

Compressibility control on gas-liquid-grain displacement dynamics

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This study probes how gas compression governs the morphodynamics of gas-liquid-grain displacement in confined granular systems, a process vital to many applications like air sparging and carbon sequestration. Combining experiments and theory, we find that facilitating compression yields effects analogous to decreasing the classic capillary number. This analogy arises from the diminished transient flow rates due to the underlying coupling between viscous liquid displacement and gas pressure buildup. A theoretical model is developed to resolve this coupling, accurately matching the reduced injection pressure and prolonged breakthrough time observed in experiments. We further introduce a compression-dependent capillary number Ca_C that effectively captures the pattern transitions. Our findings underscore the role of compressibility in gas-driven flows—typically neglected in laboratories yet becoming increasingly critical toward industrial relevance.

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I. INTRODUCTION

Multiphase flows, particularly gas displacing viscous liquids in granular and porous media, are commonly encountered in various applications including enhanced oil recovery [1], carbon geostorage [2,3], air sparging in aquifers [4], foam-driven fracturing [5], and even medical procedures like airway reopening [6]. Experimental studies on such gas-liquid displacement reveal key patterns—capillary or viscous fingering [7–15]—governed by the capillary number Ca , which balances viscous and capillary forces. These experiments usually utilize small syringes with short tubing for injection, rendering the compression of injected gas negligible. However, gas compression becomes unavoidable and impactful in practical applications, where storage tanks and pipeline networks operate at spatial and volumetric scales orders of magnitude larger than those encountered in laboratory setups. Given the high compressibility of gases ($\sim 20\,000$ times that of liquids), substantial bulk compression arises from the injection pressures, compounded by pressure gradients caused by viscous stresses in the gas itself over long flow paths. Consequently, accounting for gas compressibility effects is indispensable for accurately understanding gas-driven flows within granular materials.

Though often neglected in laboratory tests and incompressible theoretical models, gas compression generates unsteady, impulsive flow motion that is directly coupled to the displacement dynamics. Recent studies [16–20] have begun addressing this oversight, with Cuttle *et al.* [18,19]

demonstrating delayed compression-driven viscous fingering in simple fluid pairs. Furthermore, the fluid-fluid displacement frequently induces the deformation of the pore space due to grain movement [21–23]. Unlike fluid-only systems [17–19], the presence of mobile grains introduces further hydromechanical instabilities that can significantly affect the flow behavior and produce diverse patterns like channels, cavities, granular fingers, and fractures [24–31]. Despite these advances, the role of gas compressibility in the interplay between multiphase flow and granular mechanics remains underexplored. Limited research has examined how gas compression affects the displacement behavior across capillary and viscous regimes in deformable granular media, where gas-liquid-grain interactions dictate the system response.

Here, we integrate experimental and theoretical schemes to investigate the gas-liquid-grain displacement dynamics driven by the compression of a gas reservoir. We propose a dimensionless compression number C^* to quantify the fundamental coupling between compressive pressurization and viscous depressurization. Our results demonstrate that increasing C^* produces behavior analogous to lowering the capillary number Ca . We further incorporate C^* into Ca to yield a compression-dependent Ca_C , serving as a predictive tool for delineating displacement regimes. Furthermore, we introduce a simple model to capture the displacement dynamics in terms of the temporal evolution of injection pressure and flow rates.

II. METHODS

We employ a circular Hele-Shaw cell apparatus [Fig. 1(a)] consisting of two parallel plates of radius $R = 100$ mm separated by a monolayer of mobile beads of diameter $d = 2$ mm. The granular packing is initially saturated with an incompressible liquid of viscosity $\eta = 350$ mPa s and interfacial tension $\gamma = 0.021$ N/m. A coaxial needle with inner radius $r_0 = 0.75$ mm is centrally inserted at the packing for simultaneous gas injection and injection pressure P_{inj}

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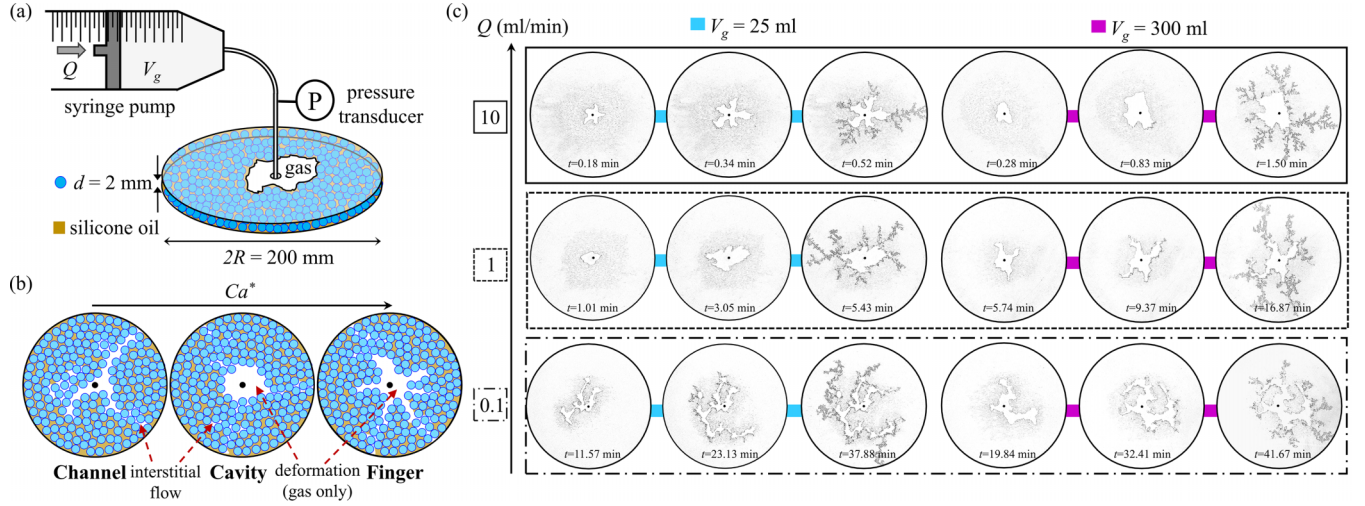


FIG. 1. (a) Schematic of laboratory Hele-Shaw cell configuration. Air with initial volume V_g is injected at nominal flow rates Q to displace a monolayer of mobile beads initially saturated with silicone oil. (b) Characteristic patterns classified by deformation shape: channel to cavity to finger with increasing Ca^* [31]. (c) Temporal evolution of representative displacement patterns until the invading air reaches the cell boundary. The three rows correspond to $\log Ca^* = [1.5, 0.5, -0.5]$ (top to bottom).

measurements via a pressure transducer. Time-lapse images are acquired from beneath the cell and postprocessed in MATLAB by subtracting the initial image (preair invasion). Detailed protocol and technical specifications are provided in Sec. A of the Supplemental Material [32], which includes Refs. [33–35].

