

Efficient control of fluxonium qubits via nonadiabatic transitions

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The fluxonium qubit is a promising platform for quantum operations due to its large anharmonicity and long coherence time. However, conventional resonant driving methods often require long operation times and complex implementation. As an alternative, nonadiabatic transitions induced by a time-dependent external flux $\phi_{\text{ext}}(t)$ can enable faster and simpler control by exploiting quantum interference between multiple transition paths. Existing approaches typically impose periodicity or symmetry on $\phi_{\text{ext}}(t)$, which can limit control efficiency. We propose a design strategy for $\phi_{\text{ext}}(t)$ that relaxes these constraints. We demonstrate the effectiveness of this approach in two key scenarios: population inversion and adiabatic passage. Numerical simulations show that our method achieves higher control efficiency than existing schemes, emphasizing the advantage of breaking periodicity and symmetry. Furthermore, by incorporating decoherence effects using the Lindblad master equation, we confirm that the proposed scheme remains robust under realistic conditions.

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I. INTRODUCTION

Superconducting qubits are regarded as one of the most promising platforms for scalable quantum computing [1–4]. A key advantage of this platform lies in its design flexibility—the energy spectrum can be engineered by tuning circuit parameters, allowing for fast gate operation due to large interaction and customized level structures tailored to specific control requirements. Among various superconducting qubit architectures, the fluxonium qubit stands out due to its large anharmonicity and long coherence times, making it a compelling platform for high-fidelity quantum operations [5–7]. These advantages arise from its unique circuit architecture: Josephson junctions shunted by a large inductance, forming a loop through which an external magnetic flux ϕ_{ext} is applied [8]. The external reduced flux ϕ_{ext} modulates the shape of the potential, making the eigenenergy spectrum dependent on ϕ_{ext} . The lowest few energy levels as a function of ϕ_{ext} are shown in Fig. 1.

When the energy gap between adjacent levels becomes minimal, it is referred to as an avoided crossing. These points are commonly known as sweet spots [9] because the first-order sensitivity to flux noise vanishes and reduces the transition rate by reduced transition dipoles or coupling to

the environment there, leading to significantly enhanced coherence times. In practical applications, the ground and first excited states at a sweet spot are typically selected as the computational space, and hence, gate operations are commonly performed at these points.

Although operating at a sweet spot offers advantages such as long coherence times and robustness against flux noise, achieving fast and high-fidelity quantum control at these points is challenging due to the small energy gaps. Conventional resonant driving techniques rely on strong coupling between states and typically require many radio frequency (rf) cycles to complete a transition, limiting their operation time and fidelity in systems with narrow level spacings, such as heavy fluxonium circuits. To overcome these limitations, alternative strategies based on nonadiabatic transitions induced by sweeping the parameter ϕ_{ext} have been explored [10–17]. In these approaches, the external reduced flux ϕ_{ext} is swept away from the sweep spot such that it passes through an avoided crossing multiple times, inducing multiple nonadiabatic transitions. This gives rise to different paths leading to the final state, and the overall transition probability at the final time is governed by interference between these nonadiabatic transition paths, analogous to a quantum interferometer. Such approaches enable efficient gate operations, achieving shorter gate times compared with conventional resonant driving on typical fluxonium circuits. This enhancement is especially evident in self-protected heavy fluxonium circuits [18–20].

In this study, following the recent developments in the approaches utilizing nonadiabatic transitions mentioned above, we aim to identify more advantageous schemes for fluxonium qubit gate operations. We specifically focus on expanding the flexibility of the control function $\phi_{\text{ext}}(t)$, as previous studies have typically assumed simple function forms, such as periodic or symmetric shapes. This increased flexibility en-

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hances the degrees of freedom in control but may also introduce additional complexity in the design process. We build on a theoretical framework proposed by Teranishi and Nakamura [21,22], which offers a systematic approach to designing control fields and provides a detailed analysis of nonadiabatic transition dynamics based on quantum interference. Although the original formulation is limited to periodic functions, its underlying principles can be generalized to non-periodic cases.

Leveraging this insight, we design nonperiodic time-dependent external reduced flux profiles $\phi_{\text{ext}}(t)$ to implement two important operations in a fluxonium qubit system: population inversion at the crossing point (sweet spot) and adiabatic passage from outside the crossing region to the crossing point (sweet spot). These operations are not only theoretically illustrative but also experimentally relevant.

Our two control schemes, population inversion and adiabatic population transfer, are key to advancing general quantum state manipulation. Our population inversion scheme, while presented with a focus on achieving unit transition probability, is directly generalizable to induce arbitrary partial population transfers between two states. In contrast, the adiabatic population transfer process is designed to induce no population change, but it plays a crucial role in modifying the phase of the quantum state. While this paper does not explicitly detail the phase dynamics during adiabatic passage, our theoretical framework can be directly generalized to precisely control the amount of this phase change. The ability to independently control both population and phase—through the combination of our generalized population transfer scheme and the phase-modifying adiabatic transfer—is particularly powerful, as it fundamentally enables arbitrary quantum state control.

To address the potential sensitivity of our scheme to flux noise, which arises from sweeping ϕ_{ext} away from the sweet spot during gate operations, we simulate the control process using the Lindblad master equation and find that the advantage of the scheme persists even when environmental effects are included, achieving an infidelity on the order of 10^{-5} .

The rest of the paper is organized as follows. In Sec. II, we briefly review the theoretical framework developed in Refs. [21,22], which forms the basis of our control strategy. In Sec. III, we examine the previously established and propose control schemes for population inversion and adiabatic passage.

II. THEORY

A. Landau-Zener model

To design fast and high-fidelity quantum control protocols using nonadiabatic transitions, it is essential to understand both the fundamental principles of nonadiabatic dynamics and the control methods associated with it. The Landau-Zener (LZ) model provides a foundational framework for describing nonadiabatic transitions due to an avoided crossing. The Hamiltonian of the LZ model is given by

$$\hat{H}(s) = \begin{pmatrix} \frac{\alpha}{2}(s(t) - s_b) & V \\ V & -\frac{\alpha}{2}(s(t) - s_b) \end{pmatrix}, \quad (1)$$

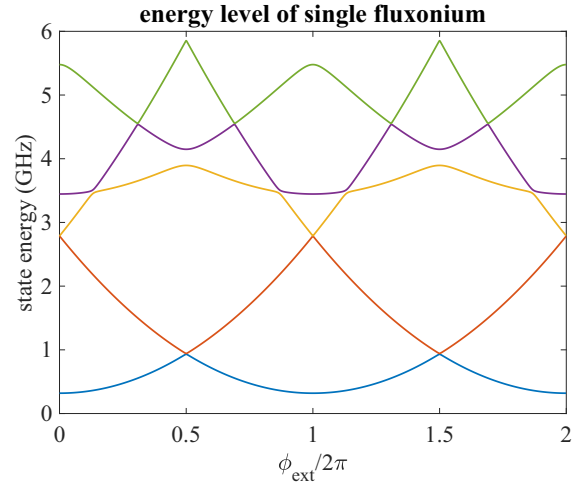


FIG. 1. Energy spectrum of the fluxonium qubit as a function of external reduced flux ϕ_{ext} , calculated using the parameters $E_C/h = 0.479$ GHz, $E_L/h = 0.132$ GHz, and $E_J/h = 3.395$ GHz in Eq. (21).

where $s(t)$ is a time-dependent parameter that controls the relative energy levels of the two states and V is the coupling strength between the two states. The eigenenergies of the system as functions of s are called the adiabatic energies, which are given by $E_{\pm} = \sqrt{\alpha^2 s^2(t) + V^2}$. Figure 2 shows the adiabatic energies as functions of s . At the point, $s(t) = s_b$, the energy gap between the adiabatic energies reaches its minimum, which is called the avoided crossing point, where the energy gap is given by $2V$. When the parameter $s(t)$ is swept through this region, nonadiabatic transitions occur. If the system is initially prepared in one of the eigenstates and the sweep starts at $t \rightarrow -\infty$, ending at $t \rightarrow \infty$, the transition probability to the other eigenstate is given by

$$P_{\text{LZ}} = e^{-2\pi(V^2/\hbar|\alpha\dot{s}|)}. \quad (2)$$

This implies that $P_{\text{LZ}} \rightarrow 0$ (the adiabatic limit) when $|V^2/\dot{s}| \rightarrow \infty$ and $P_{\text{LZ}} \rightarrow 1$ (the diabatic limit) when $|V^2/\dot{s}| \rightarrow 0$, both of which may be difficult to achieve in a real experiment. Therefore, simple schemes utilizing a single

