

Snapping and switching of elastic arches with patterned preferred curvature

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An elastic arch is an archetypal bistable system. Here, we combine elastica theory and photomechanical experiments to elucidate the mechanics of an active arch with a spatiotemporally varying preferred curvature $\bar{\kappa}(s)$. Our shallow-arch theory completely describes any such system via the decomposition of its $\bar{\kappa}(s)$ into Euler-buckling modes. Intuitively, if $\bar{\kappa}(s)$ overlaps with the fundamental mode, it snaps the arch up/down. Conversely, nonoverlapping $\bar{\kappa}(s)$ drives a second-order transition to a higher-order shape. Furthermore, the form of $\bar{\kappa}(s)$ enables control over the instability's character; we find the forms for snapping with maximum energy release and at the lowest stimulation (both binary patterns) and design forms for symmetric and asymmetric switching pathways. Analogous control can also be achieved in boundary-driven instabilities of passive arches by fabricating them with suitable $\bar{\kappa}(s)$. We thus anticipate that our results will improve switchable/snapping elements in micro-electromechanical systems, robotics, and mechanical metamaterials.

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I. INTRODUCTION

The instabilities of elastic rods have long enchanted researchers [1,2], and unify slender bodies from nanotubes [3,4] to cross-continental pipelines [5] and undersea cables [6]. Celebrated examples include Euler's buckling of a compressed column [1], the snap of Timoshenko's bimetal thermostat [7], and the coiling transitions found in tendrils [8], DNA [9], and twist-based artificial muscles [10,11]. Each such instability nucleates from a simple base state with uniform compression/twist/natural curvature, making the complex forms that emerge a compelling morphogenetic mechanism [12,13]. However, in both biology and the emerging field of shape-responsive solids, there is no fundamental restriction to translationally invariant beams: rather, patterns of growth or actuation can sculpt almost arbitrary spatiotemporal patterns of natural curvature [14,15]. Thus, it is natural to ask whether such patterns can drive, direct, and optimize instabilities.

Here, we address this question for a beam that is prebuckled into an elastic arch. Such arches are elementary bistable systems leading to applications as micro-electromechanical system switches [16], binary sensors [7], and mechanical memories [17]. Bistable switching also involves energy storage and sudden release (snapping), enabling explosive jumpers [18], catchers [19,20], and throwers [21], and powering waves through damped mechanical metamaterials [22]. Consequently, recent studies have examined many external routes to induce snapping, including boundary manipulation

[23–27], electro/magnetic fields [28,29], and even surface tension [30].

Alternatively, arch snaps can be driven internally by changing preferred curvature. Such snapping was first observed in bimetallic strips that bend on heating [7], and, more recently, using other strips that bend on electrical [31,32], thermal [21], or optical [33,34] stimulation. In such systems, inducing uniform preferred curvature suffices to snap a simply supported (pinned) arch back and forth, as the up/down states have pure negative/positive curvature, respectively [7]. Flat-clamped arches are significantly easier to fabricate, but bear opposite curvatures in their middle and sides, preventing snapping under uniform curvature stimulation. However, such arches may be snapped by stimulating the central segment [21,31–34], and snapped back by reversing the sign [31–33] or pattern [34]. These results highlight the subtlety and importance of patterned stimulation and motivate our analysis.

II. EXPERIMENTAL SYSTEM

To do so, we prepare a thermally bending nematic elastomer strip using standard mechanical programming techniques [35,36] ($L = 40$ mm, $w = 3.5$ mm, $t = 0.6$ mm; Appendix A). The resultant strip has a uniform natural curvature $\bar{\kappa}_{\max} = -5.2L^{-1}$ in the nematic phase (at room temperature), falling to approximately zero in the isotropic phase [$T > 65^\circ\text{C}$; Fig. 1(a)]. While we choose to use nematic elastomers, similar thermal benders could be fabricated using many other systems, yielding similar results.

