

Identifying statistical indicators of temporal asymmetry using a data-driven approach

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The dynamics of time-reversible systems are statistically indistinguishable when observed forward or backward in time. A rich literature of statistical methods to distinguish irreversible dynamics from the reversible dynamics of linear Gaussian processes can provide insights into underlying mechanisms and aid modeling and statistical quantification of time-series data. However, existing time-reversibility metrics have been developed individually, forming a fragmented body of research that makes it challenging to identify the most effective approaches developed to date, and the most promising directions for development. Here, we address these issues by systematically evaluating over 6000 time-series summary statistics, derived from across the time-series analysis literature, on their ability to distinguish the time irreversibility of data simulated from a diverse range of 35 systems. Our large-scale data-driven comparison highlights the effectiveness of several key families of statistics, including time-asymmetric forms of generalized autocorrelation functions, time-series symbolic sequences, and forecasting-related methods. All irreversible systems studied here could be accurately distinguished by a well-chosen time-series statistic, but no single statistic could accurately index the statistical form of irreversibility for all irreversible systems. This challenges the assumption that a given time-reversibility statistic will accurately capture time reversibility in general, and underscores the importance of tailoring statistical approaches to the time-reversal characteristics of a given system. Our results provide a unified understanding of the key algorithmic structures through which irreversibility can be effectively quantified from data, providing a foundation for connecting patterns in time series to the underlying mechanisms of the systems that generate them.

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I. INTRODUCTION

The nature of time and the existence of a privileged direction in its flow have long fascinated both philosophers [1] and scientists, inspiring fundamental questions about the origin of time [2], the neural mechanisms of time perception [3], and the evolution of the universe [4]. The concept of time reversibility has been studied across multiple disciplines, including time-series analysis [5], dynamical systems theory [6], and nonequilibrium statistical mechanics [7]. Its cross-disciplinary importance arises from the ubiquity of time-asymmetric processes in the real world, motivating studies in both abstract deterministic and stochastic systems as well as in the thermodynamics of physical systems. A deterministic dynamical system is considered reversible if its governing equations remain unchanged under time reversal [6,8] (cf. more general definitions, including for multidimensional systems [9,10]). When studied in the context of nonequilibrium physical systems, irreversibility acquires a probabilistic interpretation, traditionally expressed in ther-

modynamics through the second law of thermodynamics as entropy production at the macroscopic scale. More recently, in stochastic thermodynamics—which has emerged as a theoretical framework that extends concepts from classical thermodynamics (e.g., entropy, heat, work) to smaller-scale systems where thermal fluctuations play a dominant role in the dynamics [11]—the Kullback-Leibler divergence between the probability distribution of system trajectories evolving forward and backward in time links irreversibility (i.e., the distinguishability between the process unfolding forward versus backward in time) and entropy production, an indicator of dissipation in the underlying process [12–14].

This study focuses on the statistical interpretation of time irreversibility (hereafter referred to as “irreversibility”), which refers to the ability to infer the temporal direction of a dynamical process from its observed dynamics. That is, are the dynamics of some process statistically different from its time-reversed dynamics? Formally, a discrete-time stationary process $\mathbf{X} = \{X_t\}_{t \in \mathbb{Z}}$ is said to be *reversible* if, for every time point t and every $k \geq 1$, the joint distribution of the sequence $X_t^{(k)} \equiv (X_t, X_{t+1}, \dots, X_{t+k})$ is identical to that of its time-reversed counterpart $\tilde{X}_t^{(k)} \equiv (X_{t+k}, X_{t+k-1}, \dots, X_t)$, that is,

$$X_t^{(k)} \stackrel{d}{=} \tilde{X}_t^{(k)}, \quad \forall t \in \mathbb{Z} \quad \text{and} \quad \forall k \geq 1, \quad (1)$$

where the symbol $\stackrel{d}{=}$ denotes that $X_t^{(k)}$ and $\tilde{X}_t^{(k)}$ are identically distributed [5,15]. In other words, reversibility reflects the time-reversal symmetry of the probabilistic structure of

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a process [16] such that its statistical properties are invariant under time reversal [17].

In practice, we often seek to perform inference on the reversibility of a generative process from a single finite realization, e.g., a univariate time series $\mathbf{x} = (x_1, x_2, \dots, x_T)$. In this case, the time-reversed transformed time series $\tilde{\mathbf{x}}$ can be constructed by simply reversing the order of the data points, i.e., $\tilde{x}_t = x_{T-t+1}$ [18–21]. Under the assumption of stationarity, the reversibility condition, Eq. (1), can then be inferred by estimating the joint distributions of $\mathbf{x}_t^{(k)} = (x_t, x_{t+1}, \dots, x_{t+k})$ and $\tilde{\mathbf{x}}_t^{(k)} = (x_{t+k}, x_{t+k-1}, \dots, x_t)$, and quantifying their discrepancy using a suitable statistical distance or divergence measure (e.g., Kullback-Leibler divergence). That is, the reversibility condition in Eq. (1) can be expressed as

$$\mathbf{x}_t^{(k)} \stackrel{d}{=} \tilde{\mathbf{x}}_t^{(k)}, \quad \forall t < T - k \quad \text{and} \quad \forall k < T. \quad (2)$$

The notion of time reversibility has attracted considerable attention in the time-series analysis literature. A key result established in time-series analysis is that irreversibility reflects departures from linearity and Gaussianity in the underlying process [15]. This relationship has important implications for model development, since time-irreversible dynamics cannot be generated by canonical linear Gaussian time-series models, thus ruling out this ubiquitous class of dynamics [5,22]. Furthermore, reversibility statistics can act as powerful statistical summaries of the dynamical structure of time-series data that can be used for subsequent statistical learning applications like classification problems. For example, in medicine, reversibility metrics can capture the difference between healthy controls and patients with epilepsy [23–26], demonstrating their potential for clinical diagnosis. Reversibility has also been shown to relate to brain organization and the emergence of wakefulness [21], while serving as an indicator of abnormalities [27–29] and nonlinearity [30] in human heartbeat dynamics and hand tremor signals [31]. In economics and finance, time series (such as business cycles) often exhibit time-asymmetric structures, typically characterized by long, gradual expansions followed by sharp contractions [32,33], making tests for time reversibility useful for classification [34]. More recently, irreversibility indicators based on autocorrelation functions [35] have revealed scale-dependent time-reversal asymmetry in fully developed turbulence [36] and identified energy transfer from large to smaller structures in turbulent flows [37].

While obtaining accurate estimates of the joint probability distributions $p(\mathbf{x}_t^{(k)})$ and $p(\tilde{\mathbf{x}}_t^{(k)})$ is practicable for low k (e.g., the typical time-delay phase space corresponding to $k = 1$), estimating high-dimensional probability distributions corresponding to larger k quickly becomes impractical for realistic time-series lengths. Diverse statistical methods have been developed to more tractably perform inference on the time-reversibility condition on finite data [i.e., assess violation of the symmetry condition Eq. (2)] by quantifying changes in time-series properties under time reversal in terms of real-valued statistics [27,35,38–43], by transforming the time series into alternative representations (such as symbolic sequences or networks) [44–47] or through test statistics that can form the basis of a statistical test for reversibility [17]. In

this work, we focus on the inference of time reversibility based on a single real-valued summary statistic (or feature) extracted from a time series (i.e., a feature map $f: \mathbb{R}^T \rightarrow \mathbb{R}$) that aims to encapsulate the deviation from the time-reversibility condition Eq. (2).

We highlight two key limitations of the existing literature on time reversibility: (1) its fragmentation across multiple disciplines; and (2) the tendency for methods to be studied in isolation and evaluated on small, hand-selected sets of systems. Addressing these limitations would yield several benefits in the literature, which include clarifying conceptual relationships across results, highlighting common underlying interpretations rather than isolated outcomes, and enhancing constructive communication across diverse fields. The breadth of algorithmic contributions to the time-reversibility literature—developed and adopted over many decades and across fields, each with its own terminology, tools, and notation—has contributed to a disjoint literature leading to a practical ambiguity in how to select an appropriate statistic from this wide array of alternatives. Since these various methods have been proposed and tested individually, there is currently no systematic framework to guide researchers in selecting appropriate approaches from this vast interdisciplinary methodological literature on irreversibility. Because different formulations of time-series statistical structure have seldom been benchmarked or compared [48,49], it is not well understood which methods best capture distinct manifestations of time reversibility or how their strengths and limitations vary across systems with different dynamical structures. Furthermore, benchmarking methods on narrow and inconsistent sets of processes, most commonly deterministic systems such as the Hénon map, logistic map, and Lorenz system, can obscure their relative strengths and weaknesses in capturing the diverse ways irreversibility deviates from reversible (linear Gaussian) dynamics, including different types of temporal asymmetries and varying timescales.

In this work, we aim to address these issues by empirically evaluating a large library of interpretable statistics to identify those most effective for detecting irreversibility and by unifying existing time-series approaches to time-reversibility analysis. To this end, we systematically compare and analyze the most comprehensive collection of over 6000 time-series statistics for their ability to capture the time-reversal asymmetry of over 35 diverse processes, spanning a range of formulations and dynamical behaviors (in discrete and continuous time, and encompassing varying degrees of linearity and Gaussianity). We draw on the extensive interdisciplinary library of time-series statistics contained in the highly comparative time-series analysis package, *hctsa* [50,51], in which each feature is derived from interpretable time-series theory. This connection to theory facilitates an understanding of the types of temporal structures (and ways of quantifying them) that diverse scientists have developed to date, which are relevant to distinguishing reversibility. The highly comparative, data-driven approach to methodological comparison has previously yielded understanding and guided the selection of time-series methods for a wide range of applications—from characterizing epileptic brain dynamics [52] to distinguishing zebra finch songs [53]—and has facilitated the development of theory for challenging empirical problems like tracking the

TABLE I. Summary of simulated processes grouped into six families, with a color coding that is used throughout the paper. Each process is labeled with an abbreviation for ease of reference. The discrete- or continuous-time nature of the simulated process is indicated by “D” for discrete and “C” for continuous. The reversibility of each system is indicated by “R” for reversible and “I” for irreversible (assigned based on either referenced prior studies or from ground truth knowledge of the process; cf. Appendix A for details of specific systems).

Family of processes	Process	Label	Disc./ Cont.	Rev.
Noise	Gaussian noise [17]	GNO	D	R
	Uniform noise [17]	UNO	D	R
	Pink noise [20]	PINK	D	R
	Red noise [20]	RED	D	R
	Violet noise [20]	VIOLET	D	R
Autoregressive models	AR(1) with i.i.d. Gaussian noise [15, 17]	AR1_GNO	D	R
	Static transformation of AR(1) with i.i.d. Gaussian noise [15, 17]	STAR	D	R
	AR(1) with i.i.d. uniform noise [17]	AR1_UNO	D	I
	ARMA(1,1) with i.i.d. uniform noise	ARMA11_UNO	D	I
	AR(3) with noise sampled from a gamma distribution	AR3_GAMMA	D	I
	SETAR(2;1,1,1) ^a [56]	SETAR1	D	I
	SETAR(2;2,2,2) [57]	SETAR2	D	I
	Nonlinear AR(2) with non-Gaussian noise [57]	NAR2	D	I
Deterministic chaotic maps	Arnold’s Cat map [45]	ARNOLD	D	R
	Chirikov standard map [8]	CHIRIKOV	D	R
	Hénon map [17, 35]	HEN	D	I
	Quadratic map [47]	QUAD	D	I
	Logistic map ($r = 4$) [18, 22, 45]	LOGISTIC_4	D	I
	Logistic map ($r = 3.8284$) [18]	LOGISTIC_38	D	I
Sum of deterministic chaotic maps/flows	Sum of Hénon maps, one forward and the other reversed [17]	HENR_DIVERSE	D	R
	Sum of Hénon map and its reversed	HENR_SAME	D	R
	Sum of quadratic maps, one forward and the other reversed [47]	QUAD_RSUM	D	R
	Sum of Hénon maps [17]	HEN_SUM	D	I
	Sum of Lorenz	LORENZ_SUM	C	I
	Sum of Rössler	ROSSLER_SUM	C	I
Other deterministic models	Linear model with logistic noise source [17]	LLOG	D	I
	Modulus-1 model [17]	MOD1	D	I
	Van der Pol oscillator [35]	VDP	C	I
	Oscillator	OSCILLATOR	C	R
	Mackey-Glass [46]	MG17	C	I
Other stochastic processes	Stochastic sine map	SINE_MAP	D	I
	Ornstein-Uhlenbeck [45]	OU	C	R
	Bounded random walk	BRW	C	R
	Stochastic Van der Pol oscillator	STOCH_VDP	C	I
	Sum of stochastic Lorenz	STOCH_LORENZ_SUM	C	I

^aSelf-exciting threshold autoregressive model.

proximity to criticality in near-critical systems with unknown noise levels [54]. Here, we apply this approach to the problem of time reversibility for the first time to develop a unified understanding of the relative strengths and weaknesses of different types of time-series analysis methods across different types of dynamical processes, allowing us to synthesize disparate prior work on this topic.

