

Higher symmetry breaking and nonreciprocity in a driven-dissipative Dicke model

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Higher symmetries in interacting many-body systems often give rise to new phases and unexpected dynamical behavior. Here, we theoretically investigate a variant of the Dicke model with higher-order discrete symmetry, resulting from complex-valued coupling coefficients between quantum emitters and a bosonic mode. We propose a driven-dissipative realization of this model focusing on optomechanical response of a driven atom tweezer array comprised of n subensembles and placed within an optical cavity, with the phase of the driving field advancing stepwise between subensembles. Examining stationary points and their dynamical stability, we identify a phase diagram for $n \geq 3$ with three distinctive features: a $\mathbb{Z}_n$ ($\mathbb{Z}_{2n}$) symmetry-breaking superradiant phase for even (odd) n , a normal unbroken-symmetry phase that is dynamically unstable due to nonreciprocal forces between emitters, and a first-order phase transition separating these phases. This n -phase Dicke model may be equivalently realized in a variety of optomechanical or optomagnonic settings, where it can serve as a test bed for studying high-order symmetry breaking and nonreciprocal interactions in open systems.

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I. INTRODUCTION

Phase transitions of interacting many-body systems at thermal equilibrium are a long-standing topic of research. More recently, phase transitions in driven-dissipative many-body quantum systems have been explored [1–6], opening the question of whether emergent properties of such open systems differ from those found at equilibrium.

The Dicke model [7,8] is paradigmatic for the study of both equilibrium and driven-dissipative phase transitions. It has played a defining role in studies of quantum optical phenomena such as superradiance and lasing [7,9–11] as well as in quantum simulation of phase transitions in macroscopic [12–15] and mesoscopic [16] systems. The model describes a collection of N two-level quantum emitters identically coupled to a single-mode electromagnetic field. As a closed system, the Dicke model undergoes a second-order phase transition as the emitter-mode coupling strength increases, going from an unbroken-symmetry normal phase to a $\mathbb{Z}_2$ symmetry-breaking superradiant phase.

Driven-dissipative versions of the Dicke model have been realized experimentally. Prominent among these are systems involving atomic ensembles placed within a high-finesse optical cavity [12,17]. The two-level quantum emitter of the original Dicke model is replaced by optically driven atoms undergoing low-energy Bragg or Raman transitions,

between different mechanical or hyperfine-spin states, respectively. Under certain conditions, the driven-dissipative system reaches a steady state. As in the equilibrium Dicke model, the steady state shows a normal-to-superradiant phase transition.

Since the Dicke model considers spins that are all symmetrically coupled to the cavity, one might ask how the phenomenology changes when this condition is broken. This modification of the Dicke model has been studied in the context of disorder, where different spins have different energy-level spacings and can couple to the cavity with different strengths [18–21] and with a continuum of phases [21]. Disordered couplings among atoms placed in a multimode cavity have also been used to realize glassy systems [22,23]. In contrast, there has been little exploration of what instead happens when the complex spin-cavity coupling varies *discretely* from spin to spin as $\sum_{i=1}^N (\chi_i^* \hat{c} + \hat{c}^\dagger \chi_i) \hat{\sigma}_i^x$, where $\hat{c}$ is the electromagnetic mode field operator, σ_i is the pseudospin 1/2 vector for emitter i , and χ_i is a complex-valued emitter-mode coupling strength. We find that in this scenario, it is possible to generate systems with discrete symmetries higher than $\mathbb{Z}_2$ with a simple modification. Specifically, we consider that N emitters are divided into n groups indexed by j , and the phase of the spin-cavity coupling for each group is set to $2\pi j/n$. Similar forms of this discrete symmetry have been previously explored in Ising-like systems such as the Potts model [24] and the q -state clock model [25]. Generalizing from the $\mathbb{Z}_2$ parity symmetry of the canonical Dicke model, the system we consider now has $\mathbb{Z}_n$ symmetry for even n and $\mathbb{Z}_{2n}$ symmetry for odd n . We call this the n -phase Dicke model.