The strip is flat clamped, buckled into an arch by compressing by $\epsilon = 1.2\%$ [Fig. 1(b)], and mounted in front of a digital micromirror device-based projector (DLP 4750LC, EKB Technologies), oriented such that the natural curvature is negative (downward) [Fig. 1(c)]. The projector creates a $4\text{ cm} \times 2.25\text{ cm}$ 1080p image that entirely covers the strip, enabling arbitrary illumination patterns with an intensity of

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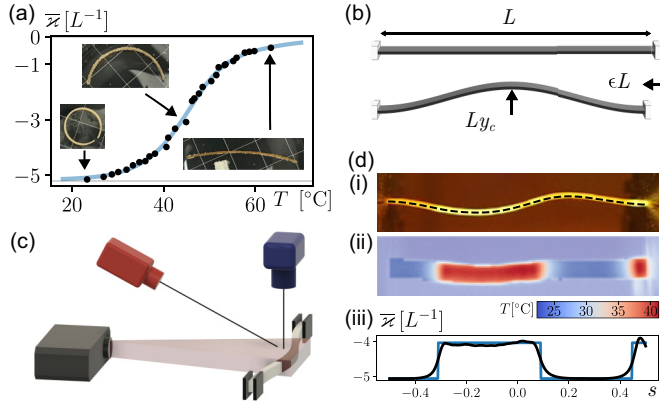


FIG. 1. Experimental system. (a) Measured spontaneous curvature $\bar{\kappa}$ of the active beam. (b) A beam of length L is clamped and compressed by ϵL into an arch, and (c) mounted in front of a projector (gray), thermal (red), and visible light (blue) cameras. (d) The projector and (i) visual and (ii) thermal feeds form a feedback loop controlling (iii) the curvature (temperature) profile (black: measured; blue: target). Brighter pixels in panel (ii) denote temperature-based background excluded from (iii) $\bar{\kappa}$ extraction.

up to 750 mW/cm². The shape and temperature of the strip are monitored by visible light and thermal cameras [Fig. 1(d)], and combined in real time (9 Hz) to reconstruct an arc-length-parametrized temperature profile $T(S)$ along the strip, and therefore $\bar{\kappa}(S)$ via the calibration curve [Fig. 1(a)].

During a particular experiment, we wish to prescribe a specific preferred curvature profile $\bar{\kappa}(S, t)$. To do so, we simply compare the current target against the observed $\bar{\kappa}(S)$ and apply bang-bang control, illuminating only the regions where the temperature is below target. Crucially, our real-time shape-tracking system enables us to robustly impose the preferred curvature profile in the Lagrangian/material arc-length description, despite the projector naturally operating in the Eulerian/laboratory frame [Fig. 1(d)].

III. SNAPPING OF CENTRALLY STIMULATED ARCH

We begin by stimulating the central 70% of the strip to induce an approximately piecewise-constant curvature profile with background $\bar{\kappa}_0$ in the outer pieces and an increasing $\bar{\kappa} = \bar{\kappa}_0 + \alpha L^{-1}$ in the center. The beam snaps down at amplitude $\alpha = 2.75$ and back up at $\alpha = -2.56$ (achieved by stimulating the edges), leading to the hysteresis loop in Fig. 2(a) and Movie 1 in the Supplemental Material [37]. The experimental data were extracted from a 50 FPS recording during postprocessing, enabling us to observe the antisymmetric character of the snapping transition.

To analyze these snaps, we model the arch as an inextensible elastica with bending stiffness B , length L , and (arc-length-parametrized) preferred curvature profile $\bar{\kappa}(S)$, such that, if it follows the plane curve with tangent angle $\theta(S)$ and local curvature $\kappa(S) = \theta'(S)$, it stores bending energy

$$E = \frac{1}{2} B \int_L (\kappa - \bar{\kappa})^2 dS. \quad (1)$$

We note this one-dimensional energy naturally emerges for our narrow strips (transversely relaxed) or very wide