The paper is organized as follows. In Sec. II, we introduce our highly comparative time-series analysis approach for detecting irreversibility. We describe how we generate synthetic data from various simulated processes and outline the procedure for extracting time-series features. We characterize features that are insensitive to the direction of time across all simulated processes and interpret high-performing statistical indicators of irreversibility in detail through Sec. III A, focusing on autocorrelation functions, symbolic measures, and forecasting-based statistics. We then compare their effectiveness in distinguishing between reversible and irre-

versible dynamics across different processes in Sec. III B, before providing concluding remarks and discussion in Sec. IV.

II. METHODS

This paper takes a highly comparative approach to understanding types of time-series analysis approaches that are useful for inferring irreversibility from time-series data. Our data-driven methodology is illustrated schematically in Fig. 1 and can be summarized in three main steps: (1) generation of time-series data by simulating stochastic processes and dynamical systems with known time-reversal properties [depicted in Fig. 1(a)]; (2) evaluation of a comprehensive set of time-series features to identify those most sensitive to temporal asymmetry [depicted in Figs. 1(b) and 1(c)]; and (3) interpretation of top-performing features to understand how

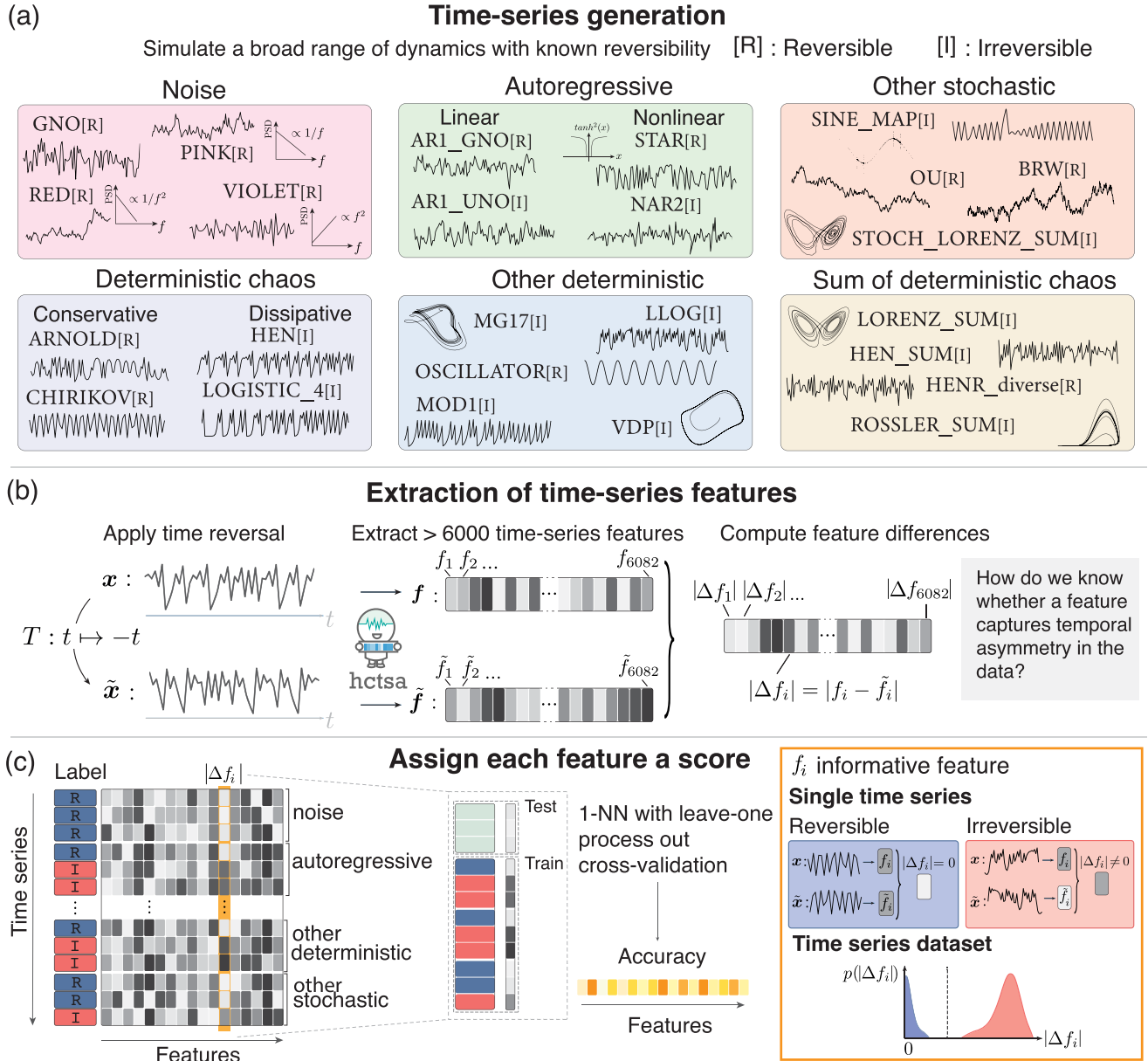


FIG. 1. Schematic of our data-driven approach to identifying high-performing time-series statistics that can accurately index irreversibility from time-series data. (a) Time series generation: To include multiple and diverse sources of irreversibility, we simulated 5000-sample time series from a comprehensive library of 35 discrete- and continuous-time processes with known reversibility properties. Each panel shows selected illustrative time series segments from a given process class, with a [R] indicating “reversible” and a [I] indicating “irreversible,” placed near the process label according to the reversibility specified in Table I. We use a consistent color coding throughout the paper to identify these families of simulated processes. (b) Extraction of time-series features: For each time series x , we constructed its time-reversed counterpart $\tilde{x}$ by reversing the order of the data points, as per prior work [18–21]. To find time-series properties that are sensitive to irreversibility, we systematically computed a large set of >6000 interpretable time-series features implemented in the *hctsa* library [51]. In this way, each time series was summarized by a set of real numbers, $f_1, \dots, f_{6082}$ that encode a broad range of its statistical properties. As an ansatz for the ability of each time-series feature f_i to index a difference in statistical properties between the original time series x and the time-reversed time series $\tilde{x}$, we quantified the absolute difference between the feature value computed on x and $\tilde{x}$, as $|\Delta f_i| = |f_i - \tilde{f}_i|$. (c) Assign each feature a score: To extract time-series properties that are sensitive to the reversibility of processes, each feature f_i was scored based on the discriminative power of its $|\Delta f_i|$ in classifying time series generated by reversible [R] and irreversible [I] processes using a 1-nearest-neighbor classifier (1-NN) with leave-one-out cross-validation; the classification accuracy served as the feature’s score. We expect that an informative feature f_i , associated with a high classification accuracy, displays $\Delta f_i \approx 0$ for reversible processes, and large deviations $|\Delta f_i| > 0$ for irreversible ones, yielding well-separated distributions across the dataset.

they quantify different types of temporal patterns associated with irreversibility (see Sec. III).

A. Model systems

For our data-driven method to meaningfully score individual time-series features on their ability to distinguish reversible from irreversible processes, we require a set of systems that is sufficiently diverse to capture as wide a range of dynamical signatures of irreversibility that are exhibited by different classes of dynamical processes. If our set of systems was instead too small or specific, features could score well by capturing idiosyncratic properties of specific processes, instead of the broader characteristic of irreversibility. To this end, we generated a collection of univariate, stationary time series simulated from 35 diverse systems: 15 reversible and 20 irreversible (containing the most comprehensive coverage of processes to date, to our knowledge). Our collection includes a broad range of both stochastic and deterministic, discrete- and continuous-time processes, subdividing processes that share similar dynamics into families, as summarized in Table I and illustrated in Fig. 1(a), where each panel corresponds to a family of simulated processes consistently color-coded throughout the paper. In selecting processes, we aimed to include as many diverse dynamical behaviors as possible, while generally aligning with the parameter settings used in previous studies to ensure a consistent reference for evaluating the reversibility of the simulated systems. In labeling each system as either “reversible” or “irreversible,” we could, in some cases, rely on prior knowledge of the process [as per the reversible independent and identically distributed (i.i.d.) noise processes], but in other cases relied on existing literature on irreversibility and systems that have been used to assess time-reversibility metrics to base the assessment. The motivation for including each type of dynamics, along with references to prior work used to assess their reversibility, is presented below, while details of the simulated systems are provided in Appendix A.

Noise. We included both uncorrelated (white noise) and long-range correlated (colored noise) stochastic processes under a common class labeled “noise” in Table I for convenience, as a class of processes that lack any deterministic component. We simulated time-series realizations of i.i.d. noise processes drawn from both a Gaussian (labeled GNO) and a uniform distribution (labeled UNO). In addition, we included three-colored noise processes, generated as time series with a given power spectrum: pink noise (labeled PINK), red noise (labeled RED), and violet noise (labeled VIOLET). Since the power spectrum is insensitive to temporal irreversibility due to its inherently time-reversal symmetric construction [55], colored noise processes are reversible.

Autoregressive processes. We analyzed time series generated from a set of autoregressive-moving-average (ARMA) models with different linearity and Gaussianity properties, focusing on two reversible and six irreversible processes (as summarized in Table I). The theoretical foundation of reversibility in linear stochastic processes—established in the seminal work of Weiss [15], which established a link be-

tween Gaussianity, linearity, and time reversibility—makes these processes canonical for studying numerical methods for indexing reversibility. As representatives of reversible processes, we considered a first-order autoregressive process with Gaussian noise (labeled AR1_GNO) and, given that the reversibility of a linear stochastic Gaussian process is preserved under any point transformation [15], a second process obtained by applying a static nonlinear transformation to the first (labeled STAR) [17]. Although nonlinearities or non-Gaussianities alone do not guarantee the irreversibility of linear stochastic processes [15,41], we accounted for modifications of standard linear Gaussian processes whose irreversibility has been established in prior studies [17,56,57]. We examined three irreversible autoregressive models with non-Gaussian noise distributions: a first-order autoregressive process AR(1) with uniform noise (labeled AR1_UNO) [15,17], a first-order autoregressive and moving average process ARMA(1,1) (labeled ARMA11_UNO) [15], and a more complex form of linear dynamics which accounts for longer memory dependencies combined with noise from a gamma distribution, implemented through a third-order autoregressive process, AR(3) (labeled AR3_GAMMA). Next, we simulated three nonlinear variants of autoregressive models by introducing nonlinearities either through regime switching [58] or through equations involving squared terms and other functional forms. Specifically, we included two variants of the self-exciting threshold autoregressive (SETAR) model previously identified as irreversible (labeled SETAR1 and SETAR2) [56,57], and a nonlinear second-order autoregressive process, AR(2) (labeled NAR2) [57].

Deterministic chaotic maps. Motivated by prior work investigating the relationship between reversibility and chaos [8], we included several examples of chaotic dynamics [17,18,22,35,45,47] (see Table I). Despite the conceptual distinction between reversible and conservative dynamics [8], we included two representative discrete-time examples of chaotic maps that are both measure-preserving and reversible, namely, the Arnold’s Cat map (labeled ARNOLD) [45] and the Chirikov standard map (labeled CHIRIKOV) [8], as well as four that are dissipative and irreversible: the Hénon map (labeled HEN) [17,35], the quadratic map (labeled QUAD) [47], and the logistic map in two chaotic regimes (labeled LOGISTIC_4 and LOGISTIC_38, respectively) [18,22,45].

Sum of deterministic chaotic maps/flows. We included linear combinations of discrete-time chaotic realizations to examine how summation affects irreversibility (e.g., from the irreversible nature of the Hénon map to the reversible nature of the linear combination between a realization and its time-reversed counterpart [17]). For discrete-time systems, this sum refers to a pointwise (time-by-time) sum, where two different realizations of the same process are added at each time instant. Specifically, we generated diverse linear combinations derived from the Hénon and quadratic maps, including both: (1) combinations of forward realizations that preserve the irreversible dynamics characteristic of these processes (labeled HEN_SUM) [17]; and (2) combinations of a forward realization with time-reversed versions, which yield reversible dynamics (labeled HENR_DIVERSE when the reversed series is a distinct realization, and HENR_SAME when the reversed series corresponds to the same realization, and

QUAD_RSUM where the reversal is generated from a diverse realization) [17,47].

To enrich our dataset with continuous-time processes, we included linear combinations of the x , y , and z components of the dynamics of two canonical examples of continuous-time chaotic flows: the Lorenz (labeled LORENZ_SUM) and Rössler (labeled ROSSLER_SUM) systems (see Table I). For these multidimensional systems, we constructed a univariate time series as a projection of the full system dynamics. By summing all components of the multidimensional system, we ensured that each variable contributes to the projected time series (noting that, per the Takens embedding theorem, the dynamics should not be highly sensitive to the specific projection [59]). Prior work has shown that individual components of these systems exhibit time-irreversible dynamics when analyzed individually [22,35,57]; therefore, the inclusion of any such component induces a violation of the time-reversibility condition in the resulting projection. This univariate time series emulates realistic measurement constraints, where the dynamics of a multidimensional system are often observed through a single variable rather than through full access to all state variables.

