

Observation of a zero-energy excitation mode in the open Dicke model

Anton Bölian^{1,*}, Phatthamon Kongkhambut^{1,2,3,*}, Christoph Georges¹, Roy D. Jara, Jr.⁴, José Vargas,¹ Jens Klinder¹, Jayson G. Cosme⁴, Hans Keßler^{1,†}, and Andreas Hemmerich^{1,‡}

¹*Institute for Quantum Physics, Universität Hamburg, 22761 Hamburg, Germany*

²*Quantum Simulation Research Laboratory, Department of Physics and Materials Science, Faculty of Science, Chiang Mai University, Chiang Mai 50200, Thailand*

³*Thailand Center of Excellence in Physics, Office of the Permanent Secretary, Ministry of Higher Education, Science, Research and Innovation, Chiang Mai 50200, Thailand*

⁴*National Institute of Physics, University of the Philippines, Diliman, Quezon City 1101, Philippines*



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Approaching phase boundaries in many-body systems can give rise to intriguing signatures in their excitation spectra. Here, we explore the excitation spectrum of a Bose-Einstein condensate strongly coupled to an optical cavity and pumped by an optical standing wave, which simulates the famous Dicke-Hepp-Lieb phase transition of the open Dicke model with dissipation arising due to photon leakage from the cavity. For weak dissipation, the excitation spectrum displays two strongly polaritonic modes. Close to the phase boundary, we observe an intriguing regime where the lower energetic of these modes, instead of showing the expected roton-type mode softening, is found to approach and persist at zero energy, well before the critical pump strength for the Dicke-Hepp-Lieb transition boundary is reached. Hence, a peculiar situation arises, where an excitation is possible at zero energy cost, but nevertheless no instability of the system is created.

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The coupling of a quantum gas to an optical cavity provides an ideal experimental scenario to study the effects of strong light-matter interactions [1,2], including cavity-induced cooling [3–5], cavity optomechanics [6–10], and quantum simulations of many-body physics [11–16]. Atom-cavity systems can also be used to emulate the open Dicke model [17], in which an ensemble of identical two-level systems is coupled to a single mode of the electromagnetic field. In cavity QED, the open Dicke model is typically realized by coupling the atomic ensemble to a single cavity mode, while the system interacts with the environment via photons leaking out of the cavity with the cavity field decay rate κ . This has been achieved with thermal atoms [18] and both fermionic [19–22] and bosonic [23–25] quantum gases. In the recoil-resolved regime, when the cavity bandwidth is smaller than the recoil frequency ω_{rec} , atom-cavity systems provide access to several distinctive phenomena, such as discrete [13,26–28] and continuous [14,29–31] time crystals.

The open Dicke model hosts a continuous phase transition, where the atoms reorganize from interacting individually with the cavity mode to forming a macroscopic dipole moment

that collectively interacts with the cavity mode [32]. It can be experimentally implemented using a Bose-Einstein condensate (BEC) strongly coupled to a longitudinal mode of an optical Fabry-Pérot type cavity, which is pumped by a retro-reflected laser beam with frequency ω_p and strength ϵ_p , aligned perpendicularly to the cavity mode. In this configuration, the phase transition is characterized by the atoms spontaneously self-organizing into one of two possible density-wave configurations above a critical pump strength ϵ_c . These density gratings enable efficient Bragg scattering of light from the transverse pump field into the cavity, and vice versa [23,33,34]. In a many-body picture, this transition can be understood as the result of tuning the strength of a light-induced infinite-range interaction (LIRI) between the atoms.

There are two disparate regimes of LIRI in an atom-cavity system. In a broad-band cavity with κ much larger than the recoil frequency ω_{rec} , the light sector dynamics is associated with a far more rapid timescale as compared to that of the matter sector such that the LIRI practically acts *instantaneously* (I-LIRI). In a recoil-resolving narrow-band cavity, where κ is on the order of ω_{rec} , the timescales for the evolution in the light and matter sectors are comparable, and hence the LIRI among the atoms is significantly *retarded* (R-LIRI regime). The excitation modes exhibit distinct polaritonic character, i.e., a composition of comparable admixtures of photonic and matter components, and higher-order excitations are strongly suppressed due to the narrow cavity bandwidth [10,33].

