

Intrinsic Floquet generation and $1/I$ quantum oscillations in a sliding charge-density wave

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Recent experiments [*Phys. Rev. B* **109**, 245123 (2024)] revealed striking inverse-current ($1/I$) quantum oscillations in quasi-one-dimensional charge-density-wave (CDW) insulators and proposed an intrinsic Floquet sideband mechanism arising from the sliding condensate. Here, we develop the complete theoretical framework underlying this proposal. We provide an exact Floquet diagonalization of the uniformly sliding CDW problem, yielding split gap edges and a ladder of Floquet sidebands with explicit unitary transformation and spectral functions. Using this exact solution, we formulate weak-probe tunneling spectroscopy and demonstrate that the local Floquet spectrum naturally yields $1/I$ oscillations as successive sideband edges cross a fixed contact chemical potential. The measured period fixes $N_{\text{eff}}\epsilon \simeq 4.80$ eV, where ϵ is the local contact-to-gap-edge offset. Combining this relation with the experimentally resolved low-temperature oscillations constrains the effective coherent channel count to be at most of order 10^3 , far below the geometric estimate $N_{\text{geom}} \sim 3 \times 10^4$, while $\epsilon = 10$ meV gives the representative value $N_{\text{eff}} \simeq 480$. This reduced coherent scale is naturally realized by a spatially nonuniform percolating network, but does not require N_{eff} independent end-to-end wires. Furthermore, using a segmented multiterminal model, we show that inelastic phase-slip dephasing near the contacts explains the experimentally observed suppression of oscillation visibility on outer voltage probes. Finally, we contrast the persistent-current-driven multiterminal geometry with a homogeneous voltage-biased two-terminal reference calculation. Our results establish a nonequilibrium transport framework for the observed $1/I$ oscillations and identify an observability condition in which the Floquet spacing must exceed collective and quasiparticle broadening.

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I. INTRODUCTION

Macroscopic quantum states such as superconductivity and charge-density waves (CDWs) offer unique platforms for exploring emergent quantum phenomena and developing next-generation quantum materials [1–4]. A CDW is a spontaneously formed, spatially periodic modulation of the electronic charge density and lattice coordinates that naturally arises in quasi-one-dimensional (quasi-1D) conductors [1,2], with related considerations applying to spin-density waves [5–7]. Above a critical depinning threshold, the CDW condensate can slide collectively, leading to highly nonlinear charge transport [4,8–13].

A particularly striking manifestation of macroscopic CDW coherence was recently reported by Le *et al.* [14], who observed robust quantum oscillations periodic in inverse applied current ($1/I$) in quasi-1D CDW insulators $(\text{TaSe}_4)_2\text{I}$ and TaS_3 . These oscillations appear in the Fröhlich conductivity regime where the depinned condensate slides coherently, and disappear once the oscillation index reaches unity, mirroring the

quantum limit of Shubnikov–de Haas oscillations. Crucially, the same work proposed that this phenomenon originates from sliding-driven inherent Floquet sidebands: Once the CDW condensate slides uniformly, its phase $\phi(t) = \phi_0 - \Omega t$ becomes explicitly time dependent, so the condensate itself generates a periodic drive. This intrinsic spatial-to-temporal conversion represents a fundamental departure from the more familiar Floquet settings, which typically require external microwave fields or ultrafast laser pulses [15–20].

Reference [14] sketched the basic Floquet mechanism and derived the leading periodicity relation $\Delta(1/I) = h/(2eN_{\text{eff}}\epsilon)$, but explicitly called for a well-developed theoretical model to address the full phenomenology. Several essential questions were left open: (1) Is the sliding-CDW Floquet problem exactly solvable, and what is the structure of its complete spectrum and Green’s functions? (2) How does the local Floquet sideband ladder manifest in different measurement geometries—weak tunneling spectroscopy versus contacted transport? (3) Why does the macroscopic terminal current produce sideband spacings far larger than naive geometric estimates would suggest? (4) Why is the $1/I$ oscillation strongly visible on inner voltage probes but heavily suppressed on outer terminal pairs? The present work develops the complete theoretical framework needed to address these questions.

Our principal contributions are as follows. First, we provide an exact Floquet diagonalization of the uniformly sliding CDW, with explicit unitary transformation, quasienergy

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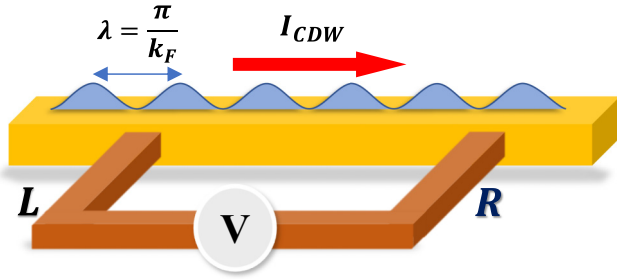


FIG. 1. Two-terminal sliding-CDW reference geometry: a finite sliding CDW region coupled to left and right metallic leads. This homogeneous voltage-biased geometry serves as a theoretical reference limit (Sec. IV) and is distinct both from the weak single-probe spectroscopy of Sec. III and from the persistent-current-driven multiterminal experiment of Ref. [14] sketched in Fig. 2.

spectrum, eigenstates, and time-averaged spectral functions (Sec. II). Second, we formulate a complete Schwinger-Keldysh nonequilibrium framework that distinguishes weak-probe spectroscopy from contacted two-terminal transport, and we identify the local time-averaged spectral function as the singular structure governing observed oscillations (Sec. III). Third, the oscillation period fixes the product $N_{\text{eff}}\epsilon$, while low-temperature resolvability supplies an additional constraint showing that the effective coherent channel count is far below the geometric count. A percolating coherent network is a natural realization of this reduced scale, although the data do not uniquely determine its microscopic geometry. Fourth, we introduce a segmented multiterminal transport model that explains the observed visibility hierarchy: Phase-slip dephasing near the current-injecting contacts confines coherent oscillations to the inner segments, accounting for why V_{2-3}^{1-4} shows clean $1/I$ oscillations while the raw outer-terminal voltage V_{1-4}^{1-4} requires background subtraction (Sec. IV). Finally, we contrast this experimentally relevant geometry with a homogeneous voltage-biased two-terminal reference calculation. The homogeneous two-terminal reference geometry is shown in Fig. 1, and the experiment-motivated segmented multiterminal geometry is shown in Fig. 2.

Beyond the specific CDW context, the framework underscores a critical material criterion for observing macroscopic Floquet coherence: The total collective and quasiparticle linewidth must remain below the Floquet spacing. A residual Fermi surface in a two-dimensional CDW provides gapless electron-electron decay channels and therefore strongly disfavors resolvable sidebands, whereas the complete excitation gap of a quasi-1D CDW insulator exponentially suppresses the electronic particle-hole bath. This gap-protected coherence offers a design principle for intrinsically driven quantum devices.

II. SLIDING CDW AND EXACT FLOQUET SOLUTION

A. Static and sliding order parameter

At low energies, the CDW amplitude mode is gapped, so we retain only the phase degree of freedom and write

$$\Delta(x, t) = \Delta e^{i\phi(x, t)}, \quad (1)$$

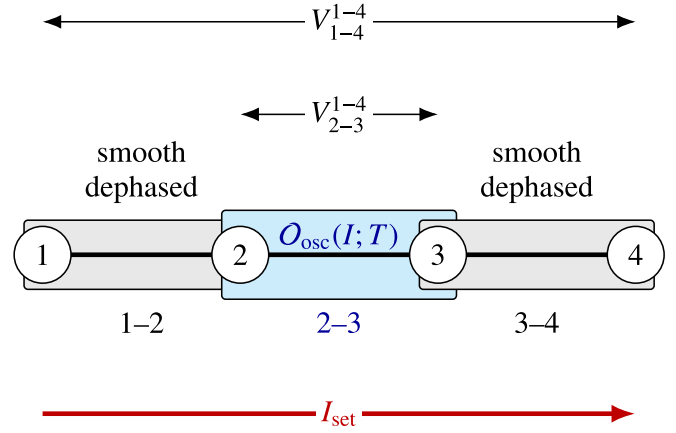


FIG. 2. Experiment-motivated segmented model for a persistent-current-driven multiterminal device [14]. One imposed current I_{set} flows through three serial segments. The central segment 2-3 hosts the coherent Floquet kernel $\mathcal{O}_{\text{osc}}(I; T)$ [defined in Eq. (H1)], whereas the outer segments contribute smooth, effectively dephased voltage drops due to phase-slip-induced inelastic scattering near the contacts. The experimentally relevant inner- and outer-terminal voltages are indicated above the device.

with $\Delta > 0$. Once a dc current exceeds the depinning threshold, the phase evolves linearly in time, $\phi(t) = \phi_0 - \Omega t$, so that the density modulation becomes

$$\rho(x, t) = \rho_0 + \rho_1 \cos(Qx - \Omega t + \phi_0), \quad (2)$$

where $Q = 2k_F$. The nearly uniform sliding state assumed here is favored by the incommensurate character of $(\text{TaSe}_4)_2\text{I}$ and TaS_3 . A commensurate CDW can acquire a lattice-locking contribution $-V_c \cos(M\phi)$ when MQ equals a reciprocal-lattice vector. In an incommensurate CDW, no low-order locking condition exists, so intrinsic lattice pinning is strongly suppressed and a nearly uniform phase rotation can emerge once disorder pinning is overcome [3,4].

Following Ref. [14], the sliding frequency is fixed by the condensate current per coherent channel. A 2π phase advance transports charge $2e$ for a spin-degenerate channel, so with $f = \Omega/(2\pi)$,

$$j_{\text{ch}} = 2ef = \frac{e\Omega}{\pi} = -\frac{e}{\pi} \dot{\phi}. \quad (3)$$

In a realistic device, the coherently participating chain segments need not be independent end-to-end wires: Transverse CDW coupling, interchain transfer, and phase slips can connect them into a percolating network. We therefore define N_{eff} operationally through $I = N_{\text{eff}} j_{\text{ch}}$. It is an effective coherent channel count governing the current-to-frequency conversion, not necessarily the literal cross section of one metallic filament. The intrinsic drive frequency is then

$$\Omega = \frac{\pi I}{eN_{\text{eff}}}. \quad (4)$$

The order parameter is therefore periodic in time even without external irradiation. Throughout the paper, Ω denotes the angular sliding frequency and $\hbar\Omega$ is the corresponding photonlike energy quantum of the periodic drive.