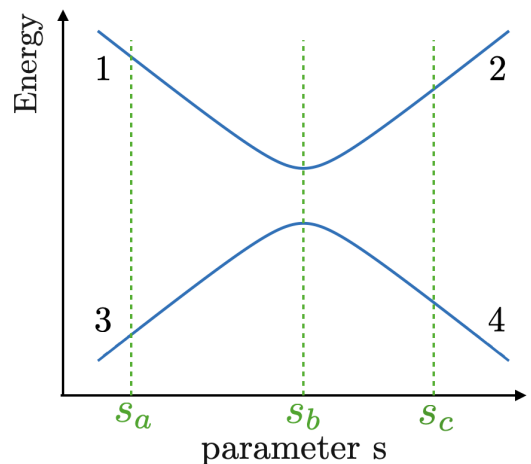


FIG. 2. Schematic illustration of the Landau-Zener model.

sweep through the avoided crossing are not sufficient for the purpose of control. To overcome this limitation, control with multiple passages, as explained below, is advantageous.

B. Quantum control via interference effect

To achieve effective control of these transitions around the avoided crossing, a control scheme involving multiple passages through the avoided crossing is required. A theory proposed by Teranishi and Nakamura [21,22] pointed out that such a control scheme can be understood by taking into account the effect of the interference among different paths of transition.

Here, we consider a two-state system, whose state vector is defined by

$$|\psi(t)\rangle = (c_1(t), c_0(t))^T, \quad (3)$$

where $c_j(t)$ are the probability amplitudes in state $|j\rangle$. The basic idea by Teranishi and Nakamura can be described with the transition matrices, which describe the transitions between adiabatic states, namely,

$$|\psi(t_2)\rangle = I_{t_1 \rightarrow t_2} |\psi(t_1)\rangle. \quad (4)$$

The transition matrix for a single passage through an avoided crossing, from $s = s_a$ at $t = t_a$ to $s = s_c$ at t_c (as labeled in Fig. 2), p , is given by a unitary matrix [22]

$$I_{a \rightarrow c} = \begin{pmatrix} I_{ee} & I_{eg} \\ I_{ge} & I_{gg} \end{pmatrix} = \begin{pmatrix} \sqrt{1-p}e^{i\phi} & -\sqrt{p}e^{i\sigma} \\ \sqrt{p}e^{-i\sigma} & \sqrt{1-p}e^{-i\phi} \end{pmatrix}, \quad (5)$$

where p is the Landau-Zener probability given by Eq. (2), and ϕ and σ are defined by

$$\begin{aligned} \phi &= \phi_0 + \sigma_{AB}/2 + \sigma_{BC}/2, \\ \sigma &= \sigma_0 - \sigma_{AB}/2 + \sigma_{BC}/2, \end{aligned} \quad (6)$$

where

$$\begin{aligned} \sigma_{AB} &= \int_{t_A}^{t_B} \Delta E(t) dt, \\ \sigma_{BC} &= \int_{t_B}^{t_C} \Delta E(t) dt. \end{aligned} \quad (7)$$

Here, t_B is the time when the adiabatic parameter reaches the avoided crossing, namely, $s(t_B) = s_B$.

The matrix element I_{nm} represents the transition amplitudes from $|m\rangle$ to $|n\rangle$. For example, the transition amplitude $I_{gg} = \sqrt{1-p}e^{-i(\phi_0 - \sigma_{AB} - \sigma_{BC})}$ can be interpreted as follows. I_{gg} is the transition amplitude for the process of staying on the ground adiabatic states, meaning that the absolute value of the transition amplitude should be $\sqrt{1-p}$, since the survival probability is $1-p$. The phase factors $-\sigma_{AB}/2$ and $-\sigma_{BC}/2$ in the exponent are the phase factors due to the adiabatic propagation along the ground states, which are given by

$$-\sigma_{AB}/2 = \int_{t_A}^{t_B} E_-(t) dt \quad (8)$$

and

$$-\sigma_{BC}/2 = \int_{t_B}^{t_C} E_-(t) dt. \quad (9)$$

Here, we use the relation $\Delta E(t) = -2E_- = 2E_+$. Then, the phase factor ϕ_0 can be interpreted as the phase shift associated with the nonadiabatic effect due to passing the avoided crossing. In the same way, I_{ge} is interpreted as the amplitude, whose absolute value is $\sqrt{p}$, and $\sigma_1/2$ and $\sigma_2/2$ in the exponent are the phase factors due to the adiabatic propagations along the ground state from t_A to t_B and along the excited state from t_B to t_C , respectively. σ_0 is the phase shift induced by passing the avoided crossing. It is well known that the backward sweep of this process, meaning sweep from s_C to s_A , is described by the transpose of I matrix, I^T .

Next, let us consider a process of one period following $s_a \rightarrow s_c \rightarrow s_a$, the transition matrix for which is given by

$$\begin{aligned} T &= I^T I = \begin{pmatrix} T_{ee} & T_{eg} \\ T_{ge} & T_{gg} \end{pmatrix} \\ &= \begin{pmatrix} [p + (1-p)e^{i2\psi}]e^{-i\sigma} & -2i\sqrt{p(1-p)}\sin\psi \\ -2i\sqrt{p(1-p)}\sin\psi & [p + (1-p)e^{-i2\psi}]e^{i\sigma} \end{pmatrix}, \end{aligned} \quad (10)$$

where

$$\psi = \phi_0 + \sigma_0 + \sigma_2, \quad \sigma = 2\sigma_0 + \sigma_2 - \sigma_1.$$

T_{eg} represents the transition amplitude from the ground state to the excited state, which can be interpreted as the interference between two transition pathways: one where the system follows $3 \rightarrow 2 \rightarrow 1$ and another where it follows $3 \rightarrow 4 \rightarrow 1$. In the first path, the system stays on the ground state until it meets the avoided crossing, followed by the nonadiabatic transition at the avoided crossing. After that it propagates along the excited state until the second avoided crossing, through which it propagates adiabatically, and stays on the excited state until the final time. Thus, the transition amplitude for this path is

$$\begin{aligned} a_1 &= (e^{i\sigma_{AB}/2}) \times (\sqrt{1-p}e^{i\phi_0}) \times (e^{i\sigma_{BC}/2}) \\ &\times (e^{i\sigma_{BC}/2}) \times (\sqrt{p}e^{i\sigma_0}) \times (e^{-i\sigma_{AB}/2}) \\ &= \sqrt{p(1-p)}e^{i(\phi_0 + \sigma_0 + \sigma_{BC})}. \end{aligned} \quad (11)$$

In the same way, we find that the transition amplitude for the second path is

$$\begin{aligned} a_2 &= (e^{i\sigma_{AB}/2}) \times (-\sqrt{p}e^{-i\sigma_0}) \times (e^{-i\sigma_{BC}/2}) \\ &\times (e^{-i\sigma_{BC}/2}) \times (\sqrt{1-p}e^{-i\phi_0}) \times (e^{-i\sigma_{AB}/2}) \\ &= -\sqrt{p(1-p)}e^{-i(\phi_0 + \sigma_0 + \sigma_{BC})}. \end{aligned} \quad (12)$$

Therefore, $|T_{eg}| = \sqrt{p(1-p)}\sin\xi = a_1 + a_2$ is interpreted as the interference of two transition pathways, analogous to the interferometer with two beam splitters, where two avoided crossings play roles of beam splitters. In the case of the interferometer with beam splitters, the phase difference is determined by the difference of the path length between two splitters. In the case of two avoided crossings, the phase difference is determined by ψ_0 and σ_0 , the adiabatic and nonadiabatic phase shifts due to the passage through the avoided

crossing, and σ_{BC} , the integral of energy gap over time between two avoided crossings.

