

Electromagnetic vectoriality beyond polarization

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For arbitrary charge-current distributions, is it possible that along all directions their radiations are nonzero and are all circularly polarized? Our current descriptions of vectorial electromagnetic waves based on polarizations and the associated polarization ellipses fail to give an answer. In this study, we introduce the concept of instantaneous optical singularities, which together with the fundamental mathematical theorem of Poincaré-Hopf immediately rules out the above possibility of nonzero and all circularly polarized radiations. Based on both the conventional polarization singularities and the instantaneous singularities, we have established a general framework to capture the singular nature of vectorial electromagnetic waves, revealing that radiation patterns without dark directions (zero radiations) must have both points of circular polarizations and lines of linear polarizations, or equivalently, patterns without circular polarizations or linear polarizations must have dark directions. The framework established is further applied to the classical problem of scattering by electromagnetic dual particles, revealing the existence of scattering dark directions that have been hidden from all other theoretical explorations. We have essentially proved that polarizations are insufficient for descriptions of vectorial electromagnetic waves and revealed the complementarity (instantaneous and steady fields and their singularities) that can provide broader and deeper insights into not only electromagnetism, but also other branches of wave physics where singularities are generic and ubiquitous. Our work can potentially fully reshape singular photonics and chiral optics.

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Introduction. Maxwell equations are expressed by time-varying vector fields, and for monochromatic waves, instantaneous field vectors oscillate cyclically in both space and time, tracing out the polarization ellipses [1]. The oscillation speed of optical (or higher-frequency) waves is orders of magnitude higher than that of currently available detectors, and thus observable physical quantities up-to-date have been essentially characterized by time-averaged parameters (e.g., the term “time-averaged” frequently appears in most chapters of the classic book [1]). To some extent, polarization ellipses (trajectories of cyclically rotating instantaneous vectors’ tips) are also time-averaged steady structures that are widely employed for descriptions of electromagnetic polarizations [1,2] (refer to Fig. 1). Their singularities of linear and circular polarizations (normal and orientation directions of ellipses are undefined, respectively) are central concepts in singular optics [3], currently bridging different vibrant disciplines of photonics [4–9].

Considering that direct observations of high-frequency field oscillations are currently inaccessible and thus time-

averaged steady quantities and structures are seemingly sufficient for descriptions of observable effects, does it mean that the perspective of instantaneous field vectors is superfluous and will not be able to provide physical insights, in particular for high-frequency electromagnetic waves? We start from the following elementary question that is important in both singular and chiral optics [3,4,10,11]: for arbitrary charge-current distributions, is it possible that along all directions their radiations are nonzero and are all circularly polarized? From the conventional perspective of polarization ellipses and polarization singularities, it is impossible to answer this question. To crack this question, we introduce the concept of instantaneous optical singularities and combine it with the Poincaré-Hopf theorem (of which a reduced scenario is the hairy ball theorem) [12,13]. We unveil how the global topology of the radiation momentum sphere (topological Euler characteristic $\chi = 2$) regulates the local distributions of the radiated fields. In the far field, the radiated waves are transverse, which are described by tangent fields on the momentum sphere: both steady directionless line fields (vectors without arrows; defined as the major or minor axes of the polarization ellipses) and cyclically oscillating directional vector fields (refer to Fig. 1). Throughout the momentum sphere, the index sum of line-field singularities of circular polarizations has to be the Euler characteristic $\chi = 2$ [14–17]. With the Poincaré-Hopf theorem also applied to the instantaneous vector fields, we reveal that singularities (instantaneous vectorial zeros of fields) always move along lines of linear polarizations in a cyclic manner; they are irrelevant to and

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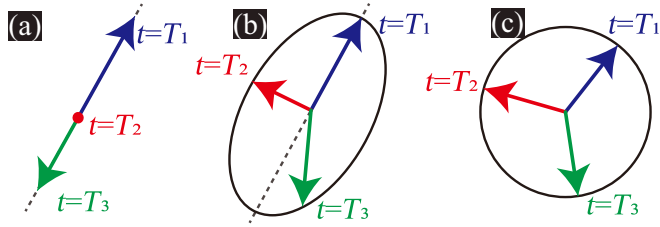