Air is injected into the cell at constant nominal volumetric rates $Q = [0.1, 1, 10]$ ml/min using a syringe pump. The balance between viscous and capillary forces is quantified by the modified capillary number $Ca^* = \eta QR/(\gamma d^3)$ [25,36,37], corresponding to $\log Ca^*$ values of $[-0.5, 0.5, 1.5]$. Three well-defined characteristic patterns form when increasing Ca^* [31]: intermittent channeling (capillary-dominant), circular cavitation (crossover), and dendritic fingers (viscous-dominant), as depicted in Fig. 1(b). To further modulate the extent of gas compression, we vary the initial volume of air in the syringe $V_g = [25, 75, 150, 300]$ ml. Importantly, varying V_g does not alter the capillary number, so its impact on the displacement phenomenon in gas-liquid-grain systems is not considered in previous studies. Twelve experimental conditions are tested with each replicated twice to ensure reproducibility (Appendix B).

III. RESULTS AND DISCUSSION

Our foremost result is an experimental phase diagram mapping the evolution of displacement morphologies from air injection in the phase space of Q and V_g [Fig. 1(c) and video in the Supplemental Material [32]]. Additional results under other experimental parameters are presented in Fig. 5 in Appendix A. Notably, the phase diagram demonstrates the impact of gas compression on reshaping the pattern-formation process under the same nominal Ca^* .

A. Quantification of displacement patterns

We first provide quantitative insight into these patterns by calculating two metrics: (1) the circularity O of deforma-

tion shape, defined as $O = (4\pi A/L^2)^{1/2}$, where L and A are, respectively, the perimeter of the deformation region (gas-only), and the area it encloses [Fig. 2(a); $O = 1$ for a perfect circle]; and (2) the fractal dimension D_f of associated fluid infiltration computed via the box-counting method [Fig. 2(b)]; this is a classic measure to distinguish capillary fingering (CF; $D_f \approx 1.82$) from viscous fingering (VF; $D_f \approx 1.62$) [35,38]. Breakdown of the analysis workflow is described in Sec. B of the Supplemental Material [32].

At $\log Ca^* = 1.5$, increasing V_g shifts O from 0.55 to 0.76 and D_f from 1.57 to 1.67, indicating that fingers become more circular with reduced branching and fluid infiltrates more intricately. This suggests that the nominal viscous regime evolves into a state resembling the crossover when increasing V_g . Similarly, the evolution of O and D_f in the crossover—shifting toward characteristics typical of the capillary regime ($O: 0.77 \rightarrow 0.53$; $D_f: 1.63 \rightarrow 1.76$)—implies that the nominal crossover is becoming more akin to the capillary regime. Upon reaching capillary flows ($\log Ca^* = -0.5$), varying V_g minimally affects the morphologies with O (< 0.4) and D_f (> 1.75) remaining stable, reflecting the limited transient volumetric compression of gas at low plunger velocities. Using circularity O and fractal dimension D_f , patterns are categorized into three nonoverlapping clusters [Fig. 2(c)]. One may notice that enlarging V_g at constant Ca^* drives the patterns in the transition of “finger to cavity to channel,” which is akin to reducing the capillary number. This effect is attributed to the competition between gas compressive forces and liquid viscous forces. Our focus below is to quantify the governing coupling and formally rationalize the compression-driven displacement dynamics.

B. Rationalization of displacement dynamics

As the gas-liquid interface advances, the compressive pressure P_C in the gas phase is competing with the viscous pressure P_V in the liquid phase. For an ideal isothermal gas ($P_C V = \text{constant}$), differentiate it with respect

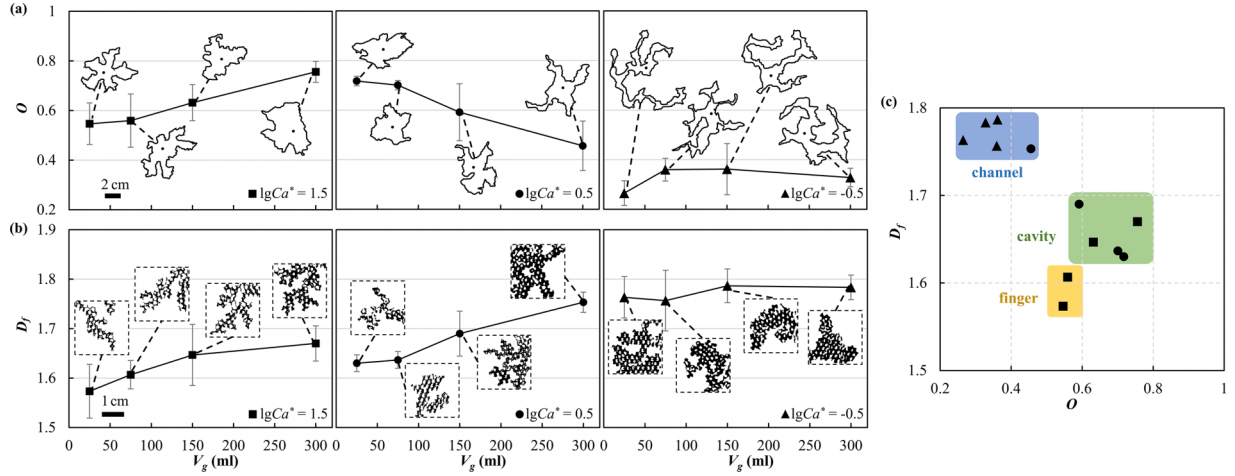


FIG. 2. (a) Circularity O of the deformation shape as a function of the gas reservoir volume V_g . (b) Fractal dimension D_f of the concurrent fluid transport pattern as a function of V_g . Insets show the corresponding representative morphologies. (c) Categorization of invasion patterns in an O - D_f plot, with shaded regions representing the three well-defined regimes.