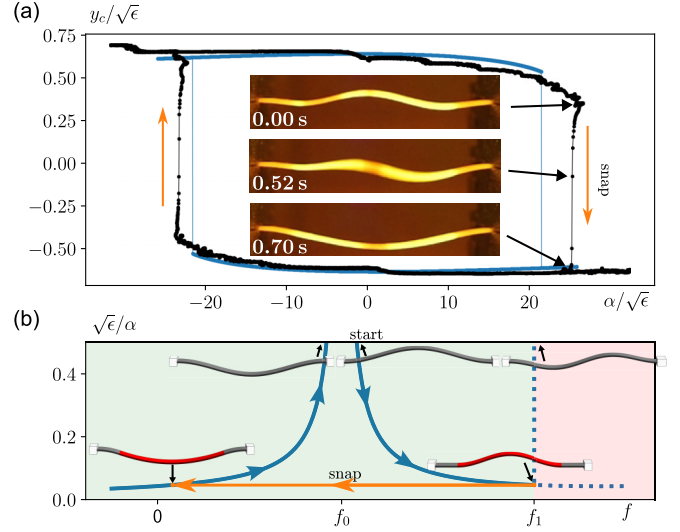


FIG. 2. Snapping of arches with active curvature. (a) Theoretical (blue) and experimental (black) hysteresis loop of center height vs curvature magnitude showing both way snapping control under the stimulation of central 70%. (b) Theoretical plot of normalized arch compression $\sqrt{\epsilon}/\alpha$ for each arch state as a function of longitudinal force f , showing compression peaks at the Euler buckling forces f_i and a transition from stable (solid) to unstable (dashed) solutions at f_1 , corresponding to the snap observed at fixed ϵ and increasing stimulation.

sheets (transversely suppressed), but intermediate widths require a full two-dimensional plate treatment [21]. Morphology and stability then follow from energy minimization. Unconstrained, the elastica can trivially follow $\bar{\kappa}$ to achieve $E = 0$, but the flat clamps impose the end-to-end displacement, $\langle \sin \theta \rangle \equiv \frac{1}{L} \int_L \sin \theta dS = 0$ (vertical) and $(1 - \cos \theta) = \epsilon$ (horizontal). Minimizing variationally subject to these constraints yields the nonlinear ordinary differential equation $B\theta'' + F \sin \theta + V \cos \theta = B\bar{\kappa}'$, where the horizontal and vertical clamp forces F and V arise as Lagrange multipliers. Flat clamping further imposes $\theta = 0$ boundary conditions, while freely rotating (pinned) ends require $\theta' = \bar{\kappa}$. Hence, pinned arches respond to uniform $\bar{\kappa}$, while clamped arches require patterns with $\bar{\kappa}'$.

To reproduce the experiment, we consider a flat-clamped arch compressed by $\epsilon = 1.2\%$ and set $\bar{\kappa}(S)$ to a binary pattern, with a uniform but growing value in the central 70% and zero otherwise. The system's nonlinearity precludes an analytic solution, but the resulting configurations and their linear stability can be readily solved using SciPy's solve_bvp (Sec. S1.1 in the Supplemental Material [37]). Alternatively, arch configurations and (damped inertial) dynamics can also be simulated using a discrete rod model (Sec. S1.2 in the Supplemental Material [37]). Both fully nonlinear approaches confirm two-way snapping via an asymmetric mode at $\alpha = \pm 2.36$; equivalent snap back can also be achieved using the dual stimulation, shown in Movie 1 in the Supplemental Material [37], as this also exactly reverses $\bar{\kappa}'$.

To gain analytical insight, we nondimensionalize ($S = Ls$, $\kappa = \kappa/L$, $F = Bf/L^2$, $V = Bv/L^2$, $E = B\epsilon/L$) and

expand to leading order in θ to obtain the light-compression equations:

$$\theta'' + f\theta + v = \bar{\kappa}', \quad \theta(\pm \frac{1}{2}) = 0, \quad \langle \theta \rangle = 0, \quad \langle \frac{1}{2}\theta^2 \rangle = \epsilon. \quad (2)$$

During the prebuckling, we have $\bar{\kappa}' = 0$. The first three equations then form Euler's classic buckling eigensystem, with modes at discrete compressive forces f_i and corresponding orthonormal mode shapes, $\langle \frac{1}{2}\theta_i\theta_j \rangle = \delta_{ij}$ with wave number $k_i = \sqrt{f_i}$ and alternating even ($\sin(k_i/2) = 0$) and odd ($\tan(k_i/2) = k_i/2$) morphologies. Finally, the nonlinear compression constraint fixes amplitude, $\theta = \pm\sqrt{\epsilon}\theta_i$, with two mirrored solutions at each f_i . The higher-order modes all require $f > f_0$, and are thus unstable to growth of θ_0 [38], so the prebuckled arch must sit bistably in one of the two $i = 0$ modes.