B. Two-branch model and exact Floquet spectrum

To model one coherent channel within this network, we retain a single band linearized around the two Fermi points, labeling the right-moving ($k > 0$) and left-moving ($k < 0$) branches by $r = +1$ and $r = -1$, respectively. This linearized two-branch approximation is valid as long as $v_F k_F \gg \Delta$. The mean-field Hamiltonian reads

$$H_{\text{CDW}} = \sum_{\mathbf{k}, \sigma} \varepsilon_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger f_{\mathbf{k}\sigma} + \frac{1}{2} \sum_{\mathbf{k}, \sigma} (\Delta e^{ir\phi} f_{\mathbf{k}+r\mathbf{Q}, \sigma}^\dagger f_{\mathbf{k}, \sigma} + \text{H.c.}), \quad (5)$$

where $f_{\mathbf{k}\sigma}^\dagger$ creates and $f_{\mathbf{k}\sigma}$ annihilates an electron with momentum $\mathbf{k}$ and spin σ , and $\phi = \phi_0 - \Omega t$. We further assume $\hbar\Omega \ll \Delta$, so that the sliding motion remains within the same low-energy two-branch description. If $\mathbf{k}$ lies on the $r = \pm 1$ branch, then $\mathbf{k} + r\mathbf{Q}$ lies on the opposite branch. Introducing the spinor field $\psi_{\mathbf{k}\sigma} = (f_{\mathbf{k}, \sigma}, f_{\mathbf{k}+r\mathbf{Q}, \sigma})^T$, the Hamiltonian takes the compact form

$$H_{\text{CDW}} = \frac{1}{2} \sum_{\mathbf{k}, \sigma} \psi_{\mathbf{k}\sigma}^\dagger H_{\mathbf{k}} \psi_{\mathbf{k}\sigma}, \quad (6)$$

with the 2×2 matrix

$$H_{\mathbf{k}} = \begin{bmatrix} \varepsilon_{\mathbf{k}} & \Delta e^{-ir\phi} \\ \Delta e^{ir\phi} & \varepsilon_{\mathbf{k}+r\mathbf{Q}} \end{bmatrix}. \quad (7)$$

In the static limit $\phi(t) = \phi_0$, one recovers the usual gapped two-branch dispersion. For a sliding CDW, however, the explicit time dependence of $H_{\mathbf{k}}$ prevents a direct static diagonalization. We therefore solve the time-dependent Schrödinger equation using the single-particle Floquet Hamiltonian [21–23]

$$H_{\mathbf{k}}^F = H_{\mathbf{k}} - i\hbar \frac{\partial}{\partial t}. \quad (8)$$

The Floquet Hamiltonian $H_{\mathbf{k}}^F$ admits exact diagonalization by the time-dependent unitary matrix

$$S_{\mathbf{k}}(t) = \begin{bmatrix} e^{-in\Omega t} & 0 \\ 0 & e^{-i(n+r)\Omega t} \end{bmatrix} \begin{bmatrix} u_{\mathbf{k}+} & u_{\mathbf{k}-} \\ v_{\mathbf{k}+} & v_{\mathbf{k}-} \end{bmatrix}, \quad (9)$$

where the eigenvectors of the static effective problem are

$$\begin{pmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{pmatrix}_+ = \begin{bmatrix} \cos \frac{\theta_{\mathbf{k}}}{2} \\ \sin \frac{\theta_{\mathbf{k}}}{2} e^{ir\phi_0} \end{bmatrix}, \quad \begin{pmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{pmatrix}_- = \begin{bmatrix} -\sin \frac{\theta_{\mathbf{k}}}{2} \\ \cos \frac{\theta_{\mathbf{k}}}{2} e^{ir\phi_0} \end{bmatrix}. \quad (10)$$

The two labels “+” and “−” refer to the sign of $\text{sgn}(\varepsilon_{\mathbf{k}} - \varepsilon_F) = \pm 1$; the partner state at $\mathbf{k} + r\mathbf{Q}$ carries the opposite sign. Diagonalization yields

$$S_{\mathbf{k}}(t)^\dagger H_{\mathbf{k}}^F S_{\mathbf{k}}(t) = \varepsilon_{\mathbf{k}} \sigma_3 + \left[a_{\mathbf{k}} + \left(n + \frac{r}{2} \right) \hbar\Omega \right] \sigma_0. \quad (11)$$

A detailed derivation of this exact two-level Floquet diagonalization is given in Appendix A. Here, we have introduced

$$a_{\mathbf{k}} = a_{\mathbf{k}+r\mathbf{Q}} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+r\mathbf{Q}}}{2}, \quad (12a)$$

$$\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{k}+r\mathbf{Q}} = \sqrt{\left(\frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+r\mathbf{Q}} - r\hbar\Omega}{2} \right)^2 + \Delta^2}, \quad (12b)$$

$$\cos \theta_{\mathbf{k}} = -\cos \theta_{\mathbf{k}+r\mathbf{Q}} = \frac{\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+r\mathbf{Q}} - r\hbar\Omega}{2\varepsilon_{\mathbf{k}}}. \quad (12c)$$

The quasienergies—the eigenvalues of $H_{\mathbf{k}}^F$ —are

$$E_{\mathbf{k}n} = \frac{\varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}+r\mathbf{Q}}}{2} + \left(n + \frac{r}{2} \right) \hbar\Omega + \text{sgn}(\varepsilon_{\mathbf{k}} - \varepsilon_F) \varepsilon_{\mathbf{k}}, \quad (13)$$

where n is an integer indexing the Floquet sideband. The corresponding eigenstates are

$$|\mathbf{k}, n\rangle = e^{-in\Omega t} (u_{\mathbf{k}} |\mathbf{k}\rangle + v_{\mathbf{k}} e^{-ir\Omega t} |\mathbf{k} + r\mathbf{Q}\rangle). \quad (14)$$

Equivalently, defining the time-dependent creation operator

$$d_{\mathbf{k}\sigma}^\dagger = u_{\mathbf{k}} f_{\mathbf{k}\sigma}^\dagger + v_{\mathbf{k}} e^{-ir\Omega t} f_{\mathbf{k}+r\mathbf{Q}, \sigma}^\dagger, \quad (15)$$

the operators $d_{\mathbf{k}\sigma}^\dagger$ create Floquet quasiparticles with quasienergies $E_{\mathbf{k}0}$. This exact solution provides the full Floquet spectrum that underlies the qualitative sideband picture sketched in Ref. [14].

C. Green's functions and Floquet spectral functions

For a Hamiltonian periodic in time with period $2\pi/\Omega$, the double-time Green's function satisfies $F(t_1, t_2) = F(t_1 + 2\pi/\Omega, t_2 + 2\pi/\Omega)$. In Wigner coordinates $\tau = (t_1 + t_2)/2$ and $t = t_1 - t_2$, its Floquet representation is

$$F(t_1, t_2) = \sum_n \int \frac{d\omega}{2\pi} e^{-i\omega t} e^{in\Omega\tau} F(\omega, n), \quad (16)$$

where $t_1 = \tau + t/2$ and $t_2 = \tau - t/2$.

For the sliding CDW, the retarded and advanced Green's functions [24] in the Floquet basis are

$$g_{\mathbf{k}}^{R/A}(\omega, n) = \frac{1}{\omega \pm i0^+ - E_{\mathbf{k}n}}, \quad (17)$$

where Eq. (14) defines the Floquet eigenbasis. The integral Floquet spectral function [25] is

$$A_n(E) := -\frac{1}{\pi} \text{Im} \sum_{\mathbf{k}} g_{\mathbf{k}}^R(E, n) = A_0(E - n\hbar\Omega), \quad (18a)$$

where $A_0(E)$ is the time-averaged density of states (DOS), derived in Appendix B:

$$A_0(E) = \frac{1}{2} \left[D\left(E + \frac{\hbar\Omega}{2}\right) + D\left(E - \frac{\hbar\Omega}{2}\right) \right], \quad (18b)$$

$$D(E) = \frac{N(0)|E - \varepsilon_F|}{\sqrt{(E - \varepsilon_F)^2 - \Delta^2}} \theta(|E - \varepsilon_F| - \Delta).$$

Here, $D(E)$ is the DOS of the static CDW state with $\Omega = 0$ and $N(0) = (\pi \hbar v_F)^{-1}$ is the Fermi-level DOS of the noninteracting band. Figure 3 illustrates how the static square-root edges split by $\pm \hbar\Omega/2$ once the CDW slides, generating the Floquet sideband structure central to the transport phenomenology developed below.

The exact Floquet solution just obtained fixes the spectrum of the isolated sliding CDW. Once metallic contacts are attached, however, one must distinguish this diagonal Floquet basis from the physical electron basis created by $\{f_{\mathbf{k}\sigma}^\dagger\}$, in which the tunneling Hamiltonian is time independent. We therefore use the exact solution as the building block for the physical-basis Green's functions: $\tilde{\mathbf{g}}$ for the isolated sliding CDW and $\tilde{\mathbf{G}}$ for the contacted system. The explicit basis transformation is summarized in Appendix C.

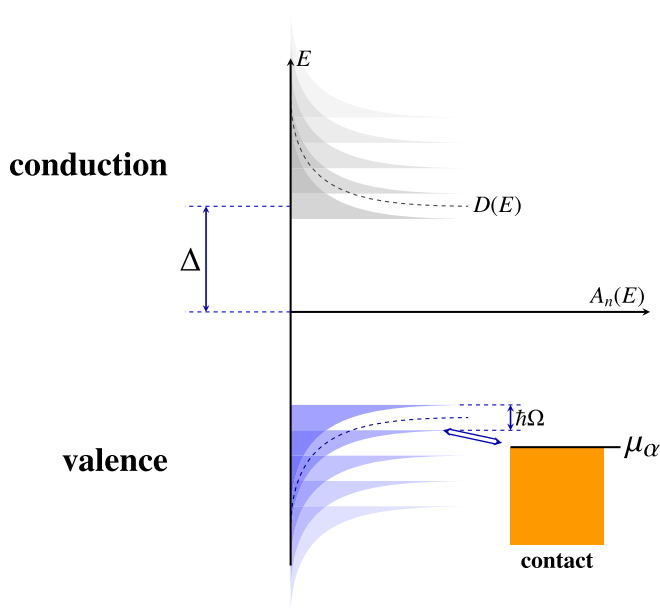


FIG. 3. Integral Floquet spectral function $A_n(E)$ of the sliding CDW. The dashed curve shows the density of states $D(E)$ of the corresponding static CDW state. Once the CDW slides, the square-root gap edges split by $\pm\hbar\Omega/2$ and form a ladder of sidebands separated by $\hbar\Omega$. The double arrow indicates tunneling between the CDW and a metallic contact at chemical potential μ_α .

III. SINGLE-CONTACT TUNNELING SPECTROSCOPY

We now develop the simplest measurement geometry in which the Floquet sideband ladder of Sec. II can be directly observed: weak single-contact tunneling spectroscopy. A metallic probe P is tunnel coupled to the sliding CDW, while the sliding steady state itself is maintained by the external dc drive and intrinsic relaxation of the CDW elsewhere in the device. This is spectroscopy rather than two-terminal transport: The probe samples the local Floquet spectrum but does not set the conserved through-current. The geometry is sketched in Fig. 4. While Ref. [14] briefly discussed sideband-assisted tunneling at the contact, we provide here the complete nonequilibrium formulation and use it to deduce the physical

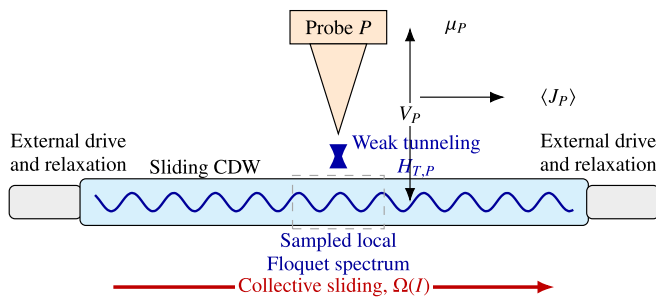


FIG. 4. Weak single-contact tunneling geometry. A metallic probe P is weakly tunnel coupled to a local region of the sliding CDW and biased by V_P relative to the local CDW potential. The probe current $\langle J_P \rangle$ samples the local Floquet spectrum, while the sliding steady state and frequency $\Omega(I)$ are maintained elsewhere by the external dc drive and intrinsic relaxation of the CDW.

nature of the sliding state through quantitative matching to experiment.

A. Setup and time-averaged probe current

The full Hamiltonian for this setup is

$$H = H_{\text{CDW}} + H_P + H_{T,P}, \quad (19a)$$

$$H_P = \sum_k \epsilon_{kP} c_{kP}^\dagger c_{kP}, \quad H_{T,P} = \sum_{k,p,\sigma} V_{kP,p} c_{kP}^\dagger f_{p\sigma} + \text{H.c.}, \quad (19b)$$

where p labels physical single-particle states in the CDW region. Because $H_{T,P}$ is time independent in the f basis, the probe self-energy is stationary and depends only on the relative time $t_1 - t_2$ (see Appendix E). The time-averaged probe current then reads

$$\langle J_P \rangle = -\frac{e}{\hbar} \text{Im} \int \frac{d\omega}{2\pi} \text{tr} \{ \Gamma^P(\omega) [\tilde{\mathbf{G}}^<(\omega, 0) + 2f_P(\omega) \tilde{\mathbf{G}}^R(\omega, 0)] \}, \quad (20)$$

where $\Gamma^P(\omega) = i[\Sigma_P^R(\omega) - \Sigma_P^A(\omega)]$. The time-averaging steps that reduce the double-time current formula to this stationary-contact expression are summarized in Appendix F. In the weak-probe limit $\tilde{\mathbf{G}} \simeq \tilde{\mathbf{g}}$, so the voltage dependence is controlled by the time-averaged local spectral function

$$A_{\text{loc}}^{\text{ave}}(\omega) = -\frac{1}{\pi} \text{Im} \text{tr} \tilde{\mathbf{g}}_{\text{loc}}^R(\omega, 0). \quad (21)$$

The nonequilibrium occupation of the sliding CDW enters through $\tilde{\mathbf{g}}^<$, whereas the singular structures in $d\langle J_P \rangle/dV_P$ are governed by Eq. (21). Thus, weak-contact tunneling spectroscopy directly probes the split gap edges and Floquet sidebands generated by the sliding CDW.