to time: $d(P_C V)/dt = P_C dV/dt + V dP_C/dt = 0$, getting $dP_C/dt = -P_C/V \times dV/dt$. We assume that the syringe is closed and then substitute $dV/dt = -Q$, getting $dP_C/dt = -P_C/V \times -Q = QP_C/V$. At $t = 0$, $P_C = P_{\text{atm}}$ and $V = V_g$, so the rate of compressive pressure buildup (pressurization) is determined as

$$dP_C/dt \sim QP_{\text{atm}}/V_g. \quad (1a)$$

For a radial Darcy flow, the velocity related to the pressure gradient is $u_r = -\kappa/\eta \times dP_V/dr$, with permeability κ and viscosity η . As the velocity is also related to the flow rate within the gap thickness b : $u_r = Q/2\pi r b$, the pressure gradient is $dP_V/dr = -\eta Q/2\pi r b \kappa$. Since $dP_V/dt = dP_V/dr \times dr/dt$ and $dr/dt = u_r$, the rate of pressure drop is $dP_V/dt = -\eta Q/2\pi r h \kappa \times Q/2\pi r d = -\eta Q^2/4\pi^2 r^2 b^2 \kappa$. As $\kappa \sim b^2$, the characteristic magnitude of the rate of the total viscous pressure drop (depressurization) is obtained as

$$dP_V/dt \sim \eta Q^2/\pi^2 R^2 \kappa^2. \quad (1b)$$

Full derivation is outlined in Sec. C of the Supplemental Material [32]. Since injection energy is partitioned between compressing the gas and displacing the liquid, Eq. (1) reflects how rapidly gas absorbs the energy and viscous liquid dissipates it. Note that directly comparing absolute pressures (P_V/P_C) fails to capture the dynamic behavior of unsteady and transient compressible flows. Instead, the rates of pressure variation are more sensitive to these transient fluctuations, and their competition dictates the regime transitions. Hence, we define a compression number C^* to reveal the relative importance of viscous resistance (energy dissipation) versus compressive effects (energy storage). C^* is physically interpreted as the ratio of the rate of viscous depressurization in the liquid-grain mixtures to that of compressive pressurization in the gas, expressed as

$$C^* = \frac{\alpha \eta Q V_g}{\pi^2 R^2 \kappa^2 P_{\text{atm}}}, \quad (2)$$

with η the liquid viscosity, Q the flow rate, V_g the gas reservoir volume, R the cell radius, d the particle diameter, P_{atm} the atmospheric pressure, Φ the porosity, and κ the

permeability estimated for monodisperse spherical beads by the Kozeny-Carman relation [39]: $\kappa = d^2 \phi^3/[180(1-\phi)^2]$. The coefficient $\alpha = V_g/\phi \pi R^2 d$ compares the initial injected gas volume to the total pore volume available for displacement. It accounts for the extent of potential partial/incomplete displacement constrained by insufficient gas, which could affect pattern formation. In principle, the compression number C^* is a measure of compression dominance, with $C^* > 1$ signaling the regime transition due to more severe compression effects. In our experiments, a large V_g provides a “gas spring” that buffers pressure buildup and thereby elevates C^* . Upon $C^* > 1$, the viscous decompression outpaces the gas compression, which leads to a significant delay in propelling the liquid and triggers the regime shift. To validate C^* , we collect our experimental data into a Ca^* - C^* phase diagram [Fig. 3(a)], with the critical demarcation $C^* = 1$ highlighted in red. The result aligns closely with the classification in Fig. 2(c), where increasing C^* plays a directly analogous role in decreasing Ca^* . The inset, plotting circularity O and fractal dimension D_f against C^* , reveals distinct variations for $C^* > 1$. At high Ca^* , our C^* is similar in spirit to the one of Cuttle *et al.* [17–19] for pure fluids, which measures how “compressible” the system is.

Furthermore, we integrate C^* into Ca^* to propose a compression-dependent capillary number Ca_C , established as

$$Ca_C = Ca^*(1 + C^*)^{-1}, \quad (3a)$$

or, more explicitly,

$$Ca_C = \frac{\eta Q R}{\gamma d^3} \left(1 + \frac{\alpha \eta Q V_g}{\pi^2 R^2 \kappa^2 P_{\text{atm}}} \right)^{-1}. \quad (3b)$$

For $C^* \ll 1$, the term $(1 + C^*)^{-1}$ approaches unity, rendering $Ca_C \approx Ca^*$ and indicating negligible compression effects. Conversely, when $C^* \gg 1$, a reduced Ca_C reflects the damping effect of compressibility. Since Q and V_g were systematically varied to assess their combined influence on the displacement dynamics, a surface plot of Ca_C on the Q - V_g plane [Fig. 3(b)] illustrates its dependence on both: Q adjusts the viscous-capillary force balance, while V_g modifies the temporal flow

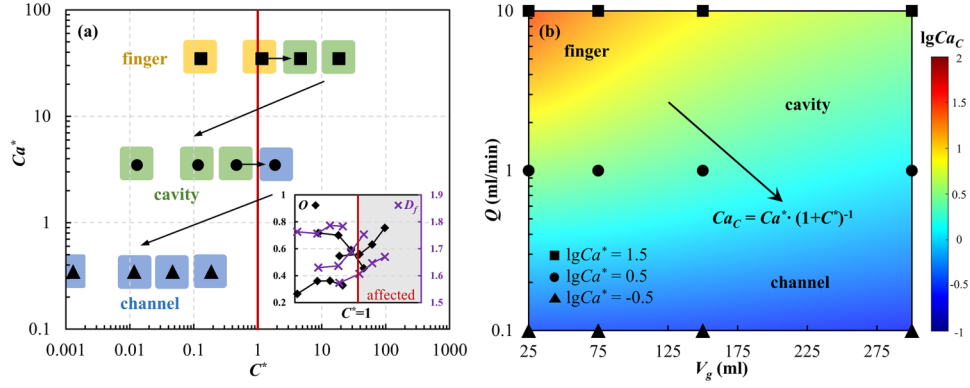


FIG. 3. Proposed controlling compression-dependent capillary number Ca_C . (a) Ca^*-C^* phase diagram shows that increasing the compression number C^* is as effective as decreasing Ca^* , with $C^* = 1$ (vertical red line) marking regime transitions. Inset: strong impact of C^* on the evolution of displacement metrics of circularity O and fractal dimension D_f . (b) Surface plot of logarithmic Ca_C on the dimensional $Q-V_g$ plane, with experimental results overlaid as solid markers. The inclusion of C^* in Ca_C robustly predicts displacement regime boundaries.

rate evolution. The compression-dependent capillary number Ca_C decreases along the trajectory indicated by the arrow in Fig. 3(b), reflecting the competition between the nominal flow rate Q and gas volume V_g . Experimental data overlaid in Fig. 3(b) corroborate the efficacy of Ca_C in predicting regime transitions. Incorporating the compression number C^* with the widely used capillary number, the compression-dependent Ca_C solely provides a predictive framework for delineating gas-liquid-grain displacement regimes.

