

Eccentric binary black holes: A new framework for numerical relativity waveform surrogates

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Mounting evidence indicates that some of the gravitational wave signals observed by ground-based observatories might arise from eccentric compact object binaries, increasing the urgency for accurate waveform models for such systems. While for noneccentric binaries, surrogate models are efficient and accurate, the additional features due to eccentricity have posed a challenge. In this work, we present a method for decomposing eccentric numerical relativity waveforms, making them amenable to surrogate modeling techniques. We parametrize the inspiral in the radial-phase domain, factoring out eccentricity-induced dephasing and thus enhancing compressibility and accuracy. This is combined with a second surrogate for the merger ringdown in the time domain and a technique to take advantage of the approximate periodicity with radial oscillations during the inspiral. We apply this procedure to the (2, 2) mode for nonspinning black hole binaries and demonstrate that the resulting surrogate, NRSurE_q4NoSpin_22, is able to faithfully reproduce the underlying numerical relativity waveforms, with maximum mismatches of 6×10^{-4} and median mismatches of 4×10^{-5} . This technique paves the way for high-accuracy parameter estimation with eccentric models, a key ingredient for astrophysical inference and tests of general relativity.

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I. INTRODUCTION

The detection of gravitational waves from binary black hole coalescences has transitioned from rare, milestone events [1–3] to routine occurrences [4–8]. Continued improvements in detector sensitivity leading to next-generation detectors

[9–17] will increase the frequency of observations, and also raises the typical signal-to-noise ratio of louder signals. This affords more opportunities to detect binaries from exotic configurations (like high mass ratio [18] or a black hole in the lower mass gap [19]), but will require ever more accurate waveform models covering an ever larger portion of parameter space to avoid systematic biases [20].

Of particular interest is the detection of binaries with non-negligible orbital eccentricity. A binary system in isolation will radiate away eccentricity throughout its inspiral [21,22], leading to a quasicircular inspiral when it enters the frequency band of ground-based detectors. However, several mechanisms could excite eccentricity in binaries in the late inspiral; these include dynamical capture scenarios [23,24], binaries resulting from triple interactions [25–31], and binary-binary

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[32,33] or binary-single [33–35] interactions. Thus, the observation of eccentric binaries would provide us with new insights into the formation of such systems, as well as the astrophysical environments in which they form.

Orbital eccentricity adds two additional dimensions to the parameter space of compact object binaries and introduces a new characteristic frequency to the gravitational wave signal (corresponding to the radial motion of the binary). Recently, there has been a significant effort in the development of waveform models incorporating this additional physics [36–54]. With these models in hand, several works have begun to perform searches for eccentric signals in data provided by Laser Interferometer Gravitational-Wave Observatory (LIGO), Virgo, and Kamioka Gravitational Wave Detector (KAGRA) [55–59], as well as reanalyzing previously detected events to search for eccentricity [36,60–65]. However, there are caveats; the increased computational cost of these models, as well as the additional dimensionality introduced by eccentricity, currently prohibits the usage of these models in routine parameter estimation pipelines [61,64]. Moreover, these models do not include complete eccentric corrections to the merger and ringdown phases, and so their ability to capture eccentric signatures for very high-mass systems is limited.

Surrogate waveform models [66–73] have shown promise in alleviating these issues. Surrogates rely on a data-driven, reduced-order modeling approach, where one interpolates between a set of basis waveforms. A typical choice of basis waveforms includes those resulting from numerical relativity (NR) simulations [74–80], which directly solve the full nonlinear Einstein equations without physical modeling approximations [74].

NR waveforms, while costly, serve as the “ground truth” for evaluating the precision of approximate waveform models and are routinely used for calibration and improvement of said models (e.g., Refs. [81–83]). Surrogate models, when trained directly on NR waveforms, are able to faithfully and efficiently reproduce the underlying waveforms to within their intrinsic errors.

However, building surrogate models for eccentric binary systems has proven challenging [71,84], because of two main issues: (1) effectively covering the eccentric parameter space with NR simulations is nontrivial and (2) surrogate modeling techniques developed for quasicircular binaries show unacceptable performance when applied to eccentric systems [84,85]. The first issue arises from sensitivity to initial conditions, and was alleviated in recent work [86–90].

