

Optomechanical separation of transverse spin and orbital energy flows in liquid crystals

Urban Mur^{1,2}, Miha Čančula¹, Hirokazu Kobayashi³, Miha Ravnik^{1,4},
Slobodan Žumer^{1,4} and Etienne Brasselet^{5,*}

¹Faculty of Mathematics and Physics, University of Ljubljana, Ljubljana, Slovenia

²Department of Engineering Science, University of Oxford, Oxford, United Kingdom

³School of System Engineering, Kochi University of Technology, Kochi, Japan

⁴Jožef Stefan Institute, Ljubljana, Slovenia

⁵Univ. Bordeaux, CNRS, LOMA, UMR 5798, Talence, France



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We report a nondissipative optomechanical approach to separate the spin and orbital components of transverse energy flows of light beams. Focusing on uniformly polarized paraxial fields, the method relies on detecting the radiation-pressure-induced reorientation of the optical axis in anisotropic media—specifically, liquid crystals. Within a perturbative analytical approach, we demonstrate the selective encoding of the transverse spin and orbital components of the Poynting vector into orthogonal orientational elastic modes, whose relative weights are mediated by the Gouy phase. The spatial distributions of spin and orbital momentum can therefore be retrieved without resorting to point-by-point measurements using absorbing or scattering probe particles. Analytical results are supported by a fully self-consistent numerical solution coupling Maxwell’s equations and Landau-de Gennes nematodynamics in three dimensions. This exact framework captures higher-order and nonparaxial effects beyond the analytical perturbative regime and allows us to validate the predicted Gouy-phase-mediated optomechanical response. These results establish a practical route toward optomechanical imaging of the internal momentum structure of light, fostering momentum-resolved optical metrology and light-matter interaction studies with structured beams.

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I. INTRODUCTION

The flow of energy in structured light has emerged as a central theme in both optics and light-matter interaction. In particular, decomposing the Poynting vector into spin and orbital (or canonical) components—reflecting the inhomogeneities associated with the polarization state and the local phase gradients, respectively—has refined our understanding of electromagnetic fields beyond the plane-wave approximation [1–3]. These contributions underpin several rapidly evolving areas of optics, including spin-orbit interactions [4], radiation-induced forces and torques [5], and the study of topological textures in light fields [6]. The theoretical framework is well established for arbitrary monochromatic electromagnetic fields in isotropic media, assuming no free charges or currents. In this case, the Poynting vector $\mathbf{P} = \frac{1}{2}\text{Re}(\mathbf{E}^* \times \mathbf{H})$ [7], where $\mathbf{E}$ and $\mathbf{H}$ are the complex electric and magnetic fields, can be written as

$$\mathbf{P} = \frac{1}{2\omega\mu} \left(\underbrace{\text{Im}[\mathbf{E}^* \cdot (\nabla)\mathbf{E}]}_{\text{orbital part (o)}} + \underbrace{\frac{1}{2}\nabla \times \text{Im}[\mathbf{E}^* \times \mathbf{E}]}_{\text{spin part (s)}} \right). \quad (1)$$

$$= \mathbf{P}_o + \mathbf{P}_s, \quad (2)$$

which defines, respectively, the orbital and spin components, $\mathbf{P}_o$ and $\mathbf{P}_s$ [2], with ω the angular frequency, μ the magnetic permeability, and $\mathbf{A} \cdot (\nabla)\mathbf{B} \equiv \sum_i A_i \nabla B_i$. This electric-field-based decomposition is valid in the paraxial regime (our focus here) where the electric-magnetic symmetry ensures consistency.

Experimentally, accessing the spatial distributions of both $\mathbf{P}_o$ and $\mathbf{P}_s$, especially their transverse components, remains challenging. Only a few techniques allow independent visualization of the transverse Poynting vector components. For instance, wave front-sensing methods are exclusively sensitive to $\mathbf{P}_o$ [8]. Optomechanical detection with micro/nano-sized probes can reveal $\mathbf{P}_s$ in the paraxial regime, and even in nonparaxial fields, such as evanescent waves [9] or tightly focused beams [10], at the cost of careful spin-orbit analysis. Yet, a device capable of selective, discretization-free, imaging internal transverse energy flows remains elusive, even for paraxial beams.

In contrast to previous approaches relying on discrete measurements, we propose a continuous platform to directly image the internal transverse energy flows in structured optical fields. Our approach exploits optically induced reorientation of the optical axis in a soft anisotropic medium—specifically, a nematic liquid crystal film—driven by an optical radiation torque density directly linked to radiation pressure, as previously noticed [11]. Building on recent experimental findings that revealed the crucial role of the longitudinal field component in light-matter coupling within uniaxial media [12–15], here we demonstrate that the azimuthal and radial components of light-induced orientational distortion encode, respectively,

*Contact author: etienne.brasselet@u-bordeaux.fr

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the spin and orbital momentum densities of the incident beam. In particular, we identify the Gouy phase as the optical parameter that controls the balance between these two elastic modes, thereby revealing a direct manifestation of Gouy-phase physics in an optomechanical response. Our work thus connects liquid-crystal optomechanics with the fundamental spin-momentum structure of light first formulated by Belinfante [16], and constitutes the optomechanical counterpart of recent acoustomechanical predictions relating radiation force and torque to canonical momentum and spin densities [17].