The proposed Ca_C indicates that compression effects are more pronounced in the viscous-dominant regime, where the gas is visually observed to expand more circularly [top row in Fig. 1(c)]. The corresponding history of injection pressure P_{inj} is measured in Fig. 4(a). Noticeably, increasing V_g delays the peak P_{inj} and extends the breakthrough time (i.e., reaching the cell outlet). This decay trend seems reasonable as larger V_g provides more available volume to be compressed, which mitigates the buildup of pressure and slows its dissipation. To rigorously rationalize the compression-controlled displacement dynamics, we now develop a theoretical model focused on the viscous regime. This model couples gas compression, fluid flow, and granular mechanics for a representative circular expansion of the invading gas within the cell. Three time-dependent unknowns are addressed: (1) the injection pressure P_{inj} ; (2) the radius of injected gas r_g ; and (3) the outflow rate Q_{out} . We first treat the air as a fixed mass of isothermal ideal gas, such that $(P_{atm} + P_{inj})V_t(t) = P_{atm}V_g(0)$, where V_t is the actual total volume of air in the system, which consists of compression in the syringe V_g and invasion into the cell V_c , with $V_t(t) = V_g(t) + V_c(t)$. Under the nominal steady injection rate Q and circular expansion, $V_g(t) = V_g(0) - Qt$ and $V_c(t) = \pi r_g(t)^2 d$, respectively. P_{inj} can be determined as

$$P_{inj}(t) = \frac{P_{atm}V_g(0)}{V_g(0) - Qt + \pi r_g(t)^2 d} - P_{atm}. \quad (4)$$

In Appendix C, we also compute an adiabatic model to show that P_{inj} is overestimated compared with the current assumption of isothermal compression. Second, we assume fluid flowing within a homogeneous granular medium of uniform porosity and permeability. The pressure diffusion equation for the incompressible defending oil in radial coordinates for

axisymmetric flow can be expressed as

$$\frac{\partial P}{\partial t} = D \left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r} \frac{\partial P}{\partial r} \right), \quad (5)$$

where D is the hydraulic diffusion coefficient (i.e., $D = \kappa/\eta\phi c_t$, with the fitted compressibility c_t normalized to unity for dimensional consistency). The outflow rate at the outlet thus can be calculated with Darcy's law as $Q_{out}(t) = -\kappa/\eta\partial P(t)/\partial r|_{r=R} \times 2\pi R d$. We subsequently perform the conservation of mass for the incompressible solid grains, which equates the cumulative volume of outflow to that of the inflow: $\partial V_s/\partial t = 0 \rightarrow \partial[(\pi R^2 d - V_{out}(t) + V_{in}(t))(1-\phi)]/\partial t = 0 \rightarrow V_{out}(t) = V_{in}(t)$, where V_s , V_{in} , and V_{out} are the volumes of solid grains, inflow, and outflow, respectively. The inflow rate is given by $Q_{in} = \partial V_{in}/\partial t = 2\pi r_g d \times \partial r_g/\partial t$, which can be expressed as

$$r_g(t) = r_g(t - dt) + \frac{Q_{out}(t)dt}{2\pi r_g(t - dt)d}. \quad (6)$$

We initialize the model by specifying P_{inj} ($r_0 \leq r \leq R$, $t = 0$) = 0, $r_g(t = 0) = r_0$, $Q_{out}(t = 0) = 0$, where $r = r_0$ is the radius of the injection needle. The three coupled governing equations [Eqs. (4)–(6)] are solved numerically using a finite-difference approximation, with an iteration solution of the unknown $r_g(t)$ until convergence is achieved at each time step.

Comparisons between the modeling results and experiments are presented in Fig. 4(a), demonstrating an excellent agreement. The model effectively reproduces the dampened and prolonged phenomenon in the evolution of pressure P_{inj} when C^* is increased. Figure 4(b) illustrates the radial diffusion of P_{inj} within the cell as a function of normalized distance r/R . As shown, the pressure distribution remains uniform up to a critical point marking the maximum extent of gas expansion. Beyond this point, P_{inj} gradually diminishes, eventually approaching zero. Experimental data, determined by approximating the deformation area as a circle, are also overlaid with a close match.

In addition, we compute the unsteady, time-dependent flow rate that was challenging to measure experimentally, as drawn in Fig. 4(c). Q_n is defined as the ratio of transient flow rate