FIG. 1. Linear, elliptic, and circular polarizations are shown in panels (a)–(c), respectively. The line field (dashed lines) is defined as the orientation line of linear polarization in panel (a), the major axis of the ellipse in panel (b), and ill-defined in panel (c). Instantaneous field vectors of which the tips trace out the polarization ellipses are shown at three instants ($t = T_{1,2,3}$) for each polarization. Only the linear polarization accommodates zero instantaneous field vectors (red dot) at the instant $t = T_2$.

never locate at points of circular or elliptic polarizations (see Fig. 1); their index sum also has to be 2, at any instant. Based on both the conventional polarization singularities and the instantaneous singularities introduced, we have established a general framework to capture the singular nature of vectorial electromagnetic waves, discovering that radiation patterns without dark directions (zero radiations) must have both points of circular polarizations and lines of linear polarizations, or equivalently, patterns without circular polarizations or linear polarizations must have dark directions. This discovery immediately rules out the aforementioned special scenario of nonzero and all circularly polarized radiations, verifying that polarizations are insufficient for descriptions of vectorial electromagnetic waves, and the complementary singularities of both instantaneous and steady fields can provide broader and deeper insights into electromagnetism.

The framework established is further applied to the classical problem of scattering by electromagnetic dual particles, revealing dark scattering directions that have been long hidden, which yield to no other theoretical explorations. From this perspective of instantaneous singularities and parameter-space global topology, we examine scatterings by electromagnetic duality-symmetric particles [18–20]. For circularly polarized (CP) incident waves, helicity conservation prohibits linearly polarized scatterings and thus instantaneous singularities can only locate on positions where the field is zero at any instant: There must be at least one dark direction (**V** point) of index 2 where the scattering (time-averaged intensity I_{sca}) is zero. We further show that when a pair of **V** points for incident left-handed and right-handed CP (LCP and RCP) waves are tuned to overlap, this direction will remain dark for arbitrary incident polarizations.

Topological framework. For electromagnetic scattering by finite scattering bodies of arbitrary geometric and optical parameters (electromagnetic radiations by arbitrary charge-current distributions), scattered (radiated) far fields are transverse and automatically tangent on the momentum sphere. For the associated line fields on the sphere, the Poincaré-Hopf theorem requires that [3,13,17])

$$\sum_i \text{Ind}(\mathbf{C}_i) = \chi = 2, \quad (1)$$

where $\mathbf{C}_i$ are isolated line-field singularities and their generic (structurally stable against perturbations) forms correspond to CP scatterings (radiations) with indexes (Hopf index) $\text{Ind} = \pm 1/2$. Nongeneric $\mathbf{C}_i$ (higher-order CP points or **V** points of other half-integer indexes not equal to $\pm 1/2$) would be broken into pairs of generic CP points upon perturbations. It is worth mentioning that though linear polarizations are also polarization singularities, they are not directly related to the Poincaré-Hopf theorem in Eq. (1). If throughout the momentum sphere there are no CP radiations, then according to Eq. (1) there must be at least one direction of zero radiation (nongeneric polarization singularity; **V** point) and the index sum of the zero(s) must be 2. A special such scenario (radiations are all linearly polarized and thus there are no CP points) has been demonstrated in our previous study [14].

We proceed to apply the Poincaré-Hopf theorem to the instantaneous vector (electric or magnetic) fields on the momentum sphere [13,17]:

$$\sum_i \text{Ind}(\mathbf{Z}_i) = 2. \quad (2)$$

Here, $\mathbf{Z}_i$ are isolated vector-field singularities located at the position where the instantaneous field is zero and thus the vector direction is not defined. $\mathbf{Z}_i$ and $\mathbf{C}_i$ differ in the following aspects: (1) $\mathbf{Z}_i$ are singularities of instantaneous vector fields and thus they can move in time (except for nongeneric forms of **V** points), while $\mathbf{C}_i$ are singularities of steady line fields and thus they are spatially fixed; (2) vector fields are directional (return to itself with at least 2π rotation) and thus $\mathbf{Z}_i$ have integer indexes (generic singularity indexes are ± 1), while line fields are directionless (π rotation is sufficient for the return) and thus $\mathbf{C}_i$ have half-integer indexes [17]; and (3) generic $\mathbf{Z}_i$ move along lines of linear polarizations in a cyclic manner, while generic $\mathbf{C}_i$ locate at CP positions. The last aspect is apparent (see Fig. 1): tips of instantaneous field vectors of CP (or elliptically polarized) waves trace out circles (or ellipses) and thus they can never reach zero at any instant; only linear polarizations (field vectors can read zero at discrete instants) accommodate vector-field singularities.

