

AdS gravastar and its signatures from dual conformal field theory

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Quantum gravity effects are expected to resolve the black hole singularity and the effects may deform the region near but outside the horizon. Applying AdS/CFT correspondence, we see their signatures from the viewpoint of dual conformal field theory. As a regularized geometry, we consider AdS gravastar constructed by gluing AdS-Schwarzschild and de Sitter spacetime. The retarded Green's functions of dual conformal field theory have bulk-cone singularities associated with null trajectories in the bulk and we obtain the singularities specific to a horizonless geometry. We also observe echoes coming from waves reflected behind the photon sphere. The existence of echoes implies the modification of geometry inside the photon sphere.

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I. INTRODUCTION

Understanding quantum gravity effects is an important open problem in theoretical physics. Quantum gravity effects are expected to resolve black hole and cosmological singularities, but it is known to be difficult to examine these problems directly within gravity theory. The holographic principle offers a way to study gravity through a dual field theory [1]. The dual field theory typically does not include gravity and is therefore more tractable than the gravity theory itself. The AdS/CFT correspondence provides a concrete realization of the holographic principle [2–4], where a gravity theory on an asymptotically anti-de Sitter (AdS) spacetime is conjectured to be described by a conformal field theory (CFT). Careful studies of the dual CFT are expected to reveal quantum effects of the gravity theory.

Let us suppose a gravity system that we would like to investigate. To apply the AdS/CFT correspondence, we first realize this gravity system in an asymptotically AdS spacetime, which may be regarded as a toy model of the original system. It is therefore important to show that this toy model captures the essential qualitative features of the original system. Once this is established, the properties of the gravity system can be studied through the dual CFT, which may have a concrete definition or even an experimental realization.

It is expected that quantum gravity effects resolve the black hole singularity, and in some proposals like the ones with firewall [5] and fuzzballs [6,7], the region near but outside the horizon is modified. AdS-Schwarzschild black hole has a photon sphere and it is difficult to examine the near-horizon region inside the photon sphere from an outside observer. As a model of regularized geometry, we consider a spacetime

with the photon sphere but without the black hole horizon. This kind of geometry may be provided by exotic compact objects (ECOs), such as boson star, wormhole, etc. (see, e.g., Ref. [8]). Among them, we consider a gravastar solution as an example of asymptotic anti-de Sitter spacetime (see, e.g., [9–12] for asymptotic flat gravastar). AdS gravastar is constructed by replacing the region inside the photon sphere by de Sitter (dS) spacetime. We observe at least two types of signatures of AdS gravastar from the dual CFT, which can be regarded as typical signatures when black hole horizon in the bulk is resolved in some way. This work is part of a series of studies, and a more detailed and extended analysis can be referred to in Refs. [13,14].

Considering a CFT on $\mathbb{R}_t \times S^{d-1}$, where $\mathbb{R}_t$ and S^{d-1} denote the time direction and $(d-1)$ -dimensional sphere, respectively. We examine the retarded Green's function of a scalar operator,

$$G_R(t, \theta) = iH(t)[\mathcal{O}(t, \theta), \mathcal{O}(0, 0)], \quad (1)$$

where $H(t)$ is the Heaviside step function and the metric of S^{d-1} is

$$d\Omega_{d-1}^2 = d\theta^2 + \sin^2 \theta d\Omega_{d-2}^2. \quad (2)$$

We evaluate the retarded Green's function from the propagation of a bulk scalar field, following Ref. [15]. There are light-cone singularities when the two insertion points of operators become lightlike separated in the boundary geometry. Along with the usual singularities, the retarded Green's function develops singularities when the bulk geodesic of the scalar particle becomes null. These are called bulk-cone singularities. See Refs. [13,14,16–25] for AdS-Schwarzschild black hole, where the null geodesic goes around the photon sphere. We find bulk-cone singularities associated with null geodesics traveling into the region inside the photon sphere for AdS gravastar. It is also known that echoes are typical signatures of gravitating objects with photon sphere but without horizon (see, e.g., Refs. [26–30] for asymptotic flat case). We observe echoes arising from the wave function of a bulk scalar field localized inside the photon sphere for AdS gravastar.

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The organization of this note is as follows. In the next section, we review the computation of the retarded Green's function from the wave function of a bulk scalar field. We employ the WKB approximation to solve the bulk wave equation. In Sec. III, we transform the expression obtained in the momentum basis in Sec. II into the coordinate basis. In this representation, bulk-cone singularities can be clearly identified. For the AdS-Schwarzschild black hole, our result agrees with that obtained in Ref. [24] up to an overall normalization factor. However, their analysis crucially relies on the presence of a black hole horizon and therefore cannot be applied to horizonless geometries. In contrast, our method applies to more general geometries, including AdS gravastars, and this technical development constitutes one of the main results of this work. In Sec. IV, we apply our method to the AdS gravastar. In addition to bulk-cone singularities specific to horizonless objects, we observe echoes associated with wave functions localized inside the photon sphere. In Sec. V, we compare our analytic results with numerical computations and confirm their consistency. Section VI is devoted to the conclusion and discussion. The Appendix collects details of the WKB analysis of the wave equations.