The paper is organized as follows. In Sec. II, we introduce the theoretical framework describing the optomechanical coupling between structured optical fields and nematic liquid crystals, and derive the perturbative analytical relations linking transverse spin and orbital energy flows to orthogonal elastic distortion modes, with particular emphasis on the role of the Gouy phase. In Sec. III, we present a fully three-dimensional (3D) numerical model combining Maxwell's equations with Landau-de Gennes (LdG) nematodynamics, allowing us to go beyond the perturbative regime and to account for nonparaxial effects as well as optomechanical feedback arising from polarization changes during the propagation of light through the medium. Section IV addresses the experimental challenge of observing the predicted optoelastic mapping, highlighting how propagation-induced polarization changes can hinder the selective imaging of transverse spin and orbital energy flows. Finally, Sec. V provides our concluding remarks and outlines possible extensions to more complex structured light fields.

II. ANALYTICS

A. Local perturbative approach

To establish analytically a quantitative connection between the transverse optical energy flow and the resulting orientational response of a nonmagnetic nematic medium (for which $\mu = \mu_0$, the vacuum magnetic permeability), we begin by recalling the expression of the electric field of a monochromatic beam exhibiting a modest degree of nonparaxiality and propagating along the z axis. This axis is taken as the optical axis of a uniformly aligned nematic liquid crystal slab, whose equilibrium director $\mathbf{n}$ —the unit vector describing the local average molecular orientation of the nematic and obeying the inversion symmetry $\mathbf{n} \equiv -\mathbf{n}$ —is initially oriented along the propagation direction, $\mathbf{n} = \mathbf{e}_z$.

To establish analytically a quantitative connection between the transverse optical energy flow and the resulting orientational response of a nonmagnetic (hence $\mu = \mu_0$, the magnetic permeability of vacuum) nematic medium, we begin by recalling the expression for the electric field of a monochromatic beam with a modest level of nonparaxiality propagating along the z axis, taken as the optical axis of a uniformly aligned nonmagnetic nematic liquid crystal slab. We consider a uniform polarization described by a two-dimensional transverse unit vector $\mathbf{e}_\perp$, and omit the global propagation factor $e^{-i\omega t + ikz}$. Moreover, in the limit of small optical anisotropy, $\epsilon_a/\epsilon_\parallel \ll 1$, where $\epsilon_a = \epsilon_\parallel - \epsilon_\perp$ is the dielectric permittivity anisotropy with $(\perp, \parallel)$ denoting the direction perpendicular and parallel

to the optical axis, the condition $\nabla \cdot \mathbf{E} = 0$ gives

$$\mathbf{E} = E_\perp \mathbf{e}_\perp + \frac{i}{k} (\mathbf{e}_\perp \cdot \nabla_\perp E_\perp) \mathbf{e}_z, \quad (3)$$

where $\mathbf{e}_z$ is the unit vector oriented along the z axis. In other words, we neglect polarization inhomogeneities arising during propagation along the optical axis, so that k refers to the ordinary wave vector $k = k_0 \sqrt{\epsilon_\perp}$, where k_0 is the wave vector in vacuum. The transfer of electromagnetic momentum to the medium is generally described by the local optical torque density that drives the orientational optical nonlinearities of liquid crystals, $\mathbf{\Gamma} = \frac{1}{2} \epsilon_0 \epsilon_a \text{Re}[(\mathbf{n} \cdot \mathbf{E}^*)(\mathbf{n} \times \mathbf{E})]$ [18], where ϵ_0 is the dielectric permittivity in vacuum. Within our framework, it reduces to

$$\mathbf{\Gamma} = \frac{1}{2k} \epsilon_0 \epsilon_a \text{Im}[(\mathbf{e}_\perp^* \cdot \nabla_\perp E_\perp^*)(\mathbf{e}_z \times \mathbf{E}_\perp)]. \quad (4)$$

This expression shows that the optical torque originates from the transverse gradients of the field amplitude and polarization, coupling the local energy flow to the director orientation. Within a local perturbative approach, without yet solving the full nonlocal light-matter problem, the corresponding director distortion $\delta \mathbf{n} = \mathbf{n}_\perp$ can be analytically estimated using a local-response approximation as

$$\delta \mathbf{n} \sim \mathbf{\Gamma} \times \mathbf{e}_z = \frac{\epsilon_0 \epsilon_a}{2k} \text{Im}[(\mathbf{e}_\perp^* \cdot \nabla_\perp E_\perp^*) E_\perp \mathbf{e}_\perp]. \quad (5)$$

Then, introducing the polarization matrix $\hat{\rho}$ in the Cartesian reference frame as

$$\hat{\rho} = \mathbf{e}_\perp \otimes \mathbf{e}_\perp^* = \frac{1}{2} (\hat{\mathbf{1}} + s_1 \hat{\sigma}_1 + s_2 \hat{\sigma}_2 + s_3 \hat{\sigma}_3), \quad (6)$$

where $\otimes$ denotes the tensorial product,

$$s_1 = |\mathbf{e}_\perp \cdot \mathbf{e}_x|^2 - |\mathbf{e}_\perp \cdot \mathbf{e}_y|^2, \quad (7)$$

$$s_2 = 2\text{Re}[(\mathbf{e}_\perp \cdot \mathbf{e}_x)^* (\mathbf{e}_\perp \cdot \mathbf{e}_y)], \quad (8)$$

$$s_3 = 2\text{Im}[(\mathbf{e}_\perp \cdot \mathbf{e}_x)^* (\mathbf{e}_\perp \cdot \mathbf{e}_y)], \quad (9)$$

are the reduced Stokes parameters [19], $\hat{\mathbf{1}}$ is 2×2 identity matrix, and

$$\hat{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \hat{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad (10)$$