Generally speaking, $\mathbf{C}_i$ and $\mathbf{Z}_i$ are different points located on different positions of circular and linear polarizations, respectively. The only exception is that they overlap at **V** points which are singularities of singularities: a **V** point can be viewed both as a special $\mathbf{C}_i$ point in Eq. (1) and as a special $\mathbf{Z}_i$ point in Eq. (2). Based on the argument of instantaneous vector fields, we can reach a similar conclusion as that for line field: if throughout the momentum sphere there are no linearly polarized radiations, then according to Eq. (2) there must be at least one direction of zero radiation and index sum of the zero(s) must be 2.

From the joint perspective of both line and vector fields, we can reach the following conclusion: scattering (radiation) patterns without dark directions must have both points of circular polarizations and lines of linear polarizations, or equivalently, patterns without points of circular polarizations or lines of linear polarizations must have dark directions. Our arguments above are fully based on the Poincaré-Hopf theorem and thus the conclusions drawn are universally applicable to arbitrary charge-current distributions, and to all scattering scenarios for scatterers of arbitrary geometric and optical parameters

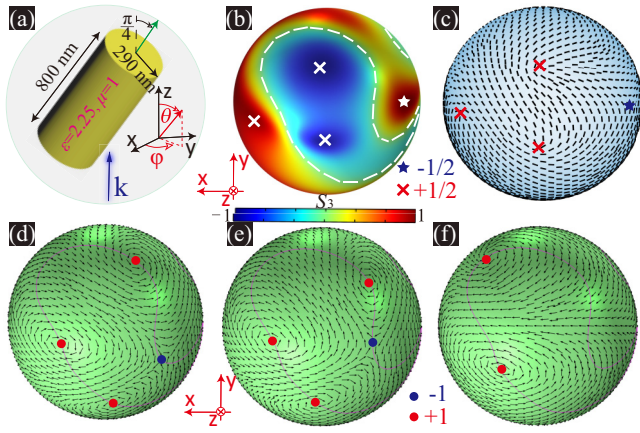


FIG. 2. (a) The scattering of a dielectric cylinder by plane waves incident along $+z$. (b) Polarization (S_3) distributions of scattered fields on the momentum sphere, with four in-view **C** points marked and indexes specified. Lines of linear polarizations are also marked by curves. The corresponding line fields [panel (c)] and instantaneous vector fields at three different instants [panels (d)–(f)] are also shown, with in-view **Z** points marked and their indexes specified.

and for arbitrarily structured incident waves. Such a discovery directly rules out the following two special scenarios: (1) nonzero and all circularly polarized radiations; and (2) nonzero and all linearly polarized radiations, explaining, e.g., why the dipolar radiations have to be zero along its orientation directions.

Application for particle scattering. Now we exemplify our discovery through an elementary scattering configuration (numerical results are obtained through comsol multiphysics) shown in Fig. 2(a): a dielectric cylinder (relative permittivity $\epsilon = 2.25$ and permeability $\mu = 1$) scatters LCP plane wave of incident wavelength λ and wave vector $\mathbf{k}$ ($k = |\mathbf{k}|$). The far-field polarization distributions (in terms of the Stokes parameter S_3 [2]; $S_3 = \pm 1$ corresponds, respectively, to LCP and RCP states; $\lambda = 810$ nm) are shown in Fig. 2(b), where four in-view line-field singularities of circular polarizations (**C** points) are marked with their indexes specified. In Fig. 2(b), we have also marked (by dashed curves) lines of linear polarizations ($S_3 = 0$). The corresponding line-field (semi-major axes of polarization ellipses) distributions are shown in Fig. 2(c), manifesting clearly both positions and indexes of the **C** points. In addition, there are two other **C** points out of view, with the index sum of all **C** points being exactly 2, see Supplemental Material [17].