In Ref. [13], we constructed a four-dimensional AdS gravastar and numerically computed the retarded Green's function for this geometry. In the current work, we develop a semianalytic technique to evaluate the retarded Green's function and apply it to AdS gravastars in general dimensions. We also present numerical results for the retarded Green's function. Further details of the analysis, as well as its extension to the AdS wormhole case, can be found in Ref. [14].

II. RETARDED GREEN'S FUNCTIONS

For $(d+1)$ -dimensional bulk geometry, we use the metric of the form

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{d-1}^2, \quad (3)$$

where the metric of S^{d-1} is given in Eq. (2). The metric function is

$$f(r) = r^2 + 1 - \frac{\mu}{r^{d-2}} \quad (4)$$

for AdS-Schwarzschild black hole. We consider the mode expansion of the bulk scalar field,

$$\phi(t, \Omega_{d-1}, r) = e^{-i\omega t} Y_{\ell m}(\Omega_{d-1}) r^{-\frac{d-1}{2}} \psi_{\omega\ell}(r), \quad (5)$$

where the spherical harmonics on S^{d-1} are denoted by $Y_{\ell m}(\Omega_{d-1})$. The wave equation for the radial part can be put into the canonical form

$$(\partial_z^2 + \kappa^2(z))\psi(z) = 0, \quad \kappa(z) = \sqrt{\omega^2 - V(z)}. \quad (6)$$

The radial coordinate z is introduced via

$$dz = -\frac{dr}{f(r)}. \quad (7)$$

For large ℓ , the potential can be approximated by

$$V(z) = \left[\ell^2 + \left(\nu^2 - \frac{1}{4} \right) r^2 \right] \frac{f(r)}{r^2}. \quad (8)$$

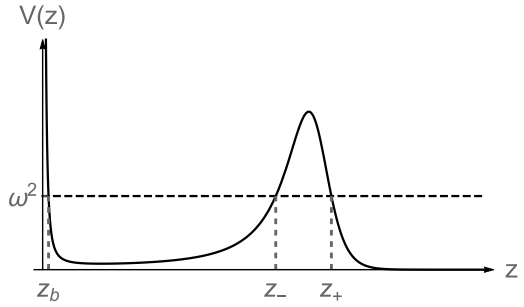


FIG. 1. The potential for AdS-Schwarzschild black hole.

Here, $\nu = \sqrt{(d/2)^2 + m^2}$ with m as the mass of the bulk scalar field and $\Delta = d/2 + \nu$ is the conformal dimension of the dual scalar operator. Note that we include the term proportional to r^2 in the potential (8), which is usually neglected (see, e.g., Ref. [24]). The extra term will play an important role in later analysis around Eq. (23). In terms of z , the AdS boundary is located at $z \rightarrow 0$, where the scalar field behaves as

$$\psi(z) \sim \mathcal{A}(\omega, \ell) z^{\frac{1}{2}-\nu} + \mathcal{B}(\omega, \ell) z^{\frac{1}{2}+\nu}. \quad (9)$$

From the asymptotic behavior, the retarded Green's function can be obtained as [15]

$$G_R(\omega, \ell) = \frac{\mathcal{B}(\omega, \ell)}{\mathcal{A}(\omega, \ell)}. \quad (10)$$

We first examine the wave equation (6) for AdS-Schwarzschild black hole. The potential is plotted in Fig. 1. The black hole horizon is located at $r = r_h$ satisfying $f(r_h) = 0$ or at $z \rightarrow \infty$. We assign the ingoing boundary condition at the horizon as $\psi(z) \sim e^{i\omega z}$. For large ℓ , the maximal of potential is realized by

$$\ell^{-2} V(r_c) \simeq 1 + \left(1 - \frac{2}{d} \right) \left(\frac{2}{d\mu} \right)^{\frac{2}{d-2}}, \quad (11)$$

where r_c is the position of photon sphere,

$$r_c = \left(\frac{d\mu}{2} \right)^{\frac{1}{d-2}}. \quad (12)$$

We solve the wave equation (6) in the WKB approximation. We consider the case where the equation $\kappa(z)^2 = 0$ has three zeros (see Fig. 1). Here, $\kappa(z)$ was introduced in Ref. (6) and the three zeros are denoted as $z_b, z_{\pm}$ ($z_b < z_- < z_+$). The potential diverges near the boundary $r \rightarrow \infty$ as $V(r) \propto r^2$, which leads to $\kappa(z)^2 = 0$ at $z = z_b$ with $z_b < 1/\ell$. We start with the wave function for $z_+ < z < \infty$ satisfying the ingoing boundary condition at $z \rightarrow \infty$. Next, we continue the wave function to the region with $0 \simeq z_b < z < z_-$. We can then read off the retarded Green's function as in Eq. (10) from the asymptotic behavior of the wave function (9). The explicit form of the retarded Green's function is given in Eq. (A6) and it is convenient to expand it in the form of

$$G_R(\omega, \ell) = \frac{\Gamma(-\nu)}{\Gamma(\nu)} \left(\frac{\omega^2 - \ell^2}{4} \right)^\nu \sum_{n=0}^{\infty} a_n e^{2inS(0, z_-)}. \quad (13)$$

