

Stochastic resetting versus thermal equilibration: Faster relaxation, different destination

Nir Sherf¹ and Rémi Goerlich²*Raymond & Beverly Sackler School of Chemistry, Tel Aviv University, Tel Aviv 6997801, Israel*Barak Hirshberg^{1,*}*Raymond & Beverly Sackler School of Chemistry, Tel Aviv University, Tel Aviv 6997801, Israel;
Center for Computational Molecular and Materials Science, Tel Aviv University, Tel Aviv 6997801, Israel;
and The Center for Physics and Chemistry of Living Systems, Tel Aviv University, Tel Aviv 6997801, Israel*Yael Roichman^{1,†}*Raymond & Beverly Sackler School of Physics and Astronomy, Tel Aviv University, Tel Aviv 6997801, Israel;
Raymond & Beverly Sackler School of Chemistry, Tel Aviv University, Tel Aviv 6997801, Israel;
and The Center for Physics and Chemistry of Living Systems, Tel Aviv University, Tel Aviv 6997801, Israel*

(Received 25 November 2025; accepted 25 April 2026; published 14 May 2026)

Stochastic resetting is known for its ability to accelerate search processes and induce nonequilibrium steady states. Here, we compare the relaxation times and resulting steady states of resetting and thermal relaxation for Brownian motion in a harmonic potential. We show that resetting always converges faster than thermal equilibration, but to a different steady state. The acceleration and the shape of the steady state are governed by a single dimensionless parameter that depends on the resetting rate, the viscosity, and the stiffness of the potential. We observe a trade-off between relaxation speed and the extent of spatial exploration as a function of this dimensionless parameter. Moreover, resetting relaxes faster even when resetting to positions arbitrarily far from the potential minimum.

DOI: [10.1103/4mtb-65my](https://doi.org/10.1103/4mtb-65my)

Introduction. In many physical, chemical, and biological systems, performance is limited by the time required to reach a steady state or complete a transition between well-defined states. The acceleration of relaxation has therefore become an active field of research: Approaches such as shortcut to adiabaticity and engineered swift equilibration demonstrate that suitably designed time-dependent manipulations of the underlying potential can substantially reduce equilibration times [1–10]. These strategies underpin applications ranging from finite-time heat engines to rapid parameter inference from steady-state observables, and place fundamental bounds on the control and performance of nonequilibrium systems.

Stochastic resetting stops a random process and reinitializes it with independent initial conditions [11–13]. It is well known to accelerate search processes [11,14–25] and has been shown to outperform equilibrium dynamics in first-passage search problems [26]. In addition, stochastic resetting enables optimization of queuing strategies [27–30], enhances

sampling in molecular simulations [31–33], and even controls the yield of chemical reactions [28,34–37].

Moreover, resetting is known to induce a nonequilibrium steady state (NESS) if the mean time between resetting events is finite [11,38–40]. Several works have focused on characterizing the resulting NESS, whose probability density distribution significantly differs from the Boltzmann distribution in thermal equilibrium and depends on the resetting rate, the resetting protocol, and the underlying potential [11,16,38,41–43].

Much less is known about the *rate of relaxation* toward the NESS, especially in comparison to thermal relaxation. For example, the full time-dependent probability distribution is analytically known only for free diffusion under resetting [41]. Since solving the dynamics is often not analytically tractable, several approximations were previously used to describe how the NESS emerges [23,40,44–47]. Usually, the stationary distribution is established near the resetting point first and then spreads out with time. Even for a diffusing particle trapped in a harmonic potential, the evolution of the steady state and the rate of relaxation remains largely unexplored [41,48].

It was recently demonstrated that stochastic resetting can accelerate a transition between two equilibrium states in a V-shaped potential [10]. This was achieved by turning off the first potential, initiating stochastic resetting for a finite time, and turning back on the second potential. It was possible because the steady-state distribution for stochastic resetting of

*Contact author: hirshb@tauex.tau.ac.il†Contact author: roichman@tauex.tau.ac.il

a freely diffusing particle has the same form as the Boltzmann equilibrium distribution in a V-shaped potential. By tuning the resetting rate, the authors obtained the same target equilibrium state, only faster. This result raises an interesting question: Does stochastic resetting always relax to its steady state faster than thermal equilibration? If so, stochastic resetting could accelerate state-to-state transformations in heat engines [49,50] or bypass thermalization in mechanical relaxations [2,51]. We emphasize that this question concerns the global relaxation of the full probability distribution toward its stationary state, rather than first-passage properties to a prescribed target, which probe fundamentally different dynamical features.