Meanwhile, we also show the instantaneous electric field vector distributions on the momentum sphere at three instants in Figs. 2(d)–2(f). In contrast to steady line fields, vector-field singularities (instantaneous vector zeros; **Z** points) cyclically move along lines of linear polarizations (also marked), with field patterns repeating themselves in every cycle $T = \lambda/c$ (c is the speed of light; more details about the birth and annihilation of opposite-index singularities on lines of linear polarizations could be found in Movie S1 in the Supplemental Material [17]). In Figs. 2(d)–2(f), only in-view **Z** points are marked with their indexes specified, and the index sum of all **Z** points is indeed 2, see Supplemental Material [17].

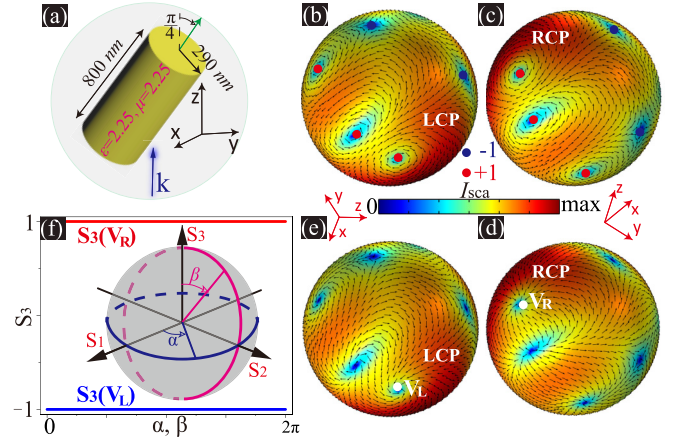


FIG. 3. (a) The scattering of plane waves along $+z$ by a self-dual cylinder. (b), (c) I_{sca} (color bar) and instantaneous field vector distributions, respectively, for LCP and RCP incidences, with in-view **V** points marked and their indexes specified. Panels (d) and (e) are the same as panels (b) and (c), but at a different instant, where two **V** points ($\mathbf{V}_L$ and $\mathbf{V}_R$) are marked. (f) The invariance of scattering polarizations on these marked points, with the inset showing the Poincaré sphere on which two great circles are parameterized by α and β , respectively.

Electromagnetic duality induced dark directions. For the results shown in Fig. 2, throughout the momentum sphere, there are no directions of zero scatterings (**V** points) and thus everywhere S_3 is not singular. This is quite expected, as codimension analysis has revealed that **V** points are not generic and generally would be absent [3,17]. That is, extra symmetries are required to achieve strictly zero scattering. In Fig. 3(a), we demonstrate scattering by another electromagnetic duality-symmetric (self-dual) cylinder ($\epsilon = \mu = 2.25$) [18–20]. One special property for self-dual particles is given as follows: With an incident CP wave, helicity conservation requires that scatterings along all directions are also CP with the same handedness as that of the incident wave [18–20]. From the perspective of steady line-field singularities, this scenario is trivial as the field is singular everywhere, while from the perspective of instantaneous vector fields, this is contrastingly nontrivial: as we have revealed, since instantaneous vector singularities do not locate on CP positions, Eq. (2) secures the existence of at least one dark direction (**V** point).

Distributions of I_{sca} and instantaneous field vectors for LCP and RCP incidences ($\lambda = 810$ nm) are shown in Figs. 3(b) and 3(c), respectively, where in-view **V** points ($\mathbf{V}_L$ and $\mathbf{V}_R$, respectively) are marked, with integer indexes also specified. Index sum of all **V** points (including out-of-view ones) is indeed 2, see Supplemental Material [17]. Different from the vector-field singularities [shown in Figs. 2(d)–2(f)] that move along lines of linear polarizations, here there are no such lines and all singularities are fixed at **V** points. This is evident from the insets of Figs. 3(d) and 3(e) that show the scattering distributions at another instant: The field vectors do rotate in time, while **V** points are fixed in space.