Here, we have introduced the WKB phase,

$$S(0, z_-) = \int_0^{z_-} \kappa(z') dz', \quad (14)$$

and assumed that $\text{Im } S(0, z_-) > 0$. The coefficients a_n are

$$a_0 = e^{-i\pi\nu}, \quad a_n = -2i(-1)^n e^{-in\pi\nu} \sin(\pi\nu), \quad (15)$$

with $n = 1, 2, 3, \dots$. As shown below, each term with $n \geq 2$ corresponds to null geodesics that bounce off the AdS boundary $n - 1$ times. For the moment, we neglect the terms suppressed by the factor $e^{-2S(z_-, z_+)}$, where we set

$$S(z_-, z_+) = \int_{z_-}^{z_+} q(z') dz', \quad (16)$$

with

$$q(z) = \sqrt{V(z) - \omega^2}. \quad (17)$$

III. BULK-CONE SINGULARITIES

In order to observe the bulk-cone singularities, we perform the following Fourier transformation:

$$G_R(t, \theta) = \frac{1}{2\pi} \sum_{\ell=0}^{\infty} \int_{-\infty+i\delta}^{\infty+i\delta} d\omega e^{-i\omega t} \times G_R(\omega, \ell) \frac{\ell + \alpha}{\alpha} C_{\ell}^{(\alpha)}(\cos \theta), \quad (18)$$

where $C_{\ell}^{(\alpha)}(\cos \theta)$ are the Gegenbauer polynomials and $\alpha = (d - 2)/2$. Following the analysis in Ref. [24], the summand over ℓ is realized by an integration over a variable denoted as k and the series expansion of Gegenbauer polynomials is applied. For $0 < \theta < \pi$, we can rewrite it as

$$G_R(t, \theta) = \sum_{j=0}^{\infty} [g_R(t, |\theta| + 2\pi j) + (-1)^{2\alpha} g_R(t, 2\pi - |\theta| + 2\pi j)], \quad (19)$$

with

$$g_R(t, \theta) = \sum_{n=0}^{\infty} g_R^{(n)}(t, \theta),$$

$$g_R^{(n)}(t, \theta) = \tilde{a}_n \int_{\infty+i\delta}^{-\infty+i\delta} d\omega \int_{-\infty+i\delta'}^{\infty+i\delta'} dk \times e^{-i\omega t + ik\theta} k^{\alpha} (\omega^2 - k^2)^{\nu} e^{2inS(0, z_-)}. \quad (20)$$

Here, we have defined

$$\tilde{a}_n = \frac{\Gamma(-\nu)}{\Gamma(\nu)} \frac{e^{-i\frac{\pi}{2}\alpha + i\pi\alpha \lfloor \frac{\theta}{\pi} \rfloor}}{2^{\alpha+1+2\nu} \pi |\sin \theta|^{\alpha} \Gamma(1 + \alpha)} a_n. \quad (21)$$

The term with $n = 0$ reproduces the light-cone singularity (see Ref. [24]). We thus focus on the terms with $n > 0$. We remark here that the integration over k is performed over just above the real axis. In Ref. [24], the authors reduced the integral over k as a sum over residues. However, this can be justified only when the contribution from large imaginary k region can be neglected. This is the case for AdS-Schwarzschild black hole

and is related to the existence of horizon. For generic geometries without horizon, the contribution from large imaginary k region cannot be neglected.

We apply the saddle point approximation for the integrals in Eq. (20), where the exponent is

$$\Phi(\omega, k) = -\omega t + k\theta + 2nS(0, z_-). \quad (22)$$

The saddle points of $\Phi(\omega, k)$ can be obtained by $(\omega, k) = (\omega_*, k_*)$, satisfying

$$\left. \frac{\partial \Phi(\omega, k)}{\partial \omega} \right|_{(\omega, k)=(\omega_*, k_*)} = \left. \frac{\partial \Phi(\omega, k)}{\partial k} \right|_{(\omega, k)=(\omega_*, k_*)} = 0. \quad (23)$$

If we neglect the term proportional to r^2 in Eq. (8), then the saddle points are degenerated and pinched singularities have to be treated as in Ref. [24]. The derivatives of the function $S(0, z_-)$ with respect to ω, k can be approximated as (see Refs. [14,20])

$$2 \frac{\partial S(0, z_-)}{\partial \omega} \simeq T(u_*) - \frac{2\bar{q}u_*}{(u_*^2 - 1)k_*},$$

$$2 \frac{\partial S(0, z_-)}{\partial k} \simeq -\Theta(u_*) + \frac{2\bar{q}}{(u_*^2 - 1)k_*}, \quad (24)$$

where we set $u_* = \omega_*/k_*$ and $\bar{q} = \sqrt{v^2 - 1/4}$. Moreover, we have introduced

$$T(u) = 2u \int_{r_*}^{\infty} \frac{dr}{f(r)} \frac{1}{\sqrt{u^2 - f(r)r^{-2}}},$$

$$\Theta(u) = 2 \int_{r_*}^{\infty} \frac{dr}{r^2} \frac{1}{\sqrt{u^2 - f(r)r^{-2}}}, \quad (25)$$

which are the arrival time and angle of null geodesic, respectively. Equation (24) can be derived in the following way. We first use the fact that $2 \frac{\partial S(0, z_-)}{\partial \omega}$ and $2 \frac{\partial S(0, z_-)}{\partial k}$ reduce to $T(u)$ and $-\Theta(u)$ in Eq. (25) for small r . We then approximate the region with large r by pure AdS, which leads to the extra terms. For the argument, we assumed that $u_*^2 - 1 = \mathcal{O}((k_*)^0)$. This is expected to hold for generic geometries, but an important exception is the pure AdS case with $u_*^2 - 1 = \mathcal{O}((k_*)^{-1})$ (see, e.g., Ref. [16]).