Equation (5) is rewritten in the Cartesian reference frame as

$$\delta \mathbf{n} \sim \frac{\epsilon_0 \epsilon_a}{2k} \text{Im}[E_\perp \hat{\rho} (\nabla_\perp E_\perp^*)]. \quad (11)$$

Finally, using the relationships $\hat{\sigma}_3 \nabla_\perp = -i \nabla_\perp \times \mathbf{e}_z$ and $s_3 = -i(\mathbf{e}_\perp^* \times \mathbf{e}_\perp) \cdot \mathbf{e}_z$, and noting that

$$\mathbf{P}_{o,\perp} = \frac{1}{2\omega\mu} \text{Im}[E_\perp^* \nabla_\perp E_\perp], \quad (12)$$

$$\mathbf{P}_{s,\perp} = \frac{s_3}{2\omega\mu} \text{Re}[E_\perp^* (\nabla_\perp E_\perp \times \mathbf{e}_z)], \quad (13)$$

are the expressions of transverse components of $\mathbf{P}_o$ and $\mathbf{P}_s$ given in Eq. (1) in the limit of negligible longitudinal component of the electric field, one obtains

$$\delta \mathbf{n} \sim -\frac{\epsilon_a}{2c\sqrt{\epsilon_\perp}} [(\hat{\mathbf{1}} + s_1 \hat{\sigma}_1 + s_2 \hat{\sigma}_2) \mathbf{P}_{o,\perp} + \mathbf{P}_{s,\perp}], \quad (14)$$

$$= \delta \mathbf{n}_o + \delta \mathbf{n}_s, \quad (15)$$

where c is the speed of light.

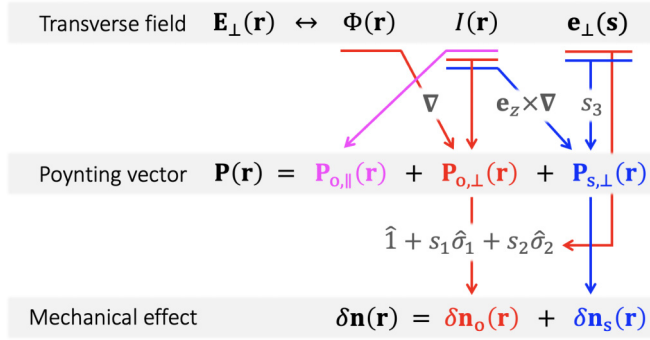


FIG. 1. Summary of how the phase (Φ), intensity (I), and polarization ($\mathbf{e}_\perp$) of the optical field determine the longitudinal and transverse components of the Poynting vector $\mathbf{P}$, and how the transverse components $\mathbf{P}_{o,\perp}$ and $\mathbf{P}_{s,\perp}$ drive the orientational response $\delta\mathbf{n}$ of the nematic medium.

Equation (14) thus provides an analytical estimate of the thresholdless local director response, explicitly relating the field-induced reorientation to both the spin and orbital components of the transverse Poynting vector, while the full nonlocal response will be treated numerically in a second step. An illustrated summary is shown in Fig. 1.

B. Gaussian beam excitation

To illustrate the analytical approach, we now consider the specific case of a Gaussian beam of power $\mathcal{P}$, described by a transverse electric field

$$\mathbf{E}_\perp(r, z) = \sqrt{\frac{4\mathcal{P}}{\pi c \epsilon_0 \epsilon_\perp^{1/2} w(z)^2}} \times \exp \left[-\frac{r^2}{w(z)^2} + i \frac{kr^2}{2R(z)} - i \Phi_{\text{Gouy}}(z) \right] \mathbf{e}_\perp, \quad (16)$$

where $w(z) = w_0 \sqrt{1 + (z/z_0)^2}$, $R(z) = z + z_0^2/z$, $z_0 = kw_0^2/2$, and $\Phi_{\text{Gouy}}(z) = \arctan(z/z_0)$ denote the beam radius, wave front radius of curvature, Rayleigh length, and Gouy phase, respectively.

From Eqs. (3) and (16), the longitudinal electric field is calculated as

$$E_z = \frac{i}{k} (\mathbf{e}_\perp \cdot \mathbf{e}_r) \frac{\partial E_\perp}{\partial r}, \quad (17)$$

from which the Poynting vector in the cylindrical coordinates basis ($\mathbf{e}_r$, $\mathbf{e}_\phi$, $\mathbf{e}_z$) can be expressed, up to the first-order terms in the small parameter $(kw)^{-1}$, as

$$\mathbf{P} = \frac{1}{2} \epsilon_0 c \sqrt{\epsilon_\perp} I \left(\underbrace{\mathbf{e}_z + \frac{r}{R} \mathbf{e}_r}_{\text{orbital part } (\parallel, \perp)} + \underbrace{\frac{2s_3 r}{kw^2} \mathbf{e}_\phi}_{\text{spin part } (\perp)} \right), \quad (18)$$

$$= \mathbf{P}_{o,\parallel} + \mathbf{P}_{o,\perp} + \mathbf{P}_{s,\perp}, \quad (19)$$

where $I(r, z)$ is the Gaussian optical intensity and s_3 quantifies the degree of circular polarization [19]. As evident from Eq. (18), $\mathbf{P}$ is purely longitudinal in the plane-wave limit