The overall transition probability after one period of sweeping is found to be

$$P = |T_{eg}|^2 = |2\sqrt{p(1-p)}\sin(\phi + \sigma)|^2 = 4p(1-p)\sin^2(\xi), \quad (13)$$

implying that perfect transition, or unit overall transition probability, is achieved when under the condition

$$p = 0.5, \quad \xi = (n + \frac{1}{2})\pi, \quad n \in \mathbb{Z}. \quad (14)$$

This implies that the condition for p can be satisfied by a finite sweep speed, in contrast to the one-passage schemes requiring an infinite sweep speed. The condition for ξ can be satisfied by adjusting the phase difference σ_{BC} , which can be controlled by adjusting the time interval between the avoided crossings or the energy gap experienced during that interval. In some realistic cases, the energy gap at the avoided crossing may be extremely large or small. In such cases, it is difficult to achieve $p = 0.5$, and the above-mentioned two-passage scheme does not work. As discussed in Ref. [22], it is still possible to achieve $P = 1$ by increasing the number of passages, even if $p \neq 0.5$. Specifically, as the number of passages through the crossing increases, the range of p values that allow for $P = 1$ expands. For example, if $p = 0.01$, complete transition achieving $P = 1$ is possible with the use of eight passages through the avoided crossing. It should be noted that the conventional control schemes utilizing the swept adiabatic parameter assume a simple functional form, such as periodic or symmetric. As shown in Sec. III, we discuss the general schemes that break the limitation and found that such generalization improves the control efficiency significantly.

C. Control with asymmetric passage

When the scheme is not symmetric, the final transition probability cannot be simplified to Eq. (13), indicating that the theoretical framework presented in Refs. [21,22] does not fully capture the system's dynamics. Therefore, we generalize the transition matrices for the paths $3 \rightarrow 2 \rightarrow 1$ and $3 \rightarrow 4 \rightarrow 1$ to analyze asymmetric-passage cases. Assume that the single-passage transition probabilities for $s_a \rightarrow s_c$ and $s_c \rightarrow s_a$ are p_1 and p_2 , respectively. The modified transition matrices are given by

$$I_1 = \begin{pmatrix} \sqrt{1-p_1}e^{i\phi_1} & -\sqrt{p_1}e^{i\sigma_1} \\ \sqrt{p_1}e^{-i\sigma_1} & \sqrt{1-p_1}e^{-i\phi_1} \end{pmatrix}, \quad (15)$$

where

$$\begin{aligned} \phi_1 &= \phi_0^{(1)} + \sigma_{AB}^{(1)}/2 + \sigma_{BC}^{(1)}/2, \\ \sigma_1 &= \sigma_0^{(1)} - \sigma_{AB}^{(1)}/2 + \sigma_{BC}^{(1)}/2. \end{aligned}$$

In the same way, we find that the transition amplitude for the second path is

$$I_2 = \begin{pmatrix} \sqrt{1-p_2}e^{i\phi_2} & -\sqrt{p_2}e^{i\sigma_2} \\ \sqrt{p_2}e^{-i\sigma_2} & \sqrt{1-p_2}e^{-i\phi_2} \end{pmatrix}, \quad (16)$$

with

$$\begin{aligned} \phi_2 &= \phi_0^{(2)} + \sigma_{AB}^{(2)}/2 + \sigma_{BC}^{(2)}/2, \\ \sigma_2 &= \sigma_0^{(2)} - \sigma_{AB}^{(2)}/2 + \sigma_{BC}^{(2)}/2. \end{aligned}$$

The final transition probability is

$$\begin{aligned} P &= |T_{eg}|^2 \\ &= |(\sqrt{1-p_2}\sqrt{p_1}e^{i(\sigma_1+\phi_2)} - \sqrt{p_2}\sqrt{1-p_1}e^{-i(\phi_1+\sigma_2)})|^2 \\ &= |e^{i\beta}(\sqrt{1-p_2}\sqrt{p_1}e^{-i\theta} - \sqrt{p_2}\sqrt{1-p_1}e^{i\theta})|^2, \end{aligned} \quad (17)$$

where

$$\begin{aligned} \beta &= \frac{1}{2}(\phi_1 + \sigma_2 - \sigma_1 - \phi_2), \\ \theta &= \frac{1}{2}(\phi_1 + \sigma_2 + \sigma_1 + \phi_2). \end{aligned}$$

The perfect transition occurs when

$$p_1 + p_2 = 1 \quad (18)$$

and

$$\theta = n\pi, \quad \text{where } n \text{ is an integer.} \quad (19)$$

This demonstrates that perfect population transfer can be achieved using the asymmetric-passage scheme. In the analogy of beam splitter, a perfect transition is achieved by a pair of splitters with splitting ratios of $x:y$ and $y:x$, respectively. This is a straightforward generalization of the symmetric scenario, as the condition $p_1 + p_2 = 1$ simplifies to $p = 0.5$ under the symmetric constraint $p_1 = p_2 = p$. Basically, as long as condition (18) is satisfied, any transition probabilities between 0 and 1 can be achieved by controlling θ .

In the previous subsection, we discussed that when the single-passage transition probability p cannot reach 0.5, symmetric or periodic schemes require increasing the number of sweeping periods to achieve perfect transfer. Our analysis shows that this requirement can be relaxed by adopting an asymmetric approach, as perfect transfer is possible without increasing the number of periods. This finding highlights the potential of the asymmetric-passage scheme to enhance control efficiency, as it achieves perfect population transfer with fewer sweeping periods than required by symmetric or periodic schemes.

D. Transition probability for sweeps starting/ending at the avoided crossing

Even though the Landau-Zener model describes the transition probability under the assumption that the initial point is $s_a \rightarrow -\infty$ and the final point is $s_c \rightarrow \infty$, the Landau-Zener formula in Eq. (2) and the transition matrix in Eq. (5) are applicable to a finite sweeping range as long as the initial and final points are sufficiently far from the avoided crossing so that adiabatic approximation is applicable. Since nonadiabatic effect appears only in the vicinity of the avoided crossing, Eqs. (2) and (5) work in most of the cases. However, in practical gate operations, the computational states are defined at the crossing point, and thus, before and after the gate operation,

the system should be at the avoided crossing. In other words, the Landau-Zener formula (2) breaks because the adiabatic approximation is not valid at initial and final times. Hence, we need to analyze the transition probability induced by a passage starting from and/or ending at the crossing point. Hereafter, we examine the transitions starting/ending at the avoided crossing.

We first define the ground and excited states at the crossing point, corresponding to $s = s_b$ in Fig. 2, as $|g\rangle$ and $|e\rangle$, respectively. The ground and excited states at $(s = s_a)$ are different from those at the avoided crossing, and denoted as $|g'\rangle$ and $|e'\rangle$, respectively. Obviously, when we sweep the parameter s from s_b to s_a sufficiently slowly, adiabatic approximation holds, and the initially prepared ground state $|g\rangle$ is perfectly conveyed to the ground state $|g'\rangle$ at s_a . When the sweep speed is not slow enough, a transition takes place, whose probability is different from the LZ formula. In order to analyze such a transition, let us take a simple example in the limit where $s_a \rightarrow -\infty$ or $\alpha s(t_a) \gg V$. In this limit, the relation between the states in two regions can be easily found. For example, the ground state at the avoided crossing can be expressed in terms of the superposition of the states at s_a , as

$$|g\rangle = \frac{1}{\sqrt{2}}(|g'\rangle + |e'\rangle), \quad (20)$$

implying that a sudden sweep from $s = s_b$ to $s = s_a$ results in a transition probability of 0.5 for the excited state at $s = s_a$. It is important to note that, according to the Landau-Zener formula (2), the transition probability increases with the sweep speed α , reaching a maximum value of 1 in the limit of an infinitely fast sweep. However, as seen in Eq. (20), when starting from the crossing point, the maximum achievable excited-state transition probability p is limited to 0.5, even in the sudden limit.

