

Dephasingless laser wakefield acceleration in a plasma waveguide

J. P. Palastro^{1,*}, K. G. Miller^{1,†}, C. D. Arrowsmith¹, R. Almeida², M. R. Edwards³, A. L. Elliott¹, A. Kiewel¹, A. Konzel¹, L. S. Mack¹, D. Ramsey¹, D. Singh³, A. G. R. Thomas⁴, and J. Vieira²

¹University of Rochester, *Laboratory for Laser Energetics*, Rochester, New York 14623-1299, USA

²GoLP/Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisbon, Portugal

³Department of Mechanical Engineering, *Stanford University*, Stanford, California 94305, USA

⁴Gérard Mourou Center for Ultrafast Optical Science, *University of Michigan*, Ann Arbor, Michigan 48109, USA



(Received 18 June 2026; accepted 26 August 2026; published 25 September 2026)

Laser wakefield accelerators (LWFAs) provide extremely large accelerating gradients for compact electron accelerators and photon sources but are limited by dephasing, where trapped electrons outrun the accelerating phase of the wakefield. Flying-focus pulses can eliminate dephasing by driving a wake at the vacuum speed of light, but these pulses involve trade-offs, such as varying spot size, long duration, or large plasma volume. Here, we show that a spatiotemporally structured laser pulse propagating in a plasma waveguide can drive a wakefield at the vacuum speed of light while maintaining a constant spot size and ultrashort duration. The pulse is formed by superposing plasma-waveguide modes with appropriately selected frequencies. Compared with flying-focus approaches, the waveguide substantially reduces the required plasma volume. Scaling laws and quasi-three-dimensional particle-in-cell simulations show that the single-stage energy gain increases linearly with the number of modes used to construct the pulse, enabling larger energy gains or shorter stages than standard LWFA.

DOI: [10.1103/69yc-j1qh](https://doi.org/10.1103/69yc-j1qh)

I. INTRODUCTION

In laser wakefield acceleration (LWFA), a high-intensity laser pulse drives a large-amplitude plasma wave that traps and accelerates electrons to high energies [1–8]. The longitudinal electric field of the plasma wave exceeds that of conventional radio-frequency accelerators by nearly 3 orders of magnitude, positioning LWFA as a potential technology for compact particle colliders and light sources. Despite this potential, LWFA faces limitations, such as dephasing, in which trapped electrons outrun the accelerating phase of the wakefield. This arises from the mismatch between the subluminal group velocity of the driving laser pulse, which determines the phase velocity of the wake, and the near-luminal velocity of the high-energy electrons. Dephasing can be overcome by accelerating electrons through a series of LWFA stages [9–13], but this introduces significant technological challenges, including timing, alignment, and preservation of electron-beam quality between stages. These challenges motivate the development of concepts that mitigate dephasing to reduce the number of stages or eliminate the need for staging altogether [14–23].

Flying-focus pulses [17,24–31] circumvent dephasing by driving a wake at the vacuum speed of light, ensuring that high-energy electrons remain in the accelerating phase of the wakefield [17,18,20–23]. These pulses feature an intensity peak that travels at a programmable velocity over distances far greater than a Rayleigh range while maintaining a near-constant profile. The “dephasingless” laser wakefield acceleration (DLWFA) enabled by flying-focus pulses allows for either larger energy gains in a stage of fixed length or equal energy gains in a shorter stage when compared to standard LWFA. Optical techniques for generating flying-focus pulses control the focal time and location of each frequency, time slice, subpulse, or annulus of the overall pulse. However, each technique involves trade-offs in the resulting pulse structure, such as a longitudinally varying spot size, extended pulse duration, or constraints on the transverse or temporal profiles.

The ultrashort flying focus [17,18,28], for instance, uses an axiparabola to focus different annuli to different longitudinal locations [32–34] and a radial echelon [17,27,28] to control the relative timing of the annuli. While the geometric aberration of the axiparabola has the benefit of extending the focal range, it also produces a longitudinally varying spot size [28,35]. Moreover, achieving the ultrashort intensity peaks optimal for LWFA necessitates an axiparabola with a relatively small, full-aperture f number (e.g., $f_{\#} \sim 5$). As a result, the volume of plasma V needed to avoid plasma refraction and achieve the desired focal velocity increases rapidly with the accelerator length L , scaling as $V \propto L^3/f_{\#}^2$. This is a critical challenge to realizing ~ 100 -GeV energy gains in a single, $L \sim 1$ -m stage [36]. Nonetheless, flying-focus-driven DLWFA has been experimentally demonstrated at shorter

*Contact author: jpal@lle.rochester.edu

†Contact author: kmill@lle.rochester.edu

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

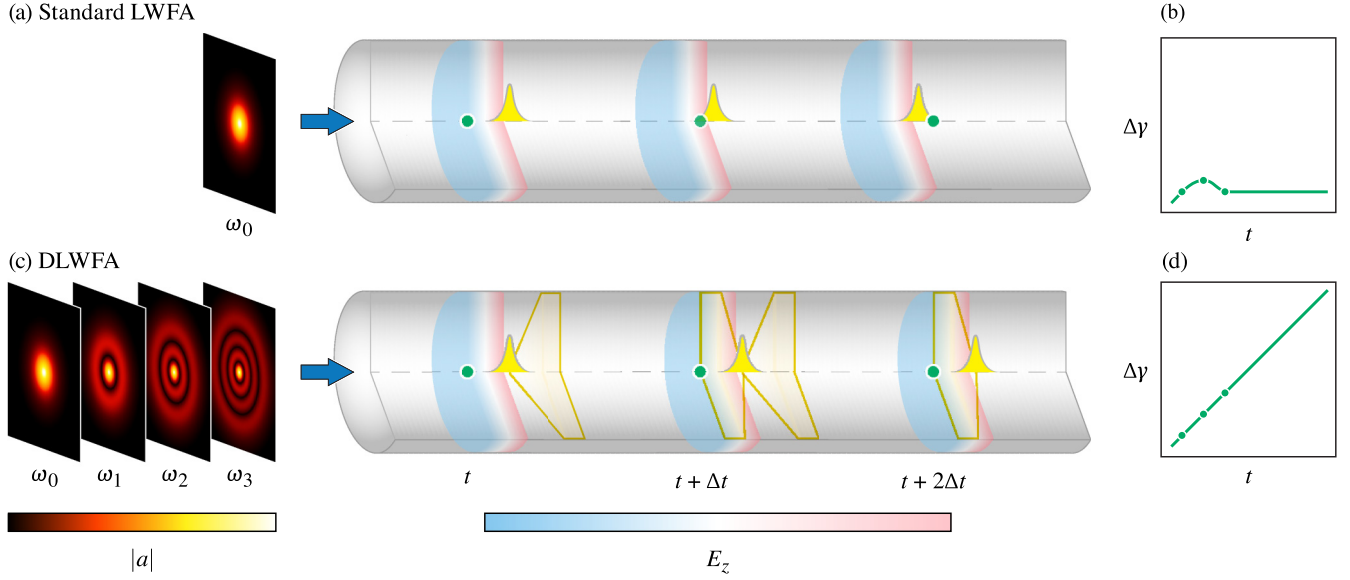


FIG. 1. Schematic of standard and dephasingless laser wakefield acceleration in a plasma waveguide. (a) In standard LWFA, an ultrashort laser pulse (yellow) with a single characteristic frequency ω_0 and Gaussian transverse profile (red scale) enters a plasma waveguide (gray) and drives a longitudinal wakefield E_z . High-energy electrons (green), initially located in the accelerating phase of the wake (blue), gain energy until they advance into a decelerating phase (red). At this point, they begin to lose energy before outrunning both the wake and the laser pulse, after which their energy plateaus, as shown in panel (b). (c) In DLWFA, a laser pulse composed of waveguide modes with distinct frequencies enters the waveguide, where modal interference produces an ultrashort intensity peak that drives a longitudinal wakefield at the vacuum speed of light. High-energy electrons remain in the accelerating phase of the wake and gain energy over the entire length of the accelerator, as shown in panel (d). The yellow structure depicts the overall envelope of the DLWFA pulse, which travels at the subluminal group velocity.

acceleration lengths, $L \sim 1$ cm, and shows promise for scaling to $L \sim 10$ cm [22].