Applying the saddle point approximation, the term with $n > 0$ in Eq. (20) can be written as

$$g_R^{(n)}(t, \theta) \simeq \tilde{a}_n k_*^{\alpha} ((\omega_*)^2 - (k_*)^2)^{\nu} \frac{2\pi i e^{i\Phi(\omega_*, k_*)}}{\sqrt{\det H}}, \quad (26)$$

with

$$H = \left(\begin{array}{cc} \frac{\partial^2 \Phi(\omega, k)}{\partial \omega \partial \omega} & \frac{\partial^2 \Phi(\omega, k)}{\partial \omega \partial k} \\ \frac{\partial^2 \Phi(\omega, k)}{\partial k \partial \omega} & \frac{\partial^2 \Phi(\omega, k)}{\partial k \partial k} \end{array} \right) \bigg|_{(\omega, k)=(\omega_*, k_*)}. \quad (27)$$

We can show that

$$\Phi(\omega_*, k_*) = 0 \quad (28)$$

up to the order $\mathcal{O}(k_*^{-1})$ corrections. Using Eq. (24) with Eq. (25), we obtain the following:

$$g_R^{(n)}(t, \theta) \simeq \tilde{a}_n \frac{(2\bar{Q}nu_*)^{\alpha+2\nu+3/2}}{(u_*^2 - 1)^{\alpha+\nu+3/2}} \frac{2\pi i}{\sqrt{2\bar{Q}n^2 \frac{d}{du_*} T(u_*)}} \times \frac{1}{(nT(u_*) - t)^{\alpha+2\nu+3/2}}. \quad (29)$$

The bulk-cone singularities are located at $t = nT(u_*)$, where u_* is obtained by solving $\theta = n\Theta(u_*)$. The corresponding null geodesic bounces off the AdS boundary $n - 1$ times. More detailed derivation will be presented in Ref. [14]. For $n = 1$, the expression coincides with the one in Ref. [24] up to an overall factor. However, we should emphasize that our arguments hold for generic geometries. In the case of pure AdS, the approximation in Eq. (24) cannot be applied and the light-cone and bulk-cone singularities coincide with each other.

IV. ADS GRAVASTAR AND ECHOES

We would like to apply the above analysis to the case of AdS gravastar. See Ref. [13] for detailed construction of four-dimensional gravastar. We first consider the AdS-Schwarzschild black hole with the horizon at $r = r_h$ and the photon sphere at $r = r_c$ in Eq. (12). We put a shell at $r = r_0$, with

$$r_h < r_0 < r_c, \quad (30)$$

and replace the core region $r < r_0$ by dS spacetime. We set the surface stress-energy tensor on the shell as

$$S = \sigma d\tau^2 + \bar{P} r_0^2 d\Omega_{d-1}^2, \quad (31)$$

where σ and $\bar{P}$ are the surface energy density and the surface tension, respectively. For simplicity, we set $\sigma = 0$. We require that the induced metric is continuous. We further assign the junction condition

$$[[\chi]] - [[\text{tr } \chi]]h = -8\pi S. \quad (32)$$

Here, χ is the extrinsic curvature on the timelike hypersurface and the bracket $[[A]] = A_+ - A_-$ denotes the jump of a quantity A across the shell. The metric of AdS gravastar can be put into the form (3), where the metric function $f(r)$ is

$$f(r) = \begin{cases} 1 - \frac{\mu}{r} - \frac{2\Lambda}{d(d-1)}r^2 & (r \geq r_0), \\ 1 - \left(\frac{8\pi\rho}{d} + \frac{2\Lambda}{d(d-1)}\right)r^2 & (r < r_0). \end{cases} \quad (33)$$

Here, the cosmological constant is given by $\Lambda = -d(d-1)/(2R_{\text{AdS}}^2) < 0$, with $R_{\text{AdS}} = 1$, and the vacuum energy of the dS region is denoted by ρ that is constant. Requiring that the metric function is continuous at the position of the shell, we have

$$r_0 = \left(\frac{d\mu}{8\pi\rho}\right)^{\frac{1}{d}}. \quad (34)$$

We solve the wave equation (6) in the WKB approximation also for the AdS gravastar constructed above. We begin with the case where the equation $\kappa(z)^2 = 0$ has two zeros at z_b, z_- ($0 \simeq z_b < z_-$). The form of potential is shown in Fig. 2.

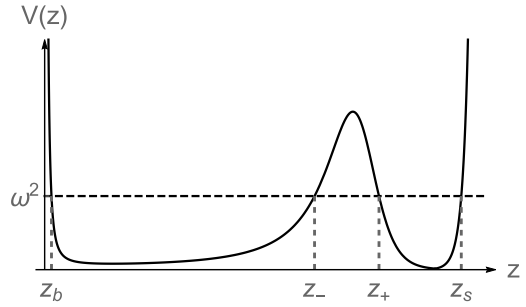


FIG. 2. The potential for AdS gravastar.