In the vicinity of a phase boundary, many-body systems can give rise to intriguing signatures in their excitation spectra. In this work, we investigate the two expected polaritonic excitation modes in the R-LIRI regime close to the phase

*These authors contributed equally to this work.

†Contact author: hkessler@physnet.uni-hamburg.de

‡Contact author: andreas.hemmerich@uni-hamburg.de

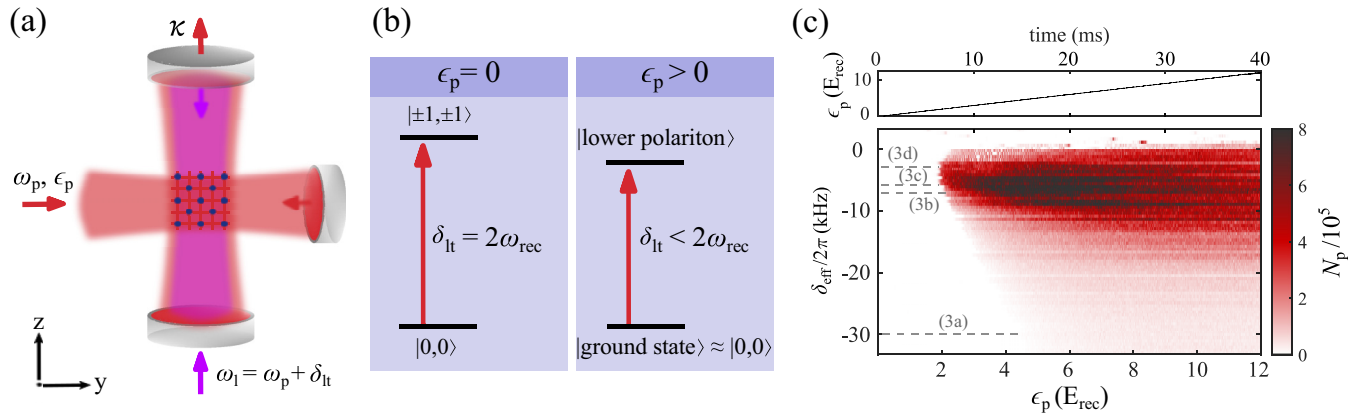


FIG. 1. (a) Illustration of the Bragg spectroscopy scheme used to probe the excitation energy of the polariton modes. Scans of the frequency detuning δ_{lt} between the on-axis longitudinal probe and the transverse pump waves are applied with variable fixed frequency ω_p and strength ϵ_p of the transverse pump. (b) Sketch of the mode softening of the lower polariton mode as the transverse pump strength increases. (c) Intracavity photon number N_p plotted against the effective detuning δ_{eff} between the transverse pump and the cavity and the transverse pump strength ϵ_p . The detunings at which the excitation spectra shown in Fig. 3 were measured [$\delta_{eff} = 2\pi \times (-30, -7.5, -6, -3)$ kHz] are marked accordingly. For negative effective detunings $\delta_{eff} < 0$, the formation of a finite intracavity light field with photon number N_p , arising above a critical value of the transverse pump strength ϵ_p , indicates that the system self-organizes into a superradiant phase.

boundary of the Dicke-Hepp-Lieb phase transition of the open Dicke model. We experimentally demonstrate an intriguing domain, where the lower energetic of these modes, instead of showing an ordinary roton-type mode softening [35], is found to approach zero energy at a pump strength ϵ_z well below the critical pump strength ϵ_c at the Dicke-Hepp-Lieb transition boundary. The excitation energy remains zero across a finite pump strength interval $[\epsilon_z, \epsilon_c]$, such that a peculiar situation arises, where an excitation is possible at zero energy cost, but nevertheless no instability of the system is created.