We next consider introducing a top-hat preferred curvature $\bar{\kappa}_{\text{rect}} = \alpha \text{rect}(s/\gamma)$ that stimulates the central fraction γ of the arch and with amplitude α . Such stimulation introduces a source term into the shape equation, and the linear system now admits an analytic particular integral $\theta_{\text{rect}}(s, f) \propto \alpha$ for every compressive force f (Sec. S2.8 in the Supplemental Material [37]). Furthermore, if f coincides with one of the original buckling forces, f_j , we must add the associated buckling form, $c\theta_j$, as a complementary function. Inserting these expressions into the compression equation, we obtain an algebraic force-amplitude relation $\sqrt{\epsilon}/\alpha = g(f)$, shown in Fig. 2(b) for $\gamma = 0.7$. This “spectrum” maps the compression generated by each solution and has a continuous background from θ_{rect} , augmented by degenerate spikes at the f_i parametrized by the constant of integration c . At a given compression and stimulation $\sqrt{\epsilon}/\alpha$ is fixed and corresponds to a horizontal line on the Fig. 2(b) plot, which starts from $\sqrt{\epsilon}/\alpha \rightarrow \infty$ and lowers as stimulation, increases (Movie 1 in the Supplemental Material [37]). Its intersections with $g(f)$ are then the solutions describing precise arch morphologies available at that $\sqrt{\epsilon}/\alpha$.

At the start of the experiment, the high horizontal line intersects only in pure buckling modes, and the arch is prebuckled in the fundamental mode ($i = 0$). As stimulation increases, the line descends and the solution “slides” down the left or right tail of the f_0 divergence, depending on whether the initial state accords (left) or opposes (right) the applied preferred curvature. According preferred curvature reduces the required compressive force, ultimately placing the arch in tension. Conversely, opposing $\bar{\kappa}$ increases the compressive force, until the solution eventually reaches the (unsplit) spike at f_1 ($\alpha = 21.52\sqrt{\epsilon}$), where it encounters a bifurcation and the arch becomes linearly unstable to growth of $\theta_1(s)$, and snaps to the stable “down” configuration at the same compression. This simple analysis thus predicts arch morphology, snapping threshold, and snapping mode. We find that all these predictions agree well with our experiment, and, at $\epsilon = 1.2\%$, essentially perfectly with our fully nonlinear discrete-rod numerics and with only minor disagreements at higher compressions (Fig. S4 in the Supplemental Material [37]). We note that our experimental system experiences minor length changes during stimulation due to changes in compressive force (Appendix B), which can be extracted from arch profiles and are reported as minor ϵ variations in the Movies of the

Supplemental Material [37]. However, the arch is sufficiently slender that our inextensible model nevertheless provides excellent agreement.

IV. RESPONSE TO GENERAL STIMULATION

More generally, any preferred curvature profile can be decomposed as a sum of buckling curvatures, $\kappa_i \equiv \theta'_i$, plus an irrelevant affine component (Sec. S2.3 in the Supplemental Material [37]). An individual component $\alpha_i\kappa_i$ then contributes a particular integral for the linear equations $\theta_{\text{PI}}(s, f) = \alpha_i\theta_i(s)f_i/(f - f_i)$ and accommodates compression (Sec. S2.1 in the Supplemental Material [37])

$$\epsilon = \alpha_i^2 \frac{f_i^2}{(f - f_i)^2}. \quad (3)$$

Mode orthogonality allows these compression contributions to be simply summed to obtain a total continuous compression spectrum with a divergence at each included f_i , augmented by degenerate spikes at any excluded f_i via the complementary functions. Thus, the even central top-hat stimulation has continuous divergences at each even buckling mode and spikes at odd ones, including f_1 where snapping commences [Fig. 2(b)].

Stimulating just the fundamental mode, $\bar{\kappa} = \alpha_0\kappa_0$, leads to a simple spectrum with only the f_0 spike split. Snapping proceeds similarly at f_1 , though now at threshold stimulation $\alpha_0 = 1.05\sqrt{\epsilon}$ ($\alpha = 26.28\sqrt{\epsilon}$). Conversely, stimulation profiles orthogonal to κ_0 (e.g., $\bar{\kappa} = \kappa_1$) yield a very different spectrum, with a degenerate spike at f_0 , as stimulation no longer distinguishes the fundamental up/down states. Such an experiment still starts prebuckled into the fundamental mode, so on stimulation the solution “slides” vertically down the f_0 spike, with a corresponding reduction in its θ_0 component, until reaching the bottom of the spike at $\alpha_1 = 0.51$ ($\alpha = 18.84\sqrt{\epsilon}$) and turning left into the $f < f_0$ continuous solutions that are pure particular integral, and hence orthogonal to θ_0 . Thus, under increasing stimulation, we expect a continuous (second-order) morphological transition from solutions that overlap with θ_0 to those that do not. Such continuous transitions describe how complex $\bar{\kappa}$ can morph an arch into normally unstable higher-order morphologies.