Here, we address this question by comparing the relaxation to steady state of a diffusing particle in a harmonic potential with and without stochastic resetting, thereby exploring an alternative route to accelerated relaxation that does not rely on time-dependent potential engineering. We find that the system under resetting always relaxes faster than its thermally equilibrated counterpart, but to a different NESS. Remarkably, this speedup is governed by a single dimensionless parameter, which depends on the stiffness of the potential, the viscosity, and the resetting rate, effectively characterizing the dynamical behavior of the system. We also find a trade-off between the speedup and the extent of spatial exploration as a function of this dimensionless parameter. Finally, we show that the approach to steady state is dictated by the slowest relaxing moment. For resetting near the potential minimum, the second moment dominates the dynamics, while for resetting far from the minimum, the first moment becomes dominant. Regardless of the resetting point, stochastic resetting systematically reduces the relaxation time relative to thermal equilibration, establishing it as a viable mechanism for accelerated steady-state preparation.

Model. We study the relaxation toward steady state for a Brownian particle confined in a one-dimensional harmonic potential under stochastic resetting (SR) and compare it to thermal relaxation. The dynamics of the trapped particle obeys the Ornstein-Uhlenbeck (O-U) process:

$$\dot{x} = -\omega_0 x + \sqrt{2D} \eta(t), \quad (1)$$

where $\omega_0 = \kappa/\gamma$ is the inverse characteristic timescale of the process, κ is the stiffness of the harmonic potential, and γ is the Stokes drag coefficient. $D = k_B T/\gamma$ is the diffusion coefficient, k_B is the Boltzmann constant, and T is the bath temperature. Here, $\eta(t)$ represents thermal white noise with zero mean. Stochastic resetting is then applied by resetting the particle's position at random time intervals taken from an exponential distribution with a constant mean rate r . At each resetting event, the particle is instantaneously returned to a predetermined position, which throughout this paper will be the same as the origin $X_r \equiv x_0$.

Without resetting, the relaxation toward equilibrium is well known [52]. Specifically, the time-dependent probability distribution $P(x, t)$ is described by a Gaussian propagator with variance $\sigma^2(t)$ and mean $\mu(t)$,

$$P(x, t) = \sqrt{\frac{1}{2\pi\sigma^2(t)}} e^{-(x-\mu(t))^2/2\sigma^2(t)}. \quad (2)$$

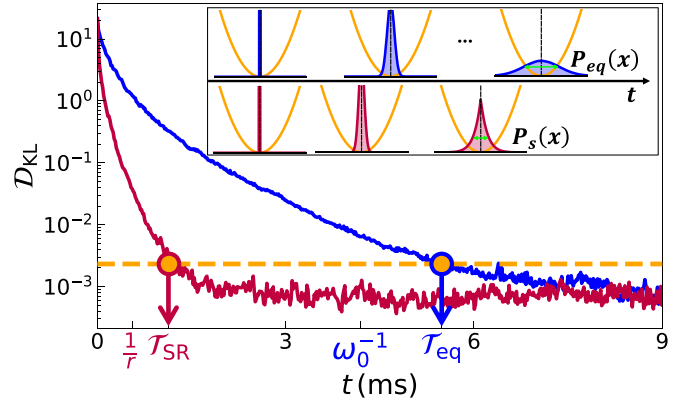


FIG. 1. KLD as a function of time for thermal equilibration (blue) and SR (red, $r = 1.8 \text{ kHz} = 7.5\omega_0$) using trap stiffness $\kappa = 2 \text{ pN}/\mu\text{m}$. The relaxation times (yellow circles) are determined when the KLD first drops below a threshold (dashed yellow line). This allows us to measure $T_{\text{eq}} = 5.49 \text{ ms} = 1.31\omega_0^{-1}$ and $T_{\text{SR}} = 1.14 \text{ ms} = 0.27\omega_0^{-1}$. The inset graphically shows the evolution of the probability distribution for both cases. The resulting steady state differs for the two processes, both in shape and in width, as is indicated by a change in standard deviation given by green arrows.