We first obtain the wave function for $z > z_-$ satisfying the regularity condition at $r \rightarrow 0$. We then continue the wave function to the region with $0 \simeq z_b < z < z_-$. From the asymptotic behavior of the wave function, we can obtain the retarded Green's function. The expression is the same as in Eq. (29), but now $S(0, z_-)$ is computed with the metric function for AdS gravastar given in Eq. (33). The bulk-cone singularities correspond to the null geodesics traveling into the gravastar region.

We then move to the case with four zeros in $\kappa(z)^2 = 0$ (see Fig. 2). We set these zeros to be located at $z = z_b, z_{\pm}, z_s$ ($0 \simeq z_b < z_- < z_+ < z_s$). We find the wave function for $z > z_s$ satisfying the regularity condition and then continue it to the region with $0 \simeq z_b < z < z_-$. From the asymptotic behavior of the wave function, we can read off the retarded Green's function as in Eq. (10). Its explicit form is given in Eq. (A13) with Eqs. (A14) and (A15), and it is convenient to expand it in the form of

$$G_R(\omega, \ell) = \frac{\Gamma(-\nu)}{\Gamma(\nu)} \left(\frac{\omega^2 - \ell^2}{4}\right)^\nu \left(\sum_{n=0}^{\infty} a_n e^{2inS(0, z_-)} + \sum_{n,p=0}^{\infty} b_{n,p} e^{2inS(0, z_-) + 2ipS(z_+, z_s)} e^{-2S(z_-, z_+)} \right) \quad (35)$$

up to the terms of order $e^{-4S(z_-, z_+)}$. Here, we set

$$S(z_+, z_s) = \int_{z_+}^{z_s} \kappa(z') dz'. \quad (36)$$

The coefficients a_n are computed as in Eq. (15) and $b_{n,p}$ are

$$b_{n,0} = i(-1)^n e^{-in\pi\nu} \sin(\pi\nu), \quad (37)$$

$$b_{n,p} = 2i(-1)^{n+p} e^{-in\pi\nu} \sin(\pi\nu),$$

with $p = 1, 2, 3, \dots$. The contribution from the terms with a_n is the same as that for AdS-Schwarzschild black hole since the region $z < z_-$ is not changed. Thus, these terms lead to bulk-cone singularities associated with null geodesics going around the photon sphere.

The terms with $b_{n,p}$ are suppressed by $e^{-2S(z_-, z_+)}$ and are regarded as tunneling effects. We perform the Fourier transformation of the expression in Eq. (35) as

$$g_R(t, \theta) = \sum_{n=0}^{\infty} g_R^{(n)}(t, \theta) + \sum_{n,p=0}^{\infty} g_R^{(n,p)}(t, \theta). \quad (38)$$

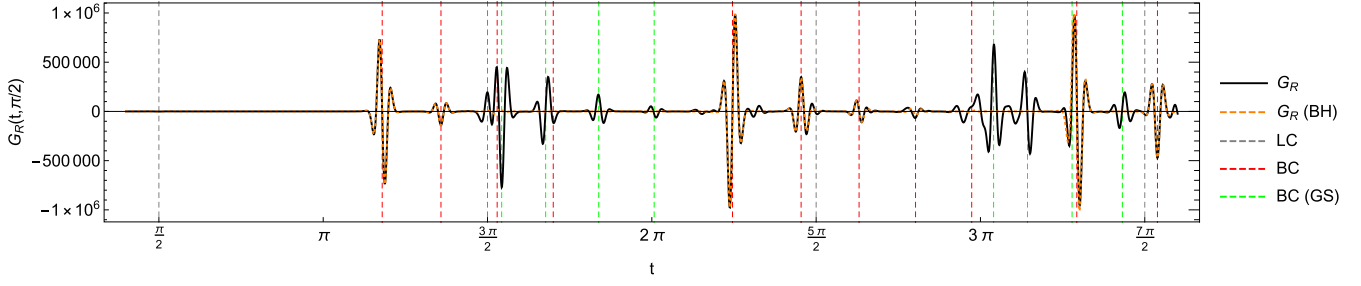


FIG. 3. The Green's function $G_R(t, \pi/2)$ for the gravastar for $d = 3$, $\mu = 1/15$, and $r_0 = 1.001r_h$. We adopt the cutoff $\omega_c = \ell_c = 35$ and the upper bound $\omega_{\max} = \ell_{\max} = 150$. The vertical dashed lines indicate the light-cone (LC) and bulk-cone (BC) singularities.