To observe such a transition, we apply a $\bar{\kappa} = \alpha_1\kappa_1$ profile to the arch and track the displacement of the central point ($s = 0$) above the clamps. Its position provides a convenient measure of overlap between the arch shape and the fundamental mode, as every higher-order mode has $y_c = 0$. As expected, both the dynamic numerical simulation and the experiment confirm that this displacement vanishes continuously above a threshold stimulation in precise agreement with theory and without any sudden snap (inset in Fig. 3(a) and Movie 2 in the Supplemental Material [37]).

Having understood stimulation in pure zeroth and first modes, we now analyze their combination $\bar{\kappa} = \alpha_0\kappa_0 + \alpha_1\kappa_1$, which leads to divergences at f_0 and f_1 as shown in Fig. 3(b). In this case, increasing stimulation amplitude at fixed α_0/α_1 , the solution slides down the right side of the f_0 peak, until we eventually encounter a minimum between the two divergences, $f_0 < f_m < f_1$, at which point, no further “up” solutions are available, and the arch must snap down. Adding

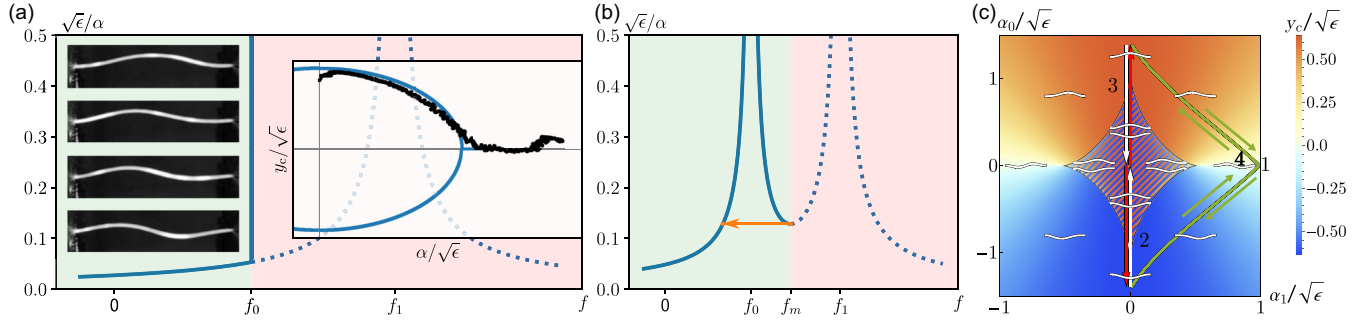


FIG. 3. Snapping condition and subcritical deformation. (a) Compression spectrum and (inset) center heights of an arch stimulated with $\bar{\kappa}_1$. (b) Snapping at $f_m < f_1$ by stimulating with $\bar{\kappa} = \alpha_0\kappa_0 + \alpha_1\kappa_1$ and (c) the corresponding phase diagram showing bistability in the dashed region and monostability outside.

κ_1 thus expedites snapping, as it directly stimulates to the previous snapping pathway.

We validate this graphical intuition by considering the stability of a base solution $\mathbf{w}_0(f) = (f, v(f), \theta(s, f))$ to small perturbations $\delta\mathbf{w}_1$. To obtain the equations of linear stability, we first substitute $\mathbf{w}_0 + \delta\mathbf{w}_1$ into Eq. (2) and linearize in δ . Comparing the linear terms with the derivative of the zeroth-order terms with respect to f confirms linear instability at f_m , to perturbations of the form $\mathbf{w}_1 = \partial_f \mathbf{w}_0(f)$ (Sec. S2.4 in the Supplemental Material [37]).

These results may be summarized on a phase diagram of stable states in $(\alpha_0/\sqrt{\epsilon}, \alpha_1/\sqrt{\epsilon})$ space, as shown in Fig. 3(c). The diagram naturally contains a large bistable region around the origin (high compression, low stimulation), corresponding to up and down arches with a large component of θ_0 . These snap to monostable at sufficient α_0 , while introducing α_1 reduces the threshold and amplitude of the snaps, until they disappear entirely via mechanical critical points at $(0, \pm(1 - f_0/f_1)^2)$. This structure highlights an additional possibility: By choosing a suitable route through the diagram, one can move from the up state at $(0, 0)$ to the corresponding down state by traveling around the critical point, and hence without any discontinuous snap. An experimental implementation of this possibility is shown in Movie 3 in the Supplemental Material [37] and described in detail in Sec. S2.9 in the Supplemental Material [37], with the followed (α_0, α_1) route marked in Fig. 3(c).