This work addresses the second issue, the ineffectualness of quasicircular surrogate approaches for eccentric binaries. We present a strategy to decomposing eccentric NR waveforms making them well suited for surrogate modeling by reparametrizing the dynamics in terms of the binary’s radial phase, i.e., the phase of the eccentricity-driven oscillation in the orbital separation vector’s magnitude. This radial-phase representation aligns oscillatory features across simulations and allows for techniques to enforce their approximate periodicity, thereby removing the apparent incoherence introduced by eccentricity. We apply this procedure to the (2, 2) mode for a set of 156 nonspinning NR simulations and show that the resulting surrogate model faithfully reproduces the underlying

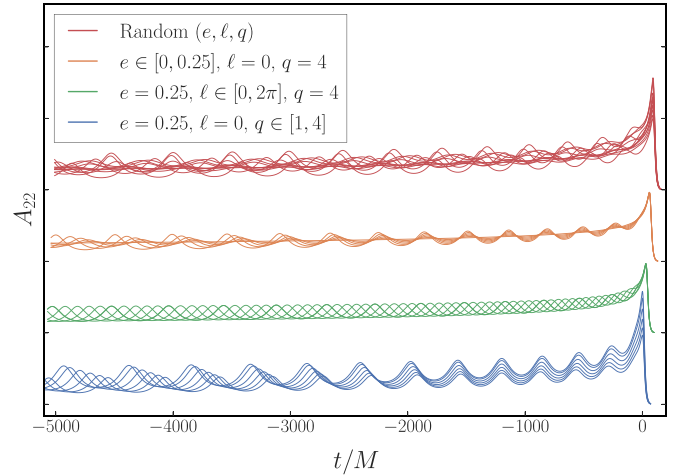


FIG. 1. Incompressibility of data pieces. We show the amplitude of the (2, 2) mode $A_{22}(t)$ for sequences of waveforms generated using the surrogate model. From top to bottom, we first vary all three parameters (eccentricity e , mean anomaly ℓ , and mass ratio q) together, and then vary e , ℓ , and q individually while keeping the other parameters constant. Each sequence of waveforms is offset vertically and horizontally for clarity.

waveforms to within their intrinsic errors, as demonstrated in Fig. 4.

II. PROBLEM OUTLINE

The effectiveness of surrogate modeling relies on finding a highly amenable decomposition of the underlying waveforms. The data pieces that are chosen to be modeled should have a minimal number of features, such that they are well described by a small number of basis functions chosen to span the waveform space efficiently. Along with this, the variation of these features across parameter space should be as simple as possible, enabling accurate regression to new parameters with only a small number of training points. For a more complete description of the surrogate construction process, we refer the reader to Refs. [66,69].

As is standard, we first decompose the complex waveform $h = h_+ - ih_\times$ into a sum of spin-weighted spherical harmonics $h_{\ell m}$. We restrict to nonspinning systems and consider only the dominant (2, 2) mode, which we further decompose into an amplitude and phase $h_{22}(t) = A_{22}(t)e^{-i\phi_{22}(t)}$. We choose $t = 0$ to correspond to the peak of $\Sigma_{m=-2}^2(A_{2m})^2$.

Figure 1 demonstrates the difficulties that arise when applying standard surrogate techniques to eccentric waveforms. Historically, surrogates for quasicircular, nonprecessing systems have involved modeling $A_{22}(t)$ and $\phi_{22}(t)$ directly as functions of time [66]. When considering eccentric systems, the main difficulty arises from the additional features and timescales introduced by orbital eccentricity, namely, the oscillations on the orbital timescale in the amplitude and phase. As can be seen in Fig. 1, these oscillations move in and out of phase across parameter space, rendering the data pieces poorly suited to efficient modeling. In fact, one of the intrinsic parameters of the system (the mean anomaly ℓ) determines the relative phasing of these oscillations, while the remaining

parameters (for this study, the mass ratio q and eccentricity e) determine the radial frequency of the system, meaning that even systems that are initially aligned can dephase. Similar features are present in $\phi_{22}(t)$.

This situation is reminiscent of issues encountered for spin-precessing binaries; if one were to naively attempt to model the gravitational waveform in the inertial frame (i.e., directly model $A_{22}(t)$ and $\phi_{22}(t)$), oscillations and mode mixing introduced by the precession of the orbital plane would render the data pieces difficult to model. As such, precessing surrogates are constructed in a frame that follows the precession of the orbital plane, to minimize precession-induced oscillations.