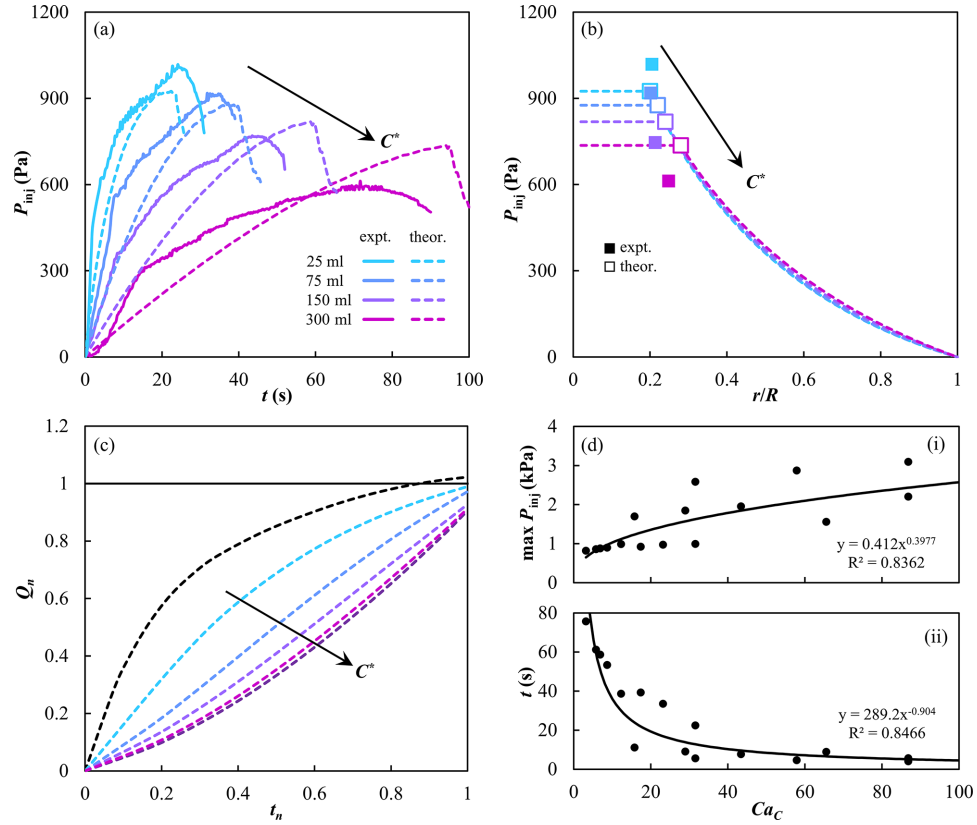


FIG. 4. Modeling results for viscous flow displacement dynamics. (a) Comparison of the evolution of P_{inj} between the experimental measurement (solid curves) and the model prediction (dashed curves) at $\log Ca^* = 1.5$. The model captures the dampened and prolonged trend with increasing C^* . (b) Spatial distribution of pressure as a function of normalized distance r/R , with turning points corresponding to the cessation of gas expansion aligned with experimental data. (c) Time-dependent flow rate normalized by the nominal value Q_n . t_n is a normalized time with the real breakthrough time over the total period. The incompressible reference is provided as a horizontal line (solid black). (d) Maximum pressure [(i), top panel] and the corresponding time [(ii), bottom panel] as functions of compression-dependent Ca_C , with curves representing the fitted power-law relationships.

to the nominal one, representing how much the instantaneous flow rate deviates from the value Q set by the pump. It reveals that systems with greater C^* exhibit a more delayed approach to the imposed flow rate. These reduced instantaneous flow rates with greater C^* are directly correlated to its comparable effect of decreasing Ca . Finally, Fig. 4(d) explores the relationship between the proposed Ca_C and key dynamic characteristics: the maximum pressure in Fig. 4(d)(i) and the corresponding time of occurrence in Fig. 4(d)(ii). A power-law relationship is robustly established for each metric, derived from a database of twenty trials. Both fits ($R^2 \approx 0.84$) satisfy the threshold ($R^2 > 0.8$ [40,41]) commonly accepted in multiphase flow studies through porous media, where variability in pore geometry and fluid dynamics often limits higher fits. Figure 4(d) clearly indicates that as Ca_C increases, the pressure builds up faster and to higher magnitudes, reflecting the system's transition toward a quasi-incompressible flow.

IV. CONCLUSIONS

In summary, we have elucidated the profound role of gas compression in controlling the gas-liquid-grain displacement dynamics, combining experimental observations

with theoretical modeling. We demonstrate that facilitating compression—achieved through using larger gas reservoir volumes—produces effects analogous to decreasing the classic capillary number Ca , accompanied by distinct reductions in pressure and prolonged breakthrough durations. We have developed a theoretical model that couples gas compression, fluid flow, and granular mechanics. The model has revealed that the diminished transient flow rates, stemming from the coupling between compressive pressurization in the gas and viscous depressurization in the liquid, play a critical role in producing unsteady displacement. Moreover, we have proposed a compression-dependent Ca_C by incorporating a nondimensional compression number C^* (compression is bound to be in effect when $C^* > 1$). The Ca_C parameter offers a predictive power for delineating displacement regimes across diverse flow and compression conditions.

Gas compression arises passively in two-phase gas-driven flows given its compressible nature. The associated unsteady flows can be advantageous or detrimental depending on the application. For instance, in cyclic gas injection (huff-and-puff) processes for enhanced oil recovery, the temporary storage of compressed gas facilitates gradual pressure buildup during injection phase, enabling controlled energy release in the production phase to help maintain reservoir pressure

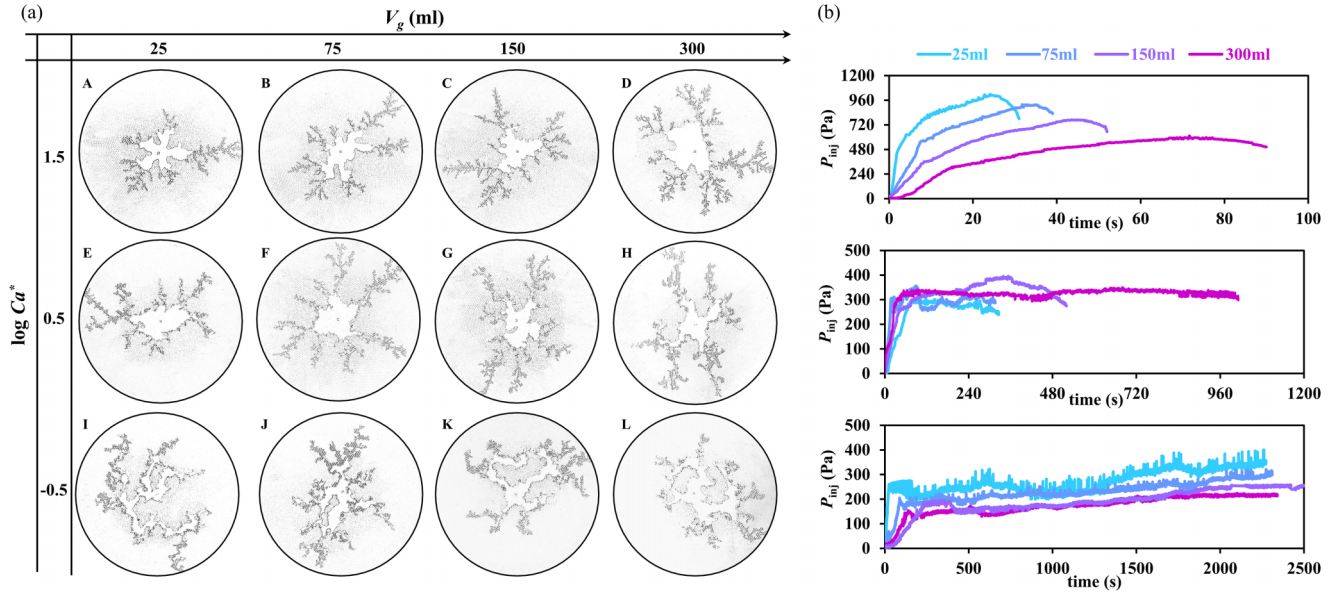


FIG. 5. (a) Experimental phase diagram of gas-liquid-grain displacement patterns for different gas reservoir volumes V_g under three capillary numbers Ca^* corresponding to viscous, transitional, and capillary regimes (top to bottom). Snapshots capture the moment the invading air reaches the cell boundary. (b) Corresponding temporal evolution of injection pressure P_{inj} , with endpoint denoting the time at which the snapshots are shown.

and improve oil displacement efficiency. Conversely, in microfluidic systems, such as those used for drug delivery or cell sorting, compressibility may induce pulsatile or burstlike flows that disrupt precise flow control. The potential for optimizing the use of gas compressibility is substantial and our study has provided a touchstone for future endeavors on such gas-driven flows in porous media.