Other deterministic models. To incorporate other nonlinear and deterministic nonchaotic processes in our analysis, we implemented two models whose irreversible character has been studied previously by Diks *et al.* [17]: a linear model with deterministic noise (labeled LLOG) and a modulus-1 process (labeled MOD1) (see Table I). We also included two kinds of continuous-time dynamic behaviors that extend beyond first-order ordinary differential equations: those described by second-order differential equations and those governed by delay differential equations. As representative of dynamics described by second-order differential equations, we included a reversible linearized oscillator (labeled OSCILLATOR), a prototypical example often used to intuitively introduce the concept of reversibility, and an irreversible Van der Pol oscillator (labeled VDP) [35]. Physical systems characterized by time delays are particularly interesting from the perspective of reversibility, as their dynamics explicitly depend on past values; accordingly, we also included the nonlinear time-delayed Mackey-Glass equations (labeled MG17) [46].

Other stochastic processes. Our dataset also encompasses stationary random walks, as well as chaotic and oscillatory dynamics corrupted by i.i.d. noise (see Table I). A discrete-time process for which we analyze the irreversibility consists of a nonlinear deterministic map perturbed by additive non-Gaussian noise (labeled SINE_MAP). In continuous time, we included two examples of reversible random walks, namely, the widely studied time-reversible Ornstein-Uhlenbeck process (labeled OU) [45] and a bounded random walk (labeled BRW). To study the effect of stochastic fluctuations on the reversibility of dynamical systems, we examine two irreversible systems—the one-dimensional projections of the multivariate Lorenz system (labeled STOCH_LORENZ_SUM), and the Van der Pol oscillator (labeled STOCH_VDP)—perturbed by noise, which we label as “irreversible” based on their underlying deterministic dynamics.

B. Time-series generation

To evaluate the performance of each time-series feature at distinguishing the diverse set of 15 reversible and 20 irreversible processes described above, we simulated a total of 100 time series from each process, yielding a dataset containing a total of 3500 time series. Continuous-time stochastic processes were simulated by numerical integration using the Euler-Maruyama method [60] with an integration time step $\Delta t = 10^{-2}$ s. Continuous-time deterministic systems were simulated by numerical integration using the Runge-Kutta-Fehlberg (RKF45) method [61] with a default integration step $\Delta t = 10^{-2}$ s [with two exceptions: $\Delta t = 1$ s for the Mackey-Glass system (MG17), which is sufficient to resolve the dynamics relative to the characteristic timescale set by the delay parameter, and $\Delta t = 10^{-3}$ s for the BRW required to ensure accuracy of the solution]. Time series were simulated using the Python language (version 3.12.4), except for colored i.i.d. noise series for which we used MATLAB 2020a.

For our main analyses, we selected a time-series length of $T = 5000$ samples for all systems, striking a compromise between being sufficiently long (to enable complex methods to capture signatures of irreversibility), while remaining realistic for many empirical applications involving finite data. We removed transients from both discrete- and continuous-time processes and downsampled the continuous-time processes by a factor m , chosen as the temporal lag at which the autocorrelation function decays to $1/e$, to ensure comparable sampling relative to the timescale of the process (see Appendix A for specific details). The sensitivity of results to this time-series length T is analyzed briefly for selected time-series features in Appendix B.

C. Feature extraction

To identify and characterize the types of time-series analysis methods that can most effectively distinguish reversible from irreversible dynamics, we systematically evaluated >7000 candidate time-series features from the *hctsa* toolbox (version 1.09) [50,51] using MATLAB 2020a. Each feature corresponds to a single real-valued statistical summary of a T -sampled long time-series property, $f : \mathbb{R}^T \rightarrow \mathbb{R}$ ($T = 5000$ here), and is typically computed after z -scoring the time series prior to feature extraction. The time-series feature-extraction process is illustrated in Fig. 1(b). For each simulated time series $\mathbf{x} = (x_1, \dots, x_T)$, we constructed its time-reversed counterpart, $\tilde{\mathbf{x}} = (\tilde{x}_1, \dots, \tilde{x}_T)$ by reversing the order of the data points, $\tilde{x}_t = x_{T+1-t}$, as per prior work [18–21]. We then z -scored time series (throughout the remainder of this work, $\mathbf{x}$ refers to z -scored time series data) and extracted the same set of 7796 features from both $\mathbf{x}$ and $\tilde{\mathbf{x}}$, yielding two feature vectors: $\mathbf{f} = \{f_i\}_{i \in \mathbb{N}}$ and $\tilde{\mathbf{f}} = \{\tilde{f}_i\}_{i \in \mathbb{N}}$, in which each entry of each feature vector corresponds to the output of some interpretable time-series analysis algorithm. After removing features that could not be successfully evaluated across the entire dataset (i.e., for all forward and all time-reversed time series), we obtained a set of 6082 time-series features used throughout the remainder of this work.

Our next goal was to identify which of the resulting set of candidate features are most effective for detecting

reversibility. We adopted the *ansatz* that the top-performing statistics can be identified via a test statistic defined for each feature f_i (with $i = 1, \dots, 6082$) as the absolute difference between the values computed on the original and time-reversed time series:

$$|\Delta f_i| \equiv |f_i - \tilde{f}_i|. \quad (3)$$

Intuitively, $|\Delta f_i|$ captures the discrepancy in a given statistical property f_i between the original and time-reversed series, and can thereby capture a source of statistical irreversibility. The construction in Eq. (3) is motivated by the similar form used in previous test statistics for reversibility (e.g., the “TR test” statistic [40], or correlation-based measures [35]). Since we expect $|\Delta f_i| \approx 0$ for reversible processes, discriminating statistics f_i for irreversibility should be able to capture deviations from the condition $\Delta f_i = 0$ in the case of irreversible dynamics. Note that since the sign of Δf_i is unrelated to the goal of assessing deviations from the $\Delta f_i \approx 0$ condition, we took $|\Delta f_i|$ as our test statistic for indexing a deviation from reversibility.

As illustrated in Fig. 1(c), we evaluated features f_i for which $|\Delta f_i|$ is highly discriminative of time series generated from reversible versus irreversible processes. For each feature f_i , we applied 1-nearest-neighbor (1-NN) classifier with leave-one-process-out cross-validation: Each time series was matched to that with the closest f_i value, excluding all time series generated from the same process to prevent overly optimistic performance estimates that could arise from a feature simply capturing characteristic properties of a given process (rather than more general statistical structure related to irreversibility). The resulting performance score for f_i was the rate at which the matches were correctly assigned to time series of the same (“reversible”/“irreversible”) label. In practice, high-performing features are those for which $|\Delta f_i|$ deviates substantially from zero for a wide range of irreversible processes, resulting in irreversible time series tending to match to other irreversible processes. We thus found this 1-NN heuristic scoring procedure to be a suitable empirical metric to identify discriminative features of irreversibility. Note that, to avoid bias from the arbitrary ordering of systems, when multiple points had the identical distance to the query point for 1-NN, the matching neighbor was selected randomly. All code required to generate the time series and reproduce the analyses presented in this paper is openly available [62]. Simulated data are also available as an accompanying resource for this paper [63].

III. RESULTS

By systematically comparing time-series features computed forward and backward in time [cf. Eq. (3)], we aim to identify the most effective methods for inferring process reversibility from a finite univariate time-series realization $\mathbf{x}$. In Sec. III A, we first identify time-series features that are invariant to time reversal (i.e., yielding $\Delta f \approx 0$ for all time series tested), then characterize the range of time-series analysis methods that are most successful at quantifying irreversibility and explain the algorithmic structures that underlie their strong performance. In Sec. III B, we demonstrate that there is no optimal summary statistic for capturing irreversibility from time-series data in general; rather, all features we tested

have strengths and weaknesses that depend on how the irreversibility of a given process manifests.

A. Time-series statistics of reversibility

Before characterizing high-performing time-series statistics of reversibility, we first aimed to identify features with algorithmic constructions that are invariant to time reversal. Understanding this trivial case provides a foundation from which to distinguish time-reversal-informative constructions. Well-known time-reversal-invariant statistics include properties of the distribution of time-series values (e.g., mean), which disregard the sequential ordering in the data, as well as two-point linear autocorrelation statistics $\langle x_t x_{t+\tau} \rangle$ for some lag τ [where $\langle \cdot \rangle$ denotes a temporal average], and statistics of the related Fourier power spectrum (cf. the Wiener-Khinchin theorem [64]) [5,35,55]. Using a heuristic criterion for time-reversal invariance, $\Delta f_i \approx 0$ to within numerical error (using the heuristic criterion $|\overline{\Delta f}| < 5\varepsilon$ here, where $\varepsilon = 2.2 \times 10^{-16}$ is the floating-point precision in Python), we identified 1414 time-reversal-insensitive time-series features (see Table S1 in the Supplemental Material for full list [65]). This list recapitulates many well-known time-symmetric feature constructions, including distributional properties (e.g., mean, median, standard deviation, and quantiles) and two-point linear autocorrelation statistics (and related properties of the power spectrum), as well as highlighting a range of conceptually related statistics, including time-symmetric constructions of generalized autocorrelations (introduced later in Sec. III A 1) and automutual information [66], as well as statistics derived from the undirected horizontal visibility graph [67].

After removing the 1414 time-reversal-invariant features characterized above, we aimed to understand the relative performance of the remaining 4668 features (listed in Table S2 in the Supplemental Material [65]) for indexing time reversibility. The distribution of accuracy values across these 4668 candidate time-series features is shown in Fig. 2(a). We see that the majority of these features are not sensitive to irreversibility, with a large peak around chance accuracy (50%). This suggests that the majority of features in the time-series analysis literature (as represented by the *hctsa* library [51]) are mainly focused on the statistics of linear, Gaussian processes, and are thus insensitive to time-reversal asymmetry. The distribution also exhibits a striking right-skewed tail, indicating the presence of a subset of high-scoring features.

To understand the types of algorithmic approaches that were most effective at indexing irreversibility, we focused on a subset of 127 features with an accuracy exceeding 72% [shown as a dashed vertical line in Fig. 2(a)]. This threshold was selected to encompass a sufficiently large set of features for analysis, while retaining a focus on those with the highest accuracy. These high-performing features (listed in full in Table S3 in the Supplemental Material [65]) encompasses a surprising range of time-series analysis algorithms, including types of statistics that have previously been used to study reversibility (e.g., estimators of higher-order moments and symbolic representations of sequential rise and fall sequences) as well as a range of approaches that have (to our knowledge) not previously been used to study