We propose a way to realize the n -phase Dicke model using ultracold atoms trapped in optical tweezers, though we expect the concepts to be applicable to other realizations. We specifically consider a pumped and dissipative form of the Dicke model that relies on optomechanical self-organization

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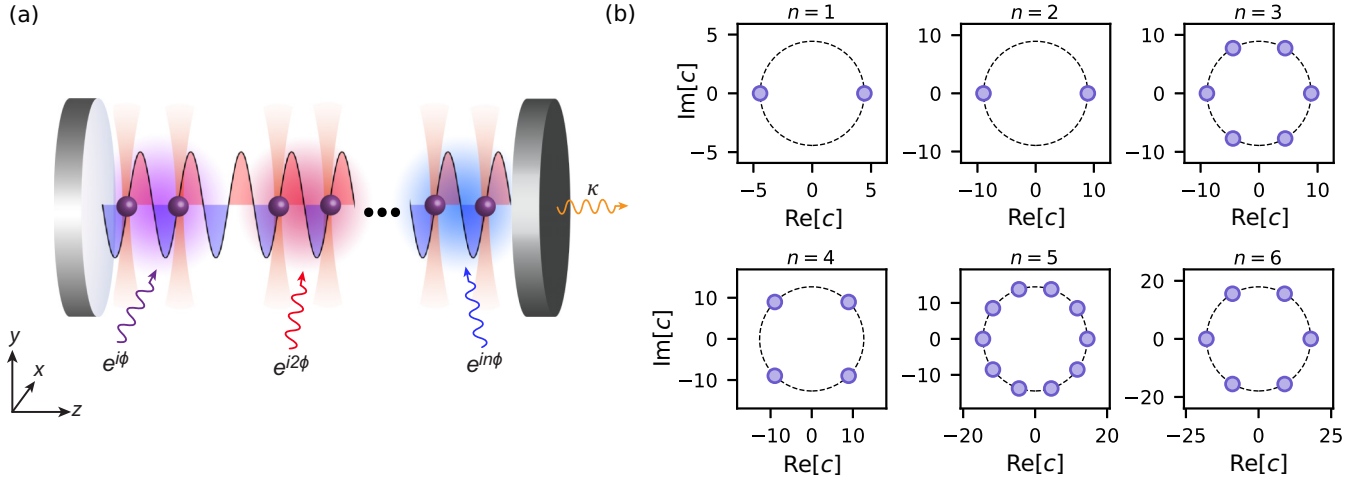


FIG. 1. (a) Schematic of the setup for the n -phase Dicke model. Atoms are held in harmonic traps at the nodes of the cavity field. The atoms are divided into n groups and each group is illuminated by a pump laser with a $\phi = 2\pi/n$ phase difference between adjacent groups. (b) Real and imaginary quadratures of the steady-state cavity field (solid circles) calculated using the atomic center-of-mass positions found through Lyapunov function minimization, shown for $n = 1$ through $n = 6$. The results show $\mathbb{Z}_n$ symmetry for even n and $\mathbb{Z}_{2n}$ symmetry for odd n . The parameters used are $\nu = 30$, $\omega_z = 2\pi \times 70$ kHz, $\Delta_{\text{pa}} = -2\pi \times 100$ MHz, $\Delta_{\text{pc}} = -2\pi \times 4$ MHz, $\kappa = 0$, $g_0 = 2\pi \times 3$ MHz, and $\Omega = 2\pi \times 20$ MHz.

of the atoms [13,14]. Here, the superradiant phase corresponds to the atoms self-organizing onto the lattice formed by the interference of the pump and cavity fields, and the atomic motion can be mapped to spins. Realization of the aforementioned $\mathbb{Z}_n$ or $\mathbb{Z}_{2n}$ symmetry occurs by adjusting separately the optical phase of the pump that drives each atom.

Analyzing this system theoretically, we characterize a phase diagram with three features. First, we find conditions at high pump strength where the cavity field and emitters together stably break the discrete $\mathbb{Z}_n$ or $\mathbb{Z}_{2n}$ symmetry, for even and odd n , respectively. Second, unlike in the canonical Dicke model, the phase transition to the stable broken symmetry state becomes first order, asymptoting to a second-order transition only when the pump light is far detuned from the cavity resonance frequency. Third, we find the normal phase to be dynamically unstable. This instability arises from nonreciprocal light-mediated interactions between the driven emitters.