B. Near-edge model and quantitative comparison with experiment

To connect most directly with experiment, we evaluate a near-edge model in which Eq. (21) is represented by a thermally broadened, Dynes-regularized split-edge envelope in a narrow window around the first positive gap edge. The sliding CDW sustains a macroscopic longitudinal voltage drop, so the local electrostatic potential acts as a rigid reference for the local CDW bands. The applied probe voltage V_P sets the probe chemical potential μ_P relative to this local environment. For the $(\text{TaSe}_4)_2\text{I}$ nanowire data of Ref. [14], the measured device voltage varies only weakly with current once the system is above depinning. As a first approximation, we therefore treat the local probe offset $\epsilon = \mu_P - E_v$ (where E_v is the local lower gap edge) as slowly varying while Ω continues to increase linearly with the imposed through-current via Eq. (4). Appendix G gives the explicit formulas for the broadened single-contact spectra and the inverse-current oscillatory contribution.

C. Effective coherent channel scale and percolating interpretation

The experimental period provides a direct constraint on the coherent current-to-frequency conversion. Using

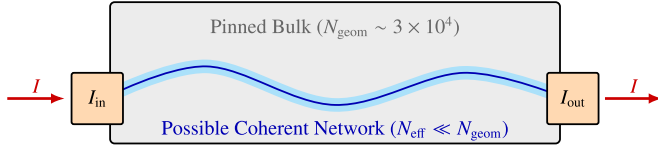


FIG. 5. Schematic percolating realization of the reduced coherent channel scale. The model-consistent N_{eff} range is far below the geometric channel estimate $N_{\text{geom}} \sim 3 \times 10^4$. The highlighted path represents coherently connected chain segments and should not be interpreted as N_{eff} independent end-to-end metallic wires or as a uniquely determined microscopic geometry.

$\Delta(1/I) = 0.00269 \mu\text{A}^{-1}$ from Ref. [14],

$$N_{\text{eff}}\epsilon = \frac{h}{2e \Delta(1/I)} \simeq 4.80 \text{ eV}. \quad (22)$$

Thus, the period alone determines the product $N_{\text{eff}}\epsilon$, not the two factors separately. The reference value $\epsilon = 10 \text{ meV}$ gives $N_{\text{eff}} \simeq 4.8 \times 10^2$ and $\hbar\Omega \simeq 0.54 \text{ meV}$ at $I = 20 \mu\text{A}$, while $\epsilon = 5\text{--}20 \text{ meV}$ gives $N_{\text{eff}} \simeq 960\text{--}240$.

The temperature-dependent visibility supplies an additional experimental constraint within the Floquet interpretation. At $I = 20 \mu\text{A}$, the measured period implies

$$\hbar\Omega [\text{meV}] = 0.0538 \epsilon [\text{meV}]. \quad (23)$$

The sideband spacing must be at least comparable to the intrinsic and thermal broadening for a resolved low-temperature sequence to remain visible. Comparing with $\gamma = 0.30 \text{ meV}$ and $3.5k_B T = 0.47 \text{ meV}$ at 1.55 K gives the conservative scale $\epsilon \gtrsim 6\text{--}9 \text{ meV}$ and hence $N_{\text{eff}} \lesssim 5 \times 10^2$ to 8×10^2 . At the opposite extreme, the near-edge construction requires the contact offset to remain within the CDW gap scale, $\epsilon \lesssim \Delta = 165 \text{ meV}$, giving $N_{\text{eff}} \gtrsim 29$. The resulting model-consistent range is therefore roughly $30 \lesssim N_{\text{eff}} \lesssim 800$, with $N_{\text{eff}} \simeq 480$ retained as a representative value rather than a parameter-free determination.

Typical device dimensions [14] yield a geometric chain count $N_{\text{geom}} \sim 3 \times 10^4$, far above the range inferred above. Taking the full geometric count would require $\epsilon \simeq 0.16 \text{ meV}$ and $\hbar\Omega(20 \mu\text{A}) \simeq 8.6 \times 10^{-3} \text{ meV}$, far below both the base-temperature thermal width and the phenomenological broadening. The experimentally relevant signal therefore cannot be governed by all geometric chains participating coherently and uniformly within this mechanism.

Several physical mechanisms can produce a reduced effective coherent scale. Surface-dominated tunneling may preferentially sample outer CDW chains; depinning may proceed through transverse domains; and a finite transverse phase-correlation length may limit the number of chain segments locked to the same sliding phase. Interchain transfer and phase slips can connect those segments into a percolating coherent network such as the schematic realization in Fig. 5. Distinguishing among these possibilities requires spatially resolved probes beyond the present theory.

The role of the reduced N_{eff} is kinematic: It permits the measured current to generate a Floquet spacing above the relevant linewidth. It should not be interpreted as squeezing an Ohmic current through a metallic area proportional to N_{eff} . Heating depends instead on collective damping and on where

phase-slip and current-conversion processes dissipate energy. A quantitative thermal consistency analysis is given in Sec. V.

D. Fixed-bias inverse-current oscillations

At fixed bias offset $\epsilon = eV - E_v$, successive sideband edges of the Floquet ladder cross the probe chemical potential whenever

$$\epsilon \simeq (m + \tfrac{1}{2})\hbar\Omega(I_m), \quad m = 0, 1, 2, \dots, \quad (24)$$

so that $1/I_m$ is linear in m . This mechanism provides a striking temporal analog of the Shubnikov–de Haas effect. In conventional magnetic quantum oscillations, periodicity in $1/B$ arises because Landau levels with energy spacing $\hbar\omega_c \propto B$ successively cross a fixed Fermi energy. Here, periodicity in $1/I$ arises because Floquet sidebands with energy spacing $\hbar\Omega \propto I$ successively cross a fixed contact mismatch energy ϵ . Figure 6 shows the resulting fixed-bias inverse-current oscillation evaluated from the near-edge model of Appendix G.

The same near-edge construction also yields the fixed-current cuts of $G(V, I)$ near the singular edge structure shown in Fig. 7. The single-contact spectra and the inverse-current oscillation are therefore complementary cuts of the same Floquet sideband ladder. Reference [14] pointed out the basic periodicity relation $\epsilon = (m + \tfrac{1}{2})\hbar\Omega(I_m)$; the present analysis additionally shows that the same ladder controls the full near-edge spectroscopy at fixed current and constrains the effective coherent channel scale through the period and resolvability.

IV. TRANSPORT THROUGH CONTACTED SLIDING-CDW DEVICES

Section III established that a weak single contact directly probes the local Floquet spectrum. The experiment of Ref. [14], however, employs a persistent current source and measures terminal-pair voltages across multiple voltage probes simultaneously. The macroscopic depinned sliding state must therefore be analyzed within a genuine multiterminal setting, where one common through-current coexists with terminal-dependent oscillation visibility. The principal puzzle is that the inner-terminal voltage $V_{2-3}^{1-4}(I)$ exhibits a clean $1/I$ oscillation, whereas the raw outer-terminal voltage $V_{1-4}^{1-4}(I)$ reveals the same kernel only after smooth-background subtraction. Reference [14] reported this visibility hierarchy but did not provide a transport-level interpretation. We now develop such an interpretation through a minimal segmented multiterminal model (Sec. IV A), and we contrast it with a homogeneous two-terminal Floquet–Landauer reference limit (Sec. IV B and Appendix I) that helps identify which features depend on spatial inhomogeneity.

A. Current-driven segmented multiterminal model

The directly relevant experimental observable is a current-driven terminal-pair voltage $V_{p-q}^{1-4}(I)$ in a multiterminal geometry [26], rather than the differential conductance of a spatially uniform voltage-biased span. Because Ref. [14] uses a persistent current source, the steady-state control variable is the common through-current and the terminal voltages are measured responses; throughout this section, “current driven”

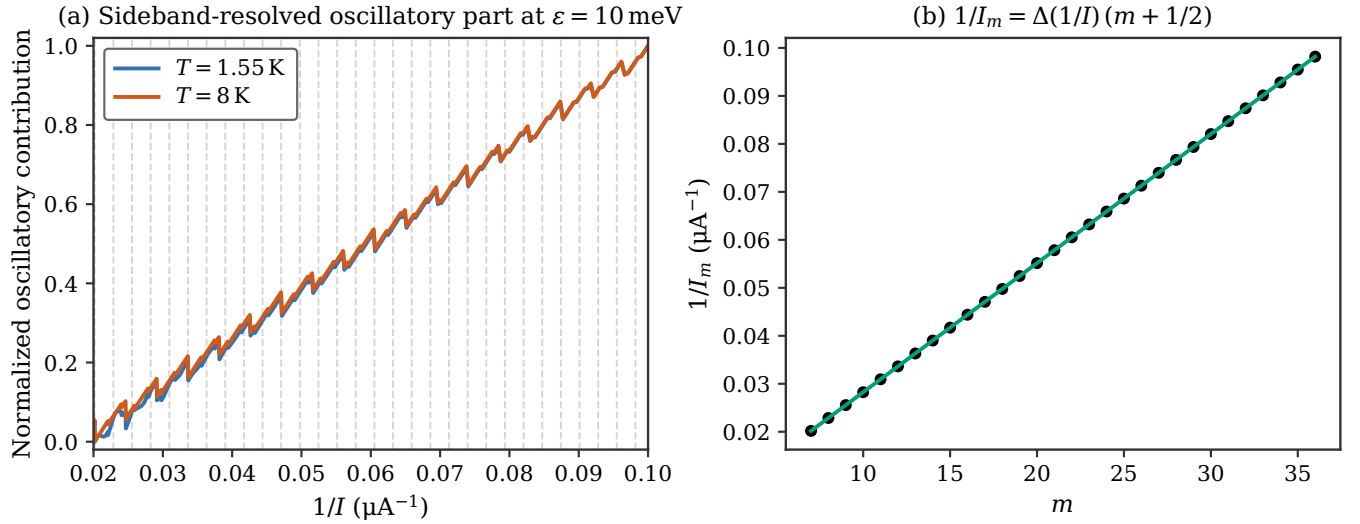


FIG. 6. Fixed-bias inverse-current oscillations generated by the Floquet sideband ladder. (a) Individually normalized oscillatory contribution $\delta G_{\text{osc}}(I, \epsilon; T)$ at the reference offset $\epsilon = 10$ meV as a function of $1/I$. (b) Corresponding crossing positions, which obey $1/I_m = \Delta(1/I)(m + 1/2)$, with $\Delta(1/I) = 0.00269 \mu\text{A}^{-1}$, matching the experimental period of Ref. [14]. The oscillatory contribution δG_{osc} is evaluated from Eq. (G8).

is used in this sense. Motivated by this geometry, we introduce a minimal segmented description in which the current path is decomposed into three series segments 1–2, 2–3, and 3–4, all carrying the same imposed current. The coherent Floquet oscillatory kernel is assumed to be generated mainly in the inner segment 2–3 of the active percolating network discussed in Sec. III. The measurement logic was already sketched in Fig. 2.