While this coprecessing-frame method is highly effective for modeling precession-induced effects, no analogous transformation exists for simplifying the structure of eccentricity-induced oscillations. As a result, we must pursue a different approach for such systems. The primary issue highlighted by Fig. 1 is the temporal incoherence of the features across the entire parameter space. To remove this incoherence, we will extract the radial phasing throughout the inspiral and use this to construct a surrogate of the waveform as a function of the *radial phase*, as opposed to time. Near merger, we can no longer reliably define a radial phase; thus, a different approach is required. We construct a separate, surrogate model for only the final ~ 2 orbits, merger and ringdown, using standard time-domain surrogate techniques [67]. We combine both surrogates to ultimately produce full inspiral-merger-ringdown (IMR) waveforms for eccentric systems.

III. INSPIRAL

In order to reparametrize the system as a function of radial phase, we first need to extract a radial phase from NR simulations. Several definitions of radial phase could be chosen: for this work, we employ the relativistic anomaly [44], which we obtain by solving the following quasi-Keplerian equations:

$$\dot{e} = -\frac{\nu e x^4}{M} \left[\frac{(121e^2 + 304)}{15(1 - e^2)^{5/2}} + 3\text{PN terms} \right], \quad (1a)$$

$$\dot{\zeta} = \frac{x^{3/2}}{M} \left[\frac{(1 + e \cos \zeta)^2}{(1 - e^2)^{3/2}} + 3\text{PN terms} \right], \quad (1b)$$

$$x = (M\Omega)^{2/3} \left[\frac{(1 - e^2)}{(1 + e \cos \zeta)^{4/3}} + 3\text{PN terms} \right]. \quad (1c)$$

Here, e denotes the quasi-Keplerian eccentricity, ζ the relativistic anomaly, $\nu = m_1 m_2 / M^2$ the symmetric mass ratio, M and Ω the total mass and instantaneous orbital frequency of the binary, and x the dimensionless orbit-averaged orbital frequency. See Ref. [37] for the complete expressions. The relativistic anomaly ζ is a smooth function of time and has a well-defined limit for $e \rightarrow 0$ (unlike the mean anomaly used in the previous work [71]). Equation (1) is currently derived up to third Post-Newtonian (PN) order for aligned spin systems and up to second for generic systems.

To solve Eq. (1), one must supply $(q, \chi_i, \Omega(t), e_0, \zeta_0)$. Since we are restricted to non-spinning systems, we set $\chi_i = 0$ and compute the mass ratio q using the Christodoulou mass of the apparent horizons [80]. For $\Omega(t)$, we use the orbital frequency computed from the coordinate centers of the indi-

vidual black holes. This choice is gauge dependent; however, we stress that it is just an internal reparametrization to aid the surrogate modeling, similar to the use of local values of the spins (which are evolved through merger via PN spin evolution equations [91]) for constructing fits for precessing systems. Any gauge dependence is ultimately removed when we construct the time-domain waveform.

For (e_0, ζ_0) , we employ the fact that the mean anomaly ℓ_{gw} (as defined in Ref. [92]) and relativistic anomaly should agree at periastron passages $\ell_{\text{gw}} = \zeta = 0$, and so extract the eccentricity e_{gw} (as defined in Ref. [93]) at the first available periastron passage using the PYTHON package `gw_eccentricity` [92]. While $e_{\text{gw}} \neq e$, we find that it is sufficiently close for our purposes. A future improvement could involve an optimization procedure to determine e_0 , using e_{gw} as an initial guess. We then integrate Eq. (1) throughout the inspiral to obtain $\zeta(t)$. We can only extract the radial phase for as long as the individual black hole positions are tracked, and even then the accuracy of the PN expansions (1) will diminish in the strong field regime near merger. Therefore, we will use the ζ parametrization only for the inspiral portion of the surrogate model, up to $t = -1200M$ before merger. To this end, we identify the first periastron passage that occurs after $t = -1200M$ for each waveform in the NR dataset and restrict the ζ parametrization up to this point. Due to length limitations of the NR waveforms, we consider the 11 radial periods immediately beforehand for modeling. In addition to modeling $A_{22}(\zeta)$ and $\phi_{22}(\zeta)$, we also model $t(\zeta)$, so that we can transform the surrogate evaluation back to the time domain. Finally, for only the inspiral surrogate, we perform a rotation and time shift to each waveform, such that $t = 0$ and $\phi_{22} = 0$ at the final periastron passage, to further simplify the data.