V. OPTIMIZED SNAPPING

Having understood the fundamentals, we now search for preferred curvature profiles that optimize snapping and switching. We first consider an arch that can attain the maximum curvature $0 < \bar{\kappa}(s) < \alpha$, and ask what pattern and ϵ should be chosen to maximize energy release, as desired in explosive applications such as jumping. To do so, we start numerically by writing the preferred curvature as a (finite) sum of buckling modes, then using SciPy to rapidly analyze the associated snap [via Eq. (3)] and optimize $\Delta\mathcal{E}$ over mode amplitudes. Optimization uncovers a binary pattern, with α stimulation in an offset central segment, $\bar{\kappa} = \alpha \text{rect}((s - l)/\gamma)$. Inspired by this result, we solve the

problem analytically for this class of profiles and optimize over stimulation width γ and its offset l . Both approaches find the same optimum that releases $\Delta\mathcal{E} = 0.661\alpha^2$ at $\alpha = 10.33\sqrt{\epsilon}$, compared to $\Delta\mathcal{E} = 0.239\alpha^2$ for pure κ_0 snapping [Fig. 4(a)].

Conversely, in switching applications, we are interested in the profile that switches at a minimum α . In the design space of offset rectangular stimuli, we find the minimum threshold of $\alpha = 9.29\sqrt{\epsilon}$, substantially below $\alpha = 26.28\sqrt{\epsilon}$ for pure κ_0 stimulation [Fig. 4(b)]. Repeating the general numerical optimization reveals the threshold may be lowered slightly further ($\alpha = 9.11\sqrt{\epsilon}$) by introducing an additional active segment at the arch's edge. Experimentally, we observe good agreement in the thresholds/hysteresis for both optimized profiles (Figs. 4(c) and 4(d) and Movies 4 and 5 in the Supplemental Material [37]).

Moreover, the threshold plot [Fig. 4(b)] of the offset rectangular profile reveals great sensitivity of central stimulation

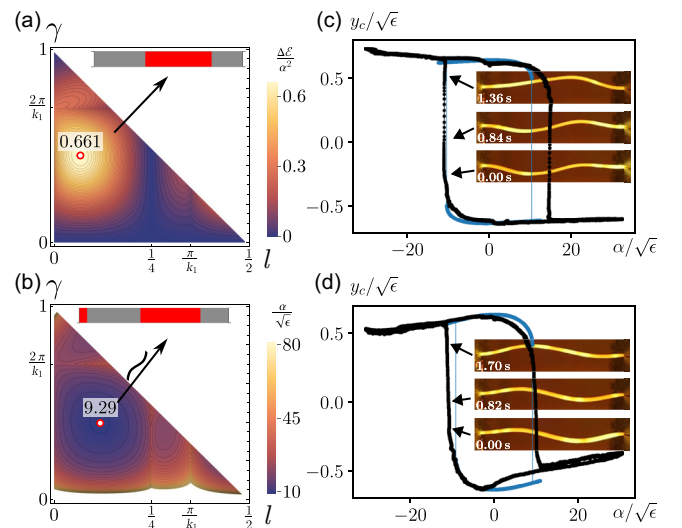


FIG. 4. Optimized stimulus profiles. (a) Elastic energy released during the snap and (b) snap-through thresholds for offset rectangular $\bar{\kappa}$, inset: optimal profiles. Comparison of center heights for (c) energy-optimized $\bar{\kappa}$ and (d) threshold-optimized $\bar{\kappa}$, theoretical design (blue) and experiment (black).

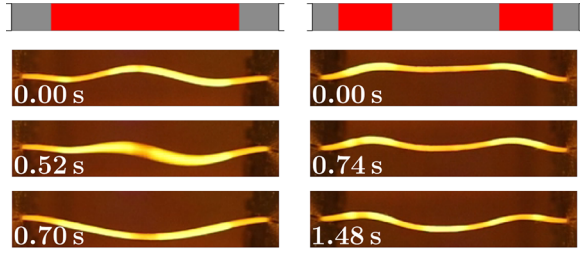


FIG. 5. Controlling symmetry of snap-through transition. Stimulation profiles $\bar{\kappa}_{\text{rect}}|_{\gamma=0.7}$ and $\bar{\kappa}_{\text{sym}}$, and the corresponding midsnap images.