The temporal evolution of a probability distribution starting at $P(x, 0) = \delta(x - x_0)$ is fully determined by the relaxation of its first two moments, the variance $\sigma^2(t) = k_B T/\kappa [1 - \exp(-2\omega_0 t)]$ and the mean $\mu(t) = x_0[1 - \exp(-\omega_0 t)]$. At long times, we obtain the Boltzmann distribution of a particle in a harmonic trap. In contrast, an explicit simple closed-form expression for the propagator of an O-U process under resetting, such as in Eq. (2), is not available. Instead, it is given implicitly through an exact renewal relation (see Appendix E). However, the steady-state analytical form $P_s(x)$ is known and given in Appendix A. Notably, it differs from the Boltzmann equilibrium distribution (see the inset of Fig. 1) [41,48,53].

To study the relaxation to steady state under resetting, we perform overdamped Langevin simulations [41,54] and subject them to SR. Resetting times are drawn from an exponential distribution with rate parameter r . The simulations are done in arbitrary units where $dt = 0.1$, $\gamma = 500$, $kT = 1$. The parameters γ , κ , and T are chosen to be consistent with a colloidal particle of radius $0.5 \mu\text{m}$ in water at $T = 300 \text{ K}$ trapped in optical tweezers. This means that the time step is equivalent to $1.677 \mu\text{s}$ and potential of $\kappa = 1$ is equivalent to $1 \text{ pN}/\mu\text{m}$, typical values that indicate we are indeed at the overdamped regime. We run at least 10^5 trajectories of 10^4 steps in all cases.

We start our analysis by defining the relaxation time to steady-state $\mathcal{T}$ through the Kullback-Leibler divergence (KLD), which measures the time-dependent distance between the evolving state $P(x, t)$ and the final steady state $P_s(x)$ as $\mathcal{D}_{\text{KL}}(t) = \int P(x, t) \ln[P(x, t)/P_s(x)] dx$ [10]. Specifically, $\mathcal{T}$ is the time at which the KLD falls below a chosen threshold (see Fig. 1 and Appendix B for details). Both $P(x, t)$ and $P_s(x)$ are evaluated numerically as histograms from an ensemble of trajectories. Note that when $r = 0$, $P_s(x) = P_{\text{eq}}(x)$. This definition allows us to compare on equal footing the relaxation times with resetting (T_{SR}) and without (T_{eq}), which both differ

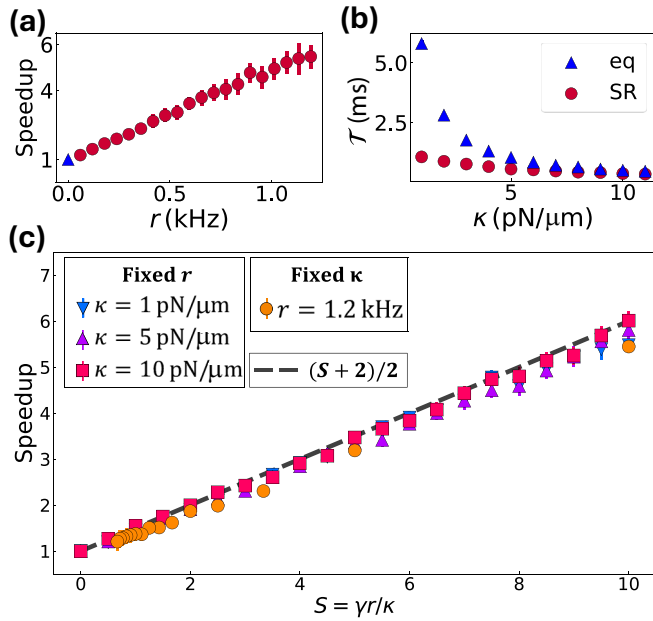


FIG. 2. (a) The speedup induced by SR, $\mathcal{T}_{\text{eq}}/\mathcal{T}_{\text{SR}}$, against r for $\kappa = 1$ pN/ μ m. Red circles for $r \neq 0$, and blue triangle underlines the thermal case $r = 0$ as shown in the legend of panel (b). (b) Relaxation times against κ for $r = 1.12$ kHz. The duration of relaxation under SR is smaller than the corresponding thermal process for all κ . This effect diminishes at large κ . (c) Speedup $\mathcal{T}_{\text{eq}}/\mathcal{T}_{\text{SR}}$, measured for various systems with different values of r and κ , collapses onto a single master curve when plotted against the dimensionless parameter S , demonstrating that relaxation dynamics are governed solely by S . The dashed black line is the ratio between the timescales of the second moments, showing that the speedup in relaxation of the full distribution can be characterized by the relaxation of the moments.