Here, $g_R^{(n)}(t, \theta)$ are obtained in Eq. (20) and $g_R^{(n,p)}(t, \theta)$ are

$$g_R^{(n,p)}(t, \theta) = \tilde{b}_{n,p} \int_{-\infty+i\delta}^{\infty+i\delta} d\omega \int_{-\infty+i\delta'}^{\infty+i\delta'} dk e^{-2S(z_-, z_+)} \times k^\alpha (\omega^2 - k^2)^\nu e^{-i\omega t} e^{2inS(0, z_-) + 2ipS(z_+, z_s)}, \quad (39)$$

with

$$\tilde{b}_{n,p} = \frac{\Gamma(-\nu)}{\Gamma(\nu)} \frac{e^{-i\frac{\pi}{2}\alpha + i\pi\alpha \lfloor \frac{\theta}{\pi} \rfloor}}{2^{\alpha+1+2\nu} \pi |\sin \theta|^\alpha \Gamma(1+\alpha)} b_{n,p}. \quad (40)$$

We evaluate the integrals of Eq. (39) in the saddle point approximation as above. We set $|S(z_-, z_+)|$ to be small, where the suppression is not so large. We look for the saddle points satisfying Eq. (23) but now with

$$\Phi(\omega, k) = -\omega t + k\theta + 2nS(0, z_-) + 2pS(z_+, z_s). \quad (41)$$

As above, we obtained

$$g_R^{(n,p)}(t, \theta) \simeq \tilde{b}_{n,p} \frac{(2n\tilde{Q}u_*)^{\alpha+2\nu+3/2}}{(u_*^2 - 1)^{\alpha+\nu+3/2}} \times \frac{2\pi i}{\sqrt{2\tilde{Q}n(n\frac{d}{du_*}T(u_*) + p\frac{d}{du_*}\tilde{T}(u_*))}} \times \frac{1}{(nT(u_*) + p\tilde{T}(u_*) - t + i\epsilon)^{\alpha+2\nu+3/2}}, \quad (42)$$

where we have defined

$$\tilde{T}(u) = 2u \int_{r(z_s)}^{r(r_+)} \frac{dr}{f(r)} \frac{1}{\sqrt{u^2 - f(r)r^{-2}}}, \quad (43)$$

$$\tilde{\Theta}(u) = 2 \int_{r(z_s)}^{r(z_+)} \frac{dr}{r^2} \frac{1}{\sqrt{u^2 - f(r)r^{-2}}}.$$

These functions correspond, respectively, to the shift of time and angle for one period of null geodesic between z_+ and z_s . Here, u_* is obtained by solving $\theta = n\Theta(u) + p\tilde{\Theta}(u)$. We may treat $S(z_-, z_+)$ in Eq. (39) in a similar way as done for $S(0, z_-)$ and $S(z_+, z_s)$, then the position of singularity is shifted as in Eq. (42) with real ϵ . The singularity is thus replaced by a bump whose center is at $t = nT(u_*) + p\tilde{T}(u_*)$. The bumps with $p > 0$ are regarded as echoes following bulk-cone singularities associated with null geodesics going around the photon sphere.

V. NUMERICAL ANALYSIS

As in Eq. (10), the retarded Green's function can be obtained from the bulk wave function satisfying Eq. (6). We have solved the equation analytically within the WKB approximation. Here, we would like to confirm the results numerically. To this end, we include a smearing factor $\exp(-\frac{\text{Re}(\omega)^2}{\omega_c^2} - \frac{\ell^2}{\ell_c^2})$ when performing the Fourier transformation (18). The Gaussian factor reduces the singularities to finite bumps in the coordinate basis. The smearing corresponds to putting some source of wave packets at $(t, \theta) = (0, 0)$ on the boundary [21,22].

A result for AdS gravastar in $d = 3$ is shown in Fig. 3 (see Refs. [13,14] for other cases). The black solid and orange dashed curves show $G_R(t, \theta)$ for AdS gravastar and AdS-Schwarzschild black hole with the same μ , respectively. Some of the bumps are common to the two cases. They are the wave packets reflected by the photon sphere potential at $r \sim r_c$. The arrival time expected from the geodesic analysis (red dashed lines) agree with the positions of the bumps. We can also see the additional bumps specific to the AdS gravastar around $3\pi/2 \lesssim t \lesssim 2\pi$ and $3\pi \lesssim t \lesssim 13\pi/4$. They are the waves passing through the gravastar interior. The geodesic analysis for the arrival time (green dashed lines) is indeed consistent with them. As expected from the power of the singularities in the semianalytic formula (29), these two kinds of bumps have the same order of magnitude in their amplitude.

Along with the bulk-cone singularities, we observe gravitational echoes in the retarded Green's function. The Planck constant for the wave equation (6) is given by the angular momentum ℓ in this case. In order to clearly see the echoes, we examine the retarded Green's function with small fixed ℓ . A result is shown in Fig. 4. See also Ref. [14] for more examples. The black solid and orange dashed curves show $G_R(t, \ell)$ of AdS gravastar and AdS-Schwarzschild black hole with the same μ , respectively. The common strong bumps are the waves reflected by the photon sphere with $n = 1, 2$. The subsequent weak signals are the echoes.