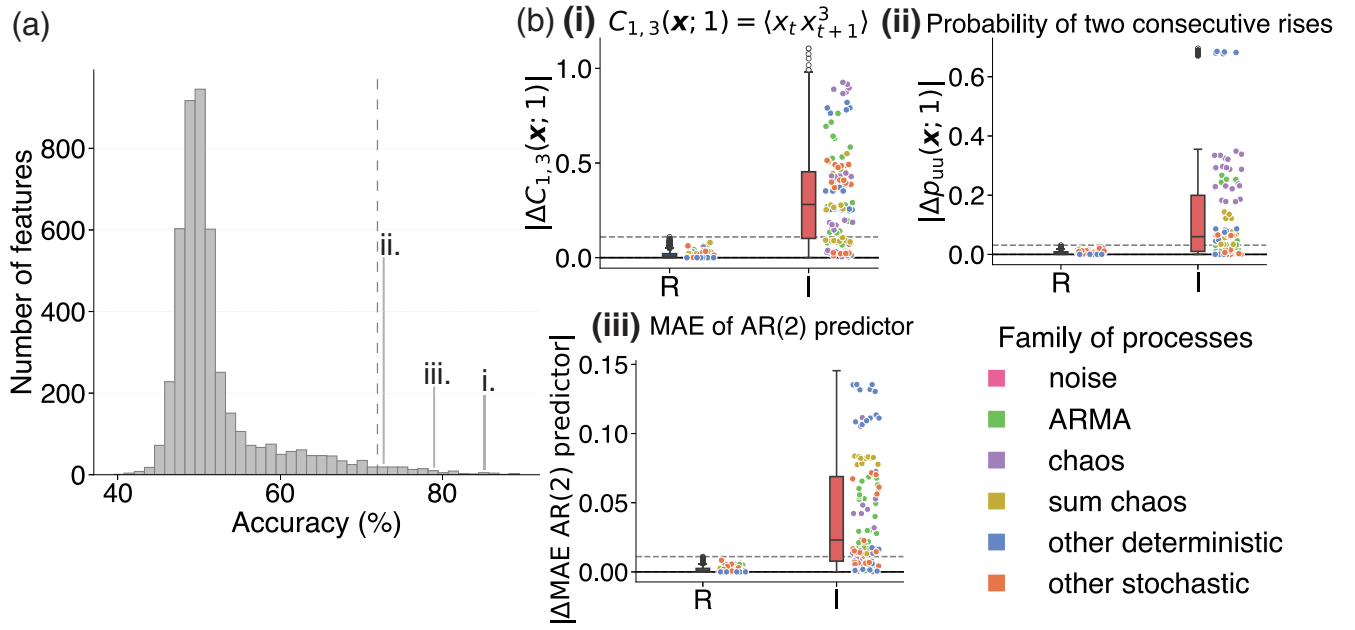


FIG. 2. Identifying high-performing and interpretable time-series features for detecting irreversibility through large-scale empirical testing of thousands of features on 35 reversible and irreversible processes. (a) Distribution of cross-validated classification accuracy (of distinguishing reversible from irreversible processes) across 4668 features (excluding 1414 that are insensitive to reversibility). Each feature f_i was assessed independently on its ability to distinguish time series generated from reversible vs irreversible processes via the absolute difference between its value computed on the time series and its time-reversed counterpart, as $|\Delta f_i|$ [Eq. (3)]. There is a tail in the accuracy distribution, pointing to a subset of high-performing features; here, we focus on the 127 features with accuracies exceeding 72%, annotated as a dashed gray vertical line (see Table S3 in the Supplemental Material for a full list [65]). (b) Distributions of $|\Delta f|$ across all 3500 time series, separated between 1500 reversible (blue) and 2000 irreversible (red) time series, are shown as box plots for three selected top-performing features: (i) the fourth-order cross-moment (an example of generalized autocorrelation-based feature) $C_{1,3}(\mathbf{x}; 1) = \langle x_t x_{t+1}^3 \rangle$; (ii) the symbolic motif feature, $p_{uu}(\mathbf{x})$, which calculates the probability of two consecutive rises in a time series, and (iii) the mean absolute error (MAE) of 1-step-ahead predictions made by a second-order autoregressive (AR) model. Next to each boxplot is a raincloud plot showing the $|\Delta f|$ value for all time series and colored according to process families (defined in Table I), with random horizontal scatter to aid visualization. Horizontal black lines indicate the zero baseline, while gray dashed lines delimit the range of values of the statistics computed from time series generated by reversible processes. The generalized autocorrelation (i), symbolic motif (ii), and MAE of a 1-step-ahead AR(2) prediction (iii) are annotated in the distribution in panel (a). These three features were chosen as demonstrative examples of broader families of easily interpretable time-reversal-sensitive time-series features, generalized autocorrelations (Sec. III A 1), symbolic motif probabilities (Sec. III A 2), and forecasting-based measures (Sec. III A 3), which are explained in detail in the main text. Their behavior mirrors that of all top-performing features, exhibiting $|\Delta f| \approx 0$ for reversible processes but deviating substantially from zero for many irreversible processes.

reversibility, including the goodness-of-fit of a range of time-series forecasting models. To better understand the behavior of high-performing statistics of irreversibility, we highlight three representative features that illustrate different statistical manifestations of irreversible dynamics: (1) the fourth-order cross-moment $\langle x_t x_{t+1}^3 \rangle$ [85% accuracy, in Fig. 2(b)(i)], which is an example of what we refer to as a “generalized autocorrelation” feature (defined in Sec. III A 1); (2) the probability of two consecutive rises in the time series [73% accuracy, denoted as $p_{uu}(\mathbf{x})$ in Fig. 2(b)(ii)]; and (3) the mean absolute error (MAE) of 1-step-ahead predictions made by a second-order AR model [79% accuracy, in Fig. 2(b)(iii)]. Figure [2(b)(i)–2(b)(iii)] show the distributions of absolute feature-value differences $|\Delta f|$, which are, as expected, tightly centered around $|\Delta f| \approx 0$ for reversible processes (reflecting their equivalence under time-reversal) and a substantial deviation $|\Delta f| > 0$ for many of the irreversible processes (reflecting an asymmetry between the feature computed on the forward versus time-reversed time series). The variability

in $|\Delta f|$ values across irreversible processes, indicated by the coloring by process in raincloud points in Fig. 2(b), reflects the sensitivity of each feature to a specific statistical form of time-reversal asymmetry that is contained in some processes but not others, and is explored in detail in Sec. III B.

Most high-performing statistics were members of the three conceptual families of time-series analysis methods: generalized autocorrelation features [defined in Sec. III A 1, like $\langle x_t x_{t+1}^3 \rangle$, in Fig. 2(b)(i)]; statistics of rise/fall patterns [like $p_{uu}(\mathbf{x})$, in Fig. 2(b)(ii)]; and forecasting-based measures [like the MAE of 1-step-ahead predictions made by a second-order autoregressive model, in Fig. 2(b)(iii)]. Accordingly, these three families of statistics are explored in depth in dedicated Secs. III A 1–III A 3. Note that, in addition to these three main families of high-performing statistics, we also identified a range of additional, miscellaneous high-performing features that we do not focus on here, including 12 “stick-angle” statistics (with accuracies as high as 89%), 27 statistics of a simulated walker trajectory driven by the

time-series values (accuracies up to 89%), and features based on the timing of extreme-value statistics (accuracies up to 79%). The remainder of this section focuses in more detail on the three main families of high-performing statistics identified by our data-driven evaluation: generalized autocorrelation functions, symbolic features, and forecasting-based statistics.

Although our main analyses focus on a single time-series length, $T = 5000$ samples, additional tests on a selected group of top-performing statistics show that results using this time-series length are broadly representative of those that would be obtained for shorter time series. In particular, we found the fourth-order cross-moment feature $\langle x_t x_{t+1}^3 \rangle$, the symbolic feature $p_{uu}(\mathbf{x})$, and the mean absolute error of an AR(2) predictor, a forecasting-based measure, to be quite robust to changes in the time-series length over the range of 10 to 5000 samples (see Fig. 5). We observe that, with around 100 samples, the Δf appears to have largely converged across all three statistics and all five processes. These preliminary results suggest that comparable outcomes can be expected with as few as approximately 100 samples.

1. Generalized autocorrelation features

Under the assumption of stationarity, generalized forms of autocorrelation extend the standard two-point linear autocorrelation by capturing different types of dependencies between local patterns in a time series. Here, we focus on two classes of features within the family of autocorrelation-based statistics, constructed from different functional forms of time averages. In our experiments, 24 features derived from this family were contained in the top-performing set described above (Sec. III A).

The first group of 12 high-performing features was of the form

$$C_{\alpha,\beta,\gamma,\dots}(\mathbf{x}; \tau_1, \tau_2, \dots) = \langle f(x_t^\alpha x_{t+\tau_1}^\beta x_{t+\tau_2}^\gamma \dots) \rangle, \quad (4)$$

for some transformation function $f(\cdot)$, being either the two-point form $\langle x_t^\alpha x_{t+\tau}^\beta \rangle$ with unequal exponents $\alpha \neq \beta$ (or with an absolute value as $\langle |x_t^\alpha x_{t+\tau}^\beta| \rangle$), or with multiple time lags, such as the three-point form $\langle x_t^\alpha x_{t+\tau_1}^\beta x_{t+\tau_2}^\gamma \rangle$. Notably, computing the (forward-time minus reversed-time) difference between statistics of this family captures, among other measures, higher-order moments (i.e., the skewness in the first difference $x_{t+\tau} - x_t$), which have been effectively employed in prior work to assess irreversibility [41]. Specific examples include $\langle x_t x_{t+1}^3 \rangle$ (85% accuracy), $\langle x_t x_{t+2}^3 \rangle$ (79%), $\langle x_t x_{t+3}^3 \rangle$ (75%), and the three-point statistic $\langle x_t^2 x_{t+1} x_{t+3} \rangle$ (75%). Interestingly, features that used an absolute value transformation $f(\cdot) = |\cdot|$ (to our knowledge not studied previously) exhibited higher performance, including $\langle |x_t^3 x_{t+1}| \rangle$ (87% accuracy) and $\langle |x_t^2 x_{t+1}| \rangle$ (87%).

A second group of 12 high-performing generalized autocorrelation features was constructed as the “generalized linear self-correlation function” formulation of Duarte Queirós and Moyano [68], given by

$$C'_{\alpha,\beta}(\mathbf{x}; \tau) \equiv \frac{\langle |x_t|^\alpha |x_{t+\tau}|^\beta \rangle - \langle |x_t|^\alpha \rangle \langle |x_{t+\tau}|^\beta \rangle}{\sqrt{\langle x_t^{2\alpha} \rangle - \langle |x_t|^\alpha \rangle^2} \sqrt{\langle x_{t+\tau}^{2\beta} \rangle - \langle |x_{t+\tau}|^\beta \rangle^2}}, \quad (5)$$

for various values of the exponents α , β , and the time-lag τ . This functional form was originally introduced to capture multiscale dependencies in traded volume in financial markets [68] but has not previously (to the best of our knowledge) been used to quantify irreversibility. Nevertheless, our comparative analysis highlighted 12 statistics of this general form, with $\tau = 1$ and various combinations of α and β (particularly $\alpha = 1, 2$ and $\beta = 2, 5, 10$), including the highest-performing statistic from this group $C'_{1,2}(\mathbf{x}; 1)$ (with 88% accuracy).

To understand why certain generalized autocorrelation measures are insensitive to time reversal (like the two-point linear autocorrelations $\langle x_t x_{t+\tau} \rangle$), while others rank among the highest-performing statistics in our empirical tests, we focused on the family of statistics given in Eq. (4) and examined the distinction between generalized autocorrelations with symmetric versus asymmetric constructions. Here, we consider the case in which the function $f(\cdot)$ is the product of time-series values, yielding a generalized form of the simple linear two-point autocorrelation:

$$C_{\alpha,\beta}(\mathbf{x}; \tau) \equiv \langle x_t^\alpha x_{t+\tau}^\beta \rangle, \quad (6)$$

for positive integer exponents α and β , and time-lag τ . In this simple case, the construction of $C_{\alpha,\beta}(\mathbf{x}; \tau)$ is asymmetric with respect to time-reversal if $\alpha \neq \beta$, while it reduces to a symmetric form when $\alpha = \beta$ (where symmetric forms are equivalent under time-reversal). It is straightforward to express $C_{\alpha,\beta}(\tilde{\mathbf{x}}; \tau)$ in terms of the original series $\mathbf{x}$ via the transformation $t \mapsto T - t - \tau + 1$ (see Appendix C), yielding an expression for the absolute difference $|\Delta C_{\alpha,\beta}(\mathbf{x}; \tau)|$:

$$\begin{aligned} |\Delta C_{\alpha,\beta}(\mathbf{x}; \tau)| &= |C_{\alpha,\beta}(\mathbf{x}; \tau) - C_{\alpha,\beta}(\tilde{\mathbf{x}}; \tau)|, \\ &= |\langle x_t^\alpha x_{t+\tau}^\beta \rangle - \langle x_{t+\tau}^\alpha x_t^\beta \rangle|. \end{aligned} \quad (7)$$

Symmetric constructions with $\alpha = \beta$ (where the linear autocorrelation, with $\alpha = \beta = 1$, is a special case) trivially have $\Delta C_{\alpha,\alpha} = 0$ for any data $\mathbf{x}$. By contrast, assigning unequal weights to present and future points, with $\alpha \neq \beta$, yields a difference $|\Delta C_{\alpha,\beta}(\mathbf{x}; \tau)|$, which is nonzero in general. In this way, asymmetric constructions of $C_{\alpha,\beta}$, via $|\Delta C_{\alpha,\beta}(\mathbf{x}; \tau)|$, can be a powerful class of statistics for distinguishing irreversible processes (with $|\Delta C_{\alpha,\beta}(\mathbf{x}; \tau)| > 0$) from reversible processes (with $|\Delta C_{\alpha,\beta}(\mathbf{x}; \tau)| \approx 0$).