Here, we introduce DLWFA driven by a spatiotemporally structured laser pulse in a plasma waveguide (Fig. 1). The approach combines the velocity control of the flying focus with the established benefits of plasma waveguides [37–46] to reduce the required plasma volume. The pulse is constructed from plasma-waveguide modes with appropriately selected frequencies, producing an intensity peak that travels at the vacuum speed of light while maintaining a constant spot size and ultrashort duration. Owing to the narrow transverse profile of the plasma waveguide, the required plasma volume can be substantially smaller than in previous flying-focus approaches. The electron energy gain and stage length increase linearly with the number of waveguide modes used to construct the pulse, enabling either larger energy gains per stage or shorter stages relative to standard LWFA. In the single-mode limit, the scalings reduce to those of standard LWFA, whereas the two-mode limit resembles beat-wave-driven LWFA [47,48]. For a pulse composed of ten modes, quasi-three-dimensional (quasi-3D) particle-in-cell (PIC) simulations demonstrate acceleration through a stage $11\times$ longer than the dephasing length, yielding a corresponding $11\times$ enhancement in the electron energy gain compared to standard LWFA at the same density.

The remainder of this article is organized as follows. Section II reviews the theory of arbitrary-velocity laser pulses in plasma waveguides [46]. Section III describes the design of a pulse and plasma waveguide for DLWFA. Section IV presents scaling laws and compares DLWFA in a plasma waveguide with standard LWFA, highlighting the advantages

and trade-offs. Section V demonstrates DLWFA with quasi-3D PIC simulations and validates the scaling laws. Section VI concludes the article with a summary of the results and a discussion of future prospects.

II. ARBITRARY-VELOCITY LASER PULSES IN PLASMA WAVEGUIDES

Consider a laser pulse propagating in the positive $\hat{z}$ direction through a preformed, parabolic plasma waveguide. The electron density of the waveguide $n(r)$ can be expressed in terms of the squared plasma frequency $\omega_p^2(r) = e^2 n(r)/m\epsilon_0$ and parametrized as

$$\omega_p^2(r) = \omega_{p0}^2 + \frac{4c^2}{w^4} r^2, \quad (1)$$

where $r = (x^2 + y^2)^{1/2}$ is the radial distance from the z axis, $\omega_{p0}^2 \equiv e^2 n(0)/m\epsilon_0$, and w is the “matched” spot size. The normalized, transverse vector potential of the pulse, $a = eA/mc$, can be decomposed into positive- and negative-frequency components $a = \frac{1}{2}(a + a^*)$, where

$$a(\mathbf{x}, t) = \frac{1}{2\pi} \int e^{-i\omega t} \tilde{a}(\mathbf{x}, \omega) d\omega, \quad (2)$$

and the integral is taken over positive frequencies. Each spectral component is a superposition of N waveguide modes:

$$\tilde{a}(\mathbf{x}, \omega) = \frac{1}{N} \sum_q \tilde{\alpha}_q(\omega) L_q \left(\frac{2r^2}{w^2} \right) \exp \left(-\frac{r^2}{w^2} \right) e^{ik_q(\omega)z}, \quad (3)$$

where q is the radial mode number, L_q is a Laguerre polynomial, and

$$k_q(\omega) = \frac{1}{c} \left(\omega^2 - \omega_{p0}^2 - \frac{4(1+2q)c^2}{w^2} \right)^{1/2}. \quad (4)$$

The coefficients

$$\tilde{\alpha}_q(\omega) = |\tilde{\alpha}_q(\omega)| e^{i\Phi_q(\omega)} \quad (5)$$

encode the spectral amplitude $|\tilde{\alpha}_q(\omega)|$ and phase $\Phi_q(\omega)$ of each mode.

A laser pulse with an intensity peak that travels at a prescribed velocity is constructed by appropriately tailoring the spectrum of each radial mode [46]. The appropriate spectra can be derived by analyzing the on-axis vector potential $a_{oa}(z, t) \equiv a(r=0, z, t)$:

$$a_{oa}(z, t) \equiv \frac{1}{2\pi N} \sum_q \int \tilde{\alpha}_q(\omega) e^{i(k_q(\omega)z - \omega t)} d\omega. \quad (6)$$

A vector potential whose amplitude travels at a longitudinal velocity v_a while maintaining its spatiotemporal profile satisfies

$$a_{oa}(z + v_a \mathcal{T}, t + \mathcal{T}) = a_{oa}(z, t) e^{i\eta \mathcal{T}}, \quad (7)$$

where $\mathcal{T}$ is an arbitrary time interval, η is a constant, and the factor $e^{i\eta \mathcal{T}}$ accounts for uniform carrier-phase evolution. Imposing this relation in Eq. (6) yields the constraint $\omega = k_q(\omega)v_a + \eta$. Because each radial mode obeys a distinct dispersion relation [Eq. (4)], no single broadband spectrum can satisfy this constraint simultaneously for all q . However, the constraint can be satisfied if each radial mode has a distinct frequency given by

$$\omega_q = \omega_0 + (k_q(\omega_q) - \kappa_0)v_a, \quad (8)$$

where $\eta = \omega_0 - \kappa_0 v_a$ has been chosen so that the frequency of the lowest-order mode ω_0 serves as a reference frequency and $\kappa_0 \equiv k_0(\omega_0)$. Geometrically, the allowed (k_q, ω_q) pairs correspond to the intersections of the dispersion hyperbolae [Eq. (4)] and the lines $\omega = k_q v_a + \eta$ [Eq. (8)]. The constraint expressed by Eq. (8) requires the spectral amplitudes to be sharply peaked at $\omega = \omega_q$. In the simplest case, as considered here, all modes share the same sharply peaked profile $|\tilde{\alpha}_q(\omega)| = \tilde{\alpha}(\omega - \omega_q)$.

III. DESIGN OF LASER PULSE AND PLASMA WAVEGUIDE FOR DLWFA

The energy gain in LWFA is determined by the longitudinal electric field of the wake E_z and the acceleration length L : $\Delta\gamma \simeq e|E_z|L/mc^2$, where γ is the Lorentz factor. Traditionally, the acceleration length of a single LWFA stage has been limited by dephasing, with electrons traveling close to the vacuum speed of light outrunning the accelerating phase of the wake after a distance

$$L_d = \frac{\pi c^2}{\omega_{p0}(c - v_w)}. \quad (9)$$

The dephasing length L_d depends sensitively on the phase velocity of the wake v_w , which is set by the velocity of the driving intensity peak. For standard laser pulses, this is

approximately their subluminal group velocity, $v_w \approx v_g < c$. The goal here is to design a pulse with an intensity peak that travels at $v_a = c$ and drives a wake with $v_w = c$. In this limit, $L_d \rightarrow \infty$, eliminating dephasing as an upper bound on the acceleration length.

When $v_a = c$, the required frequencies of the radial modes are given by

$$\omega_q = \omega_0 + \frac{4c^2 q}{(\omega_0 - c\kappa_0)w^2}. \quad (10)$$

For compatibility with the bandwidths of typical laser systems, the frequencies should satisfy $\omega_q - \omega_0 \ll \omega_0$. This condition can be rewritten in terms of the parameters of the plasma waveguide by substituting $\kappa_0 = k_0(\omega_0)$ into Eq. (10). In the underdense ($\omega_0 \gg \omega_p$) regime relevant to LWFA, the condition becomes

$$\left(\frac{\omega_{p0} w}{2c} \right)^2 \gg 1, \quad (11)$$

which states that the matched spot size should be much larger than the skin depth. With this condition satisfied, it is convenient to define the characteristic frequency spacing

$$\Omega \equiv 2\omega_0 \left(\frac{2c}{\omega_{p0} w} \right)^2. \quad (12)$$

The required frequencies then simplify to

$$\omega_q = \omega_0 + q\Omega. \quad (13)$$

Thus, an intensity peak with $v_a = c$ can be achieved by incrementally shifting the frequency of each mode relative to the reference frequency ω_0 .