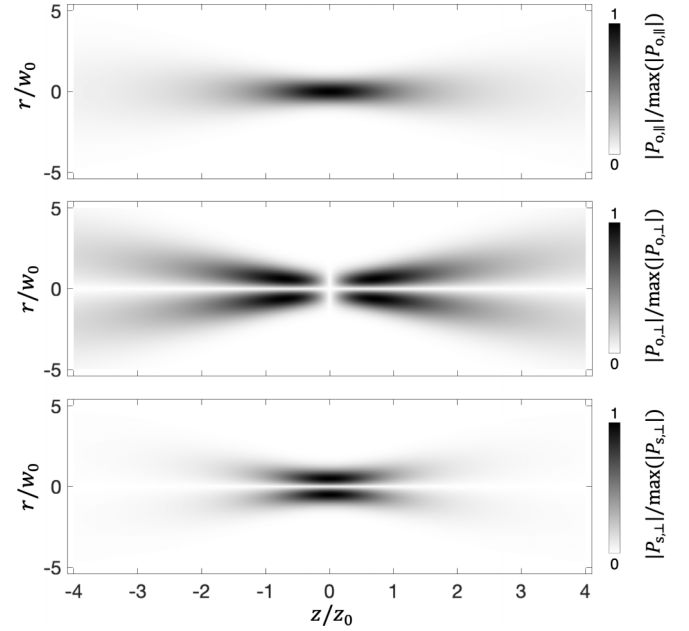


FIG. 2. Spatial profiles of the magnitude of the longitudinal orbital (top), transverse orbital (middle), and spin (bottom) components of the Poynting vector for a uniformly polarized Gaussian beam, as given by Eqs. (18) and (19). For linearly polarized beams ($s_3 = 0$), the spin contribution vanishes.

(R, w) $\rightarrow \infty$, whereas finite transverse contributions $\mathbf{P}_{o,\perp}$ and $\mathbf{P}_{s,\perp}$, inherently arising from the spatially bounded nature of a beam, appear in realistic situations. These transverse components are orthogonal and exhibit distinct spatial profiles, as illustrated in Fig. 2. Moreover, they satisfy, for all r ,

$$P_{s,\perp} \tan \Phi_{\text{Gouy}} = s_3 P_{o,\perp}. \quad (20)$$

C. Optoelastic mapping of transverse energy flows

In particular, for an incident circularly polarized Gaussian beam with helicity $\sigma = \pm 1$, $(s_1, s_2, s_3) = (0, 0, \sigma)$, Eqs. (14), (15), and (18)–(20) demonstrate that the transverse spin and orbital components of the Poynting vector selectively excite orthogonal mechanical responses $\delta\mathbf{n}_s \propto \mathbf{e}_\phi$ and $\delta\mathbf{n}_o \propto \mathbf{e}_r$ with relative strength modulated by the Gouy phase.

Such a Gouy-phase-driven optomechanical mechanism, whereby the transverse spin and orbital energy flows are selectively mapped onto orthogonal elastic modes of a nematic medium, goes beyond previous studies that focused on the role of the longitudinal field component in thresholdless three-dimensional optical reorientation [12–15]. Although earlier works demonstrated the emergence of radial and azimuthal elastic modes, neither the governing role of the Gouy phase nor its connection to transverse energy flows was recognized. More generally, and beyond the particular case of incident circular polarization, the present work reveals a previously unexplored correspondence between the three-dimensional spin-orbit structure of focused optical fields and the optomechanical response of soft anisotropic media.

D. Discussion

Owing to its elastic nature, the nematic liquid crystal exhibits a nonlocal mechanical response, whereby a local optical torque gives rise to a spatially extended director deformation governed by the elastic properties of the medium. In addition, the induced three-dimensional reorientation may lead to a gradual evolution of the optical field polarization during propagation through the sample. While such feedback can remain weak and amenable to a perturbative treatment, accounting for field evolution is nevertheless essential to assess whether the conditions required for a selective mapping between transverse spin and orbital energy flows and elastic modes are satisfied in practice.

The local perturbative approach developed in Sec. II is therefore primarily intended to establish a clear physical correspondence between $\mathbf{P}_{s,\perp}$, $\mathbf{P}_{o,\perp}$, and the optoelastic response. To evaluate its range of validity in realistic configurations and to quantify the role of propagation effects, we next turn to a full numerical treatment of the coupled light-matter nonlocal problem. This approach allows us to validate the analytical estimate given by Eq. (14) and to identify the regimes in which the predicted optomechanical trends remain robust.

III. NUMERICS

A. Methods

We perform an exact numerical solution that fully accounts for the three-dimensional nature of both the focused optical field and the reoriented nematic director, as well as their mutual coupling through Maxwell's equations. To this end, we combine the phenomenological LdG free-energy relaxation method for director reorientation—including the liquid-crystal order parameter, elastic anisotropy, rotational viscosity, and dielectric coupling to light [20]—with a custom finite-difference time-domain (FDTD) algorithm [21,22] for light propagation in spatially varying optically anisotropic media, which includes the presence of perfectly matched layer at the periphery of the simulation box to suppress optical reflections.

Time evolution of electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ is obtained by updating the Maxwell's curl equations

$$\epsilon_0 \hat{\epsilon} \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H}, \quad (21)$$

$$\mu_0 \frac{\partial \mathbf{H}}{\partial t} = -\nabla \times \mathbf{E}, \quad (22)$$

in discrete time steps, while the curls are evaluated on staggered spatial Yee grid [21]. The 3×3 matrix $\hat{\epsilon}$ is the space-varying relative dielectric permittivity tensor defined as

$$\epsilon_{ij} = \bar{\epsilon} \delta_{ij} + \frac{2}{3} \epsilon_a^{\text{mol}} Q_{ij}, \quad (23)$$

where $i, j \in (x, y, z)$, respectively, refer to the Cartesian coordinates, δ_{ij} is the Kronecker delta, $\bar{\epsilon} = (\epsilon_{\parallel} + 2\epsilon_{\perp})/3$ is the average molecular permittivity, and $\epsilon_a^{\text{mol}} = \epsilon_a/S$ with S representing nematic degree of order. The components of the liquid crystal tensorial order parameter field $\hat{Q}$ are defined as

$$Q_{ij} = \frac{1}{2} S (3n_i n_j - \delta_{ij}), \quad (24)$$

where $n_{i,j}$ denote the Cartesian components of the director, and summation over double indices is assumed.