Figure 2 presents two of the exact data pieces to be modeled. Plotted are the same 12 simulations presented in the top sequence of Fig. 1. As a function of ζ , all of the eccentricity-induced oscillations in $A_{22}(\zeta)$ and $t(\zeta)$ are mapped to a 2π period and are in phase. The varying orbital period is captured by the monotonic trend in t , while the relative phasing will be handled by how we stitch the inspiral surrogate to the merger. The final data piece, $\phi_{22}(\zeta)$, shows similar features to $t(\zeta)$, which we omit for brevity. Ultimately, data decomposed as in Fig. 2 are much more amenable to surrogate modeling than those in Fig. 1.

This simplification of the waveform should not be entirely surprising. For the modeling of eccentric orbits for black holes of large to extreme mass ratios ($q \gg 1$), where the motion of the small body evolves via ordinary differential equations, it has been well known that orbital quantities are often naturally expressed as a function of a radial phase [94–107]. In fact, in the context of waveform modeling for such systems, a decomposition in terms of “eccentric harmonics” is routinely employed, which relies upon a Fourier decomposition in terms of radial frequencies [108–111]. In contrast to this approach, we instead model the waveform directly as a function of the radial phase, without any further decomposition.

The final choice before building the inspiral surrogate lies in the precise definition of parameters assigned to each waveform, such that parametric fits can be performed across the parameter space. For the eccentric parameters, we choose

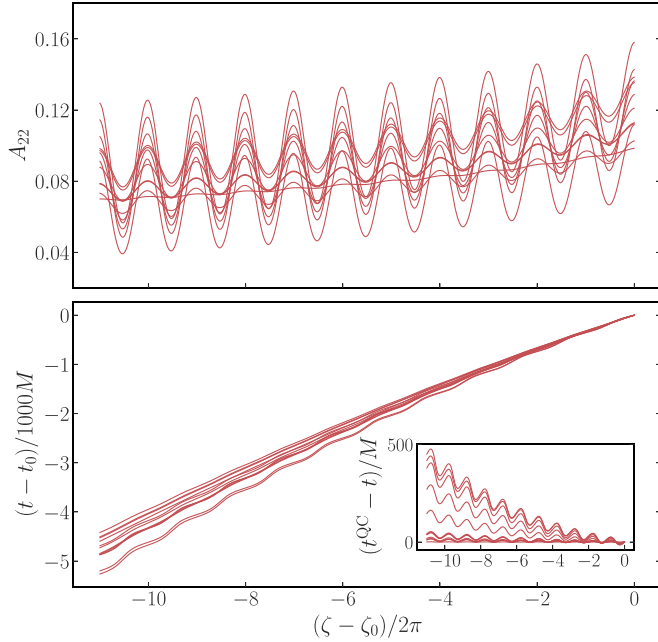


FIG. 2. Example inspiral waveform decomposition. Plotted are the $A_{22}(\zeta)$ and $t(\zeta)$ data pieces used for construction of the inspiral surrogate, for the same parameters as the top family in Fig. 1. Varying radial frequencies are captured by $t(\zeta)$, while relative phasing is handled by how the inspiral is stitched to merger. Here, t_0 (ζ_0) corresponds to the time (relativistic anomaly) at the final periastron passages considered for the inspiral surrogate. The inset of the bottom panel shows the difference $t(\zeta) - t^{\text{QC}}(\zeta)$, where the time $t^{\text{QC}}(\zeta)$ for a quasicircular inspiral is obtained by evaluating the surrogate for the same (q, ℓ_{-1200M}) , but with e_{-3000M} set to zero.

mean anomaly and eccentricity as extracted from the gravitational waveform [92]. Specifically, we choose mean anomaly ℓ_{-1200M} at time $t = -1200M$ before merger, to facilitate the attachment of the merger surrogate. Because eccentricity is difficult to extract accurately close to merger, we choose eccentricity at $t = -3000M$ (named e_{-3000M}) [115] as our parametrization variable. We parametrize the third dimension of the surrogate—mass ratio q —by $\log_{10}(q)$ as opposed to q directly [116].