II. CAVITY OPTOMECHANICAL SYSTEM

We elucidate the features of the n -phase Dicke model in the specific experimental context of cavity optomechanics with an atom tweezer array [16,26]. We consider the one-dimensional motion of a one-dimensional array of N harmonically confined ^{87}Rb atoms along the axis (z) of a single-mode Fabry-Pérot optical cavity. The harmonic traps are centered on the cavity nodes and spaced by an integer number of wavelengths. The atoms, which have an allowed dipole transition at optical frequency ω_a , are then pumped with coherent light propagating transverse to the cavity axis (along x) at frequency ω_p (wave number k and wavelength λ) and Rabi frequency Ω .

When Ω is uniform across all atoms, an optomechanical phase transition occurs at a critical pump strength Ω_c , related to the canonical Dicke phase transition, owing to competition

between the trapping potentials, which confine atoms to the cavity-field nodes, and the interference of the pump and cavity fields, which pulls atoms away from the nodes. A choice of gauge allows us to define Ω as being real and non-negative. With $\Omega < \Omega_c$, the atoms remain stably centered on the cavity nodes and do not scatter light coherently into the cavity; this corresponds to the normal phase of the Dicke model. With $\Omega > \Omega_c$, the atoms self-organize by moving collectively toward cavity antinodes with the same phase, where they emit coherently into the cavity. This superradiant mode breaks the system's $\mathbb{Z}_2$ symmetry, with the atoms displaced toward either the positive- or negative-signed cavity field antinodes, and the cavity light correspondingly having a phase of either 0 or π with respect to the pump field [16].

To generate higher-order symmetry, we now modify this setup as shown in Fig. 1(a). We suppose that a total of $N = n\nu$ trapped atoms, still each positioned on a cavity-field node, are divided into n groups. Each group j , containing ν atoms, is driven with a pump Rabi frequency of $\Omega e^{i\phi j}$, where $\phi = 2\pi/n$ and $j = \{1, 2, \dots, n\}$. We assume the pump is far detuned from the atomic resonance, i.e., that the absolute value $|\Delta_{\text{pa}}| \equiv |\omega_p - \omega_a|$ is much greater than both Ω and the excited-state decay rate, allowing us to adiabatically eliminate the atomic excited state and focus on dispersive atom-light interactions; this makes our treatment equivalent to that of other optomechanical systems of polarizable media. The Hamiltonian in the frame rotating at ω_p reads as

$$\hat{H} = -\hbar\Delta_{\text{pc}}\hat{c}^\dagger\hat{c} + \sum_{j=1}^n \left[\frac{\hat{p}_j^2}{2\mu} + \frac{\mu\omega_z^2\hat{z}_j^2}{2} + \frac{\hbar\Omega\nu}{\Delta_{\text{pa}}} g_0 \sin(k\hat{z}_j) (e^{-i\phi j}\hat{c} + e^{i\phi j}\hat{c}^\dagger) \right], \quad (1)$$

where ω_z is the tweezer trap frequency, g_0 is the vacuum Rabi frequency, and we have defined $\mu \equiv m\nu$ (m being the

atomic mass) and $\Delta_{pc} \equiv \omega_p - \omega_c$. For simplicity, we include only the center-of-mass mode $\hat{z}_j$ of each group and ignore other modes of motion. We also ignore the dispersive shift of the cavity resonance by the atoms. Note that for $n = 1$, Eq. (1) can be mapped to the canonical Dicke model in the Holstein-Primakoff representation [27] by rewriting the position and momentum operators in terms of bosonic creation and annihilation operators for the center-of-mass mode of the array.

We highlight the symmetries of the system: $\hat{H}$ preserves $\mathbb{Z}_n$ symmetry, since performing the gauge transformation $c \rightarrow e^{i\phi} c$ and simultaneous permutation of the group index j leaves $\hat{H}$ invariant. The system also has a $\mathbb{Z}_2$ symmetry in that the simultaneous transformations $\{z, p\} \rightarrow \{-z, -p\}$ and $c \rightarrow -c$ also lead to the same Hamiltonian. These combined symmetries result in $\mathbb{Z}_{2n}$ symmetry for odd n and $\mathbb{Z}_n$ symmetry for even n . In the even n case, the $\mathbb{Z}_2$ symmetry is redundant because it is equivalent to permuting the group index $n/2$ times while advancing the cavity field phase by $n\phi/2 = \pi$.