In the astrophysical context, gravitational echoes are also known as a typical signature of ECOs. The typical time interval for the echoes can be estimated as the propagation time of radial light between the ECO surface and the photon sphere

$$\Delta t_{\text{echo}} \sim 2 \int_{r_0}^{r_c} dr f(r)^{-1} \quad (44)$$

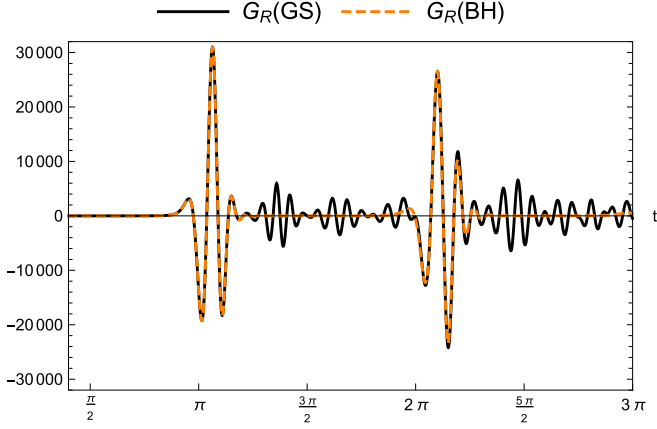


FIG. 4. Echoes of $G_R(t, \ell)$ for the gravastar with $d = 3$, $\mu = 1/50$, and $r_0 = 1.000001r_h$. We take $\ell = 1$ and $\omega_c = 15$.

(see Ref. [27]). The interval for the bumps of different n is given by

$$\Delta t_{\text{bdry}} \sim 2 \int_{r_c}^{\infty} dr f(r)^{-1}. \quad (45)$$

The time width of one wave packet is $\delta t \sim 1/\omega_c$. In the current case, these timescales, $\delta t \simeq 0.067 \ll \Delta t_{\text{echo}} \simeq 0.544 \ll \Delta t_{\text{bdry}} \simeq 3.243$, are well separated and the echo signals are clearly observed. The gravitational echoes are observed with a lower cutoff ω_c . So, they would be the important signature of AdS ECOs when we only have a restricted time resolution for the dual CFTs.

VI. CONCLUSION AND DISCUSSION

We examined the retarded Green's function of a scalar operator from the asymptotically AdS geometry formed by an ECO, i.e., AdS gravastar. We observed that there are two types of signatures for the horizonless object, which are bulk-cone singularities and echoes. We read off the CFT correlation function from the bulk wave function obtained in the WKB approximation and numerical computations. The analysis in note works for more generic geometry including AdS wormhole as analyzed in Ref. [14]. It is interesting to see what happens if we include charges and rotations to the geometry.

It is desirable to have a direct interpretation from the viewpoint of the dual CFT. In particular, we can show that the bulk-cone singularities are of the form (29), where the power of singularity is generically given by $\alpha + 2\nu + 3/2 = 2\Delta - (d - 1)/2$ independent of the detailed structure of bulk geometry. We would like to understand this universal behavior purely in terms of dual CFT, e.g., by considering the operator product expansion in terms of single and multiple stress tensors. It is also important to find the concrete setups of AdS/CFT correspondence and superstring theory. Furthermore, we would like to examine the gravitational and stringy corrections, which may resolve the bulk-cone singularities [20,24]. It is expected that the ECOs are unstable due to the localized modes near the minimum of potential and decay into a black hole (see, e.g., Ref. [31]). The dual CFT description would be useful to understand this issue in particular.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: WKB ANALYSIS

1. AdS-Schwarzschild black hole

We would like to solve the wave equation (6) in the WKB approximation for AdS-Schwarzschild black hole. We start from the wave function for $z_+ < z < \infty$. The black hole horizon is located at $z \rightarrow \infty$ and the ingoing boundary condition is required at the horizon, i.e.,

$$\psi(z) \sim e^{i\omega z} \quad (z \rightarrow \infty). \quad (A1)$$

The wave function satisfying the condition is given by

$$\psi(z) \sim \frac{1}{\sqrt{\kappa(z)}} e^{i \int_{z_+}^z dz' \kappa(z')}. \quad (A2)$$

Applying the WKB connection formula, the solution for $0 \simeq z_b < z < z_-$ can be found as

$$\psi(z) \sim \frac{C_+}{\sqrt{\kappa(z)}} e^{i \int_0^z dz' \kappa(z')} + \frac{C_-}{\sqrt{\kappa(z)}} e^{-i \int_0^z dz' \kappa(z')}, \quad (A3)$$

with

$$\begin{aligned} C_+ &= (e^{S(z_-, z_+)} + \frac{1}{4} e^{-S(z_-, z_+)}) e^{-iS(0, z_-)}, \\ C_- &= (e^{S(z_-, z_+)} - \frac{1}{4} e^{-S(z_-, z_+)}) e^{-\frac{1}{2}i\pi} e^{iS(0, z_-)}. \end{aligned} \quad (A4)$$

For $z < 1/\ell$, the asymptotic geometry is given by the AdS spacetime, and the solution is written in terms of Hankel functions as

$$\begin{aligned} \psi(z) &= C_+ \sqrt{\frac{\pi \ell z}{2}} e^{\frac{i\pi\nu}{2} + \frac{i\pi}{4}} H_\nu^{(1)}(\sqrt{\omega^2 - \ell^2} z) \\ &\quad + C_- \sqrt{\frac{\pi \ell z}{2}} e^{-\frac{i\pi\nu}{2} - \frac{i\pi}{4}} H_\nu^{(2)}(\sqrt{\omega^2 - \ell^2} z). \end{aligned} \quad (A5)$$