Indications of this phenomenon have been previously found in calculations for a broad-band cavity I-LIRI scenario [36] with the result of a tiny zero-energy pump strength interval, more than six orders of magnitude smaller than the critical pump strength $[(1 - \epsilon_z/\epsilon_c) \sim 10^{-6}]$ and as such practically not observable. In fact, experimentally, in Ref. [35], a single excitation mode with an ordinary rotonlike mode softening with $\epsilon_z \sim \epsilon_c$ was found with no discernible zero-energy domain. In a complementary theory work [37] for a narrow-band R-LIRI cavity system, a similar zero-energy excitation mode was found with zero energy attained across a much larger pump strength interval, only one order of magnitude smaller than the critical pump strength. In a more recent review article [2], the authors explicitly speculated that the domain of zero-energy excitations might be in reach in an R-LIRI cavity system. The lack of an experimental confirmation of such excitations, more than one decade after their mention by theory, reflects the extensive technical complexity to realize the R-LIRI scenario necessary for a sizable experimentally observable effect. See the Supplemental Material [38] for a discussion of the size of the zero-energy interval $[\epsilon_z, \epsilon_c]$ for I-LIRI and R-LIRI regimes.

In our experiment, a BEC of $N_a \approx 40 \times 10^3$ ^{87}Rb atoms is strongly coupled to a single mode of an optical cavity with a finesse of 4.2×10^5 . The atoms are primarily addressed with a retro-reflected laser beam with frequency ω_p , transversely oriented with respect to the cavity axis, referred to

in the following as the transverse pump. This setup is depicted in Fig. 1(a). The pump wavelength $\lambda_p = 803.26$ nm is far-red detuned with respect to the relevant atomic transitions, namely, the D_1 and D_2 lines at 794.97 and 780.24 nm, respectively; thus, we operate in the dispersive regime. At this chosen pump wavelength, the transverse pump produces an attractive standing-wave potential at the position of the atoms. In this experiment, we utilize a cavity that operates in the recoil-resolved regime, where the field decay rate of $\kappa = 2\pi \times 3.60$ kHz is comparable to the recoil frequency $\omega_{rec} = 2\pi \times 3.55$ kHz [10]. The cavity resonance is shifted as a result of the refractive index of the BEC in the cavity by an amount $\delta_- = N_a U_0/2$, where $U_0 = -2\pi \times 0.34$ Hz is the dispersive shift per atom. We therefore define an effective detuning with respect to the cavity resonance of $\delta_{eff} = \delta_c - \delta_-$, where $\delta_c = \omega_p - \omega_c$ is the detuning of the transverse pump frequency ω_p with respect to the empty cavity resonance frequency ω_c . In this experimental platform, the two-level systems at the basis of the open Dicke model are atomic momentum modes, with the zero momentum mode $|p_y, p_z\rangle \equiv |0, 0\rangle$, referred to as the BEC mode, forming the ground state, and a superposition $|\pm 1, \pm 1\rangle \equiv \frac{1}{2} \sum_{v, \mu \in \{-1, +1\}} |v\hbar k, \mu\hbar k\rangle$ of $|\pm\hbar k, \pm\hbar k\rangle$ momentum modes forming the excited state. Here, the same wavenumber $k = 2\pi/\lambda_p$ is assumed for both the transverse pump and the longitudinal probe waves, which is justified due to their comparatively small detunings on the scale of a few kHz. When the transverse pump is red-detuned with respect to the cavity resonance ($\delta_{eff} < 0$) and its pump strength ϵ_p exceeds a critical value ϵ_c , the atomic ensemble transits from the normal phase (NP), a BEC trapped in the standing-wave potential of the pump, into one of two density lattice configurations available in the superradiant (SR) phase [13,34], resulting in the buildup of a finite intracavity light field. By scanning the $\{\epsilon_p, \delta_{eff}\}$ -plane and measuring the number N_p of photons leaking out of the cavity with a heterodyne detector, we obtain the phase diagram shown in Fig. 1(c). It should be noted that the critical pump strength ϵ_c varies with δ_{eff} .