In the following, we neglect stochastic noise on the cavity field and study the evolution of the expectation values of operators under the master equation. We obtain the following equations of motion:

$$\dot{c} = (i\Delta_{pc} - \kappa)c - i \sum_{l=1}^n \frac{\nu\Omega}{\Delta_{pa}} e^{i\phi l} g_0 \sin(kz_l), \quad (2)$$

$$\dot{p}_j = -\mu\omega_z^2 z_j - \frac{\nu\Omega}{\Delta_{pa}} \hbar k g_0 \cos(kz_j) (e^{i\phi j} c^* + e^{-i\phi j} c), \quad (3)$$

$$\dot{z}_j = \frac{p_j}{\mu}. \quad (4)$$

Here, κ is the decay rate of the cavity field. We can further reduce the equations of motion to a set of $2n$ purely mechanical equations by assuming that the cavity field is in its steady state and instantaneously follows the atomic motion. The adiabatic cavity field is described by

$$c = \frac{i\nu\Omega}{\Delta_{pa}} \frac{1}{i\Delta_{pc} - \kappa} \sum_{j=1}^n e^{i\phi j} g_0 \sin(kz_j). \quad (5)$$

From Eq. (5), one can also understand the symmetry-broken states as the sets of positions $\{z_j \simeq \pm\lambda/4\}$ that maximize the magnitude of $\sum_{j=1}^n e^{i\phi j} \sin(kz_j)$. For even n , there are n such sets and for odd n there are $2n$. We verify these $\mathbb{Z}_n$ and $\mathbb{Z}_{2n}$ symmetries by performing Lyapunov function minimization in the limit $\kappa = 0$ [Fig. 1(b)] to find all the symmetry-broken steady states [28–30]. Substituting Eq. (5) into Eq. (3) yields the force equations

$$\begin{aligned} \dot{p}_j = & -\mu\omega_z^2 z_j - \frac{2\hbar\nu^2\Omega^2}{\Delta_{pa}^2} \frac{g_0^2 k \cos(kz_j)}{\Delta_{pc}^2 + \kappa^2} \sum_{l=1}^n [\Delta_{pc} \cos(\phi(j-l)) \\ & - \kappa \sin(\phi(j-l))] \sin(kz_l). \end{aligned} \quad (6)$$

III. PHASE DIAGRAM OF THE n -PHASE DICKE MODEL

To elucidate the phase diagram of the n -phase Dicke model, we first consider the infinitely dispersive cavity limit

($\Delta_{pc}/\kappa \rightarrow -\infty$), in which the interactions are dominated by the “cavity-dispersive” terms proportional to $\Delta_{pc} \cos(\phi(j-l))$. In this limit, the “cavity-reactive” terms proportional to $\kappa \sin(\phi(j-l))$ in Eq. (6) are negligible and can be ignored. The phase diagram is now analogous to that of the Dicke model: There is a stable normal phase with unbroken symmetry corresponding to $z_j = 0$ for all j when $\Omega < \Omega_c$, which continuously transitions into a superradiant, broken-symmetry phase at $\Omega = \Omega_c$, where $\Omega_c = \sqrt{\frac{\Delta_{pa}^2 |\Delta_{pc}| m \omega_z^2}{g^2 \hbar k^2 N}}$, and the superradiant phase has $\mathbb{Z}_n$ or $\mathbb{Z}_{2n}$ symmetry for even or odd n , respectively.

However, upon restoring the cavity-reactive terms, the unbroken-symmetry state becomes unstable. By linearizing Eq. (6) around ($p_j = 0, z_j = 0$) and seeking a solution of the form $\tilde{z}_j(t) = \tilde{z}_j e^{-i\omega t}$, we find that for $n > 2$, there are two eigenfrequencies given by $\omega^2 = \omega_z^2 + \frac{N\Omega^2 \hbar k^2 g_0^2}{m \Delta_{pa}^2 \sqrt{\Delta_{pc}^2 + \kappa^2}} (\cos \theta \pm i \sin \theta)$, where $\tan \theta = -\kappa/\Delta_{pc}$ [28]. We immediately see that for noninfinite $|\Delta_{pc}|$ or nonzero κ , there are eigenfrequencies with positive imaginary part, indicating the presence of exponentially growing eigenmodes. These lead the $z_j = 0$ solution to be unstable for any $\Omega > 0$.