Let us analyze the above discussion from the viewpoint of control. Here, we consider a sweeping scheme with one-period passage, namely, $(s_b \rightarrow s_a \rightarrow s_b)$. According to condition (14), the perfect transition requires $p = 0.5$, which is the upper limit of the transition probability realized by an infinitely rapid sweeping in a two-level system. In practical experiments, an ideal sudden sweep is difficult to realize. Even if a theoretically infinitely fast sweep were feasible, such a sweep does not achieve $p = 0.5$ in reality because real systems often have more than two states, and a sudden sweep can induce transitions to these higher energy levels, implying that the scheme with one-period passage may prove ineffective. The difficulty in the scheme of one-period passage can be overcome by multiple sweeping cycles as discussed earlier. In this case, the requirement of $p = 0.5$ is no longer necessary, since the allowed range of p for perfect transition broadens as the number of sweeping periods increases.

Building on the theoretical foundation discussed here, we propose control schemes applicable to a real physical system of fluxonium qubits to explore their feasibility, efficiency, and robustness in the following section. Here, we discuss two kinds of operations: population inversion and adiabatic passage, both of which should include starting and/or ending at avoided crossing.

III. RESULTS AND DISCUSSION

In this section, we present numerical simulations of several control schemes, comparing their fidelity, control time, and robustness. Initially, we focus our simulations on the closed quantum system, described by the Schrodinger equation, to examine the fidelity and control time of the control schemes. Following this, we discuss the open quantum system using the Lindblad equations to assess their robustness against the environmental effects. We take the Hamiltonian of the fluxonium qubit in the form [8]

$$\hat{H} = 4E_C\hat{n}^2 + \frac{1}{2}E_L\hat{\phi}^2 - E_J\cos(\hat{\phi} + \phi_{\text{ext}}), \quad (21)$$

where $\hat{n}$ and $\hat{\phi}$ are conjugate operators representing, respectively, the reduced charge and reduced flux, satisfying the commutation relation $[\hat{\phi}, \hat{n}] = i\hbar$. The parameters E_C , E_L , and E_J denote the charging, inductive, and Josephson energies, respectively, typically chosen such that $E_L \ll E_J$ and $1 \leq E_J/E_C \leq 10$ [23,24]. Consistent with these conditions, the system parameters employed in our simulations are $E_C/\hbar = 0.479$ GHz, $E_L/\hbar = 0.132$ GHz, and $E_J/\hbar = 3.395$ GHz. These values are relevant to a practical experimental implementation of the fluxonium qubit, as reported in Ref. [25], and ensure that the system remains well within the heavy fluxonium regime.

We solved the time-dependent Schrödinger equation using the Fourier grid method [26]. Our calculations employed 256 grid points in ϕ space. To ensure the reliability of our results, we thoroughly confirmed the convergence of the eigenenergies with varying numbers of grid points. With 256 points, we achieved an accuracy of approximately 10^{-12} for the 10th excited state and 10^{-7} for the 20th excited state. The population that escapes from the computational space is on the order of 10^{-9} , mostly leaking into the second and third excited states. Crucially, in the simulations presented in this paper, the population leaking to states higher than the third excited state was found to be negligibly small, confirming the sufficient accuracy of our calculations for the phenomena investigated.

In our simulations, we restrict the flux sweep range to $[0.25 \times 2\pi, 0.75 \times 2\pi]$, which is a moderate and realistic interval commonly used in experiments. Furthermore, our optimization results indicate that high-fidelity control can still be achieved within even narrower flux ranges. This suggests that experimental implementation of our proposed scheme not only is feasible but also does not require extreme control parameters.

A. Population inversion

First, we examine control schemes to achieve population inversion at the sweet spot, whereby the initial ground- and excited-state populations are inverted by sweeping the external reduced flux $\phi_{\text{ext}}(t)$ starting and ending at the sweet spot. We begin by introducing two existing approaches of designing the sweep schedule ϕ_{ext} : the one-cycle scheme and Schuster's scheme [25].

1. One-cycle method

In the one-cycle method, the sweeping function $\phi_{\text{ext}}(t)$ is designed as follows: Starting at the crossing point ($\phi_{\text{ext}} = \phi_a$),

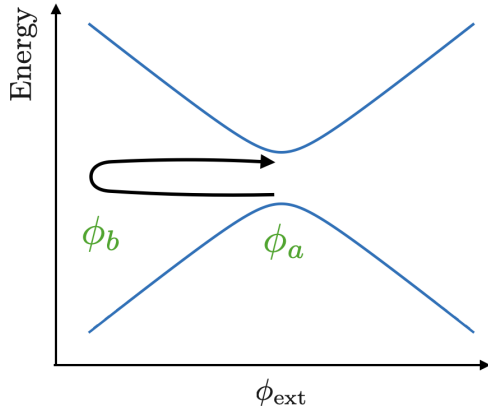


FIG. 3. The schematic plot of the one-cycle method. Sweeping from crossing point a to out of crossing point b and then sweep back to crossing point.

ϕ_{ext} is swept to a point outside the crossing ($\phi_{\text{ext}} = \phi_b$), with $\phi_b < \phi_a$. After that, ϕ_{ext} is maintained at ϕ_b for a duration Δ , followed by the back sweep from ϕ_b to the initial crossing point ϕ_a , completing the cycle. The schematic of the entire schedule is shown in Fig. 3.

The Hamiltonian (21) reveals sweet spots at $\phi = (2n + 1)\pi$ for integer n . The explicit sweeping function $\phi_{\text{ext}}(t)$ used in our simulation of the one-cycle method utilizing the sweet spot at π is chosen as

$$\phi_{\text{ext}}(t) = 0.25\pi \left[-\left(\tanh\left(\frac{t}{w}\right) + \tanh\left(\frac{1}{w}(\Delta - t)\right) \right) \right] + \pi, \quad (22)$$

the illustration of which is shown in Fig. 4(a).

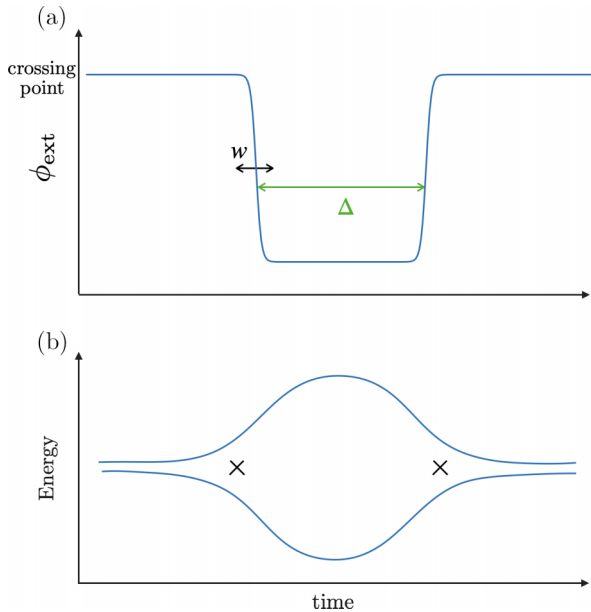


FIG. 4. Panel (a) shows the eigenenergy as a function of ϕ_{ext} , while panel (b) illustrates the eigenenergy evolution over time in the one-cycle method. The black crosses mark the crossing points where transitions occur.

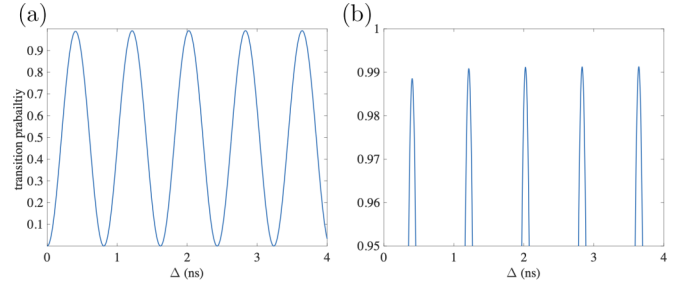


FIG. 5. The final transition probability depends on the delay time Δ with $w = 1.273$ ns. The right panel displays the same data as the left panel but with a narrower y-axis range.