To theoretically analyze the system as it approaches the phase transition, we study the excitation energy of the polaritonic modes by performing a stability analysis. In this model, we only take into account the occupation of the two lowest momentum modes, $|0, 0\rangle$ and $|\pm 1, \pm 1\rangle$, which we refer to as the two-mode stability analysis [25]. We perform this analysis for a fixed δ_{eff} to determine the excitation energy $E_{\text{exc}} = \hbar \omega_{\text{exc}}$ and excitation rate γ of the polariton modes, resulting from the real and imaginary parts of the eigenvalues of the stability matrix, respectively. Further details on the stability analysis and the stability matrix itself are provided in the Supplemental Material [38]. For zero transverse pump strength $\epsilon_p = 0$, the atomic and photonic modes are decoupled, the energy required to excite the atoms from the $|0, 0\rangle$ into the $|\pm 1, \pm 1\rangle$ momentum modes is $2\hbar \omega_{\text{rec}}$, and the energy required to excite the cavity mode is $|\hbar \delta_{\text{eff}}|$ [see Fig. 1(b)]. As the transverse pump strength increases, the off-diagonal terms in the stability matrix become nonzero, and the stability matrix obtains two eigenstates, composed of mixed atom-light excitations, referred to as the upper and lower polariton modes. The excitation energy of the lower polariton mode reaching zero is a necessary but not sufficient requirement for spontaneously transitioning into the SR phase. The second necessary criterion is a positive excitation rate, as this determines whether small deviations from the equilibrium are self-reinforcing. In the case of a positive excitation rate, the system is pulled into the SR phase. For a negative excitation rate, a transition to the SR phase is damped, causing the system to remain in the NP.

The results of the two-mode stability analysis are shown in Figs. 2(a)–2(d). In the far-detuned case $|\delta_{\text{eff}}| \gg \omega_{\text{rec}}$, approximately realized in Fig. 2(a), the lower polariton mode contains a very small admixture of the photonic component, and its excitation energy undergoes a rotonlike mode softening [35],

$$\omega_{\text{exc}} = 2 \omega_{\text{rec}} \sqrt{1 - \frac{\epsilon_p}{\epsilon_c}}, \quad (1)$$

with the excitation energy reaching zero at a pump strength nearly coinciding with the critical pump strength ϵ_c . As seen in Fig. 2(a) (red graph), the associated excitation rate γ is initially drawn to negative values before splitting into two branches very near to the critical pump strength, with the upper branch becoming positive exactly at the critical pump strength ϵ_c . This behavior resembles aforementioned previous observations in the I-LIRI regime [35,39], in which $\kappa \gg \omega_{\text{rec}}$ and $|\delta_{\text{eff}}| \gtrsim \kappa$, such that the condition of far-detuning $|\delta_{\text{eff}}| \gg \omega_{\text{rec}}$ is generally fulfilled.

In the near-resonant regime $|\delta_{\text{eff}}| \lesssim 2 \omega_{\text{rec}}$, Figs. 2(b)–2(d), the admixture of the photonic component to the polariton mode becomes significantly larger as the characteristic photon energy $\hbar|\delta_{\text{eff}}|$ approaches the relevant atomic energy $2\hbar\omega_{\text{rec}}$. While decay of the atomic momentum modes is negligible on experimental timescales, the photons decay from the cavity at a rate of $1/\kappa$. Analytical calculations using the Dicke model for the resonant case $|\delta_{\text{eff}}| = 2 \omega_{\text{rec}}$ shown in the Supplemental Material [38] suggest an interpretation that the dissipation effectively shifts the excitation rate curve by $-\kappa/2$, which means that the critical pump strength is pushed to higher values than in the case for $\kappa = 0$. This is consistent with previous results for the polariton frequencies [40,41]. This effect