The equilibrium configuration of the director field is obtained by minimization of the free energy density functional, which is expressed in terms of the liquid crystal order parameter tensor as

$$f = \frac{1}{2} K (\partial_k Q_{ij})^2 - \frac{1}{3} \epsilon_0 \epsilon_a^{\text{mol}} \langle E_i Q_{ij} E_j \rangle_t + \frac{1}{2} A (T) \text{Tr}(\hat{Q}^2) + \frac{1}{3} B \text{Tr}(\hat{Q}^3) + \frac{1}{4} C [\text{Tr}(\hat{Q}^2)]^2, \quad (25)$$

where ∂_k denotes the partial derivative with respect to the k th spatial coordinate. The first term represents the elastic free energy in a one constant approximation and K is the Frank elastic constant within the isotropic elasticity assumption. The second term is the dielectric coupling with the optical fields, where $\langle \cdot \rangle_t$ refers to time averaging over one period of the optical field oscillation. Note that this term couples to the Maxwell's equations given by Eqs. (21) and (22). The remaining terms are the Landau expansion describing the temperature-driven phase transition between nematic and isotropic state, where the coefficient $A = a(T - T_{\text{NI}}^*)$, T is the temperature, T_{NI}^* is the supercooling temperature and a , B , and C are constant material parameters taken from [20], so is S . Free energy density may also include surface terms to describe the surface anchoring. However, here we fixed the boundary conditions with $\mathbf{n} = (0, 0, 1)$ at $z \pm L/2$, which corresponds to both sides of the liquid crystals slab and open boundary condition is used in the lateral direction. The free energy is then minimized by using the Euler-Lagrange equation

$$\frac{\partial f}{\partial Q_{ij}} - \partial_k \frac{\partial f}{\partial (\partial_k Q_{ij})} = 0. \quad (26)$$

Importantly, light-matter interaction happens on two vastly different timescales: The response times of liquid crystals are mainly limited by the rotational viscosity, giving the estimated response time of ~ 1 ms, while the characteristic time of electromagnetic field changes is determined by the frequency, giving the estimation of ~ 1 fs. Therefore, to model the mutual interaction, the computation is alternating between the light propagation and free energy minimization, which are solved sequentially, until a steady state is achieved for given director orientation or light intensity, as explained in detail in Ref. [23]. Typically, a few tens of such iteration steps are needed to achieve the convergence in the equilibrium state, except in the vicinity of the Fréedericksz transition, where the number can be higher.

B. Parameters

Due to the high computational cost of this fully coupled approach, simulations are performed in a moderately sized domain. Specifically, we use a nematic slab of thickness $L = 2.4 \mu\text{m}$ and a square footprint of lateral size $W = 11.6 \mu\text{m}$ in the plane (x, y) . The simulation box is sampled with a spatial resolution of 20 nm in all directions, yielding a cubic mesh of $580 \times 580 \times 120 \simeq 4 \times 10^7$ points. The box is illuminated at normal incidence by a Gaussian beam propagating toward $z > 0$ (see Fig. 3). The optical field parameters are set to $\lambda = 532$ nm and $w_0 = 2\lambda$, typical visible-light conditions. Since the chosen degree of nonparaxiality extends beyond the paraxial description of a Gaussian beam, it is implemented by adding higher-order corrections to the transverse field in Eq. (16) [24], giving

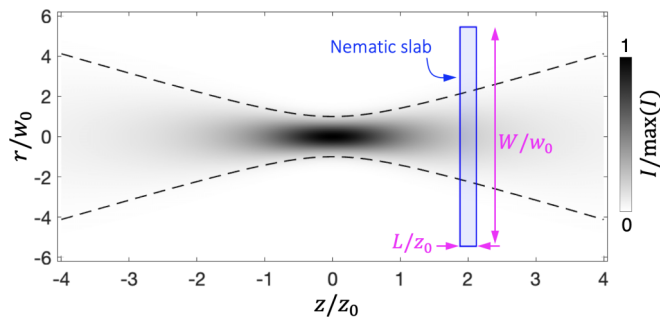


FIG. 3. On-scale illustration of the light-matter interaction geometry, the nematic slab being placed at a distance z from the focus plane of an incident Gaussian beam (here, chosen as $z = 2z_0$ for the sake of illustration) propagating toward $z > 0$. Dashed lines: $r(z) = \pm w(z)$.

an accurate full-3D incident field. The director field at equilibrium without irradiation is $\mathbf{n}_0 = \mathbf{e}_z$ in the bulk, with fixed anchoring at input and output surfaces and free lateral boundaries. The nematic used in our simulations is characterized by refractive indices $n_\perp = 1.5$ and $n_\parallel = 1.7$, a single elastic constant $K = 10$ pN, and thermodynamic parameters identical to Ref. [20].