In this minimal version, the outer segments are effectively dephased and contribute only smooth series voltages. Physically, this loss of coherence is a direct consequence of the finite phase-coherence length of the device combined with strong inelastic scattering near the current-injecting contacts. To convert the collective sliding CDW current into a normal electron current at the outer metallic leads, dissipative

phase-slip processes must occur, locally destroying the uniform Floquet phase coherence [27]. Consequently, on length scales larger than the phase-coherence length, the active network behaves as a classical series combination of coherent and incoherent subregions [28], with the strongly dephased outer segments yielding only smooth macroscopic voltage drops. Then, $V_{2-3}^{1-4}(I)$ retains a clear $1/I$ oscillation, whereas $V_{1-4}^{1-4}(I)$ is dominated by smooth background drops and reveals the same kernel only after background subtraction. Figure 8 shows this visibility effect explicitly. Appendix H gives the explicit effective construction.

This effective construction is neither intended to reproduce the full nonlinear waveform of the experiment nor the precise amplitude. Rather, it isolates how a single common current can coexist with strongly terminal-dependent oscillation

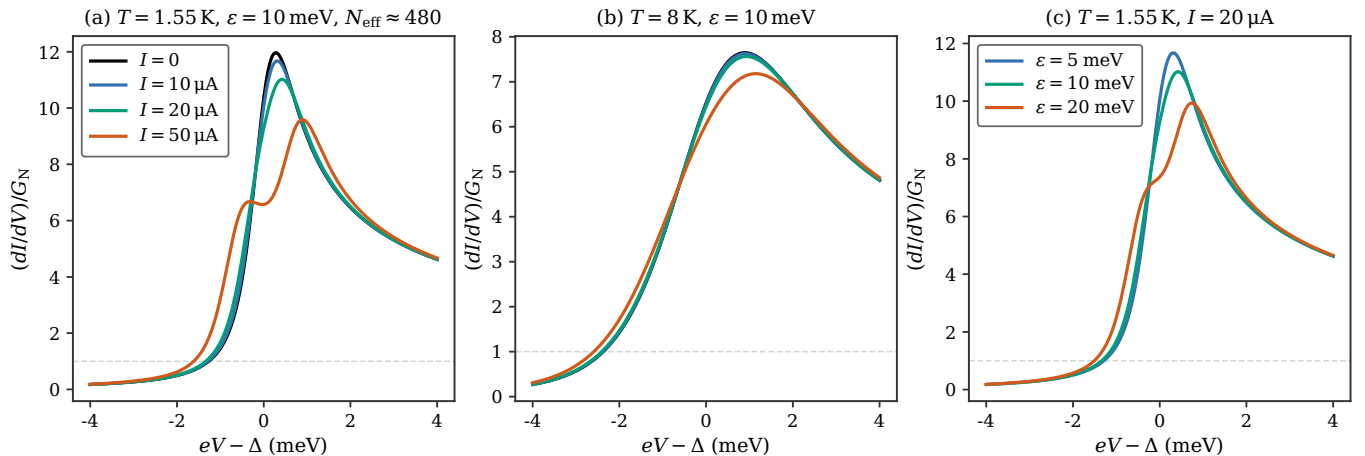


FIG. 7. Near-edge model calculation of single-contact tunneling spectra near the positive CDW gap edge, obtained by thermally convolving a Dynes-regularized split-edge envelope motivated by Eq. (21) with broadening $\gamma = 0.30$ meV. (a) Reference low-temperature spectra at $T = 1.55$ K for $\epsilon = 10$ meV, corresponding to $N_{\text{eff}} \simeq 4.8 \times 10^2$. (b) Same parameter set at $T = 8$ K. (c) Sensitivity at $T = 1.55$ K and $I = 20 \mu\text{A}$ for $\epsilon = 5, 10, 20$ meV, corresponding to $N_{\text{eff}} \simeq 960, 480, 240$. Only the positive-bias edge is shown.

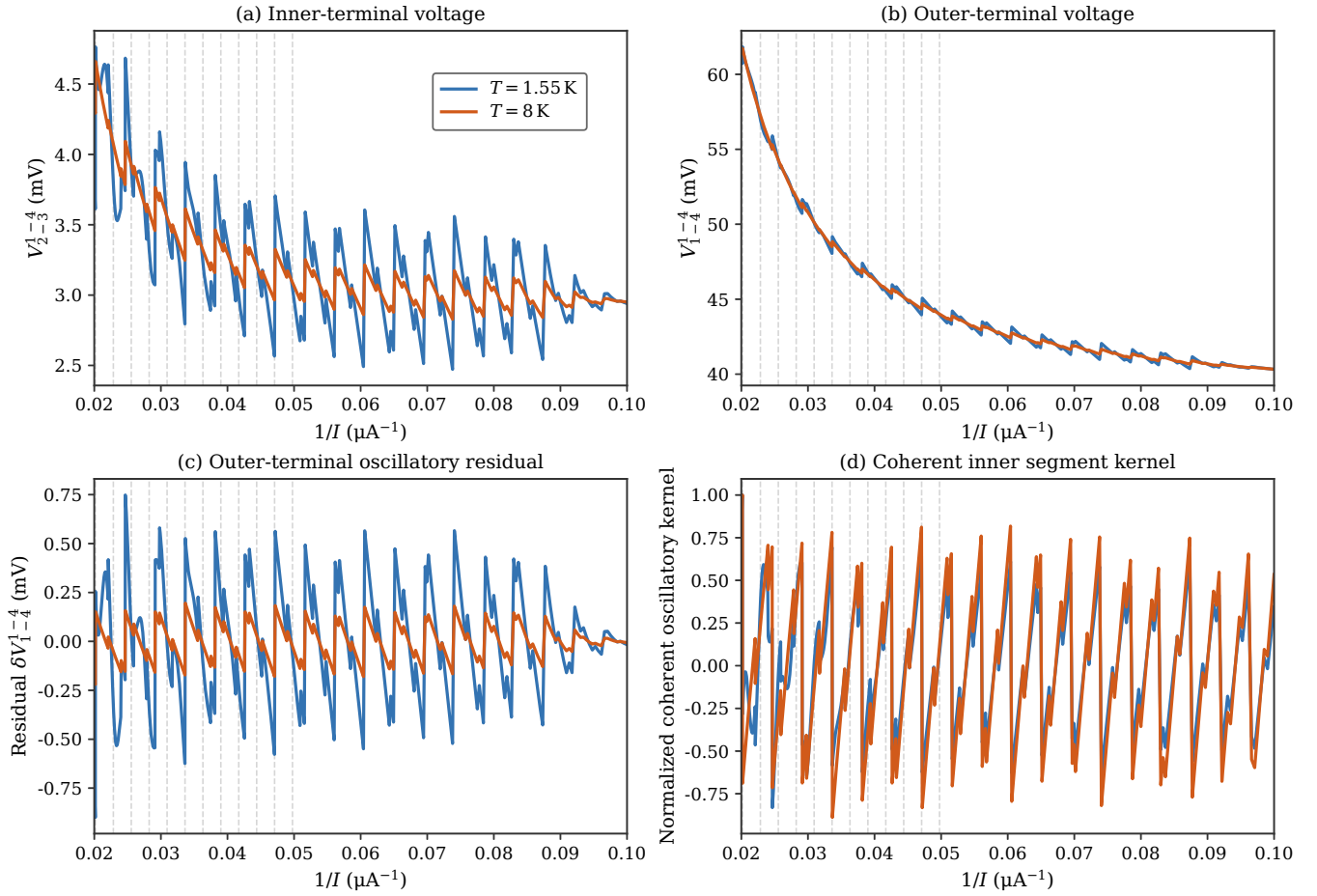


FIG. 8. Minimal current-driven segmented model for the experimental measurement geometry of Ref. [14] with current driven between terminals 1 and 4. (a) Inner-terminal voltage $V_{2-3}^{1-4}(I)$ vs $1/I$. (b) Raw outer-terminal voltage $V_{1-4}^{1-4}(I)$ vs $1/I$. (c) Oscillatory residual obtained after subtracting the smooth background from panel (b). (d) Normalized coherent kernel $\mathcal{O}_{\text{osc}}(I; T)$. In this minimal visibility model, the coherent response is concentrated in segment 2–3, while segments 1–2 and 3–4 contribute mainly smooth series voltages due to phase-slip dephasing near the contacts. Explicit definitions are given by Eqs. (H1)–(H3).

visibility through the interplay of coherent (inner) and dephased (outer) segments. The qualitative agreement with the experimentally observed hierarchy supports the underlying physical picture: Phase-slip-induced dephasing localized near the contacts, combined with a coherent network across the inner span, naturally produces the observed visibility pattern.

B. Homogeneous two-terminal reference limit

For orientation, it is useful to compare the segmented multiterminal model with the homogeneous two-terminal reference problem shown in Fig. 1. The measurement logic of the segmented multiterminal model is sketched in Fig. 2. Consider a finite sliding-CDW region $H_D(t)$ coupled to left and right reservoirs. In contrast to the weak-probe setup of Sec. III and the segmented construction above, here the dc current and the nonequilibrium distribution are determined by the contacts themselves. In the physical electron basis, the lead self-energies are stationary, while the periodicity resides in the isolated device Green's function $\tilde{\mathbf{g}}$. For a noninteracting

device in the periodic steady state [29–31], the contacted Green's functions satisfy

$$\tilde{\mathbf{G}}^{R/A} = \tilde{\mathbf{g}}^{R/A} + \tilde{\mathbf{g}}^{R/A} (\Sigma_L^{R/A} + \Sigma_R^{R/A}) \tilde{\mathbf{G}}^{R/A}, \quad (25)$$

$$\tilde{\mathbf{G}}^< = \tilde{\mathbf{G}}^R (\Sigma_L^< + \Sigma_R^<) \tilde{\mathbf{G}}^A, \quad (26)$$

where products denote matrix multiplication in the device space together with the time convolutions appropriate for the double-time Green's functions. Appendix D summarizes the corresponding Schwinger-Keldysh Dyson structure in the physical electron basis. The time-averaged current through the left contact is

$$\langle J_L \rangle = -\frac{e}{\hbar} \text{Im} \int \frac{d\omega}{2\pi} \text{tr} \{ \Gamma^L(\omega) [\tilde{\mathbf{G}}^<(\omega, 0) + 2f_L(\omega) \tilde{\mathbf{G}}^R(\omega, 0)] \}, \quad (27)$$

with an analogous expression for $\langle J_R \rangle$ and charge conservation $\langle J_L \rangle = -\langle J_R \rangle$ in the steady state.