We analyzed the simple case of two-point products for analytical tractability and to develop intuition about temporal symmetry, while remaining generalizable to multipoint products or more complex functional forms. Illustrative examples of symmetric and asymmetric forms of generalized autocorrelation statistics for three-point statistics (and generalizable to multipoint cases) are shown in Figs. 3(a) and 3(b), where we introduce a comblike representation of the functions of patterns around a given point t . This diagrammatic representation visualizes products of time-series points in the temporal average: Each data point is shown as a segment, with the number of segments in a group indicating its exponent in the statistic and the horizontal separation between groups corresponding to the temporal lag τ , as illustrated in the legend of the upper panels of Fig. 3. This representation emphasizes the temporal symmetry with respect to the midpoint of these functions in constructions with trivial time-reversal symmetry. This distinction between symmetric and asymmetric forms

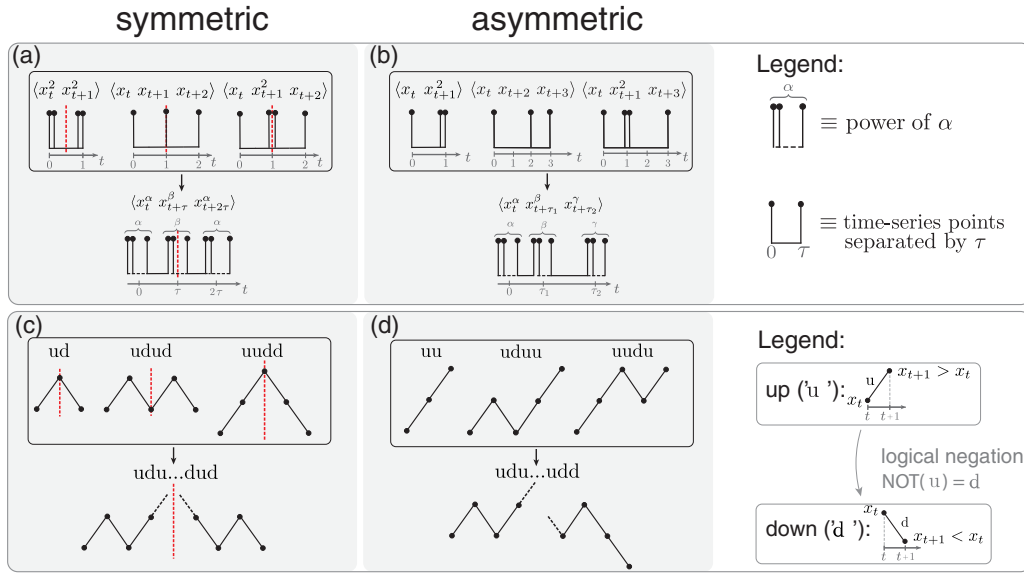


FIG. 3. Time-series features with time-symmetric constructions are invariant to time reversal, while time-asymmetric constructions can be powerful indices of time reversibility. We illustrate this concept with respect to two families of time-series statistics: (a), (b) generalized autocorrelation functions; and (c), (d): the frequency of patterns of consecutive rises ("up": u) and falls ("down": d) in a symbolized transformation of the time series. (a) We depict three examples of generalized autocorrelation functions with time-symmetric constructions, visualized diagrammatically using a comblike representation introduced here. In this representation, each vertical segment represents a time-series value at a specific time t , the spacing between groups of segments reflects the temporal lag between consecutive feature terms and the number of segments in each group represents the exponent, that is, the number of times a given time-series point contributes to the statistic. All of these features are symmetric in time with respect to their midpoint, indicated with a vertical dashed red line. (b) Three generalized autocorrelation functions with time-asymmetric constructions are depicted, with the asymmetry arising from the use of different exponents (or nonequally spaced temporal lags). (c) Three example time-symmetric sequences of successive rises ("up": u) and falls ("down": d) are depicted diagrammatically. The symmetry of these diagrams about their midpoint, depicted as a red dashed line, corresponds to the second half being the mirror image of the logic negation of the first half (indicated by the NOT operation, which transforms "u" to "d" and vice versa). (d) Three examples of time-reversal asymmetric symbolic patterns (which are therefore candidates for indexing irreversibility) are depicted.

depends on both the choice of weights assigned to each data point and the temporal lags separating successive points (see Appendix D for a general expression of the absolute difference in asymmetric three-point statistics). Examples of symmetric generalized autocorrelation measures are shown in Fig. 3(a), illustrating examples of time-symmetric constructions of a correlation statistic of the form $\langle x_t^\alpha x_{t+\tau}^\beta x_{t+2\tau}^\alpha \rangle$: $\langle x_t^2 x_{t+1}^2 \rangle$, $\langle x_t x_{t+1} x_{t+2} \rangle$, and $\langle x_t x_{t+1}^2 x_{t+2} \rangle$. In contrast, Fig. 3(b) shows examples of time-asymmetric constructions, which arise from modifying exponents (e.g., $\langle x_t x_{t+1}^2 \rangle$), unequal spacing between successive pairs of data points (e.g., $\langle x_t x_{t+2} x_{t+3} \rangle$), or a combination of both [e.g., $\langle x_t x_{t+1}^2 x_{t+3} \rangle$], which is expressed in a general asymmetric form of three-point correlation $\langle x_t^\alpha x_{t+\tau_1}^\beta x_{t+\tau_2}^\gamma \rangle$. The same reasoning behind symmetric and asymmetric constructions extends to the generalized autocorrelation $C'_{\alpha,\beta}(\mathbf{x}; \tau)$ [Eq. (5)] through the choice of the positive integer exponents α and β , where only asymmetric constructions with $\beta \neq \alpha$ can act as effective indices of irreversibility.

The best-performing features in the family of generalized autocorrelations align closely with asymmetric correlation-based statistics from prior foundational work on irreversibility. Previous work has measured irreversibility through forward-backward differences in time-averaged quantities,

corresponding to our Δf ansatz. For example, Pomeau [35] introduced cubic two-point correlation measures such as $\psi''(\tau) = \langle x_t^3 x_{t+\tau} \rangle - \langle x_t x_{t+\tau}^3 \rangle$, and Ramsey and Rothman [40] proposed the well-known "TR test" statistic for measuring asymmetry in business cycles, $\hat{\gamma}_{2,1}(\tau) = \langle x_t^2 x_{t-\tau} \rangle - \langle x_t x_{t-\tau}^2 \rangle$. However, in our empirical tests, the best-performing TR test statistic reached a maximum accuracy of just 68% (with $\tau = 2$). By contrast, we found that extending these classical measures through additional transformations not previously studied—such as taking absolute values—led to substantial boosts in performance, e.g., $\langle |x_t^3 x_{t+1}| \rangle$ and $\langle |x_t^2 x_{t+1}| \rangle$, both reached 87% accuracy.

Classical irreversibility statistics, like $t^{\text{rev}}(\tau) = \langle (x_t - x_{t-\tau})^3 \rangle$, introduced by Schreiber and Schmitz [69] (and also commonly used as a proxy for nonlinearity) can also be understood as a difference between time-average statistics evaluated from the forward and time-reversed time series, i.e., in the Δf form. Under the assumptions of stationarity and ergodicity, we can expand the cube and express this statistic in terms of the TR test statistic as

$$t^{\text{rev}}(\tau) = \langle (x_t - x_{t-\tau})^3 \rangle \quad (8)$$

$$= -3(\langle x_t^2 x_{t-\tau} \rangle - \langle x_t x_{t-\tau}^2 \rangle) \quad (9)$$

$$= -3\hat{\gamma}_{2,1}(\tau), \quad (10)$$

where, by stationarity, the cubic terms cancel. This result demonstrates that although $t^{\text{rev}}(\tau)$ is evaluated on the forward-time time series, it is equivalent to evaluating the difference between $\langle x_t^2 x_{t-\tau} \rangle$ evaluated on the forward-time and time-reversed data, and captures the same information as the TR test statistic $\hat{\gamma}_{2,1}(\tau)$ developed in the financial literature [40]. Chen *et al.* [33] showed a similar calculation using ensemble averages, thus considering that testing reversibility through $\mathbb{E}[x_t^2 x_{t-\tau}] = \mathbb{E}[x_t x_{t-\tau}^2]$ is equivalent to testing whether the third moment of the increment series $y_t = x_t - x_{t-\tau}$ vanishes.

In this section, we explained how a wide range of time-reversal asymmetric constructions of generalized forms of autocorrelation are able to index time-reversal asymmetry in time-series data, including through two-point statistics with unequal exponents (e.g., $\langle x_t^2 x_{t+1} \rangle$), higher-order statistics through the additional freedom introduced by time lags (e.g., $\langle x_t x_{t+2} x_{t+3} \rangle$), using additional operations like taking magnitudes (e.g., $\langle |x_t^2 x_{t+1}| \rangle$), and through functional forms involving time-averages like the $C'_{\alpha,\beta}(\mathbf{x}; \tau)$ statistic originally developed in the financial literature (which achieved the highest accuracy, 88%, but has not previously been used to index reversibility) [68]. This broad family of statistics was among the most successful at quantifying reversibility in our empirical tests, recapitulating existing reversibility statistics from the literature, as well as identifying forms, such as $\langle x_t^2 x_{t+1} x_{t+3} \rangle$, which generalize earlier constructions by computing, for example, absolute values of the product of time-series data points [35,39,69].

2. Symbolic features

The second key family of high-performing time-series statistics that we explore in detail is the group of “symbolic features,” which are both well-established for indexing reversibility and straightforward to analyze. These symbolic features involve transforming a time series $\mathbf{x}$ into a sequence of symbols (from a finite alphabet) using a set of coarse-graining rules [44,47,57]. Prior work has explored several symbolic approaches, including coarse-graining the time series based on value thresholds [44], encoding rules that capture relationships between successive time-series values [27], and an analysis of the distribution of all possible symbol permutations of fixed length—known as permutation patterns (or ordinal patterns) [70–72]. Among the set of symbolizing transformations included in *hctsa*, the most effective transformation for capturing irreversibility was the mapping to a two-letter alphabet based on incremental rises and falls in a time series. This transformation maps each pair of consecutive time-series values to a binary symbol {u,d} (“up” and “down”) that encodes either a rise ($x_{t+1} - x_t > 0$) or fall ($x_{t+1} - x_t < 0$) in the time series $\mathbf{x}$ for any given time t . From the resulting symbolic sequence, simple statistics can be derived that capture different temporal structures, such as the length of the longest consecutive run of a given symbol or the probability of occurrence of a given sequence. Although some symbolic features ranked among the top performers in our tests, their accuracies—generally below 74%—were lower than that of other features (which reached accuracies as high as 89%). Nevertheless, the success of these simple and interpretable measures in capturing time reversibility, together

with the established presence of these symbolic methods in the literature, motivated a deeper analysis to understand why they perform well.

We identified three high-performing symbolic features that involved counting sequence probabilities in a time series: the probability of rises, $p_u(\mathbf{x})$ (72% accuracy), which is a simple normalized index adopted in prior work [29]; and the probability of two consecutive rises (or of two consecutive falls), $p_{uu}(\mathbf{x})$ (and $p_{dd}(\mathbf{x})$) (both have 73% accuracy). We also identified five high-performing symbolic indices of irreversibility as the length of consecutive runs of a given symbol, including the excess occurrence of the pattern “uu” (or “dd”) relative to a single “u” (or “d”), i.e., $p_{uu}(\mathbf{x}) - p_u(\mathbf{x})$ or $p_{dd}(\mathbf{x}) - p_d(\mathbf{x})$ (each have 74% accuracy), as well as the mean length of consecutive rises (or falls) and the difference between the mean lengths of consecutive rises and falls (each have 73% accuracy). Figure 2(b)(ii) plots the distribution of $|\Delta p_{uu}|$, the best-performing symbolic feature measuring frequencies of specific symbolic motifs within the series, across all reversible and irreversible time series. As for all the other irreversibility statistics, $|\Delta p_{uu}| \approx 0$ for reversible processes, but irreversible processes exhibit substantial deviation from zero.

Similar to the symmetric constructions of generalized autocorrelations [Fig. 3(a)], symbolic sequences that are symmetric with respect to time reversal are invariant under time reversal (and yield $\Delta f = 0$ by construction). To clarify what we mean by such time-reversal symmetric constructions, we note that when a time series is reversed, rises become falls, and vice versa, i.e., $p_u(\mathbf{x}) = p_d(\tilde{\mathbf{x}})$. This observation can be formalized using a bitwise “logical negation” operator that maps $u \mapsto d$ and $d \mapsto u$. Accordingly, the probability of observing a given sequence in $\tilde{\mathbf{x}}$ is equal to the probability of the corresponding bitwise-negated sequence in $\mathbf{x}$, read in reverse temporal order. Symmetric sequences are those for which reading the bitwise-negated sequence in reverse yields the original sequence. Three examples of time-reversal symmetric sequences—“ud,” “udud,” and “uudd”—are depicted diagrammatically in Fig. 3(c) with lines indicating rises or falls and the midpoint represented as a red dashed line. For such features, the probability of occurrence in the original and reversed time series is identical, for example, $p_{ud}(\mathbf{x}) = p_{ud}(\tilde{\mathbf{x}})$, resulting identically in $\Delta p_{ud}(\mathbf{x}) = p_{ud}(\mathbf{x}) - p_{ud}(\tilde{\mathbf{x}}) = p_{ud}(\mathbf{x}) - p_{ud}(\mathbf{x}) = 0$.

By contrast, time-asymmetric constructions can act as a statistical indicators of time reversibility. Three such examples are illustrated in Fig. 3(d): “uu,” “uduu,” and “uudu.” For example, for the “uu” motif, $p_{uu}(\mathbf{x})$ (the probability of two successive rises in a time series, 73% accuracy), we can use the time-reversal property of rises and falls described above to write

$$\begin{aligned} \Delta p_{uu}(\mathbf{x}) &= p_{uu}(\mathbf{x}) - p_{uu}(\tilde{\mathbf{x}}) \\ &= p_{uu}(\mathbf{x}) - p_{dd}(\mathbf{x}). \end{aligned} \quad (11)$$

This provides an equivalent interpretation of $\Delta p_{uu}(\mathbf{x})$ as a measure of the difference in frequency between two successive rises versus two successive falls in the original time series $\mathbf{x}$. Since for a reversible process [with equivalent forward and time-reversed joint distributions, Eq. (2)], $p_{uu}(\mathbf{x})$ and $p_{dd}(\mathbf{x})$ have equal expectation, an imbalance between

$p_{uu}(\mathbf{x})$ and $p_{dd}(\mathbf{x})$ (which yields $\Delta p_{uu}(\mathbf{x}) \neq 0$) can thus act as an statistical indicator of irreversibility. A similar property applies to all asymmetric sequence probabilities, which are capable of capturing deviations from time reversibility via imbalances of the frequencies of a sequence (e.g., “uduu”) and its logical inversion, including the two examples shown in Fig. 3(d), with $\Delta p_{uduu}(\mathbf{x}) = p_{uduu}(\mathbf{x}) - p_{ddud}(\mathbf{x})$ and $\Delta p_{uudu}(\mathbf{x}) = p_{uudu}(\mathbf{x}) - p_{dudd}(\mathbf{x})$.