The spatiotemporal profile of the pulse is determined by the modal frequencies and spectral amplitudes $\tilde{\alpha}(\omega - \omega_q)$. The temporal structure is most clearly seen in the on-axis ($r=0$) profile

$$|a_{oa}(z, t)|^2 = \frac{1}{N^2} |\alpha(t - z/v_g)|^2 \frac{\sin^2 \left[\frac{1}{2} N \Omega (t - z/c) \right]}{\sin^2 \left[\frac{1}{2} \Omega (t - z/c) \right]}, \quad (14)$$

where $v_g = c^2 k_0(\omega_0)/\omega_0$ (see Appendix A for derivation). This expression separates the temporal envelope $\alpha(t - z/v_g)$ from the interference structure arising from the superposition of radial modes. The envelope propagates at the group velocity v_g , while the intensity peaks formed by the interference travel at c . The peaks repeat with a period $T_R = 2\pi/\Omega$, or equivalently,

$$T_R = \frac{\pi}{\omega_0} \left(\frac{\omega_{p0} w}{2c} \right)^2, \quad (15)$$

and have a full width at half maximum (FWHM) duration

$$\tau = \frac{\sigma\pi}{N\omega_0} \left(\frac{\omega_{p0} w}{2c} \right)^2, \quad (16)$$

where σ is a numerical factor that monotonically decreases from 1 to 0.88 as N becomes large. This behavior arises because the FWHM of Eq. (14) depends weakly on N beyond the dominant $1/N$ scaling explicit in Eq. (16). Due to the velocity mismatch between the peaks and the envelope, an individual

peak traverses the envelope after a distance

$$L_D = \left(\frac{cv_g}{c - v_g} \right) T \approx 2 \left(\frac{\omega_0}{\omega_{p0}} \right)^2 cT, \quad (17)$$

where T is the duration of the envelope.

A design for DLWFA driven by a luminal intensity peak arising from modal interference is guided by two considerations. First, the FWHM duration of the peak should be chosen to maximize the strength of the wakefield. In the linear regime ($a_0 \equiv \max |a| \lesssim 0.5$), this occurs for $\tau = \pi/\omega_{p0}$. Second, the recurrence period should satisfy $T_R \simeq T$ to maximize the distance that the intensity peak drives the wake before a preceding peak disrupts the downstream plasma. The first consideration determines the matched spot size through Eq. (16):

$$w = \frac{2c}{\omega_{p0}} \left(\frac{\omega_0}{\omega_{p0}} \right)^{1/2} \left(\frac{N}{\sigma} \right)^{1/2}. \quad (18)$$

While the matched spot size sets the profile of the waveguide [Eq. (1)], the FWHM spot

$$w_F = \frac{2c}{\omega_{p0}} \left(\frac{\omega_0}{\omega_{p0}} \right)^{1/2} \left(\frac{2 \ln(2)}{\sigma} \right)^{1/2} \quad (19)$$

governs the width of the wake and is smaller than w for $N > 1$ due to the coherent superposition of radial modes. The second consideration determines the stage length through Eq. (17):

$$L_D = \frac{2\pi c}{\omega_{p0}} \left(\frac{\omega_0}{\omega_{p0}} \right)^2 \frac{N}{\sigma}. \quad (20)$$

For DLWFA in a plasma waveguide, Eq. (20) replaces the dephasing length as the upper bound on the stage length. In principle, L_D can be increased arbitrarily by increasing the number of modes, albeit at the expense of greater system complexity and a broader laser bandwidth, $\Delta\omega \sim (N-1)\Omega$.

IV. COMPARISON WITH STANDARD LWFA

An ultrashort intensity peak with $\tau = \pi/\omega_{p0}$ and a mildly relativistic amplitude, $a_0 \lesssim 0.5$, drives a wake with a maximum longitudinal electric field

$$\frac{e|E_z|}{mc} = \frac{\pi}{8} \omega_{p0} a_0^2. \quad (21)$$

Through $\omega_{p0} \propto \sqrt{n_0}$, the on-axis density sets the maximum field as well as the acceleration lengths for DLWFA (D) and standard LWFA (S). To compare performance at fixed a_0 without requiring equal densities, the on-axis density, plasma frequency, FWHM spot size, and matched spot size are denoted by n_j , ω_{pj} , w_{Fj} , and w_j , respectively, with $j = D, S$. For DLWFA, the stage length is given by Eq. (20), yielding an energy gain

$$\Delta\gamma_D = \frac{\pi^2}{4} \frac{N}{\sigma} \left(\frac{\omega_0}{\omega_{pD}} \right)^2 a_0^2. \quad (22)$$

For standard LWFA, the stage length is instead limited by the dephasing length [Eq. (9)], $L_S = L_d = 2\pi c \omega_0^2 / \omega_{pS}^3$, where $v_w = v_g \approx c(1 - \omega_{pS}^2 / 2\omega_0^2)$ has been used. The corresponding

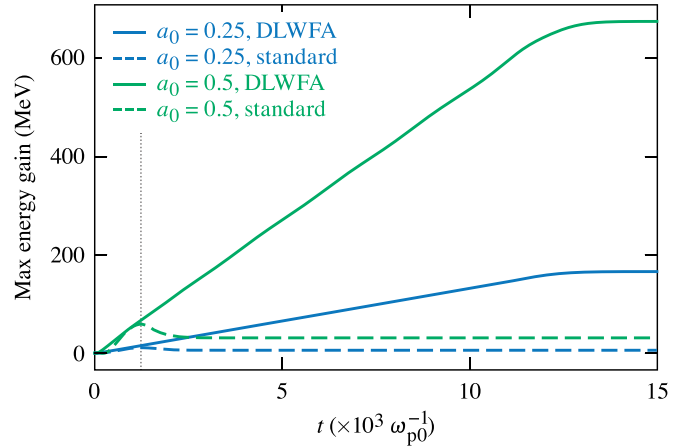


FIG. 2. Comparison of the maximum energy gain from DLWFA and standard LWFA in a plasma waveguide. With DLWFA (solid), an externally injected 100-MeV electron gains approximately $11\times$ the dephasing-limited energy gain, consistent with the $N = 10$ modes of the pulse. With standard LWFA (dashed), the electron reaches the dephasing-limited energy (dotted vertical line) before advancing into a decelerating phase and outrunning the wake. At fixed plasma density, the $11\times$ longer acceleration enabled by DLWFA requires approximately $11\times$ more laser-pulse energy. In both cases, the maximum energy gains agree with the scalings [Eqs. (22) and (23)] for $a_0 = 0.25$ (blue) and 0.5 (green). Simulation parameters are provided in Table I.

energy gain is

$$\Delta\gamma_S = \frac{\pi^2}{4} \left(\frac{\omega_0}{\omega_{pS}} \right)^2 a_0^2. \quad (23)$$

In the limit of a single mode ($N = 1$, $\sigma = 1$) and equal on-axis density ($n_D = n_S$), the energy gains are identical: $\Delta\gamma_D = \Delta\gamma_S$. Thus, the advantage of DLWFA arises from the use of multiple modes and operation at a different density.

A meaningful comparison of DLWFA and standard LWFA must also consider the energy required in their respective drive pulses. Assuming a flat-top profile for α with maximum amplitude a_0 , the energy in the DLWFA pulse is

$$\frac{U_D}{mc^2} = \frac{\pi c N}{4\sigma^2 r_e \omega_0} \left(\frac{\omega_0}{\omega_{pD}} \right)^4 a_0^2, \quad (24)$$

where r_e is the classical electron radius. For a standard LWFA pulse with the same normalized FWHM spot size, i.e., $\omega_{pD} w_{FD} = \omega_{pS} w_{FS}$, the energy is

$$\frac{U_S}{mc^2} = \frac{\pi c}{4\sigma r_e \omega_0} \left(\frac{\omega_0}{\omega_{pS}} \right)^3 \left(\frac{\omega_0}{\omega_{pD}} \right) a_0^2. \quad (25)$$

As before, the pulse energies are equal in the limit of a single mode and equal on-axis density: $U_D = U_S$.

The relative performance of DLWFA and standard LWFA depends on whether the comparison is made at fixed density, stage length, laser-pulse energy, or electron energy gain. At fixed density ($n_D = n_S$), the ratio of energy gains is

$$\frac{\Delta\gamma_D}{\Delta\gamma_S} = \frac{N}{\sigma}. \quad (26)$$

TABLE I. Laser pulse, plasma, and accelerator parameters for the figures and simulations. The parameters are motivated by commonly used laser systems and experimentally demonstrated plasma channels. The vacuum wavelength $\lambda_0 = 2\pi c/\omega_0$ and $\Lambda \equiv (\lambda_0/\omega_0)\Omega$. In the figures and simulations, time and space are normalized to $1/\omega_{p0} = 7.24$ fs and $c/\omega_{p0} = 2.17$ μm , respectively.