Equation (22) consists of two hyperbolic tangent functions, where the parameter w represents the width of each function and corresponds to the sweeping speed, with larger values indicating slower sweeps. The parameter Δ denotes the waiting time at ϕ_b . The corresponding evolution of eigenenergies over time is presented in Fig. 4(b). Initially, ϕ_{ext} is at the crossing point ϕ_a , where the energy gap is small. As ϕ_{ext} is swept to ϕ_b ($\phi_b < \phi_a$), the energy gap increases. During the waiting period at ϕ_b , the energy gap remains large. Finally, as ϕ_{ext} is swept back to the crossing point, the energy gap decreases again. Throughout this process, nonadiabatic transitions occur twice, when the external flux ϕ_{ext} is leaving and arriving at the sweet spot, which are marked by black crosses in Fig. 4(b). These two transitions imply that the overall transition matrix for this process is given by T matrix given by Eq. (10), and the overall transition probability [see Eq. (13)] should oscillate as a function of Δ , since it determines the phase difference accumulated during the adiabatic propagation in between two avoided crossings. However, the transition probability p in the matrix T is not the standard Landau-Zener probability and is limited to $p < 0.5$ as discussed in the previous section. This indicates that a rapid sweep is required to increase the overall transition probability.

Figure 5(a) shows the final transition probability after the complete one-cycle sweep as a function of Δ , with w fixed at $w = 1.273$ ns. The oscillatory behavior arises from interference effects, as we expect. Since the phase difference induced during the time interval between two avoided crossings is roughly equal to $(E_e - E_g)\Delta$, we expect the period of the oscillation, defined by the difference of Δ between two peaks in Fig. 3(a), should be $2\pi/(E_e - E_g) \approx 0.813$ ns, which is compared with the period obtained from the numerical result (0.819 ns). Since the transition probability for a single passage is not exactly 0.5, the overall transition probability of the one-cycle process does not reach unity. In order to clarify this, we plot Fig. 5(b), where the same data as panel (a) are plotted, but with a narrower vertical range, highlighting that the maximum probability remains below 1. This limitation is attributed to the deviation of the single-passage transition probability from the ideal value of 0.5.

In order to confirm the above discussion, we plot, in Fig. 6, the single-passage transition probabilities as a function of w , defined as the probability induced by the sweep function with only the first hyperbolic tangent function. Within the regime $w > 0.3$ ns, we observe a decrease in the single-passage

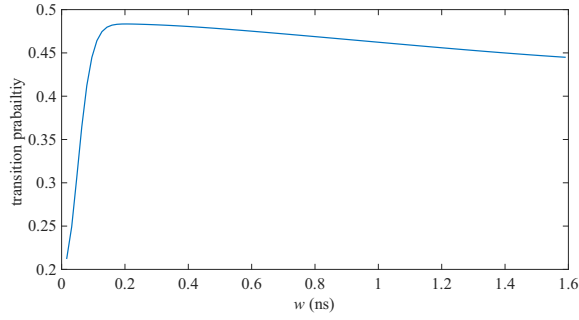


FIG. 6. The transition probability for passing through the crossing point once depends on the sweeping speed, where a larger w corresponds to a slower sweep.

transition probability as w increases (corresponding to a reduction in sweep speed). This trend is consistent with the adiabatic theorem, which predicts a diminishing probability of nonadiabatic transitions under slower sweeping conditions. Conversely for $w < 0.2$ ns, the overall transition probability is found to decrease as the sweep speed increases. This behavior is attributed to the fact that at sufficiently high sweep speeds, transitions to higher energy levels beyond the computational subspace become significant, thus reducing the probability of transition within the lower two levels. Therefore, the maximum value of one-passage transition probability achieved at $w \approx 1.1$ ns is about $p = 0.484$, which limits the fidelity of this scheme.

2. Schuster's method

Next, we examine a method proposed by Schuster [25], which extends the one-cycle method by applying it twice and incorporating a constant external flux held during the interval between the two cycles. The sweeping schedule is schematically presented in Fig. 7, showing both $\phi_{\text{ext}}(t)$ and the corresponding eigenenergy evolution over time. This method consists of three stages. The first stage involves a symmetric sweep of the external flux ϕ_{ext} : starting from the sweet spot, it is swept to ϕ_b , followed by a return sweep to ϕ_a , which is illustrated as the first peak in Fig. 7(a). In this stage, two nonadiabatic transitions occur, marked by the first two black crosses in Fig. 7(b). Following the first stage, in the second stage, ϕ_{ext} is held constant at the sweet spot over a controlled duration, depicted as the flat region in the figures. Finally, the first stage is repeated, but in an opposite direction, namely, ϕ_{ext} is increased and decreased, shown as the second peak in Fig. 7(a). As in the first cycle, two additional nonadiabatic transitions occur in this stage, as marked by the last two black crosses in panel (b).

As an implementation of this scheme, we take the following function as the sweeping function $\phi_{\text{ext}}(t)$:

$$\phi_{\text{ext}}(t) = 0.25\pi \left[- \left(\tanh\left(\frac{t - T_1}{w_1}\right) + \tanh\left(\frac{T_2 - t}{w_2}\right) \right) + \left(\tanh\left(\frac{t - T_3}{w_3}\right) + \tanh\left(\frac{T_4 - t}{w_4}\right) \right) \right] + \pi, \quad (23)$$

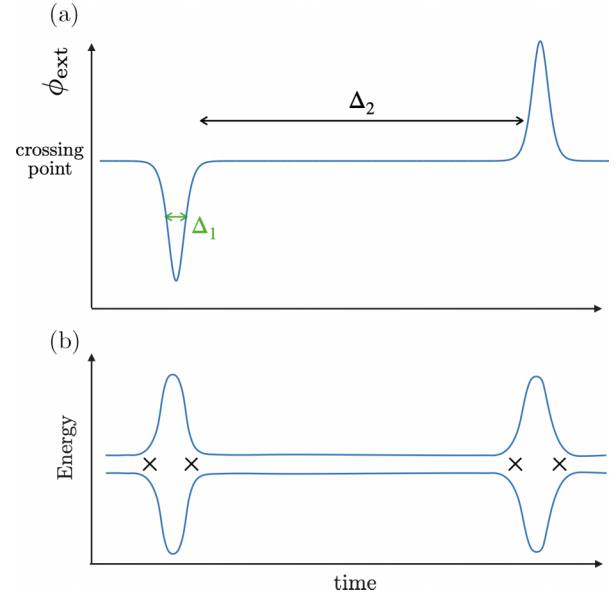


FIG. 7. Panel (a) shows the eigenenergy as a function of ϕ_{ext} , while panel (b) illustrates the eigenenergy evolution over time in Schuster's method. The black crosses mark the crossing points where transitions occur.

with

$$T_2 = T_1 + \Delta_1,$$

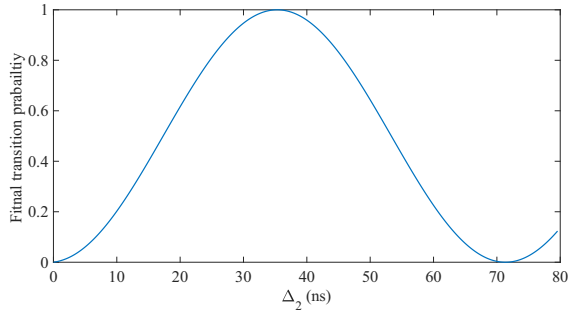
$$T_3 = T_2 + \Delta_2,$$

$$T_4 = T_3 + \Delta_3.$$

Here, we set $w_j = w$ for $j = 1 \sim 4$, and $\Delta_1 = \Delta_3$ to ensure the symmetry in Schuster's method.

The symmetric nature of Schuster's method allows the overall transition matrix T to be expressed as the product of a matrix I and its transpose, $T = I^T I$ analogous to Eq. (10), but with different definition of I matrix. Here, I matrix represents the transition matrix for the former half process, which consists of two nonadiabatic transitions in the first stage and subsequent adiabatic evolution during the former half of the second stage. Its transpose I^T similarly describes the adiabatic propagation during the latter half of the second stage followed by two nonadiabatic transitions in the third stage. This symmetric decomposition of the overall transition matrix T as a product of I and I^T enables us to use the similar conditions for complete transition discussed in the previous section, namely, the transition probability during the first half process described by I should be 0.5, and the adiabatic phase accumulated during the interval between the second and third transitions should be adjusted appropriately to achieve the constructive interference condition. In the analogy of beam splitter interferometer, the transitions by the first and third stages are analogous to the two beam splitters, whereas adiabatic propagation in the second stage corresponds to the delay stage between the two beam splitters.