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DATA AVAILABILITY

The data supporting this study's findings are available within the article. Additional raw images and processed data are available from the corresponding author upon reasonable request.

APPENDIX A: EXPERIMENTAL PHASE DIAGRAM

The initial experimental conditions remain unaltered except for the flow rate $Q = [0.1, 1, 10]$ ml/min and reservoir volume $V_g = [25, 75, 150, 300]$ ml. By varying them, we conduct a total of 12 representative tests. Our main experimental result is a visual phase diagram mapping the displacement morphologies in the phase space of Q and V_g , with time evolution of pressure P_{inj} (Fig. 5). Panels correspond to fixed Ca^* : $\log Ca^* = 1.5$ (top, viscous); $\log Ca^* = 0.5$ (middle, crossover); and $\log Ca^* = -0.5$ (bottom, capillary). We observe a contradictory variation: dendritic fingers in the viscous regime become more circular with fewer fingers as V_g increases while initially round cavities in the crossover

develop more branches with higher V_g . In viscous-dominated flows, P_{inj} follows a consistent nonmonotonic trend regardless of gas volumes: an initial rise from zero as gas compresses, peaking upon full granular compaction (i.e., no further grain movement), followed by a drop as gas tends to percolate through pores. More importantly, larger V_g significantly delays the peak P_{inj} and extends the breakthrough time. This decay trend stems from the diminished time-varying flow rates (validated in the theoretical analysis; Fig. 4). In capillary-dominated flows, despite morphological similarity, lower V_g intriguingly amplifies pressure oscillations. Capillary flow interfaces advance intermittently, controlled by pore-scale geometry-sensitive capillary pressures. As the interface navigates pore throats, the sudden filling of pore bodies causes abrupt pressure changes that are intensified in low V_g systems where $\Delta P \sim \Delta V/V_g$ would theoretically generate larger ΔP at smaller V_g for a given pore filling (i.e., ΔV).

APPENDIX B: EXPERIMENTAL REPRODUCIBILITY

The primary source of uncertainty in our experiments stems from the fabrication of nonstationary granular packings. Manually creating a perfectly homogeneous and reproducible

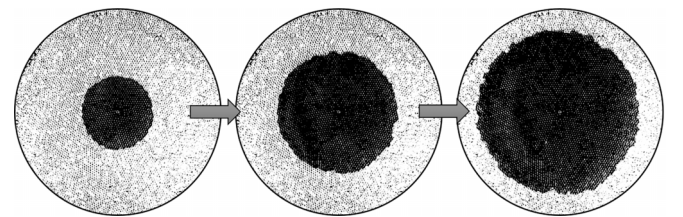


FIG. 6. An example of a sequence of images showing the slow initial saturation of viscous liquid. The radially symmetric wetting front indicates packing homogeneity and uniform injection.

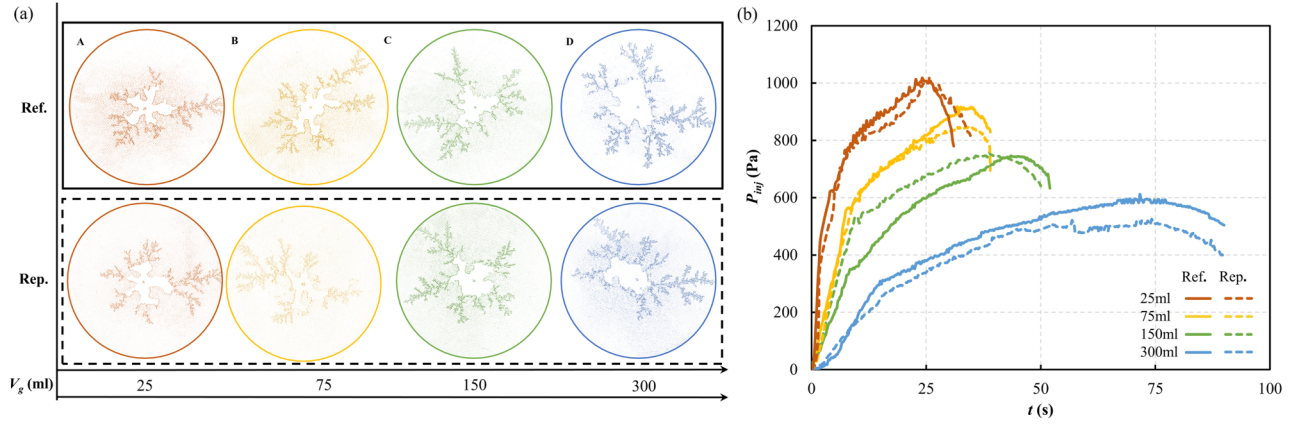


FIG. 7. Experimental reproducibility. (a) morphology evolution for experimental repetitions at $\eta = 350$ mPa s and $Q = 10$ ml/min. (b) time evolution of pressure P_{inj} . Solid curves represent the reference results, while dashed curves correspond to the repeated experiments.

packing with an identical pore aperture distribution is challenging. Following the methodology of Holtzman *et al.* [25], we evaluate system uniformity by examining the invasion front during initial saturation. As illustrated in Fig. 6, the interface front exhibits radial symmetry with a circular shape, confirming uniform injection and homogeneous packing. To minimize potential errors, we carefully exclude experiments in which the saturation pattern deviates from this circular symmetry.

After this quality control step, we conduct repeatability tests for each condition. Figure 7 presents data from two sets of experiments conducted under identical conditions ($\eta = 350$ mPa s; $Q = 10$ ml/min). The reference set corresponds to test A, B, C, and D. Comparing the two sets, we find that the bulk displacement patterns and corresponding pressure evolution exhibit high reproducibility between experiments despite some minor variations. While we have used a batch of Hele-Shaw cells with the same geometry, manual assembly introduces unavoidable differences in the packing due to inherent randomness in arranging mobile beads. These slight variations in pore structures are likely responsible for these acceptable differences.