In principle, we could now construct the inspiral surrogate. However, initial validation tests of the model indicate larger errors for $\ell_{-1200M} \approx 0$ and $\ell_{-1200M} \approx 2\pi$, which arise because the surrogate process does not incorporate the periodicity of the underlying waveforms: systems with mean anomaly ℓ_{-1200M} differing by 2π correspond to the same physical system. A similar issue would arise if one parametrized spin-precessing surrogates using the angles of the individual black hole spins. For the inspiral surrogate, building in the periodicity in ℓ_{1200M} is somewhat subtle, because the data pieces themselves are not actually periodic. Instead, systems differing by $\Delta\ell_{-1200M} = 2\pi$ correspond to the same physical configuration shifted by one radial period, as illustrated in Fig. 3. We exploit this pseudoperiodicity to enlarge the training set and to mitigate boundary effects without requiring additional simulations. Specifically, we add extra training points offset one radial period forward and backward such that the training set contains data for $\ell_{-1200M} \in$

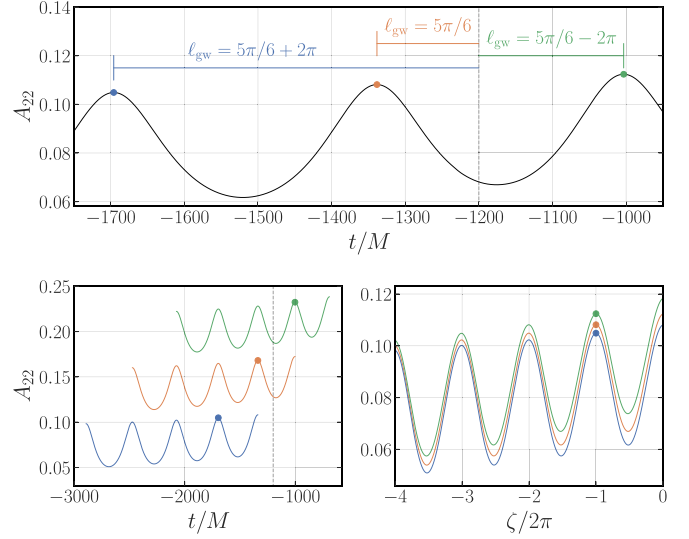


FIG. 3. Demonstration of mean-anomaly periodicity in the inspiral surrogate. Top: Illustration of how one could assign ℓ_{gw} at $t = -1200M$ for three different reference periastron locations using one waveform. Bottom Left: The corresponding portion of waveform that would be used for each case, in time domain. For visibility, we vertically offset the curves from each other and plot only the final four radial periods that would be used. Bottom Right: The corresponding data pieces that are ultimately added to the surrogate training set, ensuring that periodicity is captured for $\ell_{\text{gw}} \in [0, 2\pi]$.

$[-2\pi/3, 8\pi/3]$. We perform the parametric fit to ℓ_{-1200M} over this enlarged range; however, for evaluation of production waveforms, this fit will only be evaluated for $\ell_{-1200M} \in [0, 2\pi]$.

IV. FULL IMR MODEL

In order to produce complete waveforms describing the coalescence of two black holes, we must combine the inspiral surrogate with a separate “merger” surrogate for the final stage of the binary. Here, we choose to employ a more traditional data decomposition method, modeling $A_{22}(t)$ and $\phi_{22}(t)$ directly as a functions of time in the interval $t \in [-1300M, 75M]$. While the data may not be particularly compressible in this form, the short length of the surrogate means that a reduced basis can still be produced. Because binaries only proceed through ~ 2 orbits from $t = -1300M$ to merger, very little dephasing occurs, and fits across parameter space are sufficiently accurate. Finally, for defining parameters assigned to each waveform, we choose to use identical parameters ($\log_{10}(q)$, e_{-3000M} , and ℓ_{-1200M}) as was done for the inspiral surrogate, such that both models can be evaluated with one set of input parameters.

Again, during validation one observes that the merger surrogate faces the same issues at $\ell_{-1200M} = 0$ and 2π as the inspiral surrogate. Thus, we again extend the ℓ_{-1200M} domain of the surrogate. Here, this is simpler, as $A_{22}(t)$ and $\phi_{22}(t)$ will themselves be periodic in ℓ_{-1200M} . As such, we add copies of the data to the training set, where we add $\pm 2\pi$ to ℓ_{-1200M} to extend the domain in ℓ_{-1200M} as for the inspiral surrogate.