To understand the emergence of steady states at $z_j \neq 0$, we focus on the specific case of $n = 4$, which admits a graphical solution to the dynamically stationary states. In this case, we can reduce the system to two force equations by enforcing the symmetry $z_1 = -z_3$ and $z_2 = -z_4$. In Figs. 2(a)–2(c), we plot separately the locus of positions z_1 and z_2 where the forces $\dot{p}_1$ and $\dot{p}_2$ vanish, restricting our view to the region of position space in which symmetry-broken solutions emerge at the least distance from the cavity nodes. The intersections of these two curves denote stationary states.

Similar models of multiple atomic ensembles that couple to the cavity field with different phases have been shown to possess instabilities and dynamical solutions [31–34]. In this work, we elucidate additional aspects of the phase diagram with proximity to cavity resonance and pump strength. We identify three dynamical phases: normal, dispersive broken symmetry, and reactive broken symmetry. In the normal phase, which exists at low Ω , ($z_j = 0, p_j = 0$) is the only steady state [Fig. 2(a)]. At higher values of Ω , multiple steady states exist. In the dispersive regime ($|\Delta_{pc}| \gg \kappa$), there are nine steady states in the region $-\lambda/4 < \{z_1, z_2\} < \lambda/4$; four of these steady states break symmetry stably [stars in Fig. 2(b)], while the other five are unstable. We determine the stability by evaluating the eigenvalues of the Jacobian matrix for each steady state [28]. Closer to cavity resonance, we enter the reactive regime ($|\Delta_{pc}| \lesssim \kappa$), where there are no broken symmetry steady states for small values of $|z_1|$ and $|z_2|$, but there are near $z_1, z_2 \sim \pm 0.5\lambda$ [Fig. 2(c)]; these states are all unstable due to cavity-reactive terms dominating over cavity-dispersive terms in the equations of motion. The transitions between these phases are all discontinuous, with the exception of the transition from normal to broken symmetry in the purely dispersive limit. Numerical analyses performed for $n = 3, 5$, and 6 reveal similar phase diagrams to that shown in Fig. 2(d) [28]. Using the same parameters as in Fig. 2 while fixing $\Omega = 2\pi \times 20$ MHz and $\Delta_{pc} = -2\pi \times 4$ MHz, we numerically integrate Eqs. (2)–(4) to obtain cavity field