($l = 0$) to misalignment, with even small offsets dramatically reducing the threshold as they introduce direct coupling to the κ_1 mode. However, this sensitivity is completely suppressed if the stimulation width matches the wavelength of the first mode ($\gamma = 2\pi/k_1 \approx 0.7$), decoupling the stimulation from κ_1 and leading to an obvious plateau on the threshold plot, and enabling predictable snapping. Analogous features are observed at $l = \pi/k_1$ and $l = \pi/k_0$, associated with decoupling from κ_1 and κ_0 , respectively, the latter leading to perfect supercritical behavior. Conversely, both optimal stimuli lie in smooth energy/threshold minima, making them insensitive to imperfections.

VI. SYMMETRIC SNAP THROUGH

Finally, we explore the use of preferred curvature to modify the snapping pathway. In previous examples with symmetric stimulation (central segment, κ_0), snap through occurred at f_1 via the first (antisymmetric) mode. Similarly, if the preferred curvature contains κ_1 , even the stable stimulated arches (particular integral) have an antisymmetric θ_1 component, and when snapping occurs at the spectral minimum $f_0 < f_m < f_1$, the growing mode $\partial_f \theta(s, f)$ directly inherits this (lack of) symmetry. In both cases, we thus observe antisymmetric displacements and rotation of the center point, which may be undesirable in applications where this center point interacts with other elements. Can we instead drive a snap through that proceeds entirely symmetrically?

To achieve such symmetric snapping, we consider a symmetric profile $\bar{\kappa} = \alpha_0 \kappa_0 + \alpha_2 \kappa_2 + \dots$ that generates a symmetric particular integral with compression divergences at f_0 and f_2 , and hence a minimum in between. A snap at this minimum's bifurcation will inherit the base state's symmetry. However, it will only be observed if it precedes the standard asymmetric snap, $f_m < f_1$. Fortunately, the minimum may be moved arbitrarily close to f_0 by reducing α_0 , though at the cost of reducing the splitting of the f_0 peak and, hence, snapping strength. To demonstrate such snapping, we design a pattern composed of two segments, each spanning 20% of the arch's total length and centered at $s = \pm 0.3$, resulting in a symmetric snap through, as shown in Fig. 5 and Movie 6 in the Supplemental Material [37]. Moreover, even in the case of central stimulation from Fig. 2 for which the symmetric and antisymmetric thresholds are in proximity ($\alpha^{\text{sym}} \approx 1.1\alpha^{\text{asym}}$), during the edge stimulation in Movie 1 in the Supplemental Material

[37], we observe a symmetric snap back, most likely due to reduction in α_0 and increase in α_2 caused by thermal diffusion.

VII. DISCUSSION

Although our analysis focuses on actively curving arches, there is a duality between increases in preferred curvature and decreases in compression; therefore, our model readily extends to passive precurved arches under active boundary loading (Sec. S2.11 in the Supplemental Material [37]). Moreover, our flat-clamped results directly generalize to any combination of flat-clamped and pinned arches when analyzed in terms of the relevant buckling modes (Sec. S2.6 in the Supplemental Material [37]). However, much of the previous literature focuses on snapping of $\bar{\kappa} = 0$ arches with ends clamped at finite angles θ_L and θ_R and snaps induced by changing ϵ . To connect with these results, we note that such clamping conditions themselves generate a particular integral equivalent to a preferred curvature with mode amplitudes $\alpha_i/4 = \theta_L + \theta_R$ (i even); $\theta_L - \theta_R$ (i odd), and we may use the corresponding compression spectrum to analyze snapping. In particular, this analysis immediately explains why asymmetric clamping control results in continuous transitions, while symmetric clamping enables snap through following an antisymmetric pathway [25,26]. Furthermore, in the latter case, the symmetric transition mode lies just beyond f_1 , enabling its observation dynamically [27] if not quasistatically. However, clamping angles can provide only two degree of freedom control over snapping, thereby vastly reducing the potential for optimization.