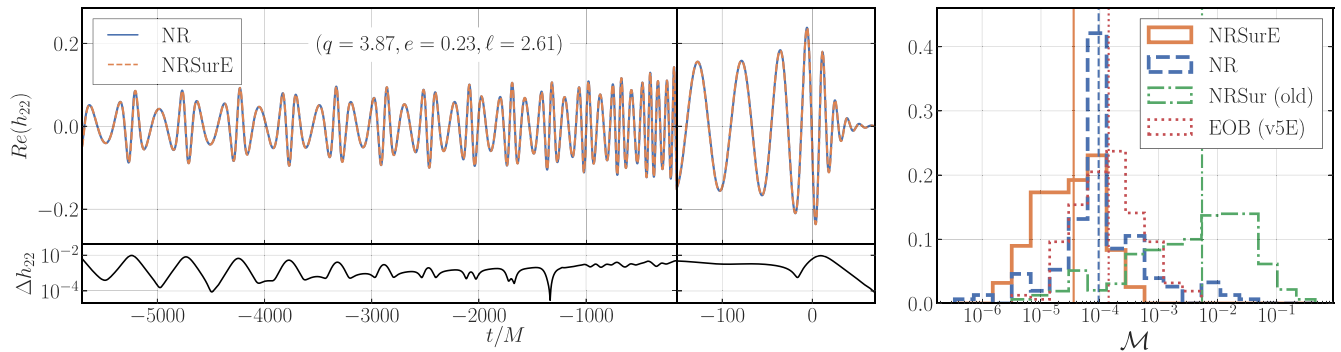


FIG. 4. Demonstration of surrogate using new waveform decomposition. Left: Worst validation case waveform, with mismatch of 6×10^{-4} . The dashed orange line is the surrogate evaluation, while the solid blue line is the corresponding NR waveform. The bottom panel shows the absolute waveform error $\Delta h_{22} = |h_{22}^{Sur} - h_{22}^{NR}|$. Right: Accuracy of NRSurE_q4NoSpin_22, compared to numerical truncation error, a traditional surrogate implementation, as well as the eccentric EOB model SEOBNRv5EHM. NRSurE: leave-eight-out cross-validation mismatches for NRSurE_q4NoSpin_22; NR: mismatches to the next lower numerical resolution of the NR waveforms; NRSur (old): leave-eight-out cross-validation mismatches using modern methods for quasicircular systems; and EOB (v5E): mismatches of the same NR simulations against SEOBNRv5EHM. Vertical lines indicate medians.

At this point, we can evaluate two surrogates to obtain the inspiral and merger portions of a waveform. Furthermore, the choices made previously ensure that there is an overlap in the time domain between the two partial waveforms. Constructing a full IMR waveform then involves aligning the partial waveforms in time and phase, and to smoothly transition between them. In order to do this efficiently, we use the fact that the radial phasing at a particular time (ℓ_{-1200M}) is known, and so impose that this is satisfied by both component surrogate evaluations. Further details can be found in the Appendix.

V. RESULTS

We demonstrate the efficacy of this decomposition by constructing a surrogate for the (2, 2) mode for nonspinning eccentric systems. We use a training set of 156 waveforms from the SXS catalog [80,90,117], with identifiers SXS:BBH:[3731-3747, 3749-3776, 3778-3820, 4293-4304, 4321-4332, 4357-4381, 4442-4455]. The training set consists of mass ratios $q \in [1, 4]$ and eccentricities $e_{-3000M} \in [10^{-3}, 0.25]$. For fits across parameter space, we employ Gaussian process regression as in Refs. [116,118]. The model, which we refer to as NRSurE_q4NoSpin_22, will be made publicly available as part of the GWSURROGATE package [119] in the near future.

As the inspiral portion of the surrogate is now constructed in the radial-phase domain, the waveform length (in terms of time) is no longer constant, but is instead a function of the input parameters. For this work, we obtain a waveform length of approximately $6000M$ (ranging from $5000M$ to $7000M$), corresponding to ~ 42 gravitational wave cycles of the (2, 2) mode.