Another consequence of helicity conservation is that, along the dark directions of $\mathbf{V}_L$ and $\mathbf{V}_R$, the scattering will remain to be, respectively, RCP ($S_3 = -1$) and LCP ($S_3 = 1$), irrespective of the incident polarizations. This is easy to comprehend,

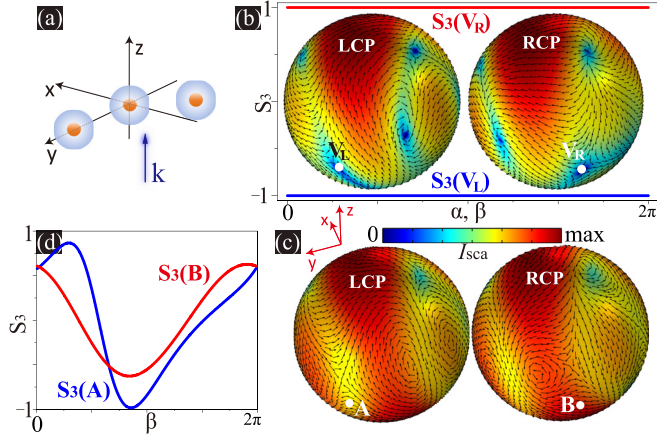


FIG. 4. (a) The scattering of plane waves along $+z$ by three self-dual core-shell particles. The two insets of panel (b) show I_{sca} and field vector distributions, respectively, for LCP and RCP incidences, with two $\mathbf{V}$ points marked. The invariant scattering polarizations with varying incident polarizations on marked points are shown in panel (b). Except that the incident wavelength is shifted to make the system not self-dual, panel (c) is the same as the two insets in panel (b), and panel (d) is the same (only β dependence is shown) as panel (b). $\mathbf{A}$ ($\mathbf{B}$) and $\mathbf{V}_L$ ($\mathbf{V}_R$) correspond to the same direction.

see Supplemental Material [17]: (1) both incident and scattered waves can be expanded into LCP and RCP components; (2) helicity conservation ensures that there are no cross couplings between them, incident LCP (RCP) components being scattered into LCP (RCP) components only; and (3) along the scattering direction of $\mathbf{V}_L$ ($\mathbf{V}_R$), for arbitrary incident polarizations, LCP (RCP) components would be absent and thus the scattered waves would invariantly be RCP (LCP). This is shown in Fig. 3(f), where we demonstrate invariant S_3 on a pair of $\mathbf{V}_L$ and $\mathbf{V}_R$ marked in Figs. 3(d) and 3(e). The incident polarizations evolve on two great circles on the Poincaré sphere [see the inset of Fig. 3(f)], which are parameterized by α [linear polarizations along x ($\alpha = 0$) and y axes ($\alpha = \pi$)] and β [LCP ($\beta = 0$) and RCP ($\beta = \pi$)], respectively.

So far, the principles we reveal have been demonstrated through a hypothetical self-dual magnetic particle ($\mu = 2.25$). We now turn to practical nonmagnetic self-dual particles, of which the magnetic responses originate from optically induced magnetism [21–24]. For spherical particles that support dominantly lowest-order dipolar responses, the effective electric and magnetic isotropic polarizabilities are [21,22,25] $\alpha_e = (3i/2k^3)a_1$ and $\alpha_m = (3i/2k^3)b_1$, where a_1 and b_1 are Mie scattering coefficients [25]. The spherical particle becomes self-dual when $a_1 = b_1$, meaning that electric and magnetic dipolar responses are balanced: $\alpha_e = \alpha_m$. Here, we employ a similar particle to that adopted in our previous studies [20,22]: Ag core-dielectric ($\epsilon = 3.4^2$) shell spherical particle with inner radius $R_1 = 40$ nm and outer radius $R_2 = 160$ nm; the permittivity of silver is extracted from the experimental data in Ref. [26]. This particle becomes self-dual at $\lambda_0 = 1115$ nm only, where $a_1 \approx b_1$ (refer to Ref. [17] for discussions about nonideal dual symmetry). Here we study the scattering by a self-dual ensemble consisting of three such particles [Fig. 4(a); see Supplemental Material Ref. [17] for