In contrast to the one-cycle method, achieving a transition probability $p = 0.5$ in the first half of the process is not challenging. This is because the first stage of Schuster's method is equivalent to the entire one-cycle method, which attains the

FIG. 8. Final transition probability vs delay time Δ_2 .

transition probability oscillating from zero to nearly unity, as demonstrated in Fig. 5. Therefore, Schuster's method offers the potential for higher fidelity compared to the one-cycle method. To achieve optimal performance, precise tuning of w , Δ_1 , and Δ_2 is required. These parameters must be chosen to ensure that the transition probabilities due to the first and third stages should be 0.5, which should be achieved by a moderate sweep speed to minimize unwanted excitations to higher energy levels.

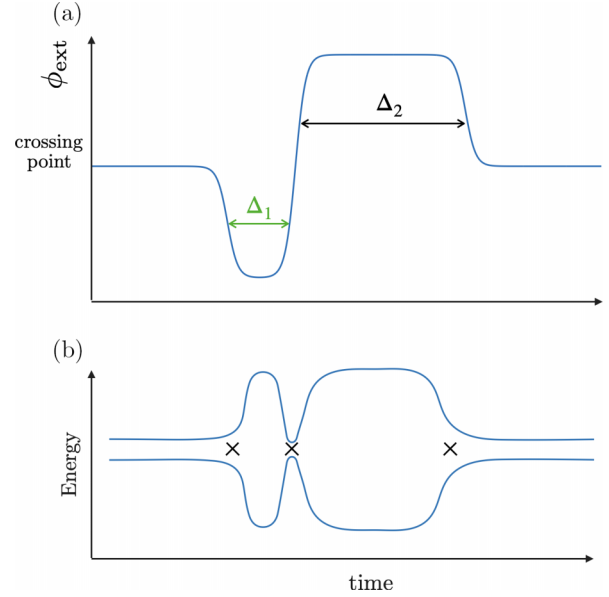
The optimization process for the overall transition probability follows these steps: (1) w is chosen to be sufficiently large to prevent excitation of higher energy levels, ensuring that the system remains within the two-level subspace. Among the possible values, the smallest w that satisfies this condition is selected to maintain the fastest feasible sweep. In our example, we choose $1 - (P_g + P_1) < 2 \times 10^{-8}$, where P_g and P_1 are the populations in the ground state and the first excited state, respectively. (2) Δ_1 is adjusted so that the transition probability after one cycle is precisely 0.5. This step ensures that the population transfer occurs reliably in each cycle. (3) The delay time Δ_2 is fine-tuned to control the phase accumulation at the crossing point, thereby maximizing constructive interference and enhancing the overall transition probability.

The optimization process is repeated multiple times to refine the parameters. The optimal infidelity, which is defined as $1 - P_1$, is achieved at $\Delta_2 \approx 35.3247$ ns, $w \approx 0.4775$ ns, and $\Delta_1 \approx 0.2027$ ns. The final transition probability as a function of Δ_2 with other parameters fixed at their optimal values exhibits oscillatory behavior as shown in Fig. 8, reflecting the interference effects discussed above. This optimal value satisfies the condition

$$(E_1 - E_g)\Delta_2/\hbar \approx \pi, \quad (24)$$

where E_1 and E_g are the eigenenergies at the crossing point. This indicates that constructive interference is achieved by controlling the waiting time Δ_2 , which depends on the energy gap at the crossing point. The optimal parameters achieve an infidelity of approximately 3×10^{-9} , significantly smaller than the infidelity of the one-cycle method, which is approximately 10^{-4} . However, Schuster's method requires significantly longer operation times compared to the one-cycle method. This is attributed to the necessity of a large Δ_2 to satisfy condition (24) due to the small energy gap at the crossing point.

Although perfect population inversion is not achieved primarily due to multilevel effects, their impact is significantly

FIG. 9. Panel (a) shows the eigenenergy as a function of ϕ_{ext} , while panel (b) illustrates the eigenenergy evolution over time in our method.

mitigated compared to the one-cycle method. Another possible source of deviation from perfect inversion may be nonadiabatic transitions induced by discontinuities in the time derivatives of $\phi_{\text{ext}}(t)$, as discussed in Ref. [27]. Investigating strategies to further suppress these effects will be an important direction for future work.

To discuss the robustness of Schuster's method, we examined the effect of small parameter deviations from their optimal values, as would occur in a real experiment. For each parameter, we introduced a small deviation δ while keeping the other parameters fixed at their optimal values, and then calculated the infidelity as a function of δ . This infidelity was fitted by the function $f(\delta) = f_{\text{opt}} + \gamma\delta^2$, where f_{opt} is the infidelity at the optimal parameters and γ (dependent on the parameter) represents the robustness against a variation.

For ω and Δ_2 , the robustness parameter γ is on the order of 10^{-5} ns^{-2} , while for Δ_1 it is on the order of 1 ns^{-2} . This indicates that the inaccuracy of Δ_1 induces a significant infidelity. For example, if it contains errors on the order of 0.001, 0.01, and 0.1 ns, the estimated infidelities are, respectively, 10^{-6} , 10^{-4} , and 10^{-2} . These results illustrate how the precision of the control parameters affects the control fidelity and provide practical guidelines for experimental implementation.

3. Asymmetric-passage method

We propose a scheme that can achieve the same order of infidelity as Schuster's scheme in a much shorter time. The key idea is that, by controlling the interference at a point away from the crossing, where the energy gap is larger, the required waiting time can be significantly reduced. This allows for faster transitions while maintaining high fidelity, making the scheme more efficient, particularly in practical implementations where minimizing operation time is critical.

Motivated by this idea, we introduce a scheme, illustrated in Fig. 9. This method consists of three main stages. First,

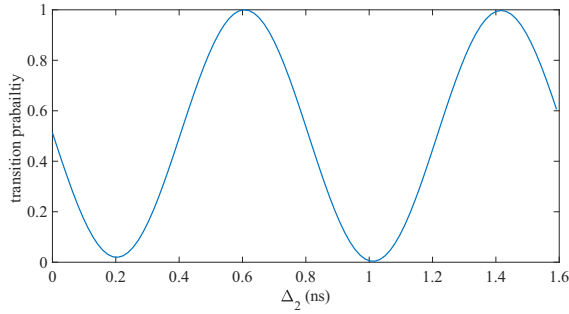


FIG. 10. Transition probability as a function of delay time Δ_2 in our scheme.

starting from the crossing point, $\phi_{\text{ext}}(t)$ is varied to reach a point with a larger energy gap, where it remains for a duration of Δ_1 . Next, $\phi_{\text{ext}}(t)$ is adjusted back toward the crossing point but does not stop; instead, it continues past the crossing to another point with a larger energy gap. The system remains at this point for a duration of Δ_2 to control the phase of the wave function. Finally, $\phi_{\text{ext}}(t)$ is swept back to the crossing point.

The time-dependent function $\phi_{\text{ext}}(t)$ is given by

$$\phi_{\text{ext}}(t) = 0.25\pi \left[- \left(\tanh\left(\frac{t-T_1}{w_1}\right) + \tanh\left(\frac{T_2-t}{w_2}\right) \right) + \left(\tanh\left(\frac{t-T_2}{w_2}\right) + \tanh\left(\frac{T_3-t}{w_3}\right) \right) \right] + \pi, \quad (25)$$

where the time parameters are defined as

$$\begin{aligned} T_2 &= T_1 + \Delta_1, \\ T_3 &= T_2 + \Delta_2. \end{aligned}$$

As discussed above, achieving a transition probability of 0.5 in the third stage is generally challenging. However, the asymmetric-passage theory implies that as long as condition (18) is satisfied, a transition probability $p_2 < 0.5$ is acceptable if $p_1 > 0.5$, which can be readily achieved in the first stage by completing one and a half cycles (i.e., 1.5 cycles). Moreover, condition (19) can be satisfied by adjusting the time delay in the second stage and the final transition probability oscillates as a function of Δ_2 , according to Eq. (17), as shown in Fig. 10.