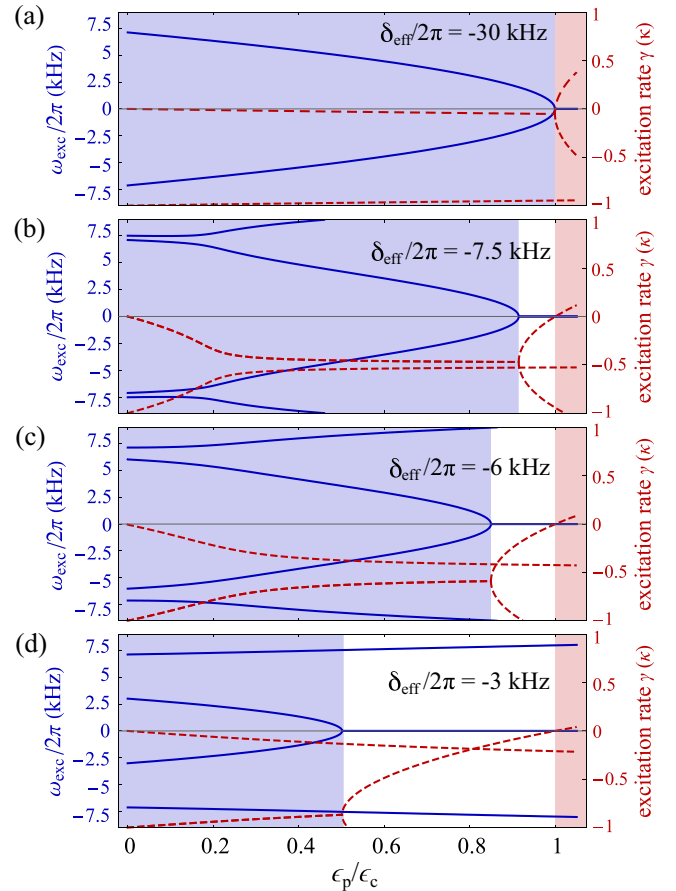


FIG. 2. Calculated excitation frequencies (blue, solid lines) and excitation rates (red, dashed lines) of the lower and upper polariton modes for different effective detunings δ_{eff} in the NP ($\epsilon_p < \epsilon_c$). In the red area, where $\epsilon_p > \epsilon_c$, a correct linearization analysis should employ a steady state within the SR phase. (a) The lower polariton mode at zero pump strength shows an excitation frequency of $\pm \omega_{\text{rec}}/\pi = \pm 7.1$ kHz, which identifies its atomic character for $\epsilon_p = 0$. As ϵ_p increases, its frequency falls to zero very close to the critical pump strength ϵ_c . The upper polariton mode resides at excitation frequencies around $\pm \delta_{\text{eff}}/2\pi = \pm 30$ kHz outside the plot range for all values of ϵ_p , and hence identifies its photonic character. The excitation rates are negative nearly all the way to $\epsilon_p = \epsilon_c$. (b) As in panel (a), at $\epsilon_p = 0$ the upper mode and the lower mode show purely photonic and atomic character, respectively. The lower polariton mode starts with an excitation frequency $\pm \omega_{\text{rec}}/\pi$ at $\epsilon_p = 0$ as in panel (a) but clearly falls to zero before the critical pump strength is reached. (c), (d) On a large part of the pump strength axis (white area), zero excitation frequency is found for the lower polariton, while all excitation rates are negative below the critical pump strength.

becomes more pronounced as $|\delta_{\text{eff}}|$ decreases and leads to a regime in which the lower polaritonic excitation frequency ω_{exc} reaches zero at a value ϵ_z significantly below ϵ_c . However, due to the dissipation-induced overdamping, $\gamma < 0$, the system remains in the homogeneous NP despite the vanishing energy cost for the excitation of the lower polariton mode. We refer to this mode as a *polaritonic zero-energy excitation mode*.

We record the excitation energy $\hbar \omega_{\text{exc}}$ of the polariton modes using a form of Bragg spectroscopy [35,42,43]. We

therefore utilize an additional beam superimposed on the cavity axis (the z axis), in addition to the previously introduced transverse pump along the y axis. This additional beam is referred to in the following as the longitudinal probe with frequency ω_l [see Fig. 1(a)]. When the frequency detuning δ_{lt} between the transverse pump and the longitudinal probe is equal to the energy required to excite one of the polariton modes, the atoms can scatter light between pump and probe, and create excitations of the polariton modes, as illustrated in Figs. 1(a) and 1(b). As the atomic component of the polariton modes is a superposition of the $|\pm 1, \pm 1\rangle$ momentum modes, this results in the buildup of a weak density grating, which fulfills the Bragg condition for constructive interference of scattering from the transverse pump into the cavity, and vice versa. In this case, this scattering process is temporary, as the modes are subject to damping, and the process is not self-reinforcing below ϵ_c . Thus, Bragg spectroscopy can be used to determine the excitation energy of the momentum modes by determining the detuning δ_{lt} at which most of the atoms are excited into the polariton mode, and correspondingly, most transverse pump light is scattered into the cavity. The highest intensity of light scattered into the cavity occurs when the detuning matches the corresponding excitation energy, $\delta_{lt} = \hbar\omega_{\text{exc}}$. The results are presented in Fig. 3. The detailed protocol for how we extract ω_{exc} from the measured intracavity photon number N_p can be found in the Supplemental Material [38].