In summary, symbolic sequences provide a simple yet powerful framework for quantifying time-reversal asymmetries in time-series data. Similar to generalized autocorrelations, this family of statistics embodies a common symmetry principle while capturing local patterns that may be temporally symmetric (and trivial) or asymmetric (and potentially useful as an index of time-reversal asymmetry).

3. Forecasting features

The third category of high-performing time-series features we focus on involves simulating a forecasting algorithm, which attempts to predict the future of the time series from its past. Forecasting algorithms explicitly distinguish the past and future of a time series, and are thus constructed with a directional asymmetry in time. Despite this, forecasting-based time-series features remain relatively unexplored for this purpose, with the exception of a small number of prior studies (such as the nonlinear prediction model used to detect irreversibility by Stone *et al.* [22]). The types of high-performing forecasting-based features identified in our data-driven comparison, resulting in 48 high-performing features, capture various goodness-of-fit metrics of a fitted forecasting model, including the MAE, and the variance and Gaussianity of the residuals.

The highest accuracy achieved by a forecasting-based feature was a measure of the Gaussianity of the residuals of a simple local mean forecaster (predicting the subsequent time step using the local mean of the three prior time points: 81% accuracy). Other high-performing forecasting-based methods include the goodness-of-fit of AR model predictions across various prediction lengths (accuracies in the range 72–78%), with the best-performing feature (78% accuracy) capturing the goodness-of-fit of 1-step-ahead errors using an AR(2) model [shown in Fig. 2(b)(iii)]. Other statistics used a range of state-space and GARCH models, and used test statistics from a Kolmogorov-Smirnov test performed on the model residuals (yielding accuracies up to 81%); or nonlinear prediction error (using a two- or three-dimensional time-delay embedding [73], achieving accuracies up to 75%).

While useful features capture a difference in predictability of a time series running forward versus backward in time, $|\Delta f| > 0$, for some irreversible processes, we were interested to further investigate the sign of the difference Δf , which indicates whether a time series is more easily predictable forward or backward in time. Intuitively, a time series generated by an irreversible process will be more predictable in the direction corresponding to the underlying rules of the governing dynamic process, which are formulated forward in time. This is consistent with the claim of Stone *et al.* [22] that “theoretically there may be nonlinear systems for which backward predictions might be better than forward predic-

tions, but in practice we have found that such systems, if they exist, must be rare.” Our results reveal an interesting subtlety to this expectation. For a sufficiently flexible prediction model, like a nonlinear prediction model (e.g., fitted in a two-dimensional time-delay embedding space [73]), we indeed found a consistent increase in the ability to accurately forecast irreversible processes forward in time (relative to their time-reversed versions). However, we found counterintuitive results from simpler models, including AR(2) and ARMA(3,1) models, where for some irreversible processes, time-reversal can lead to more accurate predictions. This behavior could be explained by the properties of different types of asymmetries in the (x_t, x_{t+1}) phase plots with respect to the $x_t = x_{t+1}$ identity line (which is a hallmark of irreversible dynamics [17]). One reason may be that the time-reversed series can have properties that better match the assumptions of a simple predictive model. For example, relative to the forward-time marginal distributions, $p(x_{t+1}|x_t)$, time reversal can yield more Gaussian marginals, $p(x_t|x_{t+1})$, which can lead to more accurate predictions by simple, strongly parametric models whose assumptions better match the statistical properties of the time-reversed data.

Our results thus highlight an underappreciated class of time-series analysis methods—well-studied algorithms for time-series forecasting—and their ability to capture time reversibility through a difference in predictability in the forward versus time-reversed directions for irreversible processes. We also report the existence of counterintuitive cases in which some irreversible processes can be more accurately predicted by simple models after time-reversal, due to the reversed series better matching the assumptions of the forecasting model.

B. Statistical signatures of irreversibility are highly process-dependent

Our results above highlight a range of time-series summary statistics that can index time-reversal asymmetry with high accuracy (up to 89%) across 35 diverse processes. However, no feature exhibited perfect separation between time series generated from reversible versus irreversible processes, suggesting that there may not exist a general-purpose real-valued summary statistic that can reliably index time reversibility. While average accuracy provides a simple index of a feature’s performance, in this section, we aimed to understand in more detail the relative strengths and weaknesses of the 127 top-performing features identified above when applied to different types of processes. This required us to disentangle the relative performance of each feature on each process. We computed this as the “left-out accuracy,” defined for each feature on each irreversible process as the proportion of the 100 time-series realizations correctly classified by the feature (during the testing phase of the 1-NN leave-one-out procedure, cf. Sec. II).

To understand the differential performance of top-performing features across processes, we computed the distribution of left-out accuracies for five representative irreversible processes (from each of the five families listed in Table I): autoregressive with uniform noise distribution (AR1_UNO); logistic map (with $r = 4$) (LOGISTIC_4); linear model with noise being a realization of a logistic map (LLOG); noise-driven sine map (SINE_MAP); and a linear projection of the

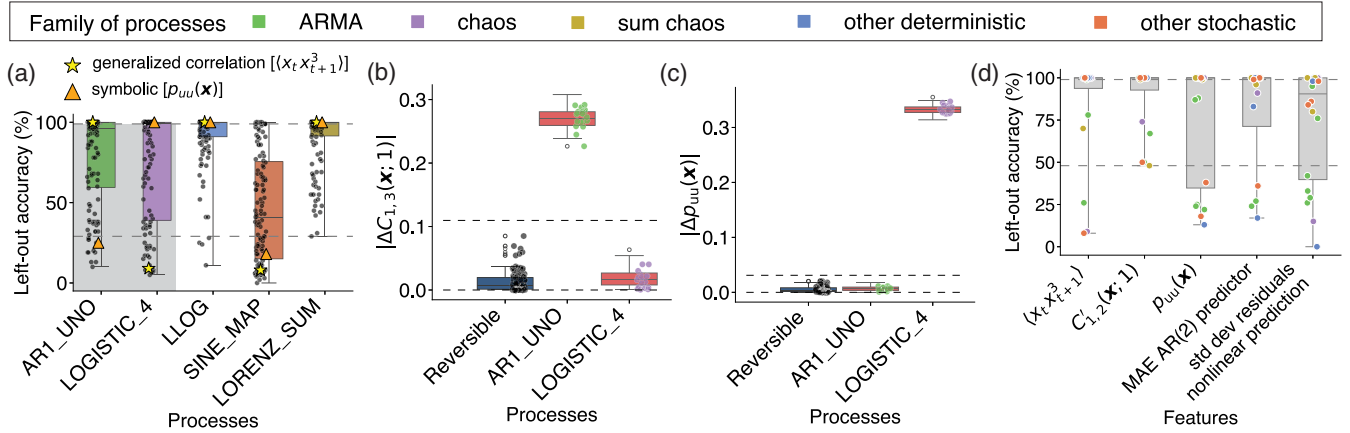


FIG. 4. All statistical time-series features have strengths and weaknesses at detecting irreversibility across different processes, demonstrating the need to tailor statistical summaries to the specific sources of irreversibility in a given process. (a) Box plot with scatter points showing the left-out accuracy of the 127 top-performing features for five representative irreversible processes: autoregressive with uniform noise distribution (AR1_UNO), logistic map (with $r = 4$) (LOGISTIC_4), linear model with logistic map noise (LLOG), noise-driven sine map (SINE_MAP), and a linear projection of the multidimensional Lorenz system (LORENZ_SUM). Simulation details of the implemented models are in Appendix A. The generalized autocorrelation feature $\langle x_t x_{t+1}^3 \rangle$ is highlighted using a yellow star and the symbolic feature $p_{uu}(x)$ using an orange triangle. For each process, we computed the minimum and maximum left-out accuracies obtained by the set of top-performing features and compared these values across all irreversible processes. We then highlighted the feature with the lowest maximum accuracy across irreversible processes using a dashed line (corresponding to 99%), and similarly marked the feature with the highest minimum left-out accuracy (29%). We plot the distribution of absolute feature differences $|\Delta f|$ across (i) all reversible time series, (ii) the autoregressive process with uniform noise (AR1_UNO), and (iii) the logistic map (with $r = 4$) (LOGISTIC_4), for two example features: (b) generalized autocorrelation $\langle x_t x_{t+1}^3 \rangle$; and (c) the symbolic sequence probability p_{uu} . In both plots, the lower solid line denotes the $|\Delta f| = 0$ baseline, while the upper dashed line delineates the maximum of the range of values observed across all time series generated from reversible processes. (d) Distribution of left-out accuracy across 2000 time series from the 20 irreversible processes for five representative features: two forms of generalized autocorrelation, namely, the fourth-order statistic $\langle x_t x_{t+1}^3 \rangle$ and the “generalized linear self-correlation function” $C'_{1,2}(x;1)$ [68], reported in Eq. (5), using $\alpha = 1$, $\beta = 2$, and $\tau = 1$; the probability of two successive increases (the “uu” pattern) in a time series, $p_{uu}(x)$; the mean absolute error of 1-step-ahead predictions made by a second-order AR model (labeled as “MAE AR(2) predictor”); and the standard deviation of the residuals from a nonlinear prediction model (labeled as “std dev residuals nonlinear prediction”).

multidimensional Lorenz system (LORENZ_SUM). Results are shown in Fig. 4(a). Different processes exhibit varying levels of difficulty, with the irreversibility of LLOG and LORENZ_SUM relatively straightforward to detect by a large number of features, to more challenging systems like the SINE_MAP where only relatively few features perform well. Nevertheless, for each of the 20 irreversible processes we tested, there was at least one feature that achieved near-perfect ($>99\%$) left-out accuracy [indicated by the upper dashed line in Fig. 4(a)]. This suggests that, the ability to select from a sufficiently diverse library of irreversibility indicators provides the flexibility to detect the specific statistical form through which irreversibility manifests in a given process.

To better understand the variability of irreversibility detection of a given reversibility indicator, we highlighted the performance two representative features (analyzed above in Figs. 2(b)(i) and 2(b)(ii)): the generalized autocorrelation $\langle x_t x_{t+1}^3 \rangle$ [marked as a yellow star in Fig. 4(a)] and the probability of the “uu” pattern, $p_{uu}(x)$ [an orange triangle in Fig. 4(a)]. We see a clear variability of performance, with both features performing well on some problems (LLOG and LORENZ_SUM) and failing to detect irreversibility on the more difficult SINE_MAP problem, but also showing complementary performance on the AR1_UNO and LOGISTIC_4 processes where only one of the two features accurately detects the irreversibility. The diverging performance of these

two features on AR1_UNO and LOGISTIC_4 is shown explicitly for $|\Delta C_{1,3}(x;1)|$ in Fig. 4(b) (where it distinguishes only AR1_UNO), and for $|\Delta p_{uu}(x)|$ in Fig. 4(c) (where it distinguishes only the LOGISTIC_4 process).

We next focus on characterizing the variability of irreversibility detection of individual features across the set of simulated processes. Figure 4(d) shows the distribution of left-out accuracies for all 20 irreversible processes across five representative top-performing features from the main categories characterized above: two generalized autocorrelation metrics, $\langle x_t x_{t+1}^3 \rangle$ and $C'_{1,2}(x;1)$ [Eq. (5)]; the probability of two consecutive rises $p_{uu}(x)$, the “MAE AR(2) predictor” (the mean absolute error of 1-step-ahead predictions made by a second-order AR model), and the “std dev residuals nonlinear prediction” (standard deviation of the residuals from a nonlinear predictor). Consistent with their high average accuracies, all features successfully captured the irreversibility of the majority of processes, but we see substantial variability in left-out accuracy across processes. In particular, for each feature there exists at least one process for which it fails to detect irreversibility (i.e., a left-out accuracy near or below the chance level of 50%). Note that our results are not specific to the 1-NN feature-scoring approach used here; we obtained similar conclusions when scoring each feature and irreversible process based on its deviation from the distribution of Δf obtained from all reversible processes (results not shown).

Taken together, our results reveal a marked degree of interprocess variation in a given feature’s ability to detect irreversibility. Each feature encapsulates a specific form of deviation from time-reversal symmetry—e.g., of a given statistical structure and on a given timescale. Using a single summary statistic represents a data-efficient simplification to directly comparing forward and time-reversed joint distributions with finite data [cf. Eq. (2)], which becomes impractical to assess, especially on longer lags. However, this summary-statistic-based approach comes at the cost of imposing a specific structural form of the irreversibility that a given statistic is sensitive to. A given feature will perform well on processes that exhibit a given type of time-reversal asymmetry (that it is sensitive to), but relatively poorly on processes that exhibit different time-asymmetric statistical structures. Our findings thus support the view of irreversibility as a multifaceted phenomenon that cannot be fully captured by any single, fixed summary statistic in general, while suggesting a promising way forward in tailoring summary statistics to a given irreversible process.