These values have been chosen to provide physically realistic conditions for prospective experimental implementations (see Sec. IV), while the present work remains primarily analytical and numerical. They place the incident beam in a weakly nonparaxial regime, which validates the assumptions underlying Eq. (3) used in the perturbative analytical treatment. Importantly, the numerical simulations themselves rely on a full three-dimensional solution of Maxwell's equations and are therefore exact within the adopted material model, while being performed under conditions that ensure quantitative consistency with the analytical expansion.

C. Benchmarking

First, we benchmark our approach by considering an incident linear polarization in the limit of a purely transverse

electric field, enforced numerically by setting $E_z \equiv 0$ at the end of each FDTD sequence before injecting the optical field into the LdG relaxation sequence. This reproduces the classical paraxial limit of the optical Fréedericksz transition [25], for which an analytical framework has existed since the early 1980s [26]. Perhaps surprisingly, our results, shown in Figs. 4(a) and 4(b), provide, to our knowledge, the first exact numerical demonstration of optical reorientation beyond the Fréedericksz transition threshold accounting for finite beam size. Above the threshold power $\mathcal{P} = \mathcal{P}_F$, a bell-shaped reorientation profile develops, with the director aligning along the input polarization and the sign of the tilt angle set by numerical noise—an analog of thermal orientational fluctuations.

Then, we solve the same problem while fully accounting for three-dimensional light-matter coupling [see Figs. 4(a) and 4(c)]. In this configuration, the reorientation becomes qualitatively different: it is thresholdless and forms a transverse pattern with two lobes of opposite tilt, as predicted by inserting Eq. (18) into Eq. (14) for an incident linear polarization state along the x axis, corresponding to $(s_1, s_2, s_3) = (1, 0, 0)$. Indeed, the perturbative analytical approach thus gives $\mathbf{n}_\perp \propto \propto x \mathbf{e}_x$. This is illustrated in Fig. 4(c) and agrees with recent experimental findings [15], providing the first complete numerical description of this phenomenon. This highlights the predictive power of the analytical model, even though it neglects feedback from light-matter coupling and the nonlocal response of the liquid crystal. In addition, the disappearance of the threshold in 3D underscores the role of off-axis plane-wave components, known to induce imperfect orientational bifurcation [25].

Having completed the benchmarking tests for linearly polarized incident fields, we confirmed consistency with known experimental observations in the deep paraxial limit and validated the thresholdless two-lobe reorientation when the longitudinal component becomes significant. We now turn to the numerical study of the optoelastic mapping previously introduced within the local, perturbative analytical framework, focusing on circularly polarized beams.

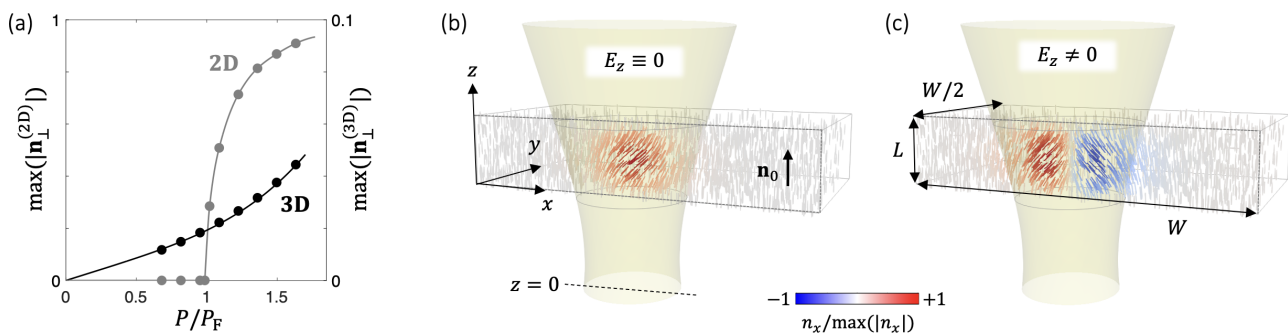


FIG. 4. (a) Simulated reorientation magnitude for a x -linearly polarized Gaussian beam with the nematic slab placed at $z = z_0$. 2D situation: $E_z \equiv 0$ and optical reorientation is associated with a threshold power $\mathcal{P}_F$ (gray color). 3D situation: $E_z \neq 0$ with a thresholdless optical reorientation (dark color). Markers: calculated points. Solid curves: guide for the eyes. Illustrated simulated optical reorientation of the director field for a x -linearly polarized incident Gaussian beam in the 2D [panel (b)] and 3D [panel (c)] configurations, at $z = z_0$. The reorientation amplitude is magnified for better visualization, and the power is taken as $\mathcal{P} \gtrsim \mathcal{P}_F$ in both cases. The beam contour is not to scale and only half of the slab is shown.