These equations define the homogeneous periodically driven two-terminal problem: The sliding CDW contributes sideband-assisted transmission channels with exchanged energies $n\hbar\Omega$, while the nonequilibrium distribution is fixed by

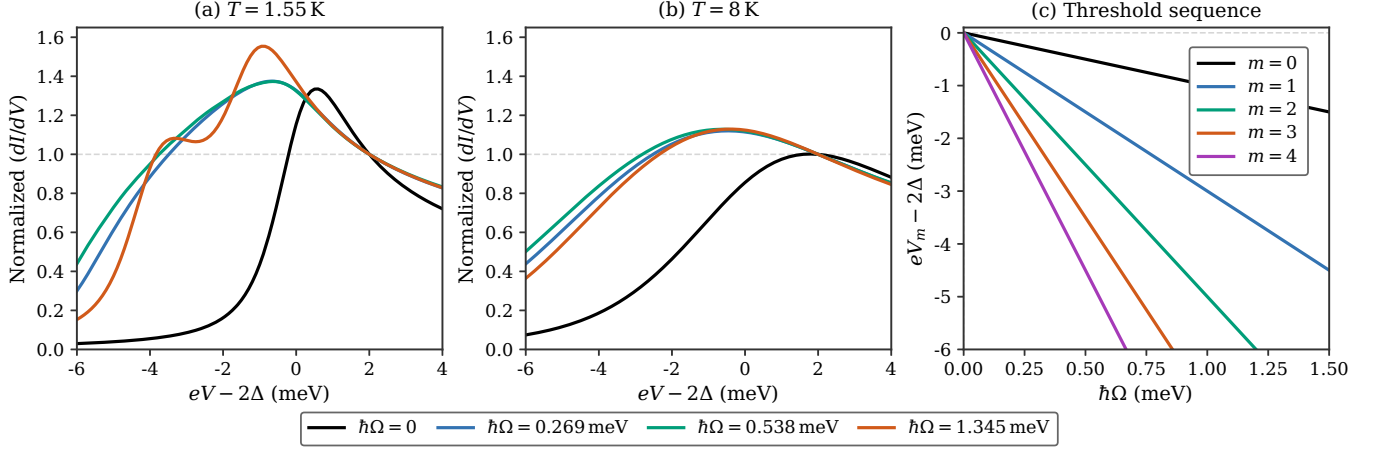


FIG. 9. Homogeneous voltage-biased reference calculation for a uniformly coherent sliding span at fixed $\hbar\Omega$. (a), (b) Normalized dI/dV traces as functions of $eV - 2\Delta$ for $\hbar\Omega = 0, 0.269, 0.538$, and 1.345 meV, at $T = 1.55$ and 8 K, respectively. Each curve is normalized by its value at $eV - 2\Delta = 2$ meV to emphasize the threshold shifts. (c) Threshold sequence $eV_m = 2\Delta - (2m + 1)\hbar\Omega$.

the two reservoirs. A fully self-consistent nonlinear $I - V$ curve would additionally require a microscopic relation between the through-current, the depinned sliding frequency, and the spatial distribution of contact voltage drops. As a reference transport calculation, we therefore keep $\hbar\Omega$ fixed and evaluate the near-threshold differential conductance around $eV \simeq 2\Delta$, where the response is controlled by the singular Floquet gap edges. The inward sideband thresholds occur at

$$eV_m \simeq 2\Delta - (2m + 1)\hbar\Omega, \quad m = 0, 1, 2, \dots, \quad (28)$$

so the response shifts linearly with $\hbar\Omega$.

Figure 9 shows the resulting fixed- $\hbar\Omega$ near-threshold conductance for the same realistic gap and broadening scales used in the single-contact estimate. This homogeneous calculation identifies the threshold structure expected from a uniformly coherent sliding span under voltage bias, but it is not the persistent-current-driven multiterminal experiment of Ref. [14]. In particular, it tracks threshold motion at fixed $\hbar\Omega$ rather than the fixed-offset sideband crossings that generate the observed $1/I$ sequence. The homogeneous two-terminal limit thus serves as a useful theoretical reference for identifying universal features of the Floquet sideband ladder under different bias conditions, but the experimentally relevant signal requires the segmented current-driven analysis of Sec. IV A. Appendix I gives the explicit near-threshold model used to evaluate Fig. 9.

V. DISCUSSION AND OUTLOOK

The exact sliding-CDW Floquet solution of Sec. II generates a single sideband ladder, but different measurements project that ladder onto different observables. A weak single contact reads it as local spectroscopy (Sec. III); once the depinned state is entered and the local contact offset varies only weakly with current, the same ladder produces a fixed-bias $1/I$ sequence through repeated sideband crossings. A persistent-current-driven multiterminal device reads it through terminal-dependent voltage drops (Sec. IV A), whereas a homogeneous voltage-biased span reads it as

threshold shifts in the (V, Ω) plane (Sec. IV B). The experiment of Ref. [14] belongs to the multiterminal category, so spatial inhomogeneity and contact dephasing are essential for understanding why the inner-terminal signal exhibits a much stronger $1/I$ oscillation than the raw outer-terminal voltage.

A. Effective coherent network and thermal consistency

The parameter N_{eff} is defined by the collective relation $I = N_{\text{eff}} e\Omega/\pi$. It counts the chain-equivalent degrees of freedom locked to the coherent current-to-frequency conversion, rather than independent end-to-end Ohmic wires. Combining the experimental period and low-temperature resolvability with the near-edge construction gives the model-consistent range $30 \lesssim N_{\text{eff}} \lesssim 800$, far below $N_{\text{geom}} \sim 3 \times 10^4$. This reduction implies that only a restricted subset of the geometric cross section contributes coherently to the observed Floquet signal. A percolating network is a natural interpretation, but its precise spatial structure is not fixed by the present data.

This interpretation leads to several tests. The extracted N_{eff} should depend more strongly on contact geometry, surface treatment, disorder, and transverse phase correlation than on the full geometric cross section. Spatially resolved probes such as scanning tunneling microscopy or microwave near-field imaging could determine whether the coherent response is filamentary, surface dominated, or distributed over disconnected transverse domains.

The thermal consistency of this interpretation requires separating collective current from ordinary Ohmic flow. In a nearly undamped Fröhlich state, the local dissipated power is controlled by the viscous phase-damping coefficient η_ϕ ,

$$p_\phi(\mathbf{r}) = \eta_\phi(\mathbf{r}) \dot{\phi}^2 = \eta_\phi(\mathbf{r}) \Omega^2, \quad (29)$$

and by the spatial distribution of phase slips and normal-current conversion. The total electrical power is ultimately dissipated, but it need not be deposited entirely inside the coherent part of the network. Residual collective energy is transferred efficiently to lattice vibrations by the electron-phonon coupling intrinsic to the CDW state. Those phonons

propagate through the electrically insulating crystal and carry heat to the substrate, metallic electrodes, and external bath. A minimal steady-state description is

$$-\nabla \times [\kappa_{\text{ph}} \nabla T(\mathbf{r})] + g_{\text{bath}}[T(\mathbf{r}) - T_b] = p_\phi(\mathbf{r}), \quad (30)$$

where κ_{ph} is the phonon thermal conductivity and g_{bath} represents heat removal through the substrate and electrodes.

A conservative consistency estimate follows directly from Ref. [14]. Near $I = 20 \mu\text{A}$, the inner-span voltage of device D3 is of order 1 V, giving $P_{\text{span}} = IV_{23} \sim 20 \mu\text{W}$. This is an upper bound on power deposited in the coherent region because the measured voltage includes phase-slip, conversion, and smooth background drops. Even if all of P_{span} were deposited locally, keeping the active region below the strong-smearing scale $T_\phi \sim 9 \text{ K}$ would require

$$R_{\text{th}}^{\text{eff}} \lesssim \frac{T_\phi - T_b}{P_{\text{span}}} \simeq 0.4 \text{ MK/W}, \quad G_{\text{th}}^{\text{eff}} \gtrsim 2.5 \mu\text{W/K}. \quad (31)$$

A peer-reviewed thermal analysis of comparable $(\text{TaSe}_4)_2\text{I}$ nanowires on an oxide substrate found a total thermal resistance of order 0.4 MK/W after including oxide and interface contributions, with conduction toward metallic contacts supplying an additional cooling channel [32]. Because that estimate concerns a different device and temperature range, it is only an order-of-magnitude benchmark, not a low-temperature fit to Ref. [14]. The experimental absence of recognizable hysteresis in D3 up to $20 \mu\text{A}$, followed by heating-related deviations above several tens of microamperes in other devices, is consistent with this thermal pathway. A device-specific temperature rise would require low-temperature interface conductance and local voltage-drop data not presently available.

B. Open puzzles and limitations

Several experimental features of Ref. [14] remain outside the present treatment and deserve future attention. First, the oscillation reentrance at elevated temperatures observed in device D3 (Appendix G of Ref. [14]), where a second oscillation sequence with a different period appears above the suppression of the low-temperature sequence, is not captured by the present framework. This reentrance suggests temperature-driven reorganization of the active coherent network or of the effective contact offset ϵ , both of which would require a more detailed microscopic model of the depinning dynamics. Second, our theory predicts a phase shift $b = 1/2$ for the linear $1/I_m$ versus m relation, while the experimental fits report device-dependent values b ranging from approximately -0.16 to 0.30 , with deviations more pronounced at high currents where heating effects also distort the periodicity. Reconciling this systematic discrepancy likely requires accounting for sideband-dependent matrix elements, broadened edge profiles, or weak current-induced renormalization of the contact offset. Third, the slight shifts of oscillation positions in transverse magnetic field observed on certain devices remain unexplained within the present framework.

Beyond these specific puzzles, our segmented current-driven model is intended as a minimal transport interpretation of the visibility hierarchy along the coherent network, not as a quantitative fit to the oscillation amplitude or to the

full sawtooth waveform of the experiment. A future material-specific microscopic theory would have to self-consistently determine how the imposed persistent current sets the local voltage drops, how phase-slip conversion near the contacts [27] feeds back on the sliding frequency, and how additional non-CDW transport channels modify the measured terminal voltages. The present framework provides a transport-level scaffold on which such material-specific microscopic theories can be built.

C. Broader implications

Within these limitations, the sequence of exact Floquet solution, weak-probe spectroscopy, segmented current-driven modeling, and homogeneous transport reference provides a coherent theoretical framework for the observed inverse-current oscillations. More broadly, resolvable sidebands require the total collective and quasiparticle linewidth to obey

$$\Gamma_\phi + \Gamma_{\text{qp}} \lesssim \hbar\Omega. \quad (32)$$

For a two-dimensional Fermi liquid with a residual Fermi surface, the canonical low-energy quasiparticle decay rate has the form [33–35]

$$\Gamma_{ee}(\xi, T) \sim C\lambda^2 \frac{\xi^2 + (\pi k_B T)^2}{E_F} \ln \left[\frac{E_F}{\max(|\xi|, k_B T)} \right], \quad (33)$$

where C is of order unity and λ is an effective interaction. A quasiparticle near a partially gapped CDW edge can decay by creating particle-hole excitations on the residual Fermi surface, so $\xi \sim \Delta_{2D}$. Representative values $E_F = 0.2\text{--}0.5 \text{ eV}$, $\Delta_{2D} = 20\text{--}50 \text{ meV}$, and $\lambda = 0.5\text{--}1$ give

$$\Gamma_{ee}^{2D} \sim 0.6\text{--}17 \text{ meV}, \quad (34)$$

which is comparable to or larger than the characteristic Floquet spacings $\hbar\Omega \simeq 0.27\text{--}1.35 \text{ meV}$ considered here. A residual Fermi surface therefore strongly disfavors resolvable sidebands, although the precise rate is material dependent. In a fully gapped quasi-one-dimensional CDW insulator, in contrast, the electronic particle-hole contribution is thermally activated, $\Gamma_{ee}^{\text{gapped}} \propto e^{-\Delta/(k_B T)}$, and is negligible for $\Delta \simeq 165 \text{ meV}$ at the temperatures of Ref. [14].

This intrinsic dc-to-ac conversion mechanism offers a compelling design principle for future quantum devices. Highly coherent, tunable high-frequency drives can in principle be generated locally via simple dc currents, without the need for external microwave architecture. The same universal mechanism may apply to other sliding ordered states, including spin-density waves and certain density-wave phases of strongly correlated electron systems. In each case, the basic prerequisite remains the same: a coherent sliding mode protected from low-energy dephasing by an excitation gap. Identifying and engineering such gap-protected coherent sliding states across material platforms is a natural direction for future work.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

All materials associated with this study are publicly available in the GitHub repository *Sliding-CDW Documents for Sliding CDW Theory* [36]. The repository contains the complete, unedited AI-assisted research transcripts used during manuscript preparation and the Python scripts, documentation, and dependency specifications used to generate the numerical results and figures.