In the far-detuned regime $|\delta_{\text{eff}}| \gg \omega_{\text{rec}}$, at an effective detuning of $\delta_{\text{eff}}/2\pi = -30$ kHz, we obtain the excitation spectrum shown in Fig. 3(a). The excitation energy of the lower polariton mode falls off in good agreement with Eq. (1), which is also captured by our two-mode stability analysis shown in Fig. 2(a). This shows the expected behavior for a small admixture of the photonic component to the lower polariton mode, with the excitation rate falling off to zero close to ϵ_c . The small deviation close to ϵ_c as compared to the results for a significantly larger κ [35] is attributed to a larger admixture of the photonic component to the lowest polariton mode in our system, which could contribute additional damping. This is further supported by our excitation spectrum measurements for an effective detuning of $\delta_{\text{eff}}/2\pi = -15$ kHz (see the Supplemental Material [38]), which are closer to the cavity resonance and differ more from the behavior predicted by Eq. (1) for the $|\delta_{\text{eff}}| \gg \omega_{\text{rec}}$ regime.

When the transverse pump is near-resonant, $|\delta_{\text{eff}}| \lesssim 2\omega_{\text{rec}}$, we see significant deviations from the prediction of Eq. (1). The energetically lowest polariton mode displays mode softening, however, such that the excitation energy reaches zero at a value ϵ_z notably below ϵ_c . This can be clearly seen in Fig. 3(b), where the excitation frequency falls close to zero at $\epsilon_z/\epsilon_c \approx 0.8$. Due to the digital bandpass filter we applied to the measured photon number, we cannot experimentally resolve frequencies below 500 Hz, indicated by red dashed lines in Figs. 3(a)–3(d). The results of the excitation frequency tending toward zero are also confirmed theoretically by the two-mode stability analysis and the dynamical multimode simulations (see the Supplemental Material [38]). This observation verifies the presence of a polaritonic zero-energy excitation mode, which is accessible due to the recoil-resolving ultralow bandwidth of our optical cavity.