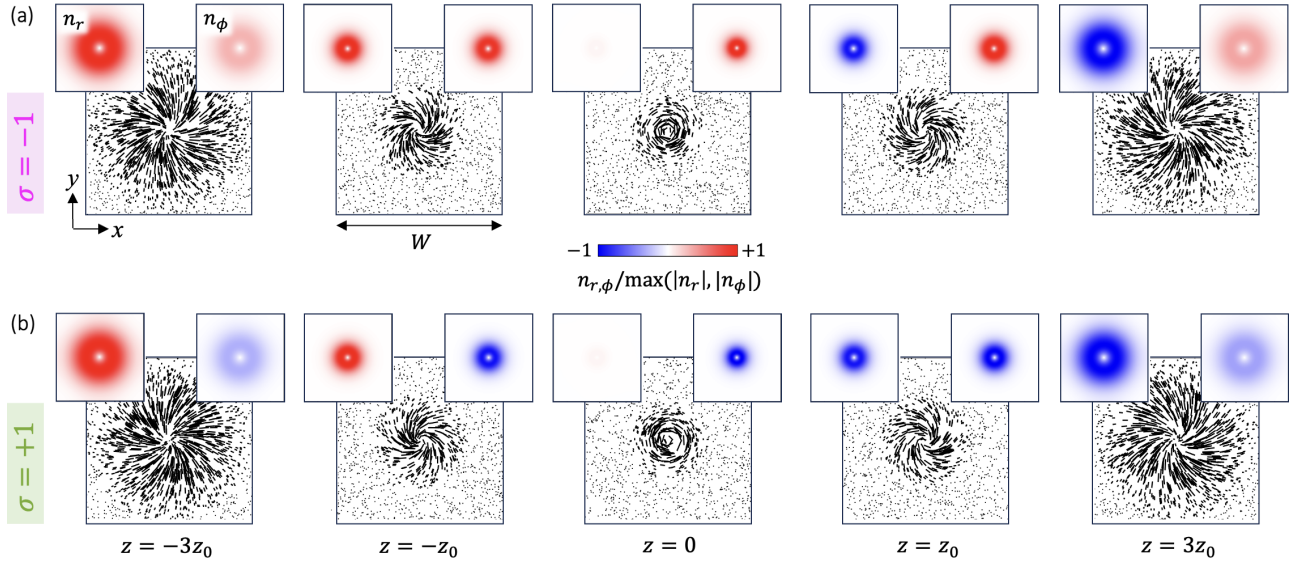


FIG. 5. Simulated transverse distribution of the director field at mid-cell for different sample positions $z/z_0 = (-3, -1, 0, 1, 3)$, incident circular polarization with helicity $\sigma = -1$ [panel (a)] and $\sigma = +1$ [panel (b)], and $\mathcal{P} \simeq 0.5\mathcal{P}_F$. Main panels: segments show the transverse reorientation pattern, their length being proportional to $|\mathbf{n}_\perp|$ and their orientation to that of $\mathbf{n}_\perp$. Upper-left and upper-right insets: transverse profiles of the radial (n_r) and azimuthal (n_ϕ) director components. The maximal reorientation amplitude is $\max(|\mathbf{n}_\perp|) \simeq (1.3, 1.5, 1.9) \times 10^{-2}$ for $z/z_0 = (0, \pm 1, \pm 3)$.

D. Validation of transverse energy flow mapping

Following Sec. II, here we test the prediction

$$\delta \mathbf{n} \cdot \mathbf{e}_r \propto -\mathbf{P}_{o,\perp}, \quad (27)$$

$$\delta \mathbf{n} \cdot \mathbf{e}_\phi \propto -\mathbf{P}_{s,\perp}, \quad (28)$$

for an incident Gaussian beam with helicity σ , leading to a reoriented director field $\mathbf{n} = (n_r, n_\phi, n_z)$ in cylindrical coordinates satisfying

$$n_r/n_\phi = \sigma \tan(\Phi_{\text{Gouy}}). \quad (29)$$

The Gouy phase dependence of the light-induced director reorientation is probed by scanning the axial position of the nematic slab within the Gaussian beam, at normalized distances $z/z_0 = (-3, -1, 0, 1, 3)$ spanning the focal, intermediate, and far-field regions. This is done for $\sigma = \pm 1$, fixing the incident optical power at $\mathcal{P}/\mathcal{P}_F = 0.5$, using for the z -dependent optical Fréedericksz transition power the analytical expression $\mathcal{P}_F(z) = \pi^3 c K[w(z)/L + \sqrt{2/\pi}]^2 n_\parallel^2 / [n_\perp (n_\parallel^2 - n_\perp^2)]$ [27].

The resulting director maps, shown in Figs. 5(a) and 5(b), reveal a clear evolution of the transverse pattern: a purely azimuthal reorientation near focus ($z = 0$), progressively turning radial as the sample moves away from the waist. At intermediate positions ($z/z_0 = \pm 1$), both components coexist, reflecting the mixed spin-orbital nature of the transverse optical momentum in this region. This transition is further quantified in Fig. 6, where the ratio between the radial and azimuthal deformation amplitudes follows Eq. (29), as shown by the solid curves defined as $\xi = \sigma \Phi_{\text{Gouy}}$. The observed agreement not only confirms that nematic reorientation provides a direct optomechanical imaging of the transverse spin and orbital energy flows of light, but also reveals that the excitation of orthogonal elastic modes is governed by the Gouy phase—a form of Gouy-phase optomechanics.

IV. EXPERIMENTAL CHALLENGE

While the present work is entirely numerical and analytical, it is important to address the experimental feasibility of observing the predicted spin-orbit optoelastic mapping. The main limitation arises from propagation-induced feedback, which increases with the reorientation amplitude and progressively alters the polarization state of light as it propagates through the nematic slab. Such feedback can ultimately hinder the selective mapping of transverse spin and orbital energy flows onto azimuthal and radial director distortions.