APPENDIX A: EXACT SOLUTION OF A GENERAL TWO-LEVEL FLOQUET SYSTEM

The exact diagonalization used in Sec. II is a special case of the following general result. Consider the time-dependent two-level Hamiltonian

$$H = \begin{bmatrix} a & be^{-i(\phi_0 - \Omega t)} \\ be^{i\phi_0} & -a \end{bmatrix}, \quad (\text{A1})$$

with real parameters a , b , Ω , and ϕ_0 . The Floquet eigenvalue problem reads

$$\left(H - i\frac{\partial}{\partial t}\right) \begin{pmatrix} u \\ v \end{pmatrix} = E \begin{pmatrix} u \\ v \end{pmatrix}. \quad (\text{A2})$$

Expanding $u(t) = \sum_n u_n e^{-in\Omega t}$ and $v(t) = \sum_n v_n e^{-in\Omega t}$, each pair (u_n, v_{n+1}) satisfies

$$\begin{bmatrix} a + n\Omega - E & be^{-i\phi_0} \\ be^{i\phi_0} & -a + (n+1)\Omega - E \end{bmatrix} \begin{pmatrix} u_n \\ v_{n+1} \end{pmatrix} = 0. \quad (\text{A3})$$

The problem closes in each (u_n, v_{n+1}) sector and yields the quasienergies

$$E_{n\pm} = \left(n + \frac{1}{2}\right)\Omega \pm \mathcal{E}, \quad \mathcal{E} = \sqrt{\left(a - \frac{\Omega}{2}\right)^2 + b^2}. \quad (\text{A4})$$

The corresponding eigenvectors diagonalize the static effective matrix

$$\begin{bmatrix} a - \frac{\Omega}{2} & be^{-i\phi_0} \\ be^{i\phi_0} & -a + \frac{\Omega}{2} \end{bmatrix} \quad (\text{A5})$$

and are independent of both t and the sideband index n :

$$\begin{pmatrix} u_n \\ v_{n+1} \end{pmatrix}_+ = \begin{bmatrix} \cos \frac{\theta}{2} e^{-i\phi_0} \\ \sin \frac{\theta}{2} \end{bmatrix}, \quad \begin{pmatrix} u_n \\ v_{n+1} \end{pmatrix}_- = \begin{bmatrix} -\sin \frac{\theta}{2} e^{-i\phi_0} \\ \cos \frac{\theta}{2} \end{bmatrix}, \quad (\text{A6})$$

with $\cos \theta = (a - \Omega/2)/\mathcal{E}$. Defining

$$S_n(t) = \begin{bmatrix} e^{-in\Omega t} & 0 \\ 0 & e^{-i(n+1)\Omega t} \end{bmatrix} \begin{bmatrix} u_{n,+} & u_{n,-} \\ v_{n+1,+} & v_{n+1,-} \end{bmatrix}, \quad (\text{A7})$$

one verifies that

$$\left(H - i\frac{\partial}{\partial t}\right) S_n = S_n \left[\mathcal{E} \sigma_3 + \left(n + \frac{1}{2}\right)\Omega \right], \quad (\text{A8})$$

so that the Floquet Hamiltonian is diagonal in the S_n basis. This result, with the identifications $a \leftrightarrow (\varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{k}+\mathbf{Q}})/2$

and $b \leftrightarrow \Delta$, underlies the sliding-CDW spectrum given by Eq. (13).

APPENDIX B: INTEGRAL FLOQUET SPECTRAL FUNCTIONS $A_n(E)$

This Appendix derives the time-averaged Floquet density of states quoted in Sec. II. Near the Fermi level, $\xi_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \varepsilon_F \simeq \hbar v_F(|\mathbf{k}| - k_F)$ and the normal-state DOS is $N(\xi) \simeq N(0) = 1/(\pi \hbar v_F)$. Then,

$$\mathcal{E}_{\mathbf{k}} = \sqrt{\left(\xi_{\mathbf{k}} - \frac{r\hbar\Omega}{2}\right)^2 + \Delta^2}. \quad (\text{B1})$$

Because $E_{\mathbf{k}n} = E_{\mathbf{k}0} + n\hbar\Omega$, the sideband-resolved spectral functions satisfy

$$A_n(E) \equiv -\frac{1}{\pi} \text{Im} \sum_{\mathbf{k}} g_{\mathbf{k}}^R(E, n) = A_0(E - n\hbar\Omega). \quad (\text{B2})$$

Evaluating the $n = 0$ component gives

$$\begin{aligned} A_0(E) &= \sum_{\mathbf{k}} \delta(E - E_{\mathbf{k}0}) = N(\xi_{\mathbf{k}}) \left| \frac{dE_{\mathbf{k}0}}{d\xi_{\mathbf{k}}} \right|_{E_{\mathbf{k}0}=E}^{-1} \\ &= \frac{N(0)|E - \varepsilon_F + \frac{1}{2}\hbar\Omega|}{2\sqrt{(E - \varepsilon_F + \frac{1}{2}\hbar\Omega)^2 - \Delta^2}} \theta \\ &\quad \times \left(\left| E - \varepsilon_F + \frac{1}{2}\hbar\Omega \right| - \Delta \right) \\ &\quad + \frac{N(0)|E - \varepsilon_F - \frac{1}{2}\hbar\Omega|}{2\sqrt{(E - \varepsilon_F - \frac{1}{2}\hbar\Omega)^2 - \Delta^2}} \theta \\ &\quad \times \left(\left| E - \varepsilon_F - \frac{1}{2}\hbar\Omega \right| - \Delta \right). \end{aligned} \quad (\text{B3})$$

Defining the static DOS by $D(E) = A_0(E)|_{\Omega=0}$, we obtain

$$D(E) = \frac{N(0)|E - \varepsilon_F|}{\sqrt{(E - \varepsilon_F)^2 - \Delta^2}} \theta(|E - \varepsilon_F| - \Delta) \quad (\text{B4})$$

and

$$A_0(E) = \frac{1}{2} \left[D\left(E + \frac{\hbar\Omega}{2}\right) + D\left(E - \frac{\hbar\Omega}{2}\right) \right]. \quad (\text{B5})$$

This is the split-edge structure illustrated in Fig. 3.

APPENDIX C: BASIS TRANSFORMATION BETWEEN FLOQUET QUASIPARTICLES AND PHYSICAL ELECTRONS

This Appendix gives the explicit transformation between the diagonal Floquet quasiparticle basis of Sec. II and the physical electron basis in which the lead self-energies (Appendix E) are stationary. The quasiparticle operators $\{d_{\mathbf{k}\sigma}^\dagger\}$ and the physical electron operators $\{f_{\mathbf{k}\sigma}^\dagger\}$ are related by the time-dependent unitary matrix

$$U_{\mathbf{k}}(t) = \begin{pmatrix} u_{\mathbf{k}} & v_{\mathbf{k}} e^{-ir\Omega t} \\ v_{\mathbf{k}+\mathbf{Q}} e^{ir\Omega t} & u_{\mathbf{k}+\mathbf{Q}} \end{pmatrix}, \quad (\text{C1})$$

so that

$$\begin{pmatrix} d_{\mathbf{k}\sigma}^\dagger \\ d_{\mathbf{k}+r\mathbf{Q},\sigma}^\dagger \end{pmatrix} = U_{\mathbf{k}}(t) \begin{pmatrix} f_{\mathbf{k}\sigma}^\dagger \\ f_{\mathbf{k}+r\mathbf{Q},\sigma}^\dagger \end{pmatrix},$$

$$\begin{pmatrix} f_{\mathbf{k}\sigma} \\ f_{\mathbf{k}+r\mathbf{Q},\sigma} \end{pmatrix} = U_{\mathbf{k}}^T(t) \begin{pmatrix} d_{\mathbf{k}\sigma} \\ d_{\mathbf{k}+r\mathbf{Q},\sigma} \end{pmatrix}. \quad (\text{C2})$$

If $\mathbf{G}_{\mathbf{k}}$ denotes the Green's function in the quasiparticle basis and $\tilde{\mathbf{G}}_{\mathbf{k}}$ the corresponding Green's function in the physical electron basis, then

$$\tilde{\mathbf{G}}_{\mathbf{k}}^{\geq}(t_1, t_2) = U_{\mathbf{k}}^T(t_1) \mathbf{G}_{\mathbf{k}}^{\geq}(t_1, t_2) U_{\mathbf{k}}^*(t_2), \quad (\text{C3})$$

and the same transformation applies to the retarded, advanced, and Keldysh components because each is a linear combination of $G^>$ and $G^<$.

Writing

$$U_{\mathbf{k}}(t) = U_{\mathbf{k}}^0 + U_{\mathbf{k}}^+ e^{-ir\Omega t} + U_{\mathbf{k}}^- e^{ir\Omega t}, \quad (\text{C4})$$

with

$$U_{\mathbf{k}}^0 = \begin{pmatrix} u_{\mathbf{k}} & 0 \\ 0 & u_{\mathbf{k}+r\mathbf{Q}} \end{pmatrix}, \quad U_{\mathbf{k}}^+ = \begin{pmatrix} 0 & v_{\mathbf{k}} \\ 0 & 0 \end{pmatrix},$$

$$U_{\mathbf{k}}^- = \begin{pmatrix} 0 & 0 \\ v_{\mathbf{k}+r\mathbf{Q}} & 0 \end{pmatrix}, \quad (\text{C5})$$

and expanding $\mathbf{G}_{\mathbf{k}}(t_1, t_2)$ in Floquet harmonics

$$\mathbf{G}_{\mathbf{k}}(t_1, t_2) = \sum_n \int \frac{d\omega}{2\pi} e^{-i\omega t} e^{in\Omega\tau} \mathbf{G}_{\mathbf{k}}(\omega, n), \quad (\text{C6})$$

one obtains the physical-basis Floquet components

$$\begin{aligned} \tilde{\mathbf{G}}_{\mathbf{k}}(\omega, n) = & U_{\mathbf{k}}^{0T} \mathbf{G}_{\mathbf{k}}(\omega, n) U_{\mathbf{k}}^{0*} + U_{\mathbf{k}}^{+T} \mathbf{G}_{\mathbf{k}}(\omega - r\Omega, n) U_{\mathbf{k}}^{+*} \\ & + U_{\mathbf{k}}^{-T} \mathbf{G}_{\mathbf{k}}(\omega + r\Omega, n) U_{\mathbf{k}}^{-*} \\ & + U_{\mathbf{k}}^{0T} \mathbf{G}_{\mathbf{k}}(\omega - r\Omega/2, n - r) U_{\mathbf{k}}^{+*} \\ & + U_{\mathbf{k}}^{+T} \mathbf{G}_{\mathbf{k}}(\omega - r\Omega/2, n + r) U_{\mathbf{k}}^{0*} \\ & + U_{\mathbf{k}}^{0T} \mathbf{G}_{\mathbf{k}}(\omega + r\Omega/2, n + r) U_{\mathbf{k}}^{-*} \\ & + U_{\mathbf{k}}^{-T} \mathbf{G}_{\mathbf{k}}(\omega + r\Omega/2, n - r) U_{\mathbf{k}}^{0*} \\ & + U_{\mathbf{k}}^{+T} \mathbf{G}_{\mathbf{k}}(\omega, n + 2r) U_{\mathbf{k}}^{-*} \\ & + U_{\mathbf{k}}^{-T} \mathbf{G}_{\mathbf{k}}(\omega, n - 2r) U_{\mathbf{k}}^{+*}. \end{aligned} \quad (\text{C7})$$

This is the quantity that enters the tunneling and transport expressions in the main text.