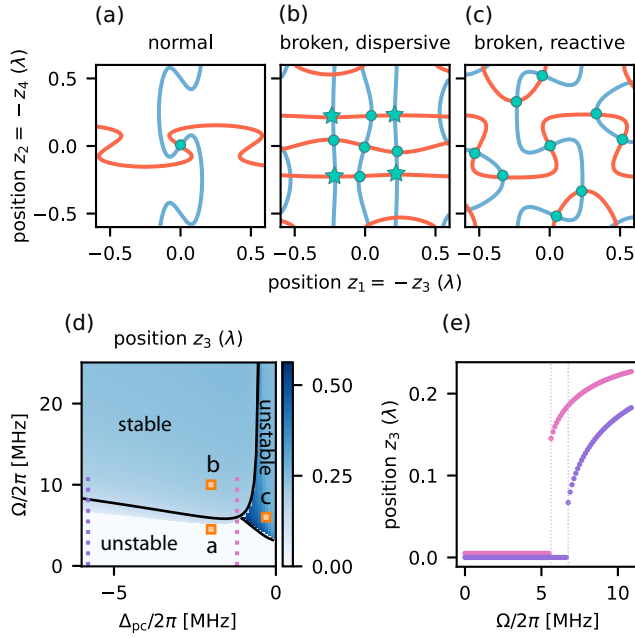


FIG. 2. (a)–(c) Contour plots of the forces $\dot{p}_1 = 0$ (blue) and $\dot{p}_2 = 0$ (red) for $n = 4$, where we have set $z_1 = -z_3$ and $z_2 = -z_4$. Steady states (teal markers) occur where the blue and red curves intersect. Jacobian eigenvalue analysis is used to determine whether each steady state is unstable (circles) or stable (stars). (d) Phase diagram for $n = 4$ showing the steady-state position of one group (z_3) at a particular symmetry-broken solution. Orange squares correspond to the parameters used in panels (a)–(c). Black lines are phase boundaries calculated by expanding Eq. (6) to third order around the symmetry-broken solutions. (e) Line cuts corresponding to the dashed lines in panel (d) showing the first-order transition as a function of Ω . Dashed lines show the location of the discontinuity for the two values of Δ_{pc} . Pink points have a slight vertical offset for visibility. In the $|\Delta_{pc}| \rightarrow \infty$ limit, the transition becomes second order (continuous). Calculations in figure are done with $\Delta_{pa} = -2\pi \times 100$ MHz, $\kappa = 2\pi \times 0.5$ MHz, $\nu = 30$, $g_0 = 2\pi \times 3$ MHz, and $\omega_z = 2\pi \times 70$ kHz.

trajectories for different initial perturbations of the atomic positions and momenta around the $z_j = 0$, $p_j = 0$ steady state. The results for $n = 3, 4, 5$ and 6 are shown in Fig. 3. The trajectories have a high density at the vertices of an n - or $2n$ -sided polygon; this confirms the $\mathbb{Z}_n$ and $\mathbb{Z}_{2n}$ symmetry breaking as well as the stability of the system's steady states for the chosen parameters. We also examine the time trajectories for $n = 4$ [Fig. 3(e)] and see that the cavity field breaks symmetry at late times. We note that the timescale for symmetry breaking appears to shorten when including the dynamics of the atomic excited state, which can also lead to instability when the cavity field is particularly strong [28].

IV. DESCRIPTION OF NONRECIPROCAL INTERACTIONS

The distinctive dynamical instabilities of the n -phase Dicke model originate from the presence of cavity reactive terms, whose effect grows when $|\Delta_{pc}|$ approaches κ . These terms represent nonreciprocal interactions in this driven-dissipative system. To elucidate the role of nonreciprocal forces, we

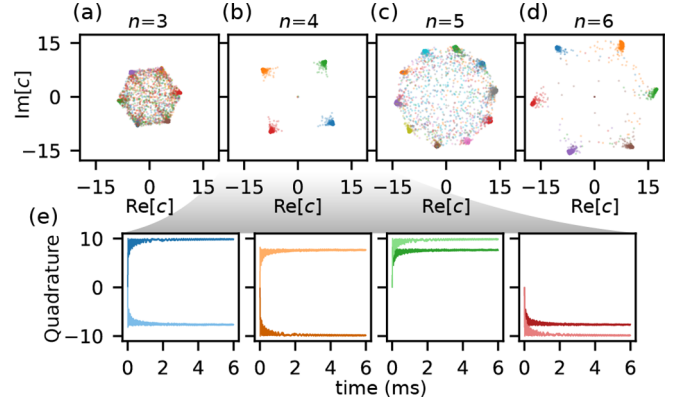


FIG. 3. Density plots of cavity field trajectories for (a) $n = 3$, (b) $n = 4$, (c) $n = 5$, and (d) $n = 6$ in a parameter regime predicted to have stable steady states. Different colors correspond to different perturbations in the initial positions of the atoms, which are the starting conditions for the numerical integration. The high concentration of single colors at the vertices of (a) a hexagon, (b) a square, (c) a decagon, and (d) a hexagon indicate that the trajectories have spontaneously broken either a $\mathbb{Z}_{2n}$ or a $\mathbb{Z}_n$ symmetry. All trajectories are integrated over a time span of 6 ms. (e) Time trajectories of the real (dark color) and imaginary (light color) quadratures of the cavity field for $n = 4$. Colors correspond to the data in panel (b). Parameters used for all data in figure are $\Omega = 2\pi \times 20$ MHz, $\Delta_{pc} = -2\pi \times 4$ MHz, $\Delta_{pa} = -2\pi \times 100$ MHz, $\kappa = 2\pi \times 0.5$ MHz, $\nu = 30$, $g_0 = 2\pi \times 3$ MHz, and $\omega_z = 2\pi \times 70$ kHz.