Pulse	Standard	Dephasingless
λ_0	1 μm	1 μm
a_0	0.25, 0.5, 0.75	0.25, 0.5, 0.75
τ	23 fs	23 fs
w_F	20 μm	20 μm
N	1	10
Λ	–	13 nm
Plasma	Standard	Dephasingless
$n(0)$	$6 \times 10^{18} \text{ cm}^{-3}$	$6 \times 10^{18} \text{ cm}^{-3}$
w	17 μm	54 μm
Accelerator	Standard	Dephasingless
L_d	2.5 mm	–
L_D	–	2.8 cm

In this case, $U_D/U_S = N/\sigma$, consistent with the longer stage length, $L_D = (N/\sigma)L_S$. The matched spot sizes satisfy $w_D = N^{1/2}w_S$. At fixed stage length ($L_D = L_S$), the energy-gain ratio becomes

$$\frac{\Delta\gamma_D}{\Delta\gamma_S} = \left(\frac{N}{\sigma}\right)^{1/3}. \quad (27)$$

Here, $U_D = U_S$ and $w_D = (\sigma^2 N)^{1/6} w_S$. Thus, comparing DLWFA and standard LWFA at fixed stage length is equivalent to comparing them at fixed laser-pulse energy. Finally, at fixed energy gain ($\Delta\gamma_D = \Delta\gamma_S$), the ratio of stage lengths is

$$\frac{L_D}{L_S} = \left(\frac{\sigma}{N}\right)^{1/2}. \quad (28)$$

Under these conditions, a DLWFA pulse requires less energy than a standard LWFA pulse: $U_D/U_S = (\sigma/N)^{1/2}$ with $w_D = \sigma^{1/2}w_S$. For all three comparisons, the pulse-energy ratio is identical to the stage-length ratio. For the latter two comparisons, the improved performance of DLWFA results from operating at higher on-axis density: $n_D = (N/\sigma)^{2/3}n_S$ for fixed stage length and $n_D = (N/\sigma)n_S$ for fixed energy gain.

V. SIMULATION RESULTS

The analysis above describes the design of DLWFA in a plasma waveguide and the associated scaling laws. To demonstrate a representative design and confirm the scaling behavior, simulations were conducted using the particle-in-cell code OSIRIS (see Table I for physical parameters and Appendix B for details). Comparisons with standard LWFA at equal on-axis density ($n_D = n_S$) show that $N = 10$ modes result in a stage length $11\times$ the dephasing length, with an energy gain enhancement of $11\times$.

Figure 2 shows that the maximum energy gain of externally injected 100-MeV test electrons is more than an order of magnitude greater in DLWFA (solid line) than in standard LWFA

(dashed line). This enhancement is consistent with the number of modes in the DLWFA pulse and holds for both $a_0 = 0.25$ (blue) and 0.5 (green). In DLWFA, the highest-energy electron gains energy over the entire length L_D , whereas in standard LWFA the electron gains energy over a dephasing length (dotted line) before entering a decelerating phase and eventually outrunning both the wake and the pulse. In both cases, the maximum energy gains from the simulations agree closely with the scaling predictions, for example, at $a_0 = 0.5$, $mc^2\Delta\gamma_D = 660$ MeV, and $mc^2\Delta\gamma_S = 58$ MeV.

Figure 3 compares the on-axis vector potential $|a_{\text{oa}}|$ and longitudinal wakefield E_z , highlighting two key features. First, at lower amplitudes, the laser pulse and accelerating field in standard LWFA slip backward in a frame moving at the speed of light, resulting in dephasing. This contrasts with DLWFA, where the pulse amplitude and accelerating field remain nearly stationary, thereby eliminating dephasing. Second, at higher amplitudes, DLWFA departs from the ideal linear regime due to self-guiding, depletion, redshifting, and temporal compression. At $a_0 = 0.5$, for example, a second trailing intensity peak emerges from the primary peak due to redshifting and mild self-guiding [cf. Figs. 3(c) and 3(e)]. Following this initial evolution, the pulse reaches a steady state, stabilizing the wake [Fig. 3(f)]. At $a_0 = 0.75$, depletion and redshifting become sufficiently strong to form a localized intensity burst with a reduced group velocity that slips backward, disrupting the wake structure. This process repeats as downstream temporal slices of the pulse constructively interfere to regenerate the moving intensity peak.

For larger values of a_0 ($\gtrsim 0.75$) and fixed density ($n_D = n_S$), the longer propagation distance in DLWFA makes the pulse more susceptible to nonlinear evolution than in standard LWFA. Despite its larger matched spot size ($w_D = N^{1/2}w_S$), the peak power of the DLWFA pulse is identical to that of the conventional pulse. This follows directly from the modal interference producing an intensity peak with a smaller FWHM spot size [Eq. (19)], such that $w_{FD} = w_{FS}$. As a consequence, the power-to-critical-power ratios are identical for the two pulses:

$$\left(\frac{P}{P_{\text{cr}}}\right)_D = \frac{\omega_{pD}^2 w_D^2}{32c^2 N} a_0^2 = \frac{\omega_{pS}^2 w_S^2}{32c^2} a_0^2 = \left(\frac{P}{P_{\text{cr}}}\right)_S, \quad (29)$$

where P_{cr} is the critical power [49]. However, because of its longer propagation distance, the DLWFA pulse accumulates more nonlinear phase and experiences stronger nonlinear refraction. In addition, as $a_0 \rightarrow 1$, the depletion length $L_{\text{pd}} = \pi c N \omega_0^2 / a_0^2 \omega_{p0}^3$ [5] decreases toward the stage length L_D . This depletion leads to redshifting, compression, and the group velocity reduction observed in Fig. 3 [50].

Figure 4 shows that the DLWFA pulse produces the same local features as the standard LWFA pulse despite its much larger matched spot size. Within a transverse region comparable to the FWHM spot size, both pulses have similar spatiotemporal profiles and drive nearly identical longitudinal fields. Outside this region, the transverse intensity modulation and limited temporal contrast of the DLWFA pulse are apparent [Fig. 4(d)]. These features arise from modal interference and are determined by the relative mode