IV. DISCUSSION

This work presents a unified organization of statistical approaches to detecting irreversibility from time-series data through the broadest comparative study of irreversibility indices to date, allowing us to characterize and identify connections between a wide array of promising numerical techniques. By systematically comparing over 6000 statistics across 3500 time series, simulated from a wide range of reversible and irreversible systems, we identified methods that (1) recapitulate the effectiveness of established reversibility statistics, including $\langle x_t x_{t+1}^3 \rangle$ [35] and $p_u(\mathbf{x})$ [29]; (2) suggest useful modifications of classical measures, e.g., using absolute values in $\langle |x_t x_{t+1}^2| \rangle$ relative to the original test statistic, the TR test statistic $\hat{\gamma}_{2,1}(1)$ [40]; (3) demonstrate that statistics originally developed for other purposes can act as powerful metrics of irreversibility, e.g., a formulation of generalized autocorrelation proposed by Duarte Queirós and Moyano [68] or the mean absolute error of 1-step-ahead predictions made by an AR(2) model, and highlight time-series analysis methods for irreversibility detection that (to our knowledge) have not previously been studied in this context, e.g., a range of forecasting-based statistics, as well as “walker” and extreme-value statistics (Sec. III A). The comprehensive and unified overview of methodological approaches for irreversibility are a strength of the high-comparative data-driven approach taken here, which complements theory-driven approaches. Our results demonstrate that irreversibility is a multifaceted concept that requires tailored statistical approaches to accurately detect and quantify it for a given system.

We provide a deeper understanding of our findings by analyzing the algorithmic implementation of high-performing statistics, illustrated with a diagrammatic representation for generalized autocorrelation and symbolic statistics (Fig. 3), showing that their asymmetric construction allows them to behave as nontrivial identifiers of irreversibility. Our analysis highlights relationships between diverse formulations of irreversibility, bridging methods developed across different scientific fields. We highlight important similarities

between generalized autocorrelation statistics (Sec. III A 1) and symbolic sequences derived from time-series increments (Sec. III A 2): In the case of generalized autocorrelations, asymmetry arises through the choice of weights or temporal lags, while for symbolic features it emerges from asymmetric patterns in successive rises and falls. In both cases, trivial statistics correspond to time-symmetric local patterns, and deviations from this symmetry give rise to families of statistics with relatively high predictive performance. This finding aligns with the observation of Steinberg [55] for continuous signals, namely, that “appropriate measures being those that depend on the direction of the *time arrow*.” The variety of constructions identified here resonates with the considerations of Pomeau [35], who emphasized the potentially infinite formulations of asymmetric autocorrelation functions as effective proxies for detecting irreversibility.

Furthermore, the breadth of comparison conducted here, of both analysis methods and systems, allowed us to demonstrate that there is no general “best statistic” that is effective at detecting irreversibility in all systems; but rather that each statistic captures a specific manifestation of irreversibility, expressed as an asymmetry in some underlying property. At the same time, we show that (given the leave-one-process-out nearest-neighbor matching heuristic used here) the large algorithmic library of time-series statistics contained in *hctsa* [51] is sufficiently comprehensive to identify an accurate statistic for capturing time irreversibility for all simulated processes. This suggests that the summary statistic could in general be tailored to the specific statistical properties that characterize the time-reversal asymmetry of a given process. The limited information captured by any individual time-series statistic helps explain why previous studies, often focusing on single approaches, sometimes reached contrasting conclusions about the same processes [48].

Our results suggest a range of recommendations. We argue that future work should aim to benchmark the behavior of different irreversibility statistics across a broad range of systems with different irreversibility signatures (rather than focusing on a small number of handpicked systems). And rather than developing statistics, future research may more productively focus on developing methods to efficiently tailor statistical indices of irreversibility to a given system of interest. This will be especially useful for emerging applications of time-irreversibility metrics, including in applications of nonequilibrium statistical mechanics, and in inferring and understanding how time-asymmetric structures in time-series data result from underlying dissipative and nonlinear mechanisms in complex physical systems.

Recognizing the strengths and weaknesses of each time-series feature lays the groundwork for future investigations into explicit hypothesis tests, facilitated by the development of an appropriate null distribution [17]. This would extend the 1-NN approach used here as a practical heuristic index, under the assumption that the set of reversible and irreversible processes is sufficiently comprehensive. While this heuristic was adequate for our current purpose, which primarily focuses on comparative analysis, future work could aim to establish a formal testing framework that more precisely characterizes statistical deviations from reversibility. For example, an empirical treatment could involve defining a broad set of reversible

processes as the null distribution for reversibility, and then quantifying the deviation for a given time series (and test statistic) using a permutation test. While we have briefly confirmed relatively similar behavior of key irreversibility indices as a function of time-series length (as shown in Fig. 5), future work could more comprehensively evaluate the generalization of our results to short, noisy time series (where different features may exhibit different levels of robustness to time-series length and additive noise). Our analysis here was also restricted to univariate time series; future work could investigate extending this approach to multivariate data, building on approaches such as cross-correlation analysis widely used in neuroscience [74], noting that a comprehensive library of pairwise statistics has recently been developed [75]. Finally, while our range of systems analyzed here is comprehensive, it is not exhaustive, and our specific quantitative and qualitative results depend on these choices. Future work could therefore evaluate a broader range of processes, including those containing more complex timescales [49].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [62,63].

APPENDIX A: SIMULATION DETAILS OF IMPLEMENTED MODELS

Here, we present the simulated processes in greater detail, including the governing equations and specifying the parameter choices made in this work. In the summary Table I, we indicated with a “D” the discrete-time and with a “C” the continuous-time processes to emphasize that distinct simulation strategies are required for continuous-time systems that include greater attention to initial condition selection, downsampling procedures, and integration parameters. For discrete-time processes, we initially simulated 10 000 samples and then discarded the first 5000 samples, resulting in the $T = 5000$ sample time series analyzed here. For each continuous-time system, we simulated the dynamics over a sufficiently long time interval to discard the initial transient that exceeded the longest characteristic timescale (estimated as the first zero-crossing of the autocorrelation function). To ensure that all processes were sampled on a relatively comparable timescale relative to the dynamics of the process (and avoid substantial undersampling or oversampling, which can bias many time-series statistics operating over relatively short temporal lags), we downsampled time series generated from continuous-time processes by a factor m , chosen as the temporal lag at which the autocorrelation function decays to $1/e$. While this approach of estimating a “correlation length” is more naturally suited to stochastic processes, using this common processing

heuristic for all continuous-time processes allowed us to effectively tackle the problem of setting a sampling rate that similarly resolves the relevant dynamical correlations for both deterministic and stochastic systems. After downsampling, we retained the first $T = 5000$ samples of each series.

Noise. Time-series realizations of i.i.d. noise processes were drawn from both a Gaussian $x_t \sim \mathcal{N}(0, 1)$ (labeled GNO) and a uniform distribution $x_t \sim \mathcal{U}(-0.5, 0.5)$ (labeled UNO). Three colored noise processes were generated using Zhivomirov’s algorithm [76], implemented in MATLAB. As mentioned in Sec. II A, we simulated realizations of pink noise, characterized by a power spectral density (PSD) proportional to $1/f$ (labeled PINK), where f is frequency; red noise with a PSD proportional to $1/f^2$ (labeled RED); and violet noise with a PSD proportional to f^2 (labeled VIOLET).

Autoregressive models. We simulated a first-order autoregressive process, AR(1), given by

$$x_t = a x_{t-1} + \varepsilon_t, \quad (\text{A1})$$

where $a = 0.5$ and $\varepsilon_t \sim \mathcal{N}(0, 1)$ is i.i.d. Gaussian noise (labeled AR1_GNO). Additionally, we considered the static nonlinear transformation of another AR(1) process given by Eq. (A1) with $a = 0.6$, as $y_t = \tanh^2(x_t)$ (labeled STAR). We adopted the same or closely related parameter choices reported in Diks *et al.* [17].

Next, we simulated autoregressive processes with various forms of non-Gaussianities and nonlinearities. Linear non-Gaussian stochastic processes simulated here include

(1) the AR(1) in Eq. (A1) with noise terms sampled from a uniform distribution, $\varepsilon_t \sim \mathcal{U}(-0.5, 0.5)$ (labeled AR1_UNO). We decided to maintain the same parameter choice ($a = 0.5$) for consistency with AR1_GNO, in order to compare two processes with the same deterministic component and, therefore, isolate the noise contribution;

(2) an autoregressive ARMA(1,1) process:

$$x_t = 0.6 x_{t-1} + \varepsilon_t + 0.4 \varepsilon_{t-1}, \quad (\text{A2})$$

with noise sampled from a uniform distribution $\varepsilon_t \sim \mathcal{U}(-0.5, 0.5)$ (labeled ARMA11_UNO);

(3) an autoregressive processes with Gamma noise distribution, defined as

$$x_t = 0.3 x_{t-3} - 0.2 x_{t-2} + 0.1 x_{t-1} + \epsilon_t, \quad (\text{A3})$$

with manually specified parameters and noise sampled from gamma distribution, $\epsilon_t \sim \Gamma(1, 0.3)$ (labeled AR3_GAMMA). The initial condition was drawn from a uniform distribution over $[0, 1)$.

We also considered three nonlinear autoregressive processes. The first form of nonlinearity was introduced through self-exciting threshold autoregressive (SETAR) models, which switch between k different autoregressive regimes based on the past d values of the time series [58]. A SETAR model is denoted SETAR($k; d, p_1, \dots, p_k$), where k represents the number of regimes, d is the delay parameter that controls the switch between regimes, and $p_1, \dots, p_k$ are the orders of the autoregressive models within each of the k regimes. We simulated two variants of this process, namely,

the SETAR(2; 1, 1, 1) model defined as [56] (labeled SETAR1)

$$x_t = \begin{cases} -0.9x_{t-1} + \epsilon_t, & \text{if } x_{t-1} \geq 1, \\ -0.4x_{t-1} + \epsilon_t, & \text{if } x_{t-1} < 1, \end{cases} \quad (\text{A4})$$

and the following specific SETAR(2; 2, 2, 2) process introduced by Martínez *et al.* [57]:

$$x_t = \begin{cases} 0.62 + 1.25x_{t-1} - 0.43x_{t-2} + 0.0381\epsilon_t, & \text{if } x_{t-2} \leq 3.25, \\ 2.25 + 1.52x_{t-1} - 1.24x_{t-2} + 0.0626\epsilon_t, & \text{if } x_{t-2} > 3.25, \end{cases} \quad (\text{A5})$$

where $\epsilon_t \sim \mathcal{N}(0, 1)$ are i.i.d. (labeled SETAR2). For the initial conditions, the first point of the SETAR1 process and the first two points of the SETAR2 process were sampled from a standard normal distribution. The second form of nonlinear process is defined by the following equation:

$$x_{t+1} = 0.5x_t - 0.3x_{t-1} + 0.1y_{t-1} + 0.1x_{t-1}^2 + 0.4y_t^2 + 0.0025\eta_t, \quad (\text{A6})$$

where

$$y_t = \sin(4\pi t) + \sin(6\pi t) + 0.0025\xi_t, \quad (\text{A7})$$

which is driven by both Laplacian $\eta_t \sim \text{Laplace}(0, 1)$ (sharper peak and heavier tails comped to Gaussian distribution) and bimodal Gaussian $\xi_t \sim 0.5\mathcal{N}(0.63, 1) + 0.5\mathcal{N}(-0.63, 1)$ noise (labeled NAR2). The first two points of both the x and y components were set to zero.

Deterministic chaotic maps. We simulated the conservative (measure-preserving) Arnold's Cat map [77] (labeled ARNOLD), defined by

$$\begin{aligned} x_{t+1} &= x_t + y_t \bmod 1, \\ y_{t+1} &= x_t + 2y_t \bmod 1, \end{aligned} \quad (\text{A8})$$

and the Chirikov standard map [8] (labeled CHIRIKOV):

$$\begin{aligned} \theta_{t+1} &= \theta_t + p_t + \frac{K}{2\pi} \sin(2\pi\theta_t) \bmod 1, \\ p_{t+1} &= \theta_{t+1} - \theta_t \bmod 1, \end{aligned} \quad (\text{A9})$$

with parameter $K = 0.971635$. For our analysis, we considered the x component of the Arnold's Cat map and the p component of the Chirikov standard map. As representatives of dissipative chaotic maps, we examined the x component of the well-studied standard Hénon map [78] ($a = 1.4$ and $b = 0.3$) (labeled HEN):

$$x_{t+1} = 1 - ax_t^2 + y_t, \quad y_{t+1} = bx_t, \quad (\text{A10})$$

as well as two additional one-dimensional chaotic systems. The first one is a quadratic map [79] given by the equation $x_{t+1} = 1 - ax_t^2$, with choice of the parameter $a = 1.8$ (labeled QUAD). The second, the logistic map [80], is given by $x_{t+1} = rx_t(1 - x_t)$, which we simulated both in fully chaotic regime ($r = 4$) (labeled LOGISTIC_4) and in the period-3 window ($r = 3.8284$), where it exhibits intermittency (labeled LOGISTIC_38). For all chaotic maps, initial conditions were randomly sampled from a uniform distribution over the unit interval, $[0, 1)$.