APPENDIX C: ISOTHERMAL VERSUS ADIABATIC

In our theoretical model, the assumption of isothermal compression treats gas temperature remaining constant due to adequate heat exchange with the environment through the syringe walls. However, the rapid compression at small V_g may induce temperature fluctuations that are not accounted for in our model. Our assumption therefore requires some justification. For the polyethylene syringes used experimentally, the thermal diffusivity is $\alpha \approx 0.1$ mm²/s. The typical timescale of thermal diffusion is defined as $t_d = L^2/\alpha$ with L -syringe wall thickness $\rightarrow t_d = 1.5^2/0.1 = 29$ s. We can then compute the Fourier number $Fo = t_c/t_d$, where t_c denotes the compression time, equivalent to the actual duration of the experiment. A value of $Fo > 1$ indicates that heat generated during compression dissipates rapidly into the environment, thereby maintaining a near-constant temperature and approximating an isothermal process. We determine that, for

$V_g = [25, 75, 150, 300]$ ml at $Q = 10$ ml/min, the corresponding $Fo \approx [1.4, 1.7, 2.3, 4]$. Hence, we may reasonably handle compression to be isothermal for all, but non-negligible temperature changes could arise in small V_g with $Fo \approx 1$.

Alternatively, we could replace the isothermal compression with an *adiabatic* counterpart, assuming the system is perfectly insulated with no heat transfer to the environment. In an adiabatic scenario, the relationship $PV^\gamma = \text{constant}$ holds (where the adiabatic index $\gamma > 1$, typically 1.4 for air), unlike $\gamma = 1$ in isothermal condition. Equation (4) then becomes

$$P_{inj}(t) = \frac{P_{atm} V_g(0)^{1.4}}{[V_g(0) - Qt + \pi r_g(t)^2 d]^{1.4}} - P_{atm}. \quad (C1)$$

Figure 8 compares the peak P_{inj} calculated under both isothermal and adiabatic conditions. The adiabatic model consistently predicts a higher peak P_{inj} than its isothermal

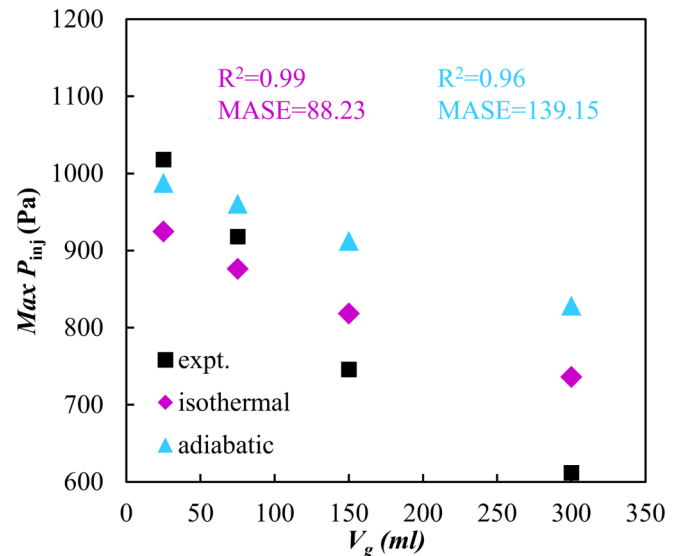


FIG. 8. Comparison of peak pressure P_{inj} under isothermal (pink diamond) and adiabatic (blue triangle) predictions against the experimental measurements for $Q = 10$ ml/min. The isothermal model exhibits superior overall fit, supporting its use for capturing compressibility-driven flow dynamics.

counterpart, as the trapped heat increases molecular collisions and thereby elevates pressure. To evaluate model performance, we calculate the coefficient of determination (R^2) and root mean squared error (RMSE). While the adiabatic model shows closer agreement with the measurement at the smallest V_g , the isothermal model demonstrates superior overall accuracy, evidenced by higher R^2 and lower RMSE. In practice, the compression process is neither purely

isothermal nor purely adiabatic but rather polytropic, occupying an intermediate state between the two extremes. Nonetheless, adopting a polytropic model would introduce undue complexity, diverting attention from our study's central focus on impact of compressibility. Consequently, we retain the isothermal model that balances accuracy and simplicity, ensuring a clear exposition of the compression-controlled dynamics.