Applying the asymptotic behaviors of Hankel functions, we can read off the retarded Green's function as

$$\begin{aligned} G_R(\omega, \ell) &= \frac{\Gamma(-\nu)}{\Gamma(\nu)} \\ &\quad \times \left(\frac{\omega^2 - \ell^2}{4} \right)^\nu \frac{C_+ e^{-\frac{i\pi\nu}{2} + \frac{i\pi}{4}} - C_- e^{\frac{i\pi\nu}{2} - \frac{i\pi}{4}}}{C_+ e^{\frac{i\pi\nu}{2} + \frac{i\pi}{4}} - C_- e^{-\frac{i\pi\nu}{2} - \frac{i\pi}{4}}}. \end{aligned} \quad (A6)$$

Expanding in $e^{2iS(0, z_-)}$ and $e^{-2S(z_-, z_+)}$, we obtain Eq. (13) with Eq. (15).

2. AdS gravastar

We examine the wave equation (6) for the AdS gravastar. Here, we consider the case with four zeros of $\kappa^2(z) = 0$. We set these zeros to be located at $z = z_b, z_{\pm}, z_s$ ($0 \simeq z_b < z_- < z_+ < z_s$). The wave function for $z > z_s$ is

$$\psi_{\omega\ell}(z) \sim \frac{1}{\sqrt{q(z)}} e^{-\int_{z_s}^z dz' q(z')}. \quad (\text{A7})$$

We connect it to the region $z_+ < z < z_s$ as

$$\begin{aligned} \psi(z) \sim & \frac{e^{\frac{i\pi}{4} - iS(z_+, z_s)}}{\sqrt{\kappa(z)}} e^{i \int_{z_+}^z dz' \kappa(z')} \\ & + \frac{e^{-\frac{i\pi}{4} + iS(z_+, z_s)}}{\sqrt{\kappa(z)}} e^{-i \int_{z_+}^z dz' \kappa(z')}. \end{aligned} \quad (\text{A8})$$

We further continue the wave function to the region $0 \simeq z_b < z_-$ as

$$\psi(z) \sim \frac{C_+}{\sqrt{\kappa(z)}} e^{i \int_0^z dz' \kappa(z')} + \frac{C_-}{\sqrt{\kappa(z)}} e^{-i \int_0^z dz' \kappa(z')}, \quad (\text{A9})$$

with

$$C_+ = e^{\frac{i\pi}{4} - iS(z_+, z_s)} C_+^{(1)} + e^{-\frac{i\pi}{4} + iS(z_+, z_s)} C_-^{(2)}, \quad (\text{A10})$$

$$C_- = e^{\frac{i\pi}{4} - iS(z_+, z_s)} C_-^{(1)} + e^{-\frac{i\pi}{4} + iS(z_+, z_s)} C_+^{(2)}. \quad (\text{A11})$$

Here, we set

$$\begin{aligned} C_+^{(1)} &= (e^{S(z_-, z_+)} + \frac{1}{4} e^{-S(z_-, z_+)}) e^{-iS(0, z_-)}, \\ C_-^{(1)} &= (e^{S(z_-, z_+)} - \frac{1}{4} e^{-S(z_-, z_+)}) e^{-\frac{1}{2} i\pi} e^{iS(0, z_-)}, \\ C_+^{(2)} &= (e^{S(z_-, z_+)} + \frac{1}{4} e^{-S(z_-, z_+)}) e^{iS(0, z_-)}, \\ C_-^{(2)} &= (e^{S(z_-, z_+)} - \frac{1}{4} e^{-S(z_-, z_+)}) e^{\frac{1}{2} i\pi} e^{-iS(0, z_-)}. \end{aligned} \quad (\text{A12})$$

The retarded Green's function is given by Eq. (A6) for the wave function with the asymptotic behavior [Eq. (A3)]. We thus find

$$G_R(\omega, \ell) = \frac{\Gamma(-\nu)}{\Gamma(\nu)} \left(\frac{\omega^2 - \ell^2}{4} \right)^\nu \frac{P}{Q}, \quad (\text{A13})$$

with

$$\begin{aligned} P &= (1 + e^{2iS(z_+, z_s)}) \cos \left(S(0, z_-) + \frac{\pi\nu}{2} \right) \\ &\quad - \frac{i}{4} e^{-2S(z_-, z_+)} (1 - e^{2iS(z_+, z_s)}) \sin \left(S(0, z_-) + \frac{\pi\nu}{2} \right) \end{aligned} \quad (\text{A14})$$

and

$$\begin{aligned} Q &= (1 + e^{2iS(z_+, z_s)}) \cos \left(S(0, z_-) - \frac{\pi\nu}{2} \right) \\ &\quad - \frac{i}{4} e^{-2S(z_-, z_+)} (1 - e^{2iS(z_+, z_s)}) \sin \left(S(0, z_-) - \frac{\pi\nu}{2} \right). \end{aligned} \quad (\text{A15})$$

Expanding in $e^{2iS(0, z_-)}$, $e^{2iS(z_+, z_s)}$, and $e^{-2S(z_-, z_+)}$, we obtain Eq. (35) with Eqs. (15) and (37).

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