Sum of deterministic chaotic maps/flows. We constructed time series by combining two independent realizations of the

x component (one realization taken in forward time and another in reverse-time order, constructed by flipping the second in time) of two discrete-time systems: (1) the Hénon map (labeled HENR_DIVERSE); and (2) quadratic map (labeled QUAD_RSUM). As a special case for the Hénon map, we also simulated and included a process obtained by summing a forward-time realization with its time-reversed counterpart, generated by flipping the same time series (labeled HENR_SAME). In addition, we summed pairs of independent forward realizations of the x component of the standard Hénon map (labeled HEN_SUM).

We included continuous-time processes in our analysis, considering a linear projection $x + y + z$ of two multidimensional systems: the Lorenz and Rössler systems. We simulated realizations of the Lorenz system [81] with equations ($\sigma = 10$, $\rho = 28$, $\beta = 8/3$):

$$\frac{dx}{dt} = \sigma(y - x), \quad \frac{dy}{dt} = x(\rho - z) - y, \quad \frac{dz}{dt} = xy - \beta z, \quad (\text{A11})$$

with initial conditions sampled from uniform distributions $x_0, y_0 \sim \mathcal{U}(-8, 8)$ and $z_0 \sim \mathcal{U}(0, 10)$ (labeled LORENZ_SUM). The second chaotic flow analyzed is the Rössler system [82] given by ($a = 0.2$, $b = 0.2$, and $c = 5.7$)

$$\frac{dx}{dt} = -y - z, \quad \frac{dy}{dt} = x + ay, \quad \frac{dz}{dt} = b + z(x - c), \quad (\text{A12})$$

for which the initial conditions were sampled from the uniform distribution $x_0, y_0 \sim \mathcal{U}(-8, 8)$ and $z_0 = 0$ (labeled ROSSLER_SUM).

Other deterministic models. We generated time series from two discrete-time deterministic models:

(1) the linear model with deterministic noise source, $x_{t+1} = ax_t + \epsilon_t$, where $a = 0.5$ and ϵ_t is a realization of a logistic model $\epsilon_t = 4\epsilon_{t-1}(1 - \epsilon_{t-1})$ (labeled LLOG) that we refer to as "logistic noise;"

(2) the process $x_t = ax_{t-1} \bmod 1$, where $a = \sqrt{2}$ is the parameter that rules the reversible nature of the process (labeled MOD1).

Additionally, we included two continuous-time systems described by second-order differential equations:

(1) a linearized oscillator with dynamics described by the equation:

$$\frac{dx^2}{dt^2} + x = 0, \quad (\text{A13})$$

with initial conditions sampled from a uniform distribution $x_0, y_0 \sim \mathcal{U}(-2, 2)$ (labeled OSCILLATOR);

(2) the Van der Pol oscillator [83] specified by

$$\frac{d^2x}{dt^2} - \mu(1 - x^2) \frac{dx}{dt} + x = 0, \quad (\text{A14})$$

where $\mu = 1$, to include effects due to nonlinear damping and initial conditions sampled uniformly at random from the $[-2, 2]$ interval (labeled VDP).

We simulated dynamics Mackey-Glass equations, dynamics characterized by temporal delays [84]. Specifically, we analyzed the following delay differential equation:

$$\frac{dx}{dt} = \frac{\beta_0 x(t - \tau)}{1 + x(t - \tau)^n} - \gamma x(t), \quad (\text{A15})$$

where we set $\beta_0 = 0.2$, $\gamma = 0.1$, $n = 10$, and the delay $\tau = 17$ (labeled MG17). The initial condition was sampled from a uniform distribution, $x_0 \sim \mathcal{U}(0, 1.5)$.

Other stochastic processes. As a discrete-time system, we simulated a noise-driven sine map:

$$x_{t+1} = \mu \sin(x_t) + y_t \eta_t, \quad (\text{A16})$$

where $\mu = 2.4$ and y_t is Bernoulli random variable that is 1 with probability 0.01 and 0 otherwise and $\eta_t \sim \mathcal{U}(-2, 2)$ [85] (labeled SINE_MAP).

In continuous time, we simulated the Ornstein-Uhlenbeck process given by

$$dx = -\theta x dt + \sigma dW, \quad (\text{A17})$$

with $\theta = 0.8$, $\sigma = 0.3$, W a Wiener process, and initial condition $x_0 = 0$ (labeled OU).

We also included BRW, which are random walks adjusted by a function that biases the random walk downward toward a neighborhood of the stationary mean τ when the process deviates excessively from it [86]. The introduction of this adjustment function guarantees the global stationarity of the generated time series, limiting the trend behavior typical of random walks. The continuous-time version of BRW simulated here is given by

$$dx = e^k (e^{-\alpha_1(x-\tau)} - e^{\alpha_2(x-\tau)}) dt + e^{\sigma/2 + \beta/2x^2} dW, \quad (\text{A18})$$

with parameters $k = -2$, $\alpha_1 = \alpha_2 = 2$, $\sigma = 4$, $\beta = 0.1$, $x_0 = 0$, and W a standard Wiener process (labeled BRW). Finally, we included stochastic variants of the Lorenz and Van der Pol systems. The stochastic Lorenz system (labeled STOCH_LORENZ_SUM) was obtained by adding independent Wiener processes, η_x, η_y , and η_z to the deterministic dynamics of each coordinate in Eq. (A11), and the Van der Pol oscillator (labeled STOCH_VDP) was generated by adding a standard Wiener process W to Eq. (A14). Both systems were initialized as in the deterministic case, and the noise intensity was set by scaling the Wiener processes by a factor of 1.5.

APPENDIX B: ROBUSTNESS OF TIME-SERIES STATISTICS FOR REVERSIBILITY TO THE TIME-SERIES LENGTH

The main analysis in this work was carried out considering 5000-sample time series. To assess the robustness of the selected features with respect to time-series

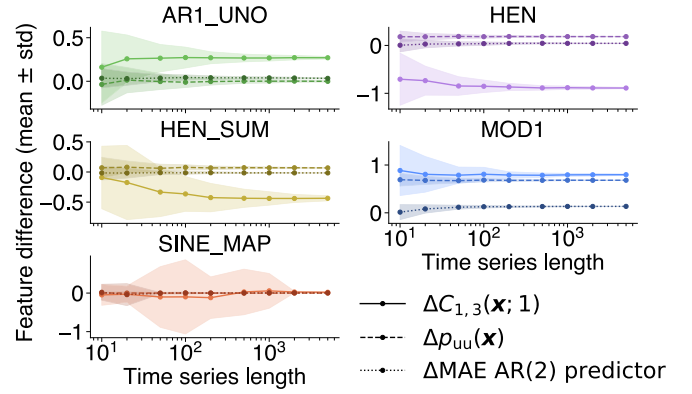


FIG. 5. Time-series features $\langle x_t^3 x_{t+1} \rangle$, $p_{uu}(\mathbf{x})$, and MAE of an AR(2) predictor as a function of the time-series length for a representative process from each family. We show the mean and standard deviation of the feature difference of the generalized autocorrelation $\langle x_t^3 x_{t+1} \rangle$ (light colors, solid line) and the probability of two successive rises $p_{uu}(\mathbf{x})$ (medium colors, dashed line) and MAE of an AR(2) predictor (dark colors, dotted line) for time series of lengths $T = [10, 20, 50, 100, 200, 500, 1000, 2000, 5000]$ (x axis, logarithmic scale), computed over 100 realizations of each process. Each panel reports a representative process per family, with colors indicating process families (see Table I).

length, we computed the mean and standard deviation of the feature differences Δf for a subset of the top-performing statistics discussed in Sec. III A, across nine different lengths of discrete-time processes: $T \in \{10, 20, 50, 100, 200, 500, 1000, 2000, 5000\}$. Results are depicted in Fig. 5 for the generalized autocorrelation $\langle x_t^3 x_{t+1} \rangle$ (light colors, solid line), the symbolic $p_{uu}(\mathbf{x})$ (medium colors, dashed line) statistics, and the MAE of an AR(2) predictor (dark colors, dotted line) for a representative process per family (see Table I). Overall, we observe a larger variability in the generalized autocorrelation compared to the symbolic and the forecasting-based statistics, which exhibits a relatively flat trend for $T > 50$. For shorter time series, errors in estimating the statistics increase, leading to higher variability in the results, particularly for the generalized autocorrelation.

APPENDIX C: ABSOLUTE DIFFERENCE OF ASYMMETRIC TWO-POINT GENERALIZED AUTOCORRELATION

Here, we compute explicitly the absolute difference for the general formulation of a two-point generalized autocorrelation statistic $C_{\alpha,\beta}(\mathbf{x}; \tau) = \langle x_t^\alpha x_{t+\tau}^\beta \rangle$ using the shifted time-reversal transformation $t \mapsto T - t - \tau + 1 \equiv s$, which preserves the number of points in the time average ($t, s \in [1, T - 1]$). We can express the two-point autocorrelation $C_{\alpha,\beta}(\tilde{\mathbf{x}}; \tau)$ in terms of the original time series $\mathbf{x}$ as

$$\begin{aligned} C_{\alpha,\beta}(\tilde{\mathbf{x}}; \tau) &= \langle \tilde{x}_t^\alpha \tilde{x}_{t+\tau}^\beta \rangle = \frac{1}{T - \tau - 1} \sum_{t=1}^{T-\tau} x_{T-t+1}^\alpha x_{T-t-\tau+1}^\beta \\ &= \frac{1}{T - \tau - 1} \sum_{s=1}^{T-\tau} x_{s+\tau}^\alpha x_s^\beta = \langle x_{t+\tau}^\alpha x_t^\beta \rangle. \end{aligned} \quad (\text{C1})$$

Consequently, the absolute difference can be written as

$$\begin{aligned} |\Delta C_{\alpha,\beta}(\mathbf{x}; \tau)| &= |C_{\alpha,\beta}(\mathbf{x}; \tau) - C_{\alpha,\beta}(\tilde{\mathbf{x}}; \tau)|, \\ &= |\langle x_t^\alpha x_{t+\tau}^\beta \rangle - \langle x_{t+\tau}^\alpha x_t^\beta \rangle|. \end{aligned} \quad (\text{C2})$$

APPENDIX D: ABSOLUTE DIFFERENCE OF ASYMMETRIC THREE-POINT GENERALIZED AUTOCORRELATION

Here, we compute explicitly the absolute difference for the general case of a three-point generalized autocorrelation statistic, $|\Delta C_{\alpha,\beta,\gamma}(\mathbf{x}; \tau_1, \tau_2)|$. The calculation is similar to the reasoning that brought to Eq. (7), but here we emphasize the dependence on both the choice of the temporal lags τ_1, τ_2 and of the exponents α, β, γ to build symmetric constructions. We consider the general form of a three-point generalized autocorrelation

$$C_{\alpha,\beta,\gamma}(\mathbf{x}; \tau_1, \tau_2) = \langle x_t^\alpha x_{t+\tau_1}^\beta x_{t+\tau_2}^\gamma \rangle \quad (\text{D1})$$

for positive, integer exponents α, β , and γ , and time lags τ_1, τ_2 , such that $\tau_2 \neq 2\tau_1$ if $\alpha = \gamma$. Since we have two time

lags τ_1 and τ_2 , an appropriate shifting transformation is $t \mapsto T - t - \tau_{\max} + 1$, where $\tau_{\max} = \max(\tau_1, \tau_2)$. By writing $C_{\alpha,\beta,\gamma}(\tilde{\mathbf{x}}; \tau_1, \tau_2)$ in terms of $\mathbf{x}$, as for the two-point generalized autocorrelation, the absolute feature difference gets

$$\begin{aligned} |\Delta C_{\alpha,\beta,\gamma}(\mathbf{x}; \tau_1, \tau_2)| &= |C_{\alpha,\beta,\gamma}(\mathbf{x}; \tau_1, \tau_2) - C_{\alpha,\beta,\gamma}(\tilde{\mathbf{x}}; \tau_1, \tau_2)| \\ &= |\langle x_t^\alpha x_{t+\tau_1}^\beta x_{t+\tau_2}^\gamma \rangle - \langle x_{t+\tau_{\max}}^\alpha x_{t+(\tau_{\max}-\tau_1)}^\beta x_{t+(\tau_{\max}-\tau_2)}^\gamma \rangle|. \end{aligned} \quad (\text{D2})$$

This formulation extends Eq. (7) and can itself be generalized to longer temporal scales in two ways: (1) by considering higher-order products, which incorporate longer histories, or (2) by evaluating pairs or triplets of observations separated by larger temporal lags. Therefore, the test statistic $|\Delta C_{\alpha,\beta,\gamma}(\mathbf{x}; \tau_1, \tau_2)|$ captures deviations from reversibility by quantifying the asymmetry in the data structure arising from the combined effects of unequal weights and differing temporal lags between data points.

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