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- [1] F. M. Orr and J. J. Taber, Use of carbon dioxide in enhanced oil recovery, *Science* **224**, 563 (1984).
 - [2] J. M. Nordbotten and M. A. Celia, *Geological Storage of CO₂. Modeling Approaches for Large-Scale Simulation* (John Wiley & Sons, Inc., Hoboken, New Jersey, 2011).
 - [3] M. L. Szulczewski, C. W. MacMinn, H. J. Herzog, and R. Juanes, Lifetime of carbon capture and storage as a climate-change mitigation technology, *Proc. Natl. Acad. Sci. USA*, **109**, 5185 (2012).
 - [4] J. E. McCray, Mathematical modeling of air sparging for subsurface remediation: State of the art, *J. Hazard. Mater.* **72**, 237 (2000).
 - [5] C. Y. Lai, B. Rallabandi, A. Perazzo, Z. Zheng, S. E. Smiddy, and H. A. Stone, Foam-driven fracture, *Proc. Natl. Acad. Sci. USA* **115**, 8082 (2018).
 - [6] M. Baudoin, Y. Song, P. Manneville, and C. N. Baroud, Airway reopening through catastrophic events in a hierarchical network, *Proc. Natl. Acad. Sci. USA* **110**, 859 (2013).
 - [7] L. Paterson, Diffusion-limited aggregation and two-fluid displacements in porous media, *Phys. Rev. Lett.* **52**, 1621 (1984).
 - [8] C. W. Park and G. M. Homsy, The instability of long fingers in Hele-Shaw flows, *Phys. Fluids* **28**, 1583 (1985).
 - [9] R. Lenormand, E. Touboul, and C. Zarcone, Numerical models and experiments on immiscible displacements in porous media, *J. Fluid Mech.* **189**, 165 (1988).
 - [10] R. Hu, T. Lan, G.-J. Wei, and Y.-F. Chen, Phase diagram of quasi-static immiscible displacement in disordered porous media, *J. Fluid Mech.* **875**, 448 (2019).
 - [11] S. Suo, M. Liu, and Y. Gan, Fingering patterns in hierarchical porous media, *Phys. Rev. Fluids* **5**, 034301 (2020).
 - [12] B. Zhao, C. W. MacMinn, and R. Juanes, Wettability control on multiphase flow in patterned microfluidics, *Proc. Natl. Acad. Sci. USA* **113**, 10251 (2016).
 - [13] B. K. Primkulov, A. A. Pahlavan, X. Fu, B. Zhao, C. W. MacMinn, and R. Juanes, Signatures of fluid-fluid displacement in porous media: Wettability, patterns and pressures, *J. Fluid Mech.* **875**, R4 (2019).
 - [14] Z. Wang, K. Chauhan, J.-M. Pereira, and Y. Gan, Disorder characterization of porous media and its effect on fluid displacement, *Phys. Rev. Fluids* **4**, 034305 (2019).
 - [15] Y. Qiu, K. Xu, A. A. Pahlavan, and R. Juanes, Wetting transition and fluid trapping in a microfluidic fracture, *Proc. Natl. Acad. Sci. USA*, **120**, e2303515120 (2023).
 - [16] C. Lee, B. Zhao, R. Abouatallah, R. Wang, and A. Bazylak, Compressible-gas invasion into liquid-saturated porous media: Application to polymer-electrolyte-membrane electrolyzers, *Phys. Rev. Appl.* **11**, 054029 (2019).
 - [17] C. Cuttle and C. W. MacMinn, Dynamics of compression-driven gas-liquid displacement in a capillary tube, *Phys. Rev. Lett.* **130**, 114001 (2023).
 - [18] C. Cuttle, L. C. Morrow, and C. W. MacMinn, Compression-driven viscous fingering in a radial Hele-Shaw cell, *Phys. Rev. Fluids* **8**, 113904 (2023).
 - [19] L. C. Morrow, C. Cuttle, and C. W. MacMinn, Gas compression systematically delays the onset of viscous fingering, *Phys. Rev. Lett.* **131**, 224002 (2023).
 - [20] G. G. Peng, C. Cuttle, C. W. MacMinn, and D. Pihler-Puzović, Axisymmetric gas-liquid displacement flow under a confined elastic slab, *Phys. Rev. Fluids* **8**, 094005 (2023).
 - [21] Z. Sun, Z. Li, D. N. Espinoza, and M. T. Balhoff, Fluid-driven fractures in granular media: Insights from numerical investigations, *Phys. Rev. E* **101**, 042903 (2020).
 - [22] F. J. Carrillo and I. C. Bourg, Capillary and viscous fracturing during drainage in porous media, *Phys. Rev. E* **103**, 063106 (2021).
 - [23] H. Shin and J. C. Santamarina, Desiccation cracks in saturated fine-grained soils: Particle-level phenomena and effective-stress analysis, *Géotechnique* **61**, 961 (2011).
 - [24] B. Sandnes, E. G. Flekkøy, H. A. Knudsen, K. J. Måløy, and H. See, Patterns and flow in frictional fluid dynamics, *Nat. Commun.* **2**, 288 (2011).
 - [25] R. Holtzman, M. L. Szulczewski, and R. Juanes, Capillary fracturing in granular media, *Phys. Rev. Lett.* **108**, 264504 (2012).
 - [26] G. Ramos, G. Varas, J.-C. Géminard, and V. Vidal, Gas-induced fluidization of mobile liquid-saturated grains, *Phys. Rev. E* **92**, 062210 (2015).
 - [27] S. Lee, J. Lee, R. Le Mestrel, F. Xu, and C. W. MacMinn, Migration, trapping, and venting of gas in a soft granular material, *Phys. Rev. Fluids* **5**, 084307 (2020).
 - [28] R. Juanes, Y. Meng, and B. K. Primkulov, Multiphase flow and granular mechanics, *Phys. Rev. Fluids* **5**, 110516 (2020).
 - [29] Y. Meng, B. K. Primkulov, Z. Yang, C. Y. Kwok, and R. Juanes, Jamming transition and emergence of fracturing in wet granular media, *Phys. Rev. Res.* **2**, 022012 (2020).
 - [30] F. Ke, C. Y. Kwok, and K. Duan, Impact of intergranular bonds on morphology transition of two-phase fluid-induced deformation, *Phys. Rev. Fluids* **10**, 013902 (2025).
 - [31] F. Ke, Q. Dai, C. Y. Kwok, and K. Duan, Visualization and characterization of capillary-to-viscous transition in fluid-induced granular deformation, *Phys. Fluids* **36**, 123333 (2024).
 - [32] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/5cmb-kgt3> for a detailed description of the experimental procedure, quantitative analysis, theoretical derivation, and videos of the representative experiments.

- [33] C. Chevalier, M. Ben Amar, D. Bonn, and A. Lindner, Inertial effects on Saffman–Taylor viscous fingering, *J. Fluid Mech.* **552**, 83 (2006).
- [34] D. Crandall, G. Ahmadi, M. Ferer, and D. H. Smith, Distribution and occurrence of localized-bursts in two-phase flow through porous media, *Physica A* **388**, 574 (2009).
- [35] A. Islam, S. Chevalier, I. Ben Salem, Y. Bernabe, R. Juanes, and M. Sassi, Characterization of the crossover from capillary invasion to viscous fingering to fracturing during drainage in a vertical 2D porous medium, *Int. J. Multiphase Flow* **58**, 279 (2014).
- [36] M. Trojer, M. L. Szulczewski, and R. Juanes, Stabilizing fluid-fluid displacements in porous media through wettability alteration, *Phys. Rev. Appl.* **3**, 054008 (2015).
- [37] Y. Meng, W. Li, and R. Juanes, Fracturing in wet granular media illuminated by photoporomechanics, *Phys. Rev. Appl.* **18**, 064081 (2022).
- [38] R. Holtzman and R. Juanes, Crossover from fingering to fracturing in deformable disordered media, *Phys. Rev. E* **82**, 046305 (2010).
- [39] H. J. Butt, K. Graf, and M. Kappl, *Physics and Chemistry of Interfaces* (Wiley-VCH, Weinheim, 2003).
- [40] R. J. Lenhard and J. C. Parker, Experimental validation of the theory of extending two-phase saturation-pressure relations to three-fluid phase systems for monotonic drainage paths, *Water Resour. Res.* **24**, 373 (1988).
- [41] J. Wang, J. Lao, X. Luo, Y. Zhou, and H. Song, Intelligent prediction of water-CO₂ relative permeability in heterogeneous porous media towards carbon sequestration in saline aquifers, *Water* **17**, 1598 (2025).