APPENDIX D: SCHWINGER-KELDYSH FORMULATION AND DYSON EQUATION

This Appendix summarizes the Schwinger-Keldysh structure used throughout the main text [30,31,37–40]. Only the retarded, advanced, and Keldysh components are needed. In the Larkin-Ovchinnikov representation,

$$\mathbf{G} = \begin{bmatrix} G^R & G^K \\ 0 & G^A \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \Sigma^R & \Sigma^K \\ 0 & \Sigma^A \end{bmatrix} \quad (\text{D1})$$

with $G^< = (G^K - G^R + G^A)/2$. Dyson's equation $\mathbf{G} = \mathbf{G}_0 + \mathbf{G}_0 \Sigma \mathbf{G}$ implies

$$G^{R/A} = G_0^{R/A} + G_0^{R/A} \Sigma^{R/A} G^{R/A}, \quad (\text{D2a})$$

$$G^K = (1 + G^R \Sigma^R) G_0^K (1 + \Sigma^A G^A) + G^R \Sigma^K G^A. \quad (\text{D2b})$$

For the transport problem of Sec. IV, Dyson's equation must be applied in the physical electron basis created by $\{f_p^\dagger\}$, because the tunneling Hamiltonian to the contacts is stationary in this basis. Denote by $\tilde{\mathbf{g}}$ the Green's functions of the isolated sliding CDW and by $\tilde{\mathbf{G}}$ the Green's functions after the contacts are attached. Then, $\tilde{\mathbf{G}} = \tilde{\mathbf{g}} + \tilde{\mathbf{g}} \Sigma \tilde{\mathbf{G}}$ becomes

$$\tilde{\mathbf{G}}^{R/A} = \tilde{\mathbf{g}}^{R/A} + \tilde{\mathbf{g}}^{R/A} \Sigma^{R/A} \tilde{\mathbf{G}}^{R/A}, \quad (\text{D3a})$$

$$\tilde{\mathbf{G}}^K = (1 + \tilde{\mathbf{G}}^R \Sigma^R) \tilde{\mathbf{g}}^K (1 + \Sigma^A \tilde{\mathbf{G}}^A) + \tilde{\mathbf{G}}^R \Sigma^K \tilde{\mathbf{G}}^A. \quad (\text{D3b})$$

When the periodic steady state is fixed by the contacts or by additional relaxation processes, the second term in the Keldysh equation is the relevant one, recovering the familiar relation $\tilde{\mathbf{G}}^K \simeq \tilde{\mathbf{G}}^R \Sigma^K \tilde{\mathbf{G}}^A$.

APPENDIX E: LEAD SELF-ENERGY IN THE PHYSICAL ELECTRON BASIS

This Appendix derives the stationary lead self-energy used in Secs. III and IV. Let p and q label physical single-particle states in the CDW region. The contact Hamiltonian is

$$H_{\alpha} = \sum_k \epsilon_{k\alpha} c_{k\alpha}^\dagger c_{k\alpha}, \quad (\text{E1a})$$

$$H_{T,\alpha} = \sum_{k,p,\sigma} V_{k\alpha,p} c_{k\alpha}^\dagger f_{p\sigma} + \text{H.c.} \quad (\text{E1b})$$

Because the couplings $V_{k\alpha,p}$ are time independent in the f basis, the corresponding self-energy is stationary, with matrix elements

$$\Sigma_{\alpha,pq}^{R(A,K)}(t_1, t_2) = \sum_k V_{k\alpha,p}^* g_{k\alpha}^{R(A,K)}(t_1 - t_2) V_{k\alpha,q}, \quad (\text{E2})$$

or, after Fourier transformation,

$$\Sigma_{\alpha,pq}^{R(A,K)}(\omega) = \sum_k V_{k\alpha,p}^* g_{k\alpha}^{R(A,K)}(\omega) V_{k\alpha,q}. \quad (\text{E3})$$

For an equilibrium contact,

$$g_{k\alpha}^{R/A}(\omega) = \frac{1}{\omega - \epsilon_{k\alpha} \pm i0^+},$$

$$g_{k\alpha}^K(\omega) = 2\pi i [2f_{\alpha}(\epsilon_{k\alpha}) - 1] \delta(\omega - \epsilon_{k\alpha}), \quad (\text{E4})$$

where $f_{\alpha}(\omega) = n_F(\omega - \mu_{\alpha})$. Therefore,

$$\Sigma_{\alpha,pq}^{R/A}(\omega) = \Lambda_{\alpha,pq}(\omega) \mp \frac{i}{2} \Gamma_{\alpha,pq}(\omega), \quad (\text{E5a})$$

$$\Sigma_{\alpha,pq}^K(\omega) = i[2f_{\alpha}(\omega) - 1] \Gamma_{\alpha,pq}(\omega), \quad (\text{E5b})$$

with the linewidth matrix

$$\Gamma_{\alpha,pq}(\omega) = 2\pi \sum_k V_{k\alpha,p}^* V_{k\alpha,q} \delta(\omega - \epsilon_{k\alpha}). \quad (\text{E6})$$

The total self-energy is $\Sigma = \sum_{\alpha} \Sigma_{\alpha}$. In the wide-band limit, one sets $\Lambda_{\alpha}(\omega) \rightarrow 0$ and $\Gamma_{\alpha}(\omega) \rightarrow \Gamma_{\alpha}$.

APPENDIX F: TIME AVERAGING THE CURRENT FORMULA

This Appendix details the reduction of the double-time current expression to the stationary-contact form used in

Eqs. (20) and (27). The time-dependent current through contact α is

$$J_\alpha(t) = -\frac{2e}{\hbar} \int_{-\infty}^t dt_1 \text{Im} \int \frac{d\epsilon}{2\pi} \text{tr}\{e^{-i\epsilon(t_1-t)} \Gamma^\alpha(\epsilon, t_1, t) \times [\tilde{\mathbf{G}}^<(t, t_1) + f_\alpha(\epsilon) \tilde{\mathbf{G}}^R(t, t_1)]\},$$

and the time-averaged current is

$$\langle J_\alpha(t) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt J_\alpha(t). \quad (\text{F1})$$

The retarded term extends naturally over the full square domain because $\tilde{\mathbf{G}}^R(t, t_1) = 0$ for $t_1 > t$. For the lesser term, the symmetry $\tilde{\mathbf{G}}^{<(>)}(t_1, t_2)^\dagger = -\tilde{\mathbf{G}}^{<(>)}(t_2, t_1)$ together with $\Gamma^\alpha(\epsilon, t_1, t_2)^\dagger = \Gamma^\alpha(\epsilon, t_2, t_1)$ implies that the imaginary part of the integrand is symmetric under $t \leftrightarrow t_1$. The triangular domain can therefore be symmetrized, yielding

$$\begin{aligned} \langle J_\alpha(t) \rangle = & -\frac{e}{\hbar} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt \int_{-T/2}^{T/2} dt_1 \text{Im} \\ & \times \int \frac{d\epsilon}{2\pi} \text{tr}\{e^{-i\epsilon(t_1-t)} \Gamma^\alpha(\epsilon, t_1, t) [\tilde{\mathbf{G}}^<(t, t_1) \\ & + 2f_\alpha(\epsilon) \tilde{\mathbf{G}}^R(t, t_1)]\}. \end{aligned} \quad (\text{F2})$$

Introducing the relative time $s = t_1 - t$ and the center-of-mass time $\tau = (t + t_1)/2$, and using the double Fourier expansions

$$\Gamma^\alpha(\epsilon, t_1, t) = \sum_n \int \frac{d\omega}{2\pi} e^{-i\omega s} e^{in\Omega\tau} \Gamma^\alpha(\epsilon, \omega, n), \quad (\text{F3a})$$

$$\tilde{\mathbf{G}}^{<(>)}(t, t_1) = \sum_l \int \frac{d\omega'}{2\pi} e^{i\omega' s} e^{il\Omega\tau} \tilde{\mathbf{G}}^{<(>)}(\omega', l), \quad (\text{F3b})$$

substitution gives

$$\begin{aligned} \langle J_\alpha(t) \rangle = & -\frac{e}{\hbar} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{n,l} \text{Im} \int \frac{d\epsilon}{2\pi} \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} \int d\tau \int ds \\ & \times e^{-i(\epsilon+\omega-\omega')s} e^{i(n+l)\Omega\tau} \text{tr}\{\Gamma^\alpha(\epsilon, \omega, n) [\tilde{\mathbf{G}}^<(\omega', l) \\ & + 2f_\alpha(\epsilon) \tilde{\mathbf{G}}^R(\omega', l)]\}. \end{aligned} \quad (\text{F4})$$

Using

$$\begin{aligned} \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} d\tau e^{i(n+l)\Omega\tau} &= \delta_{n,-l}, \\ \int_{-\infty}^{\infty} ds e^{-i(\epsilon+\omega-\omega')s} &= 2\pi \delta(\omega' - \omega - \epsilon), \end{aligned} \quad (\text{F5})$$

the τ, s , and ω' integrations yield

$$\begin{aligned} \langle J_\alpha(t) \rangle = & -\frac{e}{\hbar} \sum_n \text{Im} \int \frac{d\omega}{2\pi} \int \frac{d\epsilon}{2\pi} \text{tr}\{\Gamma^\alpha(\epsilon, \omega, n) \\ & \times [\tilde{\mathbf{G}}^<(\omega + \epsilon, -n) + 2f_\alpha(\epsilon) \tilde{\mathbf{G}}^R(\omega + \epsilon, -n)]\}. \end{aligned} \quad (\text{F6})$$

For a stationary contact, $\Gamma^\alpha(\epsilon, t_1, t_2)$ is independent of the center time and only the $n = 0$ harmonic survives. The expression then reduces to

$$\langle J_\alpha \rangle = -\frac{e}{\hbar} \text{Im} \int \frac{d\epsilon}{2\pi} \text{tr}\{\Gamma^\alpha(\epsilon) [\tilde{\mathbf{G}}^<(\epsilon, 0) + 2f_\alpha(\epsilon) \tilde{\mathbf{G}}^R(\epsilon, 0)]\}, \quad (\text{F7})$$

which is the form used in Eqs. (20) and (27).

APPENDIX G: SINGLE-CONTACT TUNNELING SPECTRA AND INVERSE-CURRENT OSCILLATIONS dI/dV

This Appendix provides the explicit formulas underlying Figs. 6 and 7, and demonstrates that the near-edge spectra at fixed current and the fixed-bias inverse-current oscillation are two complementary cuts of the same Floquet sideband ladder.

For the weak-probe spectroscopy of Sec. III, the normalized differential conductance is approximated by

$$\frac{G(V, I)}{G_N} \simeq \int dE A_{\text{loc}}^{\text{ave}}(E; I) \left[-\frac{\partial n_F(E - eV)}{\partial E} \right]. \quad (\text{G1})$$

To regularize the square-root singularity at the CDW gap edge, we introduce a Dynes broadening γ and write

$$D_\gamma(E) = \text{Re} \frac{E + i\gamma}{\sqrt{(E + i\gamma)^2 - \Delta^2}}, \quad (\text{G2})$$

so that the time-averaged spectrum in the sliding state is

$$A_{\text{loc}}^{\text{ave}}(E; I) \simeq \frac{1}{2} \left[D_\gamma \left(E + \frac{\hbar\Omega(I)}{2} \right) + D_\gamma \left(E - \frac{\hbar\Omega(I)}{2} \right) \right]. \quad (\text{G3})$$

This is the near-edge envelope used in Fig. 7: It captures the split singularities of the sliding-CDW DOS while retaining a phenomenological broadening suitable for comparison with experiment.

For the $(\text{TaSe}_4)_2\text{I}$ nanowire data of Ref. [14], we take $\Delta = 165$ meV, temperatures $T = 1.55$ and 8 K, and the experimental oscillation period $\Delta(1/I) = 0.00269 \mu\text{A}^{-1}$. These values fix the product

$$N_{\text{eff}} \epsilon = \frac{h}{2e \Delta(1/I)} \simeq 4.80 \text{ eV}, \quad (\text{G4})$$

where $\epsilon = \mu - E_v$ is the local nonequilibrium alignment between the probe chemical potential and the lower CDW gap edge, and N_{eff} is the effective coherent channel count defined by the current-to-frequency relation. This offset includes band bending, screening, and the voltage distribution in the driven device and is not identical to a bare work-function difference or room-temperature contact barrier. Therefore,

$$N_{\text{eff}} \simeq \frac{4.80 \text{ eV}}{\epsilon}, \quad \hbar\Omega [\text{meV}] \simeq 0.00269 I [\mu\text{A}] \epsilon [\text{meV}]. \quad (\text{G5})$$

For the reference choice $\epsilon = 10$ meV used in Fig. 7, this gives $N_{\text{eff}} \simeq 4.8 \times 10^2$, together with $\hbar\Omega = 0.269, 0.538$, and 1.345 meV at $I = 10, 20$, and 50 μA , respectively. For $\epsilon = 5, 10$, and 20 meV, the corresponding channel numbers are about $9.6 \times 10^2, 4.8 \times 10^2$, and 2.4×10^2 . With $\gamma = 0.30$ meV and thermal widths $3.5k_B T = 0.47$ meV at 1.55 K and 2.41 meV at 8 K, the reference value $\hbar\Omega(20 \mu\text{A}) \simeq 0.54$ meV is barely resolved at base temperature and strongly smeared at 8 K, consistent with Fig. 7. Requiring $\hbar\Omega(20 \mu\text{A})$ to be at least comparable to γ and the base-temperature thermal width gives $\epsilon \gtrsim 6$ – 9 meV and hence $N_{\text{eff}} \lesssim 5 \times 10^2$ – 8×10^2 . The near-edge construction also requires $\epsilon \lesssim \Delta = 165$ meV, giving $N_{\text{eff}} \gtrsim 29$.