Overall, our approach thus provides a comprehensive and straightforward understanding of snapping in arches, illuminates prior results on naturally straight passive arches, and elucidates the more complex behavior of arches with preferred curvature, whether fixed at fabrication or induced by stimulation. Fabricated curvature enables tuning of thresholds, energy release, and mode shape, paving the way for improved snapping in metamaterials and other externally snapped devices. Arches with dynamic preferred curvature drive their own snaps, and can additionally sustain unexpected sequences of transitions and instabilities, including morphing between higher-order forms, nonreciprocal motion, and switching without snapping at all. Our experimental realizations of these phenomena are all demonstrated in a single arch, hinting at how such control may enable complex robotic functions and highlighting how mechanical constraints enrich and complicate active morphing.

Our work also inverts a common paradigm: We do not form patterns from instabilities, but rather instabilities from patterns. Looking ahead, this inversion has broad applicability beyond snapping arches, leading to similar opportunities for optimization and enrichment. Encouragingly, in biology, it is increasingly understood that evolution often harnesses prepatterning to regulate and guide buckling morphogenesis [39–41]; similarly, the celebrated snapping in Venus flytraps [19] and hummingbird beaks [20] will inevitably have already undergone evolutionary optimization over stimulation patterns and initial shape. We thus anticipate a rich seam of research on the interaction of patterns and instabilities, and

hope the elementary system analyzed here provides a useful foundation.

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M.Z. conducted the theoretical and numerical investigation, A.K. conducted the experiments, and M.Z. and J.S.B. conceptualized the work. All authors contributed to writing the manuscript.

DATA AVAILABILITY

The data that support the findings of this article and the software required for reproduction of the results of this publication are openly available on Zenodo [42].

APPENDIX A: SAMPLE PREPARATION

Mesogenic diacrylates RM257 (CAS: 174063-87-7) and C6BAPE (CAS: 151464-39-0) were acquired from Daken Chemical. Dithiol chain extender EDDT (CAS: 14970-87-7), tetrathiol crosslinker PETMP (CAS: 7575-23-7), catalyst dipropylamine DPA (CAS: 142-84-7), photoinitiator Irgacure 2959 (CAS: 106797-53-9), absorbing dye Brilliant Yellow (CAS: 3051-11-4), and solvent chloroform (CAS: 67-66-3) were acquired from Sigma Aldrich. All reagents were used as received.

We fabricated liquid crystal elastomer samples via mechanical programming, following a protocol analogous to Refs. [35,36], with molar ratios of RM257, C6BAPE,

EDDET, and PETMP of 0.55-0.55-0.75-0.125, to maintain an excess of acrylate bonds. Photoinitiator was added in 1.5 wt % together with 0.25% photodye and 20 wt % solvent to form an isotropic solution. First-stage polymerization was initiated by adding a 10 wt % catalyst solution (5 vol % DPA in chloroform) in a 700- μm -thick glass cell and then left to complete at room temperature for 18 h, fixing the flat geometry as the isotropic configuration. The sample was taken out of the cell, the solvent evaporated at 100°C for 2 h, yielding a 600- μm -thick sheet, which was cut into a strip measuring 48 mm $\times$ 3.5 mm, with an excess length for clamping. The strip was then mechanically programmed by wrapping it around a cylindrical mold of radius $R_{\text{prog}} = 7.6\text{ mm}$ fixed in via exposure to UV light, imprinting uniform natural curvature $\kappa_{\text{prog}} = 1.3\text{ cm}^{-1}$ at room temperature. Upon heating, the material undergoes nematic-to-isotropic phase change and nearly recovers the flat isotropic geometry at $T > 65^\circ\text{C}$.

APPENDIX B: OBSERVED COMPRESSION VARIATION

Experimental data analysis detects a small variation in ϵ throughout the experiment due to the extensibility of the active beam. The length change due to compressive force is given by

$$\Delta\epsilon = -\frac{Yt^3 w/12}{Yt w L} \frac{f}{L} = -\frac{t^2 f}{12L^2}, \quad (\text{B1})$$

where Y is the Young's modulus, t the beam thickness, L its length, and f the nondimensionalized compressive force.

For the dimensions of our beam ($t = 0.6\text{ mm}$, $L = 40\text{ mm}$) and prior to any stimulation; i.e., $f = f_0$, we calculate $\Delta\epsilon = -0.075\%$ and extract $\epsilon = 1.2\%$ from Movie 1 of the Supplemental Material [37]. At the snap through, we have $f = f_1$ giving $\Delta\epsilon = -0.15\%$ compared to the extracted $\epsilon = 1.14\%$, while after the snap $f \approx 0$ gives $\Delta\epsilon = 0\%$ and $\epsilon = 1.28\%$, giving satisfactory agreement between the theory and experiments.

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