To validate the model, we perform a leave-eight-out cross-validation study, where we divide the training set randomly into groups of eight cases. We build a series of surrogates, each excluding one group of waveforms from their training set, which is then validated against the excluded waveforms. The effectiveness of this modeling approach is demonstrated in the right panel of Fig. 4. Plotted are the mismatches from

the leave-eight-out study, along with the mismatches obtained comparing the two highest resolutions for each system, which serves as a conservative estimate for the accuracy of the training waveforms. We see that the surrogate is able to faithfully reproduce the underlying NR waveforms, reducing the largest mismatches by several orders of magnitude and the median mismatch by a factor of ~ 3 . This indicates that the limiting factor in the accuracy of the surrogate is likely the accuracy of the underlying NR waveforms, and not the surrogate modeling techniques (similar to what was seen in Ref. [67]). As an extra illustration of the accuracy of the model, the left panel of Fig. 4 shows the worst validation case waveform, corresponding to one of the largest mass ratios and eccentricities in the training set (i.e., a boundary case). In general, we find that mismatches increase near parameter space boundaries (except near mean anomaly boundaries, owing to our handling of periodicity in ℓ), consistent with findings in previous surrogate models. The inclusion of systems with larger eccentricities and mass ratios would likely improve the accuracy of the model in these regimes.

To further emphasize the importance of this framework, Fig. 4 also presents the mismatches obtained from a surrogate constructed using modern techniques for quasicircular systems [67] [indicated as NRSur (old)]. This results in mismatches several orders of magnitude worse than the eccentric surrogate presented here. In addition, Fig. 4 also plots mismatches obtained using the semianalytic waveform model SEOBNRv5EHM. Given that it is not straightforward to translate $(e_{-3000M}, \ell_{-1200M})$ to the input parameters of SEOBNRv5EHM, we follow the procedure of Ref. [37] to obtain input parameters that minimize the mismatch against NR. Even with this additional optimization, we find that NRSurE_q4NoSpin_22 achieves mismatches almost an order of magnitude smaller, with the median improving by a factor of ~ 4 .

Finally, while this work focuses on the accuracy and versatility of the surrogate model, another appealing aspect of surrogate models is their computational efficiency. We find that NRSurE_q4NoSpin_22 generates waveforms (interpolated to a grid spacing of $0.1M$) in ~ 11 ms on a single Intel®

Xeon® Platinum 8360Y CPU @ 2.40 GHz CPU. As a comparison, generating the same waveform using SEOBNRv5EHM takes ~ 280 ms, yielding a speedup of a factor of ~ 25 .

VI. CONCLUSION

We have presented an approach for decomposing eccentric waveforms, enabling accurate and efficient construction of full IMR surrogate models for eccentric NR waveforms. During the inspiral, we compute the radial-phase ζ and construct surrogates as function of ζ for the waveform and the time coordinate. This removes the dephasing of eccentricity-induced oscillations and enables reconstruction of NR waveforms to within their intrinsic errors.

While the $(2, 2)$ mode predicted by the model is accurate to within the errors of the underlying NR waveforms, it is still restricted in its length. This can be alleviated by hybridization of the NR waveforms with analytical approximants [68,120]. In particular, waveform models that already solve the quasi-Keplerian equations (such as SEOBNRv5EHM [37]) would be a natural choice. Regardless, by ensuring that future simulation campaigns focus on length in terms of *radial periods* as opposed to frequency/time, we can ensure that the surrogate is sufficiently long for most applications.

Another application of this work is the speeding up of other waveform models for parameter estimation. Currently, most eccentric waveform models are prohibitively slow to evaluate for low-mass systems, making their use in standard parameter estimation routines challenging [61,64]. None of the procedure presented relies on the underlying waveforms being produced by NR simulations (and, in fact, extraction of the radial phase is often easier for semianalytic models than in NR). This has recently been demonstrated in Ref. [121].

Finally, this work has been restricted to the nonspinning binary black hole subspace. Given that the waveform morphology does not change significantly for aligned spins, and that Eq. (1) already contain spin terms, extension to the case of generic *nonprecessing* systems should be straightforward. Still, the generalization to completely generic systems remains an open problem. Several works have shown that (for quasicircular systems), in the coprecessing frame, standard surrogate modeling techniques can be used to construct waveforms that are then “twisted up” to obtain the inertial frame waveform [67]. This suggests that a similar procedure may be applicable for eccentric systems, and will be explored in future work.

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DATA AVAILABILITY

The simulations used to construct and validate the model will be made available in Ref. [117], following a short embargo.