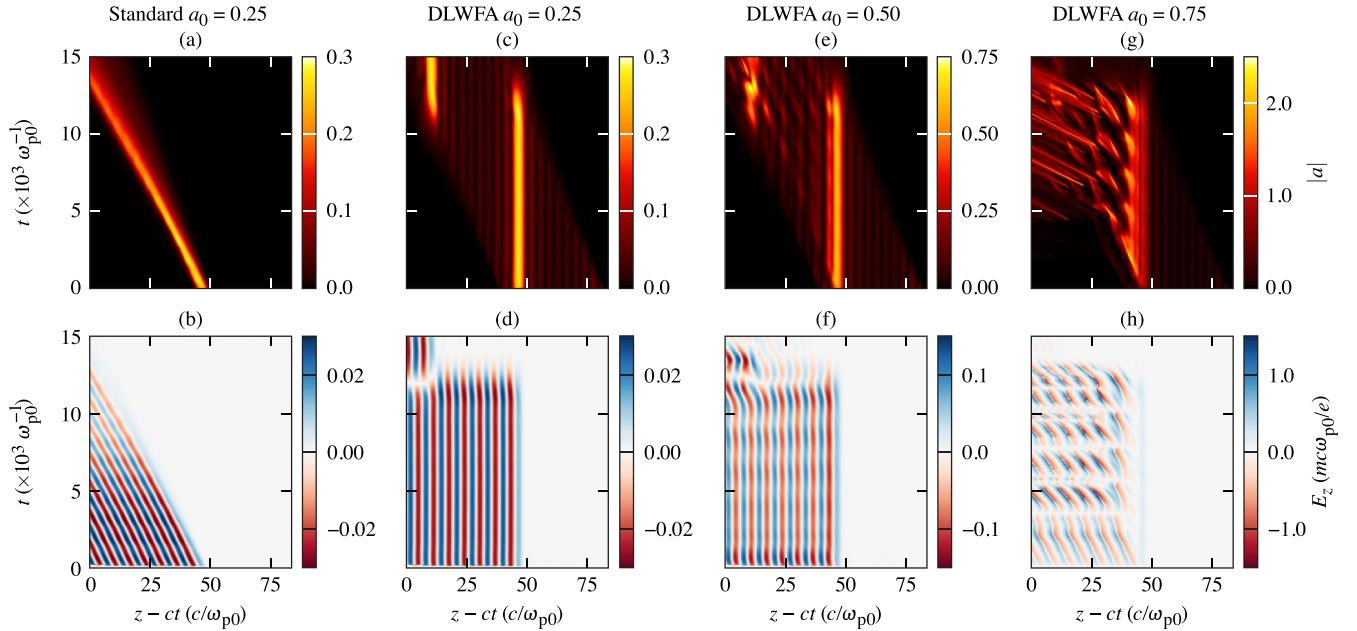


FIG. 3. On-axis ($r = 0$) structure and evolution of the driving laser pulse and wakefield in a frame moving at the vacuum speed of light. (a) A standard pulse with $a_0 = 0.25$ drives the wake shown in panel (b) whose phase fronts slip backward due to the subluminal group velocity. (c)–(f) The luminal intensity peak of DLWFA pulses with (c) $a_0 = 0.25$ and (e) $a_0 = 0.5$ drive wakes with near-stationary phase fronts, eliminating dephasing. The first recurrence of the moving intensity peak appears as the vertical feature originating at $t \approx 10^4 \omega_{p0}^{-1}$ and $z - ct = 11 c/\omega_{p0}$ in panels (c) and (e), consistent with Eq. (15). In panel (e), at $a_0 = 0.5$, self-focusing and depletion begin to modify the pulse profile and corresponding wake. (g) At $a_0 = 0.75$, self-focusing and depletion significantly distort the trajectory of intensity peak and the associated wake. Simulation parameters are provided in Table I.

amplitudes and phases. The transverse intensity modulations modify the wake structure, producing multiple transverse periods [Fig. 4(f)]. At high values of a_0 , the pedestal due to the limited temporal contrast could excite a wake in advance of the primary intensity peak, potentially disrupting propagation. More generally, the temporal and transverse profiles of the DLWFA pulse can be further engineered using selected transverse modes or by tailoring the mode amplitudes and phases to improve the temporal contrast and optimize the pulse structure.

VI. CONCLUSIONS AND PROSPECTS

A laser pulse composed of plasma-waveguide modes with appropriately chosen frequencies can drive a wakefield at the vacuum speed of light, enabling electron energy gains beyond the dephasing limit. The acceleration length of this DLWFA scheme increases with the number of modes in the pulse and depends on the electron density. This contrasts with standard LWFA, where the length depends only on density. The scaling with the number of modes provides additional flexibility to optimize either for higher energy gains at fixed stage length or for identical energy gains in shorter stages. The energy required in the DLWFA pulse scales proportionally with the stage length, such that, in principle, the pulse does not incur an energy penalty relative to standard LWFA.

The parameters for the simulation examples were chosen to verify the scaling laws while keeping the computational cost manageable. As a result, despite the $11\times$ increased energy gain enabled by DLWFA, the absolute energy gain was

660 MeV. The potential impact of DLWFA for high-energy colliders or radiation sources can be illustrated by comparison with a 100-GeV standard LWFA design consisting of ten 10-GeV stages. For $a_0 = 0.5$ and $\lambda_0 = 1 \mu\text{m}$, such a design requires a density $n_S = 3.5 \times 10^{16} \text{cm}^{-3}$ and a stage length $L_S = L_d = 5.7 \text{m}$ [Eq. (23)]. Using a ten-mode pulse, DLWFA could achieve 100 GeV in only five stages operating at $n_D = 2 \times 10^{17} \text{cm}^{-3}$, with a stage length $L_D = 4.7 \text{m}$ [Eqs. (20) and (22)]. The higher operating density reduces the total pulse energy by a factor of 2.4, $5U_D = (10U_S)/2.4$, and may also mitigate beamstrahlung at the interaction point [51]. These advantages come at the expense of a larger matched spot size: $w_D = 690 \mu\text{m}$ compared with $w_S \simeq 160 \mu\text{m}$. However, this could also be a benefit by improving tolerances and reducing sensitivity to misalignment.

In addition to overcoming dephasing, a primary motivation for DLWFA in a plasma waveguide is to retain the velocity control afforded by flying-focus pulses while avoiding the large plasma volumes required to suppress refraction. To prevent refraction from altering the trajectory of an ultrashort flying focus or reducing energy coupling, the plasma volume V should approximately fill the full-aperture $f_\#$ cone of the axiparabola over the acceleration length. This yields the scaling $V \propto \pi L^3/(12f_\#^2)$. In contrast, a plasma waveguide confines the interaction to a narrow cylinder with $V \propto \pi N w^2 L = \sigma(\omega_{p0}/\omega_0)^3 L^3/\pi$ [see Eqs. (18) and (20)], where the factor of N ensures that the waveguide radius ($\propto \sqrt{N}w$) is large enough to confine the $N - 1$ mode. Because $\omega_{p0}/\omega_0 \ll 1$, the waveguide could substantially reduce the required volume and ionization energy.

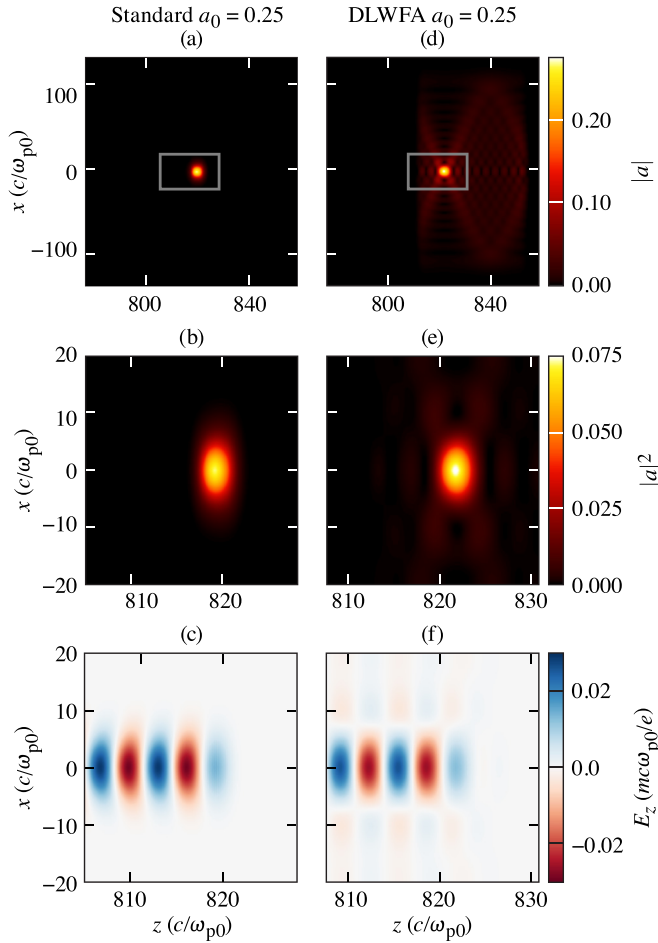


FIG. 4. Comparison of the spatiotemporal structure of the driving laser pulse and wakefield in standard LWFA and DLWFA at fixed density, $n_s = n_D$. (a), (b) For a standard pulse, both the matched spot size w_S and FWHM spot size w_{FS} are small, leading to a wake of comparable transverse width as shown in panel (c). (d), (e) For a DLWFA pulse, the matched spot size is large, $w_D = N^{1/2}w_S$, but the FWHM spot size is small, $w_{FD} = w_{FS}$. As a result, the DLWFA pulse drives a wake with nearly the same transverse width as the standard pulse, as seen in panel (f). In both cases, $a_0 = 0.25$. Panels (b) and (e) show magnified views of $|a|^2$ in the region outlined by the white box in panels (a) and (d). The structure of the modal interference and its effect on the wake are apparent in panels (d) and (f). Simulation parameters are provided in Table I.

Realizing the advantages of DLWFA in a plasma waveguide requires generating a highly structured laser pulse. As discussed in Ref. [46], a pulse composed of multiple spatial modes with distinct frequencies can be assembled using recently developed techniques for spatiotemporal structuring [52–54]. One approach is to disperse a broadband pulse into several frequency bands that are independently tailored using metasurface optics or a spatial light modulator [52–54]. Alternatively, the multiplexed outputs of a high-power fiber laser [55] could be independently manipulated before coherent combination. Recent advances in plasma and gas-phase optics [56] suggest a third route, in which an appropriately designed diffractive element separates different frequency bands of a high-power pulse into distinct spatial modes.

