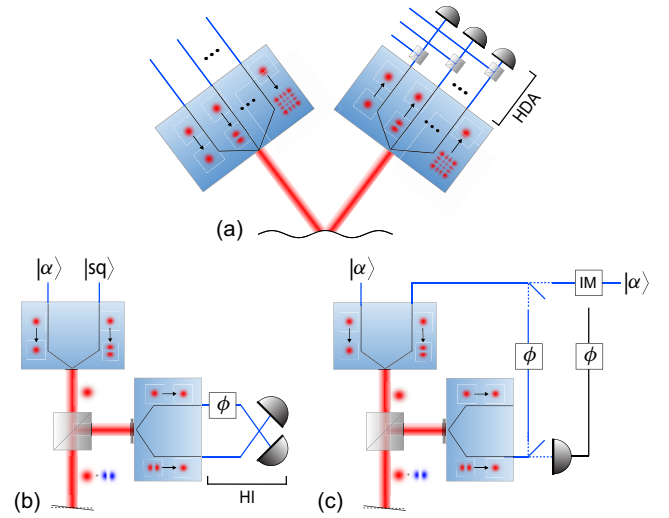


FIG. 4. Imaging-based quantum optomechanics using SPADE. Objects are as labeled in Figs. 1 and 2. (a) Transceiver model for imaging of a mechanical oscillator with a multimode transmitter and receiver produced by SPADE: HDA (homodyne detector array). Examples of quantum-enhanced measurement (b) and feedback control (c) of a torsion oscillator with an HG-mode SPADE: $|\alpha\rangle$ (coherent state), $|sq\rangle$ (squeezed state), ϕ (phase shifter), HI (homodyne interferometer), and IM (intensity modulator). See main text for details.

is the bath phonon number. For our nominal measurement efficiency and strength, $n_m = 1300$ is achievable from room temperature ($n_{\text{th}} = 1.2 \times 10^8$) and $n_m = 0.9$ is achievable in the backaction limit ($n_{\text{BA}} \gg n_{\text{th}}$).

Looking forward, we envision feedback cooling as one element in a broader program to improve the performance of SPADE-based displacement sensing, explore its radiation pressure quantum limits using nanomechanical resonators, and apply these findings to fundamental experiments and practical applications. A transceiver model of this “imaging-based” quantum optomechanics program is sketched in Fig. 4(a), in which a mechanical oscillator is imaged using a multimode transmitter and receiver based on a pair of SPADEs. Using our base case of HG₀₀ illumination of a torsion oscillator as an example, Figs. 4(a) and 4(b) show how quantum-enhanced readout, measurement-based feedback [35,36], or coherent feedback [37,38] could be implemented, by injecting squeezed light, the measurement record, or the scattered field into the HG₁₀ port of the input field, respectively. These examples are readily extended to higher-order single- or multimode squeezed light coupled to multimode mechanical resonators (such as the drum modes of a membrane [22]), providing a playground for testing entanglement-enhanced [39,40] and quantum-inspired [12,41] imaging protocols. They also have a wide application space, from multimode ground-state preparation [17] to label-free mass spectroscopy [15].

The authors thank Aman Agrawal for fabricating the devices, Andrew Land for engineering key elements of the optical setup, and Nico Deshler and Allison Rubenok for

helpful discussions. We also thank Mitul Dey Chowdhury for the photograph in Fig. 2. This work was supported by the National Science Foundation (NSF) through Awards No. 2239735 and No. 2330310. C.M.P. acknowledges support from the ARCS Foundation. W.H. and S.G. acknowledge support from the Office of Naval Research (ONR) through

Contract No. N00014-19-1-2189 and the Air Force Office of Scientific Research (AFOSR) through Contract No. FA9550-22-1-0180.

The data that support the findings of this article are openly available [42].

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- [23] Specifically, $\langle u_1 | u_2 \rangle = \iint u_1^*(x, y) u_2(x, y) dx dy$, where $x - y$ is the plane transverse to the optical axis.
- [24] In the main text and in the Appendix, $S_x[\Omega]$ refers to the signal-sided power spectral density at Fourier frequency Ω , with units $[x]^2/\text{Hz}$ and normalization $\int_0^\infty S_x[\Omega] d\Omega / (2\pi) = \langle x^2 \rangle$.
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- [31] For details about the thermal noise calibration technique and uncertainties, see Supplemental Material of Refs. [27,30]. For optical lever measurements made on similar devices, thermal noise and direct calibration typically differ by $\sim 10\%$.
- [32] We have observed heating rates of $\lesssim 5$ K/mW at $\lambda = 850$ nm for ribbons with identical dimensions [27,30] and $\lesssim 1$ K/mW at 1550 nm for trampolines with tenfold narrower tethers [43].

- This low heating stems from the low absorbance of Si_3N_4 [43] and large width of the ribbon (increasing thermal conductance).
- [33] In practice, optimal sensitivity is achieved by increasing the beam diameter $2w_0$ to match the ribbon width w_r (i.e., minimizing θ_D). In this case, a detailed analysis reveals that the ratio of the imprecision to the peak zero-point displacement spectral density scales as $S_\theta^{\text{imp}}/S_\theta^{\text{ZP}} \propto h_r^3/(w_r Q_0)$, where h_r is the ribbon width and Q_0 is the intrinsic material Q [27,30].
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Correction: An incorrect version of Figure 2 was used for publication and has now been replaced with the correct version.