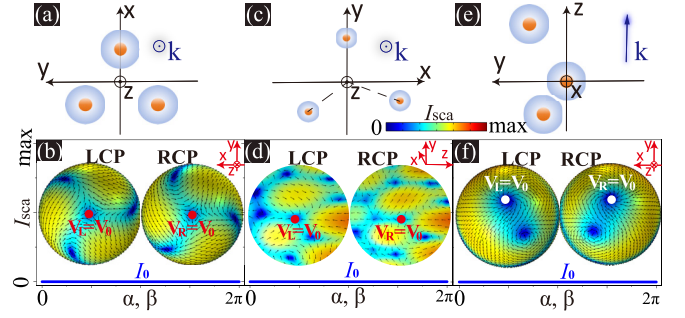


FIG. 5. Scattering of plane waves along $+z$ by three tripartite ensembles: (a) The configuration is rotationally symmetric with rotation axis parallel to z axis; the interparticle couplings are weaker in panel (c) with far-separated particles and stronger in panel (e) with closely packed particles (see Supplemental Material Ref. [17] for detailed position information). The corresponding invariant zero scattering with varying incident polarizations along the direction of $\mathbf{V}_0$ (marked in insets that show I_{sca} and vector-field distributions with CP incidences) is shown in panels (b), (d), and (f), respectively. For the insets in panel (d), only field distributions on a spherical disk of the momentum sphere (centered around the direction close to $-\mathbf{x}$) are shown to better visualize the overlapping of $\mathbf{V}_L$ and $\mathbf{V}_R$.

detailed position information, as is also the case for ensembles in Fig. 5]. Scattering distributions at one instant for $\lambda = \lambda_0$ are shown as insets in Fig. 4(b), displaying a pair of $\mathbf{V}$ points. The invariance of S_3 on both points with varying incident polarizations is summarized in Fig. 4(b). For comparison, we shift the incident wavelength to $\lambda_0 = 1040$ nm, where the system is not self-dual anymore ($a_1 \neq b_1$). As shown in Fig. 4(c), along the directions of $\mathbf{A}$ and $\mathbf{B}$ [identical directions as those of $\mathbf{V}_L$ and $\mathbf{V}_R$ in Fig. 4(b), respectively], here the scattering is not zero anymore. We further demonstrate the evolution of S_3 along those directions in Fig. 4(d) for different incident polarizations. As expected, S_3 varies and is not fixed at ± 1 anymore.

It is clear from Figs. 3 and 4 that $\mathbf{V}_L$ and $\mathbf{V}_R$ generally are spatially separated (never located on the forward direction, as required by the optical theorem [25]). When they are tuned to overlap at point $\mathbf{V}_0 = \mathbf{V}_L = \mathbf{V}_R$, on $\mathbf{V}_0$ the scattered field would contain neither LCP nor RCP components, and thus have to be zero for arbitrary incident polarizations. Such $\mathbf{V}$ -point overlapping can be secured by an extra geometric symmetry [19,20]: For a self-dual scattering system that exhibits no less than three-fold rotation symmetry, with CP waves incident along the rotation axis, $\mathbf{V}_L$ and $\mathbf{V}_R$ would overlap on the backscattering direction. One such scattering configuration is shown in Fig. 5(a) and $\mathbf{V}_0$ is marked in the insets of Fig. 5(b). The directional scattering intensity on $\mathbf{V}_0$ (I_0) is invariantly zero for varying incident polarizations, as verified by Fig. 5(b).

For an ensemble of randomly distributed self-dual dipolar particles ($\alpha_e = \alpha_m = \alpha_0$), the scattering can be described by the coupled dipole theory [22,27,28]:

$$\begin{aligned} \mathbf{p}_i &= \alpha_0 \mathbf{E}_i^0 + \alpha_0 \sum_{j \neq i} (\mathbf{E}_i^{\mathbf{p}_j} + \mathbf{E}_i^{\mathbf{m}_j}), \\ \mathbf{m}_i &= \alpha_0 \mathbf{H}_i^0 + \alpha_0 \sum_{j \neq i} (\mathbf{H}_i^{\mathbf{p}_j} + \mathbf{H}_i^{\mathbf{m}_j}), \end{aligned} \quad (3)$$