As presented, DLWFA in a plasma waveguide operates in the linear regime of laser wakefield acceleration and therefore cannot take advantage of self-injection [57] or ionization injection [58,59]. Several injection techniques do, however, remain viable. These include external injection from a linac [60] or an all-optical plasma photoinjector [61], density-down-ramp injection [62], and colliding-pulse injection [63]. Alternatively, the modal interference pattern or waveguide profile could be tailored to initially reduce the wake phase velocity, relaxing the requirements for self-injection. These injection strategies, together with extensions to the nonlinear regime, will be explored in future work.

A laser pulse with an intensity peak that travels at or near the vacuum speed of light in a plasma channel can also be used to overcome dephasing in other LWFA schemes, such as multipulse LWFA [64,65]. Following the design procedure described in Sec. III, a configuration in which multiple luminal intensity peaks resonantly excite the wake would be guided by two considerations: (1) choosing the FWHM duration to nearly maximize the wake strength, $\tau = \pi\sigma/\omega_{p0}$, and (2) setting the recurrence period to $T_R = 2\pi\ell/\omega_{p0}$, where ℓ is an integer. Together with Eqs. (15) and (16), these conditions yield the simple relation $N = 2\ell$. The resulting pulse would combine resonant wakefield enhancement with dephasing-free acceleration. More generally, the pulses described here may prove useful in the wide range of plasma-based applications that benefit from the flying focus.

ACKNOWLEDGMENTS

The authors would like to thank Jessica Shaw, Jeremy Pigeon, and Dustin Froula for discussions. The work of J.P.P., K.G.M., C.D.A., A.L.E., A.K., L.S.M., and D.R. was supported by the Office of Fusion Energy Sciences under Award No. DE-SC0021057, the Department of Energy National Nuclear Security Administration under Award No. DE-NA0004144, the University of Rochester, and the New York State Energy Research and Development Authority. The work of A.G.R.T. was supported by the U.S. National Science Foundation under Award No. 2512014.

This report was prepared as an account of work sponsored by an agency of the US Government. Neither the US Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the US Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the US Government or any agency thereof.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: SPATIOTEMPORAL PROFILE

The spatiotemporal profile of the laser pulse is obtained by substituting Eq. (10) into Eq. (2) and Taylor expanding $k_q(\omega)$ about $\omega = \omega_q$ to first order. This yields

$$|a(\mathbf{x}, t)|^2 = \frac{1}{N^2} |\alpha(t - z/v_g)|^2 \times \left| \sum_{q=0}^{N-1} L_q \left(\frac{2r^2}{w^2} \right) e^{-r^2/w^2} \exp[-iq\Omega(t - z/c)] \right|^2, \quad (\text{A1})$$

where $\alpha(t) = \frac{1}{2\pi} \int \tilde{\alpha}(\omega) e^{-i\omega(t-t_d)} d\omega$ and the phases $\Phi_q = q\Omega t_d$ have been chosen to allow for a desired delay t_d . In the regime considered here [Eq. (8)], the group velocities of the modes, $v_{g,q} = c^2 k_q(\omega_q)/\omega_q$, are nearly equal, so that $v_{g,q} \approx v_{g,0} \equiv v_g$. For simplicity, the sum is taken over consecutive modes, although this is not required. On axis ($r = 0$), $L_q(\frac{2r^2}{w^2}) e^{-r^2/w^2} = 1$ and the summation reduces to a finite geometric series. Evaluating the series leads to Eq. (14). Note that, due to the nonzero frequency spread of $\tilde{\alpha}(\omega)$, the shape-invariant property described by Eq. (8) only occurs within the uniform intensity region of the temporal envelope $\alpha(z - v_g t)$.

APPENDIX B: SIMULATION DETAILS

The simulations presented in this work were performed using OSIRIS [66]. A moving window and quasi-3D geometry were employed, with the fields of the laser pulse and plasma decomposed into a truncated expansion of azimuthal modes [67]. A customized field solver that mitigates errors from numerical dispersion and the time staggering of the electromagnetic fields was used [68]. The domain of the moving window extended 178 μm longitudinally and 323 μm radially, corresponding to 4980×1490 cells in z and r , respectively. The zeroth- and first-order azimuthal modes were retained, which was sufficient to capture the salient dynamics for the linearly polarized laser pulse. The time step was 70.9 as, and the total simulation time was 109 ps. The electrons were represented using 16 particles per cell. The longitudinal density profile of the plasma had an initial 152 μm upramp followed by a uniform region. The temporal profile of the DLWFA pulse, α , consisted of a 36-fs rise, a 258-fs flattop, and a 36-fs fall, resulting in an intensity FWHM of 286 fs. The rise and fall were sixth-order polynomials with vanishing derivatives at their end points. To test the scaling laws and generate Fig. 2, 100 MeV test electrons were initialized uniformly in z and r behind the laser pulse, and out to a radius of $r = 16 c/\omega_{p0}$. The test electrons had zero energy spread and no divergence.

-
- [1] T. Tajima and J. M. Dawson, Laser electron accelerator, *Phys. Rev. Lett.* **43**, 267 (1979).
 - [2] J. Faure, Y. Glinec, A. Pukhov, S. Kiselev, S. Gordienko, E. Zavalzsky, J.-P. Rousseau, P. d'Oliveira, and V. Malka, A laser-plasma accelerator producing monoenergetic electron beams, *Nature (London)* **431**, 541 (2004).
 - [3] C. G. R. Geddes, C. Toth, J. van Tilborg, E. Esarey, C. B. Schroeder, D. Bruhwiler, C. Nieter, J. Cary, and W. P. Leemans, High-quality electron beams from a laser wakefield accelerator using plasma-channel guiding, *Nature (London)* **431**, 538 (2004).
 - [4] S. P. D. Mangles, C. D. Murphy, Z. Najmudin, A. G. R. Thomas, J. L. Collier, A. E. Dangor, E. J. Divall, P. S. Foster, J. G. Gallacher, C. J. Hooke, D. A. Jaroszynski, A. J. Langley, W. B. Mori, Z. Najmudin, P. A. Norreys, F. S. Tsung, R. Viskup, B. R. Walton, and K. Krushelnick, Monoenergetic beams of relativistic electrons from intense laser-plasma interactions, *Nature (London)* **431**, 535 (2004).
 - [5] W. Lu, M. Tzoufras, C. Joshi, F. Tsung, W. Mori, J. Vieira, R. Fonseca, and L. Silva, Generating multi-GeV electron bunches using single stage laser wakefield acceleration in a 3D nonlinear regime, *Phys. Rev. ST Accel. Beams* **10**, 061301 (2007).
 - [6] E. Esarey, C. B. Schroeder, and W. P. Leemans, Physics of laser-driven plasma-based electron accelerators, *Rev. Mod. Phys.* **81**, 1229 (2009).
 - [7] B. Miao, J. E. Shrock, L. Feder, R. C. Hollinger, J. Morrison, R. Nedbailo, A. Picksley, H. Song, S. Wang, J. J. Rocca, and H. M. Milchberg, Multi-GeV electron bunches from an all-optical laser wakefield accelerator, *Phys. Rev. X* **12**, 031038 (2022).
 - [8] A. Picksley, J. Stackhouse, C. Benedetti, K. Nakamura, H. E. Tsai, R. Li, B. Miao, J. E. Shrock, E. Rockafellow, H. M. Milchberg, C. B. Schroeder, J. van Tilborg, E. Esarey, C. G. R. Geddes, and A. J. Gonsalves, Matched guiding and controlled injection in dark-current-free, 10-GeV-class, channel-guided laser-plasma accelerators, *Phys. Rev. Lett.* **133**, 255001 (2024).
 - [9] S. Steinke, J. van Tilborg, C. Benedetti, C. Geddes, C. Schroeder, J. Daniels, K. Swanson, A. Gonsalves, K. Nakamura, N. Matlis, *et al.*, Multistage coupling of independent laser-plasma accelerators, *Nature (London)* **530**, 190 (2016).
 - [10] J. Luo, M. Chen, W. Y. Wu, S. M. Weng, Z. M. Sheng, C. B. Schroeder, D. A. Jaroszynski, E. Esarey, W. P. Leemans, W. B. Mori, and J. Zhang, Multistage coupling of laser-wakefield accelerators with curved plasma channels, *Phys. Rev. Lett.* **120**, 154801 (2018).
 - [11] V. B. Pathak, H. T. Kim, J. Vieira, L. O. Silva, and C. H. Nam, All optical dual stage laser wakefield acceleration driven by two-color laser pulses, *Sci. Rep.* **8**, 11772 (2018).
 - [12] A. G. R. Thomas and D. Seipt, Modeling chromatic emittance growth in staged plasma wakefield acceleration to 1 TeV using nonlinear transfer matrices, *Phys. Rev. Accel. Beams* **24**, 104602 (2021).
 - [13] C. Schroeder, F. Albert, C. Benedetti, J. Bromage, D. Bruhwiler, S. Bulanov, E. Campbell, N. Cook, B. Cros, M. Downer, *et al.*, Linear colliders based on laser-plasma accelerators, *J. Instrum.* **18**, T06001 (2023).
 - [14] P. Sprangle, B. Hafizi, J. R. Peñaño, R. F. Hubbard, A. Ting, C. I. Moore, D. F. Gordon, A. Zigler, D. Kaganovich, and T. M. Antonsen, Wakefield generation and GeV acceleration in tapered plasma channels, *Phys. Rev. E* **63**, 056405 (2001).