linearize Eq. (6) and rewrite it in terms of bosonic ladder operators $\hat{b}_j$, which annihilate tweezer phonons in the j th group of atoms. Defining $\mathbf{b} = (\hat{b}_1, \hat{b}_2, \dots, \hat{b}_n)^T$, this results in the equation of motion $\dot{\mathbf{b}} = -i\mathbf{H}_{\text{eff}}\mathbf{b}$, where $\mathbf{H}_{\text{eff}}$ is an effective Hamiltonian whose matrix elements are given by

$$H_{\text{eff},jl} = \begin{cases} \omega_z + \frac{Ck\Delta_{pc}}{\mu\omega_z}, & j = l, \\ \frac{Ck[\Delta_{pc}\cos(\phi(j-l)) - \kappa\sin(\phi(j-l))]}{\mu\omega_z}, & j \neq l, \end{cases} \quad (7)$$

where we have defined $C \equiv \frac{2\hbar\nu^2\Omega^2}{\Delta_{pa}^2} \frac{g_0^2 k}{\Delta_{pc}^2 + \kappa^2}$ for convenience. We see that $H_{\text{eff},jl} \neq H_{\text{eff},lj}$ unless $\kappa\sin(\phi(j-l)) = 0$ for all values of j and l . Since $\phi = 2\pi/n$, this results in $\mathbf{H}_{\text{eff}}$ being non-Hermitian when $n > 2$ and $\kappa > 0$. The fact that $H_{\text{eff},jl} \neq H_{\text{eff},lj}$ means that the groups of atoms interact nonreciprocally [35]: The amplitude with which phonons tunnel from group j to group l is not equal to the amplitude with which phonons tunnel from group l to group j .

Though the interactions described by H_{eff} are generically nonreciprocal, they do not realize ideal nonreciprocal interactions in the sense that it is not possible to have both $H_{\text{eff},jl} = 0$ and $H_{\text{eff},lj} \neq 0$ for $l \neq j$. However, we now show that one can achieve ideal nonreciprocity if we take $n = 2$ and no longer fix the phase difference ϕ to be $2\pi/n$. To avoid notational confusion, we define this new, variable phase difference as φ . The effective Hamiltonian has off-diagonal matrix elements $H_{\text{eff},12} = Ck(\Delta_{pc}\cos\varphi + \kappa\sin\varphi)/(\mu\omega_z)$ and $H_{\text{eff},21} = Ck(\Delta_{pc}\cos\varphi - \kappa\sin\varphi)/(\mu\omega_z)$.

To illustrate how to achieve a perfectly nonreciprocal interaction, suppose we would like group 1 to be completely decoupled from the motion of group 2, but not vice versa.

Achieving this requires $H_{\text{eff},12} = 0$, which is satisfied for $\varphi = \tan^{-1}(-\Delta_{\text{pc}}/\kappa)$. We note that this is equivalent to setting $\varphi + \theta = \pm\pi/2$, which has the simple physical interpretation of requiring the cavity field emitted by group 2 to be $\pi/2$ out of phase with the pump field at group 1. As such, $\varphi + \theta$ is analogous to a synthetic flux found in other systems with nonreciprocal interactions [36–38]. Such nonreciprocity has been considered in systems of nanoparticles held close together in optical tweezers [39–42] and an SiN membrane positioned in an optical cavity [38]. Here, we have shown that a similar type of interaction can be achieved with trapped atoms in an optical cavity.

V. CONCLUSION

We have investigated a variant of the driven-dissipative Dicke model, in which one can realize $\mathbb{Z}_n$ or $\mathbb{Z}_{2n}$ symmetry breaking by pumping an optomechanical array with n optical phases. For a strong pump, the system stably breaks symmetry in a certain parameter regime, while for a weak pump, it is described by complex eigenfrequencies and nonreciprocal interactions. Our results contribute to the growing literature on instabilities and dynamical solutions in variants of the Dicke model [31–34,43]. We have also discussed how to achieve one-way phonon propagation in this system by harnessing control over nonreciprocal interactions. Although we have examined this Dicke model variant in the context of cold atoms, we expect the concepts also to apply to cavity-coupled arrays of other types of mechanical elements, such as thin membranes [38], optomechanical crystals [44], and optically suspended nanoparticles [45]. One might also consider cavity coupling to the internal spin states of atoms, rather than

their mechanical degrees of freedom. Engineering of nonreciprocal spin excitation dynamics may have applications in quantum information science [36,46]. This model additionally has broad prospects for studies of the interplay between symmetries and nonreciprocity in the vicinity of a phase transition [32,47–49].

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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