To connect these fixed-current spectra with the observed $1/I$ oscillation, the full Floquet sideband ladder must be restored. Because $A_n(E) = A_0(E - n\hbar\Omega)$ and $A_0(E)$ has edge singularities at $E_v \pm \hbar\Omega/2$, the positive-bias singularities occur at

$$E_v + \left(m + \frac{1}{2}\right)\hbar\Omega, \quad m = 0, 1, 2, \dots \quad (\text{G6})$$

At a fixed bias offset $\epsilon = eV - E_v$, the oscillatory part of the single-contact response is governed by repeated crossings of these edge singularities. A convenient way to isolate the singular contribution of one broadened positive edge is

$$\delta G_{\text{edge}}(\delta; T, \gamma) = \int dE [D_\gamma(E) - 1] \left[-\frac{\partial n_F(E - \Delta - \delta)}{\partial E} \right], \quad (\text{G7})$$

where δ measures detuning from the positive gap edge and subtraction of 1 removes the smooth normal-state background. Repeating this near-edge profile along the sideband ladder gives the fixed-bias oscillatory contribution

$$\delta G_{\text{osc}}(I, \epsilon; T) \simeq \sum_{m=0}^{\infty} \delta G_{\text{edge}}\left(\epsilon - \left(m + \frac{1}{2}\right)\hbar\Omega(I); T, \gamma\right). \quad (\text{G8})$$

Equation (G8) omits the smooth background and possible sideband-dependent matrix elements, but provides a direct calculation of the singular oscillatory part generated by the Floquet ladder. The resonance condition reads

$$\epsilon = \left(m + \frac{1}{2}\right)\hbar\Omega(I_m) = \left(m + \frac{1}{2}\right)\frac{\hbar I_m}{2eN_{\text{eff}}}, \quad (\text{G9})$$

which gives

$$\frac{1}{I_m} = \frac{\hbar}{2eN_{\text{eff}}\epsilon} \left(m + \frac{1}{2}\right), \quad \Delta(1/I) = \frac{\hbar}{2eN_{\text{eff}}\epsilon}. \quad (\text{G10})$$

For $\epsilon = 10$ meV, Eq. (G10) yields $\Delta(1/I) = 0.00269 \mu\text{A}^{-1}$, exactly the experimental value of Ref. [14]. Figure 6 shows the corresponding fixed-bias oscillatory contribution evaluated from Eq. (G8) with the same reference broadening $\gamma = 0.30$ meV used in Fig. 7. The peaks are equally spaced in $1/I$, and the higher temperature suppresses and smears the oscillatory sideband structure through the thermal convolution in Eq. (G7).

It is instructive to compare N_{eff} with a crude geometric estimate. Reference [14] reports a typical cross section smaller than $0.03 \mu\text{m}^2$ for device D1. Taking one effective chain per

nm^2 gives $N_{\text{geom}} \sim 3 \times 10^4$. The measured period would then imply $\epsilon \sim 0.16$ meV and $\hbar\Omega(20 \mu\text{A}) \sim 8.6 \times 10^{-3}$ meV, far below both γ and $3.5k_B T$. The full geometric count is therefore incompatible with the observed resolvability within the proposed Floquet mechanism. The data constrain a much smaller effective coherent scale but do not by themselves distinguish a narrow filament from surface selectivity, transverse domains, or a finite phase-correlation network.

APPENDIX H: CURRENT-DRIVEN SEGMENTED TRANSPORT FOR TERMINAL-PAIR VOLTAGES

This Appendix implements the measurement logic of Ref. [14] at the minimal effective level used in Fig. 8. There a persistent current source fixes one common through-current in the whole device, while the voltage is measured between selected terminal pairs. To model that geometry, we keep the oscillatory kernel generated by the Floquet sideband ladder of Appendix G, but distribute the measured voltage over three series segments 1–2, 2–3, and 3–4. The label “current driven” is used deliberately: The theory takes the net through-current as the control parameter and assigns the terminal voltages through an effective series decomposition, which is the steady-state idealization of the experimental persistent-current setup.

Define the normalized coherent oscillatory kernel

$$\mathcal{O}_{\text{osc}}(I; T) = \frac{1}{\mathcal{N}_T} \sum_{m=0}^{\infty} \delta G_{\text{edge}}\left(\epsilon - \left(m + \frac{1}{2}\right)\hbar\Omega(I); T, \gamma\right), \quad (\text{H1})$$

where δG_{edge} is the broadened edge profile of Eq. (G7), $\epsilon = 10$ meV is the reference contact mismatch, and the normalization $\mathcal{N}_T$ ensures $\max_I |\mathcal{O}_{\text{osc}}(I; T)| = 1$ within the plotted current window $10 \leq I \leq 50 \mu\text{A}$. Equation (H1) inherits the same inverse-current spacing $\Delta(1/I) = 0.00269 \mu\text{A}^{-1}$ as Eq. (G10); only its assignment to different voltage probes is changed here.

The three segment voltages are written as

$$\begin{aligned} V_{12}(I; T) &= V_{12}^{\text{bg}}(I) - A_{12}(T)\mathcal{O}_{\text{osc}}(I; T), \\ V_{23}(I; T) &= V_{23}^{\text{bg}}(I) - A_{23}(T)\mathcal{O}_{\text{osc}}(I; T), \\ V_{34}(I; T) &= V_{34}^{\text{bg}}(I) - A_{34}(T)\mathcal{O}_{\text{osc}}(I; T), \end{aligned} \quad (\text{H2})$$

with smooth background voltages $V_{ij}^{\text{bg}}(I)$ and temperature-dependent oscillation amplitudes $A_{ij}(T)$. For current driven between terminals 1 and 4, the measured inner- and outer-terminal voltages are

$$\begin{aligned} V_{2-3}^{1-4}(I) &= V_{23}(I; T), \\ V_{1-4}^{1-4}(I) &= V_{12}(I; T) + V_{23}(I; T) + V_{34}(I; T). \end{aligned} \quad (\text{H3})$$

Figure 8 uses the minimal visibility choice:

$$\begin{aligned} A_{12}(T) &= A_{34}(T) = 0, \\ A_{23}(1.55 \text{ K}) &= 0.90 \text{ mV}, \\ A_{23}(8 \text{ K}) &= 0.22 \text{ mV}, \end{aligned} \quad (\text{H4})$$

corresponding to a coherent inner segment together with effectively dephased outer segments. The smooth backgrounds are taken as representative low-order polynomials,

$$V_{12}^{\text{bg}}(I) = 16.0 + 0.22I + 0.0025(I - 24)^2 \text{ mV}, \quad (\text{H5})$$

$$V_{23}^{\text{bg}}(I) = 2.5 + 0.030I + 0.0008(I - 24)^2 \text{ mV}, \quad (\text{H6})$$

$$V_{34}^{\text{bg}}(I) = V_{12}^{\text{bg}}(I), \quad (\text{H7})$$

with I measured in microamperes. These background terms are not universal microscopic predictions; they simply represent the smooth series voltage of the outer regions. In this minimal limit, the raw outer-terminal voltage is dominated by $V_{12}^{\text{bg}} + V_{34}^{\text{bg}}$, while the coherent $1/I$ oscillation is inherited from the inner segment through V_{23} . Subtracting the smooth background from $V_{1-4}^{1-4}(I)$ therefore recovers the same oscillatory kernel that is directly visible in $V_{2-3}^{1-4}(I)$. The segmented model thus cleanly separates two ingredients that are entangled in the experiment: One common current fixes the Floquet frequency, while spatially nonuniform coherence determines the terminal dependence of the observed visibility.

APPENDIX I: NEAR-THRESHOLD TWO-TERMINAL CONDUCTANCE AT FIXED SLIDING FREQUENCY

This Appendix provides the homogeneous reference limit used to generate Fig. 9. The goal is not to reproduce the persistent-current-driven experiment of Ref. [14], but to show how the Floquet sideband ladder appears when a finite sliding span is voltage biased between two reservoirs. For the genuine two-terminal device, the stationary-contact current of Eq. (F7) can be recast into the usual Floquet-Landauer form for a noninteracting periodically driven region,

$$\bar{I}(V, \Omega) = \frac{e}{h} \sum_n \int dE T_n(E; \Omega) [f_L(E) - f_R(E)], \quad (\text{I1})$$

where $T_n(E; \Omega)$ is the transmission probability from an incoming state at energy E to an outgoing state at $E + n\hbar\Omega$. A full microscopic evaluation of T_n for a finite depinned device is beyond the present parameter-estimate calculation, because one must specify the finite sliding region, its channel count, and the self-consistent relation between Ω and the through-

current. Here, we keep $\hbar\Omega$ as an externally specified sliding frequency and isolate the near-threshold structure controlled by the Floquet gap edges.

Near the positive gap edge, the same broadened singular profile used in the single-contact analysis provides the line shape of one threshold,

$$g_{\text{edge}}(\delta; T, \gamma) = \int dE D_\gamma(E) \left[-\frac{\partial n_F(E - \Delta - \delta)}{\partial E} \right], \quad (\text{I2})$$

where δ measures detuning from the positive edge. In a symmetric two-terminal bias window, dI/dV near $eV \simeq 2\Delta$ is controlled by the transmission at the edges of the bias window, $E \simeq \pm eV/2$. The Floquet ladder therefore produces inward threshold replicas at

$$E_m = \Delta - (m + \frac{1}{2})\hbar\Omega, \quad eV_m = 2E_m = 2\Delta - (2m + 1)\hbar\Omega. \quad (\text{I3})$$

Keeping only the thresholds inside a narrow near-threshold voltage window, the differential conductance is estimated as

$$\frac{dI}{dV}(V, \Omega; T) \propto \sum_{m \in \mathcal{M}_{\text{vis}}} g_{\text{edge}}\left(\frac{eV}{2} - \Delta + \left(m + \frac{1}{2}\right)\hbar\Omega; T, \gamma\right), \quad (\text{I4})$$

where $\mathcal{M}_{\text{vis}}$ denotes the few inward thresholds visible in the plotted voltage range. Equation (I4) is the two-terminal counterpart of the single-contact sideband sum: Instead of scanning current at fixed bias, one scans bias at fixed $\hbar\Omega$, so the ladder appears as threshold shifts linear in $\hbar\Omega$.

Figure 9 evaluates Eq. (I4) with the realistic reference parameters $\Delta = 165$ meV, $\gamma = 0.30$ meV, and $T = 1.55$ or 8 K. The representative values $\hbar\Omega = 0.269, 0.538$, and 1.345 meV are the same sliding frequencies used in the single-contact estimate, but here treated as fixed inputs rather than tied self-consistently to a through-current. The low-temperature curves display weak shoulders below 2Δ at the thresholds of Eq. (I3), while the high-temperature curves show that these structures are readily washed out once $3.5k_B T$ exceeds the sideband spacing. The homogeneous two-terminal case is thus naturally discussed in terms of threshold lines in the (V, Ω) plane; converting those lines into the terminal-pair voltages of the experiment requires the current-driven construction of Appendix H.

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