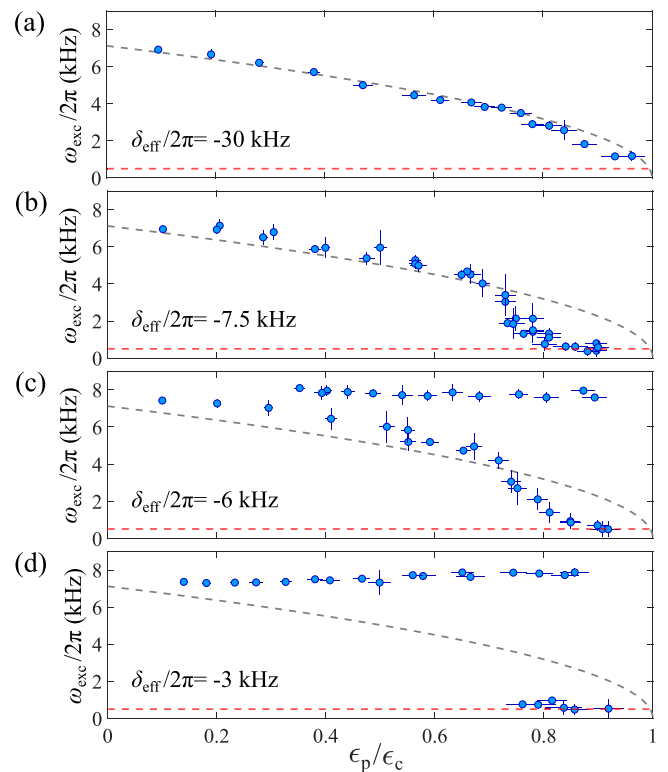


FIG. 3. Excitation spectra measured for different effective detunings δ_{eff} according to Fig. 2. The lowest frequency that can be accurately resolved with our experimental protocol, 500 Hz, is marked by a red dashed line, and the square-root behavior predicted for the far-detuned regime as per Eq. (1) is shown in gray. (a) The mode softening of the lower polariton mode as the transverse pump strength ϵ_p approaches ϵ_c . (b) The formation of an overdamped excitation mode, as the excitation energy of the lower polariton mode falls off to zero and remains at zero before ϵ_p reaches the critical pump strength ϵ_c . In panels (c) and (d), for $\epsilon_p = 0$, the upper (lower) polariton mode acquires pure atomic (photonic) character. Away from $\epsilon_p = 0$, both modes possess sufficient atomic admixture to be observable with our detection protocol. Horizontal error bars represent the uncertainty of the critical pump strength. The vertical error bars represent the standard deviation of the excitation energies around the mean as well as the respective uncertainties from the data evaluation method, whereas the horizontal error bars result from the uncertainty of the ratio of the transverse pump strength ϵ_p relative to ϵ_c . The exact methodology for determining these error bars is described in more detail in the Supplemental Material [38].

Since the fundamental cavity mode excitation in the absence of a pump is $\hbar\delta_{\text{eff}}$, it is interesting to explore the case when the cavity photon excitation energy is slightly below the atomic excitation energy, which is the case for $\delta_{\text{eff}}/2\pi = -6$ kHz. This results in a strong mixture of atomic and photonic components for both upper and lower polaritonic modes as the transverse pump strength increases. In Fig. 3(c), we observe two polaritonic modes with different excitation energies that deviate significantly from each other beyond $\epsilon_p/\epsilon_c = 0.3$, with one remaining approximately constant and the other tending toward zero at $\epsilon_z/\epsilon_c \approx 0.80$. We interpret this result as the observation of both upper and lower polaritonic modes. While the upper polaritonic mode was

also predicted for $\delta_{\text{eff}}/2\pi = -7.5$ kHz in Fig. 2(b), we were unable to verify it experimentally because our measurement protocol is primarily sensitive to polaritonic modes with a significant atomic character. Therefore, the detection of both the upper and lower polariton modes throughout the excitation spectrum in Fig. 2(c) confirms their strong mixtures of atomic and photonic sectors. This is further corroborated by the absence of a multipeak structure in the excitation curves from the dynamical multimode simulations, which are presented in the Supplemental Material [38].

For smaller detunings, $|\delta_{\text{eff}}| < 2\omega_{\text{rec}}$, for example, at $\delta_{\text{eff}}/2\pi = -3$ kHz, the deviations from the prediction of Eq. (1) become even more pronounced, as shown in Fig. 3(d). Here, we observe an upper polariton mode with an excitation frequency that begins at $\omega_{\text{exc}} \approx 2\omega_{\text{rec}}$ at $\epsilon_p = 0$ and increases from there to around 8 kHz. At a transverse pump strength of $\epsilon_p/\epsilon_c \geq 0.75$, a second mode appears with an excitation frequency that remains around zero and elicits a significantly stronger response than the upper polariton mode, which significantly weakens past this point for the entire range of parameters, where the lower polariton mode can be detected. Below this pump strength, this mode, which is predicted in the two-mode stability analysis in Fig. 2(d), is not observable with the applied experimental protocol. While this may provide some indication as to the evolution of the atomic and photonic components of the polaritonic modes with ϵ_p/ϵ_c in the near-resonant regime, more extensive simulations, including a full eigenfunction analysis, would be required to confirm this, which goes beyond the scope of this work.

In conclusion, in the normal phase of the open Dicke model, implemented in an atom-cavity platform, we experimentally observed a polaritonic zero-energy excitation

mode, for which, despite vanishing excitation energy, no instability of the normal phase arises. Theoretical analysis confirms that the weak dissipation in the photonic sector, with the intracavity field decay rate being smaller than the photon recoil frequency, is responsible for the observations. We speculate that our observations may prove advantageous for critical quantum metrology applications [44,45], since an infinitely steep excitation energy change becomes accessible at ϵ_z , well separated from the critical point ϵ_c where a phase transition occurs.

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Data availability. The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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