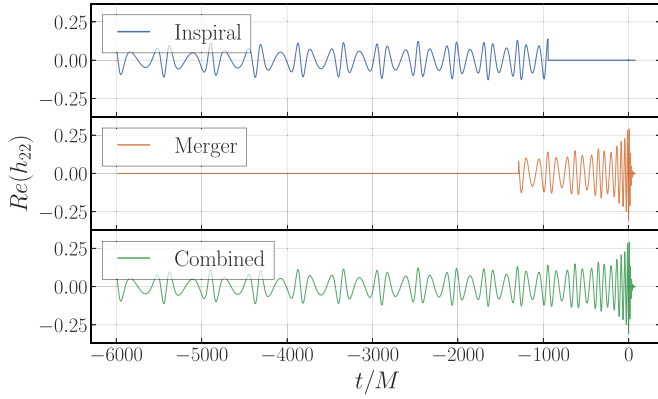


FIG. 5. Example evaluations of the inspiral, merger, and full IMR surrogate. Top: The real part of h_{22} after evaluating the surrogate, performing the time shift and rotating the waveform. Middle: The same for the merger surrogate evaluation. Bottom: The full IMR waveform after stitching both component surrogates together.

APPENDIX: STITCHING SURROGATE EVALUATIONS TOGETHER

First, by construction, we know that $t = -1200M$ is within the domain of both surrogates: for the merger model, we extend as far back as $t = -1300M$, while for the inspiral model the final epoch considered is the closest periastron passage after $t = -1200M$. This means that there will always be at least $100M$ ($-1300M < t < -1200M$) of overlap between the surrogates. However, stitching the surrogate evaluations together requires a mapping between the time coordinates of both surrogate evaluations (as we set $t = 0$ to be the final modeled epoch in the inspiral surrogate). For clarity, we will denote the time outputted by the inspiral surrogate as t' . This time shift will be a function of the input parameters of the surrogate, and so is not known *a priori*. However, we can utilize the fact that both surrogates are parametrized by the mean anomaly at $t = -1200M$. While ζ will not agree with ℓ_{gw}

throughout an orbit, it will agree at periastron passages, where $\ell_{\text{gw}} = \zeta = 2n\pi$ [modulo small dephasing obtained from integrating Eq. (1)]. We also know that, by construction, $t = -1200M$ must fall in the final period modeled by the inspiral surrogate (i.e., $\zeta \in [-2\pi, 0]$). Thus, we can extract $t'(-2\pi)$ and $t'(0)$, and use this to determine the t' that corresponds to $t = -1200M$ (labeled t'_{-1200M}) as

$$t'_{-1200M} = \frac{\ell_{-1200M}}{2\pi} (t'(0) - t'(-2\pi)) + t'(-2\pi). \quad (\text{A1})$$

We now align the time grids by performing the shift

$$t' \rightarrow t' - t'_{-1200M} - 1200M. \quad (\text{A2})$$

Additionally, we rotate both waveforms such that $\phi_{22} = 0$ at $t = -1200M$ to align the phases. Figure 5 presents an example of the two surrogate evaluations after performing the time shift and rotation. The top panel shows the inspiral surrogate evaluation, while the middle panel shows the merger surrogate evaluation.

Now that the two surrogate evaluations are time and phase aligned, all that remains is to stitch them together, so that we smoothly transition from inspiral to merger. To this end, we use a Planck windowing function, which is a smooth function that transitions from 0 to 1 over a specifiable interval. Defining $x = t - t_0 + \sigma/2$, the Planck window is given by

$$w(t; t_0, \sigma) = \begin{cases} 0, & t \leq t_0 - \sigma/2, \\ (1 + e^{\frac{x}{\sigma} - \frac{\sigma}{x}})^{-1}, & t_0 - \sigma/2 < t < t_0 + \sigma/2, \\ 1, & t \geq t_0 + \sigma/2. \end{cases} \quad (\text{A3})$$

We can then obtain the final IMR waveform as

$$h_{22}^{\text{IMR}}(t) = w(t; -1230M, 10M)h_{22}^{\text{merger}}(t) + (1 - w(t; -1230M, 10M))h_{22}^{\text{inspiral}}(t). \quad (\text{A4})$$

An example resulting waveform is presented in the bottom panel of Fig. 5.

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