- [15] S. J. Yoon, J. P. Palastro, and H. M. Milchberg, Quasi-phase-matched laser wakefield acceleration, *Phys. Rev. Lett.* **112**, 134803 (2014).
- [16] A. Debus, R. Pausch, A. Huebl, K. Steiniger, R. Widera, T. E. Cowan, U. Schramm, and M. Bussmann, Circumventing the dephasing and depletion limits of laser-wakefield acceleration, *Phys. Rev. X* **9**, 031044 (2019).
- [17] J. P. Palastro, J. L. Shaw, P. Franke, D. Ramsey, T. T. Simpson, and D. H. Froula, Dephasingless laser wakefield acceleration, *Phys. Rev. Lett.* **124**, 134802 (2020).
- [18] C. Caizergues, S. Smartsev, V. Malka, and C. Thaury, Phase-locked laser-wakefield electron acceleration, *Nat. Photon.* **14**, 475 (2020).
- [19] J. P. Palastro, B. Malaca, J. Vieira, D. Ramsey, T. T. Simpson, P. Franke, J. L. Shaw, and D. H. Froula, Laser-plasma acceleration beyond wave breaking, *Phys. Plasmas* **28**, 013109 (2021).
- [20] K. G. Miller, J. R. Pierce, M. V. Ambat, J. L. Shaw, K. Weichman, W. B. Mori, D. H. Froula, and J. P. Palastro, Dephasingless laser wakefield acceleration in the bubble regime, *Sci. Rep.* **13**, 21306 (2023).
- [21] J. R. Pierce, K. G. Miller, F. Li, J. P. Palastro, and W. B. Mori, Laser wakefield acceleration driven by a discrete flying focus, *Phys. Rev. Accel. Beams* **28**, 101303 (2025).
- [22] C. D. Arrowsmith, K. G. Miller, M. V. Ambat, S.-W. Bahk, I. A. Begishev, J. Bromage, S. Bucht, N. Dauphin, C. Dorrer, C. Jeon, *et al.*, Dephasingless laser wakefield acceleration of electrons using a flying focus, *Nat. Phys.* **22**, 1513 (2026).
- [23] A. Liberman, A. Golovanov, S. Smartsev, A.-M. Talposi, S. Tata, and V. Malka, First electron acceleration in a tunable-velocity laser wakefield, *Phys. Rev. Res.* **8**, L022001 (2026).
- [24] A. Sainte-Marie, O. Gobert, and F. Quéré, Controlling the velocity of ultrashort light pulses in vacuum through spatio-temporal couplings, *Optica* **4**, 1298 (2017).
- [25] D. H. Froula, D. Turnbull, A. S. Davies, T. J. Kessler, D. Haberberger, J. P. Palastro, S.-W. Bahk, I. A. Begishev, R. Boni, S. Bucht, *et al.*, Spatiotemporal control of laser intensity, *Nat. Photon.* **12**, 262 (2018).
- [26] S. W. Jolly, O. Gobert, A. Jeandet, and F. Quéré, Controlling the velocity of a femtosecond laser pulse using refractive lenses, *Opt. Express* **28**, 4888 (2020).
- [27] M. Ambat, J. Shaw, J. Pigeon, K. Miller, T. Simpson, D. Froula, and J. P. Palastro, Programmable-trajectory ultrafast flying focus pulses, *Opt. Express* **31**, 31354 (2023).
- [28] J. Pigeon, P. Franke, M. Lim Pac Chong, J. Katz, R. Boni, C. Dorrer, J. P. Palastro, and D. Froula, Ultrabroadband flying-focus using an axiparabola-echelon pair, *Opt. Express* **32**, 576 (2024).
- [29] D. Li, K. G. Miller, J. R. Pierce, W. B. Mori, A. G. R. Thomas, and J. P. Palastro, Spatiotemporal control of high-intensity laser pulses with a plasma lens, *Phys. Rev. Res.* **6**, 013272 (2024).
- [30] M. Piccardo, M. O. Cernaianu, J. P. Palastro, A. Arefiev, C. Thaury, J. Vieira, D. H. Froula, and V. Malka, Trends in relativistic laser-matter interaction: The promises of structured light, *Optica* **12**, 732 (2025).
- [31] S. Cao, D. Singh, L. S. Mack, J. P. Palastro, and M. R. Edwards, Flying focus with arbitrary directionality for spatiotemporal control of laser pulses *Phys. Rev. Appl.* **25**, 064060 (2026).
- [32] S. Smartsev, C. Caizergues, K. Oubrierie, J. Gautier, J.-P. Goddet, A. Tafzi, K. T. Phuoc, V. Malka, and C. Thaury, Axiparabola: A long-focal-depth, high-resolution mirror for broadband high-intensity lasers, *Opt. Lett.* **44**, 3414 (2019).
- [33] P.-F. Geng, M. Chen, X.-Z. Zhu, W.-Y. Liu, Z.-M. Sheng, and J. Zhang, Propagation of axiparabola-focused laser pulses in uniform plasmas, *Phys. Plasmas* **29**, 112301 (2022).
- [34] K. Oubrierie, I. A. Andriyash, R. Lahaye, S. Smartsev, V. Malka, and C. Thaury, Axiparabola: A new tool for high-intensity optics, *J. Opt.* **24**, 045503 (2022).
- [35] A. Liberman, A. Golovanov, S. Smartsev, S. Tata, I. A. Andriyash, S. Benracassa, E. Y. Levine, Y. Wan, E. Kroupp, and V. Malka, Direct observation of a wakefield generated with structured light, *Nat. Commun.* **16**, 10957 (2025).
- [36] J. L. Shaw, M. V. Ambat, K. G. Miller, R. Boni, I. A. LaBelle, W. B. Mori, J. J. Pigeon, A. Rigatti, I. A. Settle, L. S. Mack, J. P. Palastro, and D. H. Froula, Path to a single-stage, 100-GeV electron beam via a flying-focus-driven laser-plasma accelerator, *Phys. Plasmas* **32**, 083107 (2025).
- [37] C. G. Durfee and H. M. Milchberg, Light pipe for high intensity laser pulses, *Phys. Rev. Lett.* **71**, 2409 (1993).
- [38] Y. Ehrlich, C. Cohen, A. Zigler, J. Krall, P. Sprangle, and E. Esarey, Guiding of high intensity laser pulses in straight and curved plasma channel experiments, *Phys. Rev. Lett.* **77**, 4186 (1996).
- [39] T. Ditmire, R. A. Smith, and M. H. R. Hutchinson, Plasma waveguide formation in predissociated clustering gases, *Opt. Lett.* **23**, 322 (1998).
- [40] T. R. Clark and H. M. Milchberg, Optical mode structure of the plasma waveguide, *Phys. Rev. E* **61**, 1954 (2000).
- [41] V. Kumarappan, K. Y. Kim, and H. M. Milchberg, Guiding of intense laser pulses in plasma waveguides produced from efficient, femtosecond end-pumped heating of clustered gases, *Phys. Rev. Lett.* **94**, 205004 (2005).
- [42] C. G. R. Geddes, C. Toth, J. van Tilborg, E. Esarey, C. B. Schroeder, J. Cary, and W. P. Leemans, Guiding of relativistic laser pulses by preformed plasma channels, *Phys. Rev. Lett.* **95**, 145002 (2005).
- [43] B. D. Layer, A. York, T. M. Antonsen, S. Varma, Y.-H. Chen, and H. M. Milchberg, Ultrahigh-intensity optical slow-wave structure, *Phys. Rev. Lett.* **99**, 035001 (2007).
- [44] R. J. Shalloo, C. Arran, L. Corner, J. Holloway, J. Jonnerby, R. Walczak, H. M. Milchberg, and S. M. Hooker, Hydrodynamic optical-field-ionized plasma channels, *Phys. Rev. E* **97**, 053203 (2018).
- [45] B. Miao, L. Feder, J. E. Shrock, A. Goffin, and H. M. Milchberg, Optical guiding in meter-scale plasma waveguides, *Phys. Rev. Lett.* **125**, 074801 (2020).
- [46] J. P. Palastro, K. G. Miller, M. R. Edwards, A. L. Elliott, L. S. Mack, D. Singh, and A. G. R. Thomas, Arbitrary-velocity laser pulses in plasma waveguides, *Phys. Rev. Res.* **7**, 023249 (2025).
- [47] M. Everett, A. Lal, D. Gordon, C. E. Clayton, K. A. Marsh, and C. Joshi, Trapped electron acceleration by a laser-driven relativistic plasma wave, *Nature (London)* **368**, 527 (1994).
- [48] A. Pukhov, N. E. Andreev, A. A. Golovanov, I. I. Artemenko, and I. Y. Kostyukov, Laser-plasma wake velocity control by multi-mode beatwave excitation in a channel, *Plasma* **6**, 29 (2023).
- [49] G.-Z. Sun, E. Ott, Y. C. Lee, and P. Guzdar, Self-focusing of short intense pulses in plasmas, *Phys. Fluids* **30**, 526 (1987).
- [50] W. Zhu, J. P. Palastro, and T. M. Antonsen, Pulsed mid-infrared radiation from spectral broadening in