After parameter optimization, the transition probability yields an infidelity of approximately 2×10^{-9} . The optimized parameters are $w \approx 0.4762$ ns, $w_4 \approx 1.2981$ ns, $\Delta_1 \approx 0.2027$ ns, and $\Delta_2 \approx 0.6063$ ns. This infidelity is comparable to that of Schuster's method. Figure 11(a) shows the optimized $\phi_{\text{ext}}(t)$ in our scheme, compared to that of Schuster's. The operation time is approximately 12 ns, which is significantly shorter than that of Schuster's method, measured at approximately 45 ns.

Figure 11(b) illustrates the nonadiabatic transition probability as a function of time. In Schuster's scheme, the transition probability reaches 0.5 after the first cycle, but a waiting period between the first and second cycles is required for phase accumulation at the crossing point before the probability reaches 1. In contrast, our scheme achieves a perfect transition almost immediately, with minimal waiting time. This is because our method takes advantage of a large energy

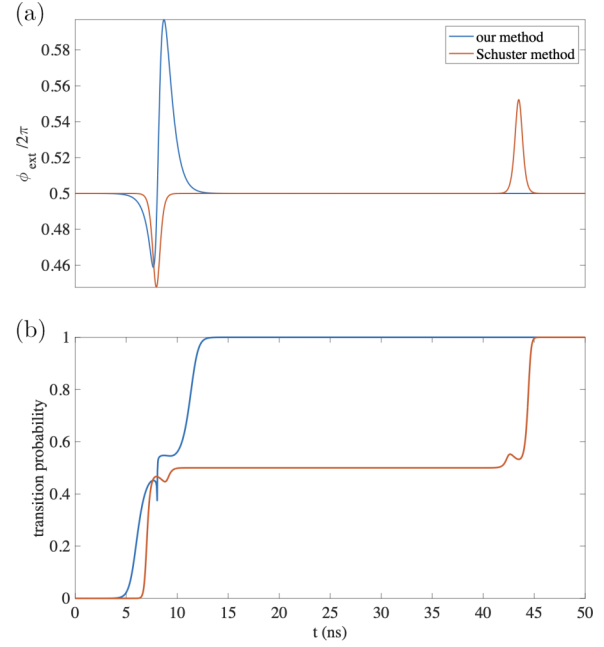


FIG. 11. Panel (a) shows the optimal $\phi_{\text{ext}}(t)$ compared to Schuster's method, while panel (b) displays the transition probability as a function of time.

gap to control the interference, whereas Schuster's scheme relies on the small gap at the crossing point. This implies that if these two schemes are applied to a heavier fluxonium, where the gap at the crossing point is smaller, Schuster's method would require a longer time, whereas our scheme maintains a short operation time even in heavier fluxonium.

We evaluated the robustness parameters discussed above. The robustness parameters γ for w and w_4 are found to be on the order of 10^{-4} ns $^{-2}$, while those for Δ_1 and Δ_2 are on the order of 10^{-1} ns $^{-2}$. These results indicate that Δ_1 and Δ_2 are the parameters most sensitive to infidelity. Errors in these parameters on the order of 0.001, 0.01, and 0.1 ns result in infidelities on the order of 10^{-7} , 10^{-5} , and 10^{-3} , respectively.

B. Adiabatic passage

We now demonstrate how to implement adiabatic passage efficiently by bringing the ground state from outside the crossing region to the crossing point while minimizing unwanted excitations. Since maintaining adiabaticity typically requires a slow sweep of ϕ_{ext} , as shown in Fig. 12(a), the effectiveness of this scheme depends on the nonadiabatic coupling between adiabatic states i and j , given by [28]

$$d_{ij} = - \frac{(\hat{w}^{-1} \dot{\hat{H}} \hat{w})_{ij}}{E_i - E_j}, \quad (26)$$

where $\hat{H}$ is the system Hamiltonian, $\hat{w}$ is the matrix that diagonalizes $\hat{H}$, and E_i , E_j are the instantaneous eigenenergies. The term $(\hat{w}^{-1} \dot{\hat{H}} \hat{w})_{ij}$ represents the matrix elements of the time derivative of the Hamiltonian in the adiabatic basis. Equation (26) implies that a smaller energy gap requires a slower sweep to suppress the transition.

To overcome this limitation, we propose a scheme that leverages interference effects to cancel the net excitation at

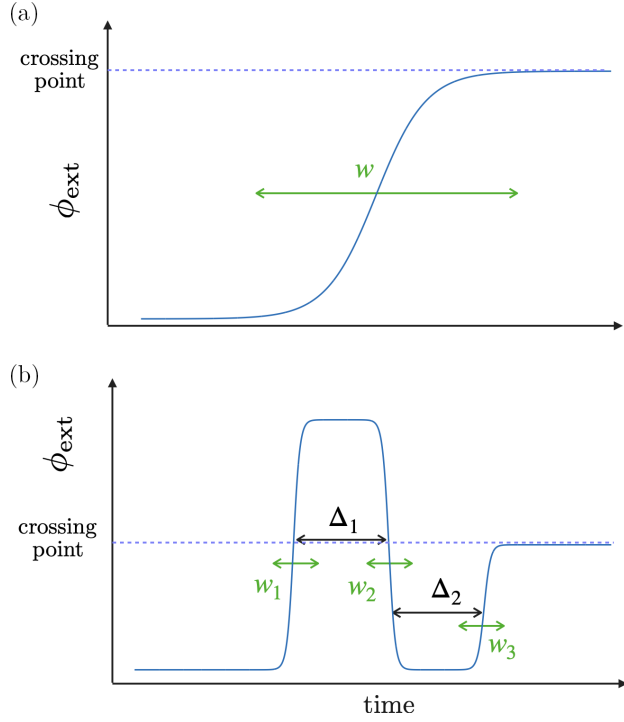


FIG. 12. Schematic illustration of (a) the conventional adiabatic passage scheme and (b) the interference-based scheme.

the end of the process, enabling a significantly faster adiabatic passage. Our scheme is illustrated in Fig. 12(b), which introduces a back-and-forth sweeping motion to induce transitions that interfere destructively, ultimately eliminating the population in the excited state. The scheme consists of two stages. In the first stage, ϕ_{ext} is swept from outside the crossing region, passes through the sweet spot (i.e., the crossing point), and is then swept back to the same side from which it started. In the second stage, ϕ_{ext} is swept from outside the crossing to the crossing point, with the transition probability determined by the parameter w_3 .

A key point of our scheme is the use of controlled interference. In contrast to the slow-sweep scheme, which suppresses transitions at all times during the process, our scheme allows them to occur but is carefully designed so that destructive interference cancels out the final transition probability. Specifically, the first stage introduces a transition probability, followed by a second stage that induces another transition with the same transition probability as the first stage. The time interval between the two transitions is carefully tuned so that the accumulated phase leads to destructive interference, thereby eliminating the final excited-state population. This interference-based mechanism is expected to achieve a transitionless sweep of the adiabatic parameter in significantly less time than the conventional slow-sweep scheme.