where $\mathbf{p}_i$ and $\mathbf{m}_i$ are electric and magnetic dipolar moments supported by the i th particle centered at $\mathbf{r}_i$, respectively; $\mathbf{E}_i^0$ and $\mathbf{H}_i^0$ are the incident electric and magnetic fields at $\mathbf{r}_i$, respectively; and $\mathbf{E}_i^{p_j}$ ($\mathbf{E}_i^{m_j}$) and $\mathbf{H}_i^{p_j}$ ($\mathbf{H}_i^{m_j}$) are electric and magnetic fields radiated by the electric dipole $\mathbf{p}_j$ (magnetic dipole $\mathbf{m}_j$) at $\mathbf{r}_i$, respectively. When the particles are separated by sufficiently large distances so that the couplings among them are negligible, $|\mathbf{E}_i^{p_j}| \ll |\mathbf{E}_i^0|$ and $|\mathbf{H}_i^{p_j}| \ll |\mathbf{H}_i^0|$ for all i and j , Eq. (3) is reduced to $|\mathbf{p}_i| = |\mathbf{m}_i|$ and $\mathbf{p}_i \perp \mathbf{m}_i$ for all i . That is, for incident plane wave of arbitrary polarization, each particle supports a pair of orthogonal and equal electric and magnetic dipolar moments (Kerker particles [29–37]) and the backward scattering is zero for both individual particles and the whole ensemble.

When the couplings among the particles are non-negligible and at the same time the scattering configuration does not exhibit the rotation symmetry required, there is no mechanism to guarantee overlapped $\mathbf{V}_L$ and $\mathbf{V}_R$ at the backward direction or any other directions (see Fig. 4). Nevertheless, accidental overlapping is still accessible by tuning the incident direction and particle distributions. Two such scattering configurations are shown in Fig. 5(c) [Fig. 5(e)], where the particles are well separated (closely packed) see Supplemental Material [17] with weaker (stronger) interparticle couplings. For both scenarios, overlapped $\mathbf{V}_0$ are obtained [marked in the insets of Figs. 5(d) and 5(f)] and neither locate in the backward direction. As expected, the directional scattering intensity on $\mathbf{V}_0$ for both scenarios remains to be zero, irrespective of the incident polarizations [Figs. 5(d) and 5(f)]. In fact, the existence of overlapped $\mathbf{V}_0$ can be verified from a single incidence: When the incident polarization is noncircular (contains both LCP and RCP components), the dark direction must correspond to an overlapped $\mathbf{V}_0$, since the scattered LCP and RCP components do not interfere, and thus they both have to be zero, see Supplemental Material [17]. That is, for self-dual scattering systems, the dark direction for one noncircular incident polar-

ization would remain dark for all incident polarizations, see Supplemental Material [17].

Conclusions. It is uncovered that polarizations underdescribe vectorial electromagnetic waves and both steady and instantaneous parameters and structures are required to capture the subtleties of vectorial electromagnetism. We reveal that the Poincaré-Hopf theorem regulates the distributions of not only steady line fields but also instantaneous vector fields, for which the generic singularities are, respectively, fixed $\mathbf{C}$ points and cyclically moving (along lines of linear polarizations) vector zeros ($\mathbf{Z}$ points), with the index sum being identically the Euler characteristic 2. Besides the global application throughout the momentum sphere, instantaneous singularities can also be applied locally to an arbitrarily chosen loop, see Supplemental Material [17]. Our discovery is independent of optical or geometric parameters of the scatterers or structures of incident waves. As a result, our demonstrations with self-dual particles are only a special application of it. The complementarity (instantaneous and steady fields) we reveal can provide broader and deeper insights into not only electromagnetism, but also other branches of wave physics where singularities are generic and ubiquitous. To some extent, in this work, we are not discussing principles that obey Maxwell equations, but rather we are discussing principles that Maxwell equations themselves have to obey. Our work has the potential to fully reshape not only singular optics, but also chiral optics [10,11,40], and other associated disciplines of topological and non-Hermitian photonics that are highly related to the vectorial nature of electromagnetic waves.

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