To illustrate this effect, we numerically investigate configurations where an almost purely azimuthal ($z = 0$) or radial ($z = \pm 3z_0$) reorientation is expected in the low-reorientation regime, as shown in Fig. 5. To enhance propagation effects, we gradually increase the incident optical power $\mathcal{P}$. The resulting director maps, displayed in Fig. 7 for $\mathcal{P} \simeq 1.5\mathcal{P}_F$, show that

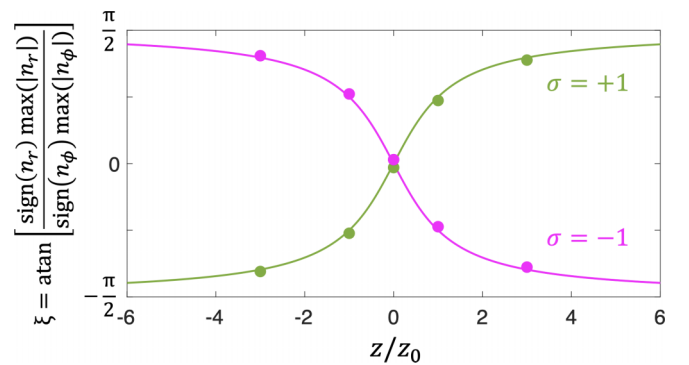


FIG. 6. Relative weights of the radial and azimuthal components extracted from Fig. 5 and represented by the parameter ξ . Markers: numerically calculated values; solid curves: analytically expected behavior $\xi = \sigma \Phi_{\text{Gouy}}$.

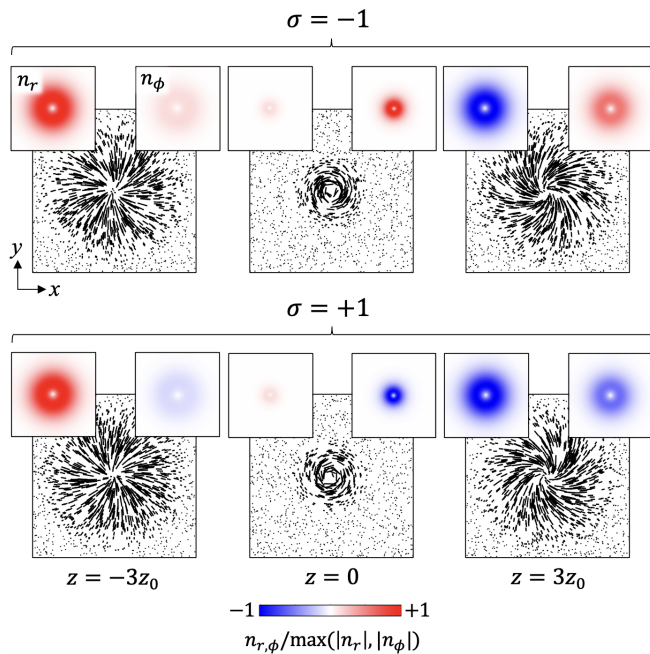


FIG. 7. Same as in Fig. 5 but for a larger incident optical power, $\mathcal{P} \simeq 1.5\mathcal{P}_F$, for $z/z_0 = (-3, 0, 3)$ and $\sigma = \pm 1$. The maximal reorientation amplitude is $\max(|\mathbf{n}_\perp|) \simeq (0.6, 1.3) \times 10^{-1}$ for $z/z_0 = (0, \pm 3)$.

the transverse director patterns are significantly altered and no longer reflect a clear separation between spin and orbital transverse energy flows. Indeed, the slightly twisted pattern observed at $z = -3z_0$ evolves toward a more radial configuration, while that at $z = 3z_0$ becomes clearly twisted-radial, independently of the incident helicity $\sigma = \pm 1$. Qualitatively, this behavior can be attributed to a local rotation of the transverse director field around the z axis, with a sense set by σ , arising from a net spin angular momentum transfer as the polarization state evolves during propagation. As the reorientation amplitude increases with optical power, this mechanism tends to unwind the residual twist at $z = -3z_0$ while reinforcing it at $z = 3z_0$, which breaks the optoelastic mapping.

In addition, we attempted preliminary experimental realizations using sample thicknesses in the range 20–50 μm , under conditions consistent with those considered in the present work in terms of the incident beam divergence and the nematic material. These attempts were ultimately unsuccessful in producing a clear optoelastic mapping, highlighting the experimental challenges of observing the predicted spin-orbit separation. Propagation-induced feedback was sufficiently strong to prevent drawing clear conclusions, and while thinner samples could help mitigate these effects, they introduce other practical challenges. In particular, the characteristic intensity required to induce reorientation, as estimated from the Fréedericksz threshold, scales as $1/L^2$, which can lead to issues

such as photodegradation or damage to the boundary conditions. Using nematic materials with lower birefringence could also help reduce these limitations. These practical considerations may help guide the design of future experiments.

V. CONCLUDING REMARKS

We have introduced a nondissipative optomechanical framework for revealing the transverse spin and orbital components of optical energy flows in structured light using a nematic liquid crystal film. Azimuthal and radial director deformations provide a direct mechanical readout of the spin and orbital (canonical) parts of the transverse Poynting vector, establishing an explicit optoelastic mapping between internal electromagnetic momentum flows and mechanical degrees of freedom in a soft anisotropic medium. By combining perturbative analytical estimates with full numerical simulations coupling Maxwell's equations and Landau-de Gennes nematodynamics in three dimensions, we demonstrate that this mapping is robust across focal, intermediate, and far-field regions of a Gaussian beam, while also identifying the regimes in which propagation-induced optomechanical feedback becomes significant. This enables a quantitative assessment of experimental constraints, clarifying the conditions for observing optomechanical imaging of intrinsic transverse momentum flows.

Overall, our approach integrates structured-light theory with nonlinear liquid-crystal optomechanics and establishes a practical route toward momentum-resolved optomechanical metrology in soft anisotropic media. Looking forward, it may be extended to more complex scalar and vectorial structured fields with topological features such as knots, Möbius strips, and skyrmions, opening perspectives for optical manipulation, photonic device design, and the mechanical probing of light's internal momentum structure.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

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