We choose the following function to govern our scheme:

$$\begin{aligned} \phi_{\text{ext}}(t) = & 0.1\pi \left(\tanh\left(\frac{t-T_1}{w_1}\right) + \tanh\left(\frac{T_2-t}{w_1}\right) \right) \\ & + 0.05\pi \left(\tanh\left(\frac{t-T_3}{w_2}\right) + 1 \right) + 0.9\pi, \end{aligned} \quad (27)$$

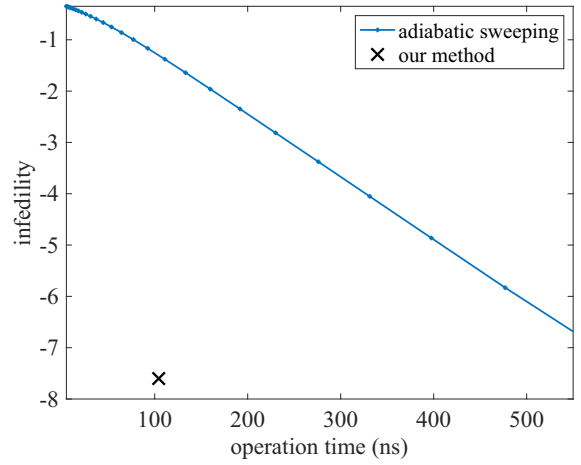


FIG. 13. Comparison of control efficiency between our scheme and the conventional slow-sweep method. The infidelity is plotted as a function of operation time, defined as $T_{\text{op}} = 5 \times w$ for the slow-sweep method and $T_{\text{op}} = 5 \times (w_1 + w_2 + w_3) + \Delta_1 + \Delta_2$ for our scheme.

where the time parameters are defined as

$$\begin{aligned} T_2 &= T_1 + \Delta_1, \\ T_3 &= T_2 + \Delta_2. \end{aligned} \quad (28)$$

To optimize the parameters, we first fix w_1 and w_2 . The transition probability for stage 1 then depends on Δ_1 , while the transition probability for stage 2 depends on w_3 . We select values of Δ_1 and w_3 such that the resulting transition probabilities are equal. Then, we fine-tune Δ_2 to induce destructive interference, effectively canceling the population in the excited state. In this way, we determined the optimal parameters: $w_1 = w_2 = 6.2500$ ns, $w_3 = 0.9649$ ns, $\Delta_1 = 7.880$ ns, and $\Delta_2 = 16.9463$ ns, which result in an infidelity below 10^{-7} .

To compare the two schemes, we define the operation time as $T_{\text{op}} = 5 \times w$ for the slow-sweep method and $T_{\text{op}} = 5 \times (w_1 + w_2 + w_3) + \Delta_1 + \Delta_2$ for our method. The resulting infidelities as functions of operation time are shown in Fig. 13. The blue dotted line represents the slow-sweep scheme, in which increasing the operation time, corresponding to a slower sweep, leads to a decrease in infidelity. For the same operation time, our method achieves an infidelity up to five orders of magnitude lower than that of the slow-sweep method. If high-precision control of $\phi_{\text{ext}}(t)$ is not feasible due to experimental limitations, the slow-sweep method remains a viable option, albeit at the cost of significantly longer operation times.

C. Robustness against decoherence

In the above sections, we proposed schemes for population inversion and adiabatic passage, which perform better than conventional methods. Fast and high-fidelity control is achieved by utilizing interference, controlled through the accumulation of a desired phase difference at a point with a large energy gap. Since the coherence time at such points is typically shorter than that at sweet spots, our method may sacrifice some robustness against environmental effects. To

assess the impact of environmental noise on our scheme, we simulate the dynamics of an open quantum system by solving the Lindblad master equation:

$$\dot{\rho} = -\frac{i}{\hbar}[\hat{H}, \rho] + \sum_{k=1, \phi} \left(L_k \rho L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho - \frac{1}{2} \rho L_k^\dagger L_k \right). \quad (29)$$

Here, ρ is the density matrix of the system and $\hat{H}$ represents the system Hamiltonian. The terms containing L_k describe the effects of dissipation and dephasing due to the interaction with the environment. Specifically, the jump operators are defined as

$$L_1 = \Gamma_1 \sigma_-, \quad L_\phi = \Gamma_\phi \sigma_z. \quad (30)$$

The operator L_1 accounts for energy relaxation, leading to transitions from the excited state to the ground state, while L_ϕ represents pure dephasing, which introduces phase fluctuations without energy exchange [29].

The decay rates Γ_1 and Γ_ϕ are directly related to the system's relaxation time T_1 and dephasing time T_2 , expressed as [30]

$$\begin{aligned} \Gamma_1 &= \frac{1}{T_1}, \\ \Gamma_\phi &= \frac{1}{T_\phi}, \\ \frac{1}{T_2} &= \frac{1}{2T_1} + \frac{1}{T_\phi}. \end{aligned} \quad (31)$$

The values of Γ_1 and Γ_ϕ used in our simulation are derived from experimentally measured coherence times of the fluxonium qubit, as reported in Ref. [25]. Specifically, T_1 is taken as the shortest relaxation time across all values of ϕ_{ext} , while T_2 is modeled as a function of ϕ_{ext} based on the experimental data. The measured range of the external flux in Ref. [25] is $\phi_{\text{ext}} = 0.5 \pm \delta\phi_{\text{ext}}$, where $|\delta\phi_{\text{ext}}| < 4 \times 10^{-4}$. For the range $|\delta\phi_{\text{ext}}| > 10^{-4}$, we use the value of $T_2 = 10 \mu\text{s}$, which is a typical value of flux noise dominated in the fluxon transition regime [23, 25]. By incorporating these decay terms, we calculate the infidelity of the control schemes, Schuster's, asymmetric-passage scheme, and adiabatic passage, where parameters used in these simulations are the optimized parameters in the open quantum simulation.

For population inversion, the infidelity of Schuster's scheme is approximately 1.6×10^{-3} when including the noise effect, while our scheme achieves a lower infidelity of around 1.1×10^{-4} . Although our scheme in Fig. 11 appears to deviate further from the crossing point compared to Schuster's scheme, its total operation time is shorter, making it less susceptible to noise effects. This suggests that even when operating outside the crossing region, the fidelity loss can remain insignificant as long as the operation is completed within a short time.

For adiabatic passage, our scheme achieves an infidelity of approximately 3.8×10^{-5} . This indicates that even when

considering the effects of noise, our scheme is more efficient than a simple slow-sweep approach. As shown in Fig. 13, achieving an infidelity on the order of 10^{-5} with slow sweeping requires more than 400 ns, which is about four times longer than the operation time of our scheme.

These results highlight an important trade-off between stability and speed. While maintaining stability at the crossing point is generally preferred, our findings suggest that minimizing operation time can serve as an effective alternative strategy for preserving fidelity. Depending on the specific experimental constraints, optimizing for shorter operation times may provide a viable approach for improving nonadiabatic transitions.

IV. CONCLUSION

We have developed a quantum control strategy for fluxonium qubits by extending the theoretical framework proposed by Teranishi and Nakamura, eliminating the conventional requirement of periodic or symmetric parameter sweeps. This approach enables more efficient and flexible nonadiabatic transition-based control methods by breaking traditional constraints.

To demonstrate the applicability and effectiveness of our method, we investigated two key control scenarios: population inversion and adiabatic passage. For population inversion, we analyzed the limitations of one-cycle control and the recent approach proposed by Schuster [25]. Based on the analysis, we proposed a scheme utilizing asymmetric sweeping of the adiabatic parameter, which achieves a comparable level of infidelity while significantly reducing the operation time, surpassing both the rf period and the durations required by existing methods. Our approach of asymmetric sweeping is applicable to adiabatic passage as well. It exhibits an infidelity five orders of magnitude smaller than that of the conventional slow-sweeping method within a comparable operation time.

We further verified the robustness of the schemes we proposed under realistic decoherence by incorporating environmental effects through the Lindblad master equation, using the values of T_1 and T_2 relevant to the experiment. Since decoherence effects are minimized at the sweet spot, our schemes, spending most of the time out of the sweet spot, can be significantly affected by stronger decoherence. However, our simulations show that the short control times in our schemes effectively compensate for the increased decoherence. These results suggest that removing symmetry and periodicity constraints in nonadiabatic control schemes not only enhances efficiency but also maintains high fidelity under experimentally relevant noise conditions.

Our work offers a promising direction for fast and robust gate operations in multilevel superconducting circuits. While the current work does not explicitly construct universal gate operations, the underlying control framework is general, relying on interference among nonadiabatic paths and controlling the final state via adiabatic phase accumulation. Although phase control is not explicitly demonstrated in this study, our theoretical framework allows the adiabatic phase accumulated along the control path to be precisely tuned, enabling quantum phase manipulation independently of population transfer.

Therefore, this framework can, in principle, be extended to realize universal quantum gate operations. Moreover, the proposed nonadiabatic transition-based method is both fast and high fidelity according to our simulations, indicating strong potential as a platform for high-speed, high-fidelity quantum control. In addition, exploring optimal sweep schedules under experimental constraints, or applying the scheme to other multilevel superconducting platforms, such as transmons with leakage suppression, presents compelling directions for future research.

Although we have demonstrated the method using fluxonium qubits, the underlying theory is general and applicable to a broad range of systems with avoided level crossings.

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are openly available [31].

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