- laser wakefield simulations, *Phys. Plasmas* **20**, 073103 (2013).
- [51] C. B. Schroeder, E. Esarey, C. G. R. Geddes, C. Benedetti, and W. P. Leemans, Physics considerations for laser-plasma linear colliders, *Phys. Rev. ST Accel. Beams* **13**, 101301 (2010).
- [52] D. Cruz-Delgado, S. Yerolatsitis, N. K. Fontaine, D. N. Christodoulides, R. Amezcua-Correa, and M. A. Bandres, Synthesis of ultrafast wavepackets with tailored spatiotemporal properties, *Nat. Photon.* **16**, 686 (2022).
- [53] M. Piccardi, M. de Oliveira, V. R. Policht, M. Russo, B. Ardingi, M. Corti, G. Valentini, J. Vieira, C. Manzoni, G. Cerullo, and A. Ambrosio, Broadband control of topological–spectral correlations in space–time beams, *Nat. Photon.* **17**, 822 (2023).
- [54] Q. Zhan, Spatiotemporal sculpturing of light: A tutorial, *Adv. Opt. Photon.* **16**, 163 (2024).
- [55] A. Rainville, M. Whittlesey, C. Pasquale, Y. Jing, M. Chen, S. Chen, H. Pei, J. Ruppe, T. Zhou, Q. Du, Z. Zhang, G. Chang, F. X. Kärtner, and A. Galvanauskas, Near-complete extraction of maximum stored energy from large-core fibers using coherent pulse stacking amplification of femtosecond pulses, *Optica* **11**, 1540 (2024).
- [56] M. R. Edwards, V. R. Munirov, A. Singh, N. M. Fasano, E. Kur, N. Lemos, J. M. Mikhailova, J. S. Wurtele, and P. Michel, Holographic plasma lenses, *Phys. Rev. Lett.* **128**, 065003 (2022).
- [57] D. H. Froula, C. E. Clayton, T. Döppner, K. A. Marsh, C. P. J. Barty, L. Divol, R. A. Fonseca, S. H. Glenzer, C. Joshi, W. Lu, *et al.*, Measurements of the critical power for self-injection of electrons in a laser wakefield accelerator, *Phys. Rev. Lett.* **103**, 215006 (2009).
- [58] A. Pak, K. A. Marsh, S. F. Martins, W. Lu, W. B. Mori, and C. Joshi, Injection and trapping of tunnel-ionized electrons into laser-produced wakes, *Phys. Rev. Lett.* **104**, 025003 (2010).
- [59] C. McGuffey, A. G. R. Thomas, W. Schumaker, T. Matsuoka, V. Chvykov, F. J. Dollar, G. Kalintchenko, V. Yanovsky, A. Maksimchuk, K. Krushelnick, V. Y. Bychenkov, I. V. Glazyrin, and A. V. Karpeev, Ionization induced trapping in a laser wakefield accelerator, *Phys. Rev. Lett.* **104**, 025004 (2010).
- [60] Y. Wu, J. Hua, Z. Zhou, J. Zhang, S. Liu, B. Peng, Y. Fang, X. Ning, Z. Nie, F. Li, C. Zhang, C.-H. Pai, Y. Du, W. Lu, W. B. Mori, and C. Joshi, High-throughput injection–acceleration of electron bunches from a linear accelerator to a laser wakefield accelerator, *Nat. Phys.* **17**, 801 (2021).
- [61] A. Jain, J. Yan, J. R. Pierce, T. T. Simpson, M. Polyanskiy, W. Li, M. Babzien, M. Palmer, M. Downer, R. Samulyak, C. Joshi, W. B. Mori, J. P. Palastro, and N. Vafaei-Najafabadi, Collider-quality electron bunches from an all-optical plasma photoinjector, *Phys. Rev. Res.* **8**, 023213 (2026).
- [62] A. J. Gonsalves, K. Nakamura, C. Lin, D. Panasenkov, S. Shiraishi, T. Sokollik, C. Benedetti, C. B. Schroeder, C. G. R. Geddes, J. van Tilborg, J. Osterhoff, E. Esarey, C. Toth, and W. P. Leemans, Tunable laser plasma accelerator based on longitudinal density tailoring, *Nat. Phys.* **7**, 862 (2011).
- [63] J. Faure, C. Rechatin, A. Norlin, A. Lifschitz, Y. Glinec, and V. Malka, Controlled injection and acceleration of electrons in plasma wakefields by colliding laser pulses, *Nature (London)* **444**, 737 (2006).
- [64] S. Hooker, R. Bartolini, S. Mangles, A. Tünnermann, L. Corner, J. Limpert, A. Seryi, and R. Walczak, Multi-pulse laser wakefield acceleration: A new route to efficient, high-repetition-rate plasma accelerators and high flux radiation sources, *J. Phys. B: At. Mol. Opt. Phys.* **47**, 234003 (2014).
- [65] O. Jakobsson, S. M. Hooker, and R. Walczak, GeV-scale accelerators driven by plasma-modulated pulses from kilohertz lasers, *Phys. Rev. Lett.* **127**, 184801 (2021).
- [66] R. A. Fonseca, L. O. Silva, F. S. Tsung, V. K. Decyk, W. Lu, C. Ren, W. B. Mori, S. Deng, S. Lee, T. Katsouleas, and J. C. Adam, OSIRIS: A three-dimensional, fully relativistic particle in cell code for modeling plasma based accelerators, in *Lecture Notes in Computer Science* (Springer, Berlin, 2002), pp. 342–351.
- [67] A. Davidson, A. Tableman, W. An, F. Tsung, W. Lu, J. Vieira, R. Fonseca, L. Silva, and W. Mori, Implementation of a hybrid particle code with a PIC description in r – z and a gridless description in ϕ into OSIRIS, *J. Comput. Phys.* **281**, 1063 (2015).
- [68] F. Li, K. G. Miller, X. Xu, F. S. Tsung, V. K. Decyk, W. An, R. A. Fonseca, and W. B. Mori, A new field solver for modeling of relativistic particle-laser interactions using the particle-in-cell algorithm, *Comput. Phys. Commun.* **258**, 107580 (2021).