from the characteristic timescales ω_0^{-1} and r^{-1} . Although $\mathcal{T}$ is defined operationally via a KLD threshold, the late-time approach to stationarity is governed by the slowest relaxing moment of the distribution. As shown below (Fig. 2), for centered resetting, the second moment decays with a rate determined solely by the dimensionless parameter S . The resulting speedup therefore reflects this intrinsic late-time dynamical rate. Moreover, Appendix B shows that the speedup is not sensitive for both the KLD threshold used [Fig. 6(a)] and the distance metric itself [Fig. 6(b)].

We stress that the comparison is performed between the convergence times of each dynamics to its respective stationary state. Stochastic resetting does not accelerate equilibration toward the Boltzmann distribution; instead, it modifies the stationary state and accelerates relaxation toward the resetting-induced nonequilibrium steady state. This distinction is essential for a meaningful quantification of relaxation speedup, as it separates acceleration of dynamics from convergence to a common target distribution.

Resetting accelerates relaxation. Figure 1 shows the $\mathcal{D}_{\text{KL}}(t)$ without resetting (blue line) and with resetting at a rate $r = 1.8$ kHz $= 7.5\omega_0$ (red line). Here, we define the *speedup* induced by resetting as the ratio $\mathcal{T}_{\text{eq}}/\mathcal{T}_{\text{SR}}$. For this choice of parameters, we find that resetting significantly accelerates the arrival to steady state. As seen in the inset, the final steady

state is also modified by resetting. In the following, we test the effect of the physical parameters of the system (resetting rate, trap stiffness, and resetting position) on the speedup.

Figure 2(a) shows the speedup as a function of r for a fixed κ . We find that SR always accelerates relaxation since the speedup increases monotonically with the resetting rate. Using parameters typical for an experiment of resetting in an optical trap [24,53,55], we see a speedup of up to ~ 5 . This effect persists even when resetting to a Gaussian distribution around x_0 , which emulates the mechanism of resetting using a harmonic trap (see Appendix C). In Fig. 2(b), we plot the speedup as a function of the potential stiffness κ for a given r . We find that acceleration persists for all κ . However, the speedup diminishes with increasing κ as the difference between $\mathcal{T}_{\text{eq}}$ and $\mathcal{T}_{\text{SR}}$ decreases and the external potential dynamics dominate.

To gain further insight on the behavior of the speedup, we combine the main dynamical parameters into a single dimensionless number $S = \gamma r / \kappa$. This parameter effectively measures the interplay between the resetting and diffusive processes on the system's evolution and NESS. For small S values, the evolution is dominated by the harmonic potential; for large S , it is dominated by the resetting. In Fig. 2(c), we plot the measured speedup as a function of S . Remarkably, we see that values of the speedup, measured from simulations with varying values of r and κ , all collapse onto a single master curve. This universal scaling demonstrates that the speedup in relaxation is fully determined by S .

To elucidate this universality, we show that the Fokker-Planck equation for a Brownian particle in a harmonic potential under resetting with rate r can be recast into a dimensionless form, governed by S :

$$\partial_{\tilde{t}} P(\tilde{x}, \tilde{t}) = \partial_{\tilde{x}} (\tilde{x} P(\tilde{x}, \tilde{t})) - S \left[(1 - \partial_{\tilde{x}}^2) P(\tilde{x}, \tilde{t}) - \delta(\tilde{x}_0 - \tilde{x}) \right], \quad (3)$$

with dimensionless parameters $\tilde{x} = x/\alpha$, where $\alpha = \sqrt{D/r}$ is the characteristic length scale of the free diffusing particle under resetting [11], and $\tilde{t} = \omega_0 t$. Equation (3) shows that indeed the evolution and NESS are both determined by S alone.

We now turn to study the speedup dependence on S analytically. Solving Eq. (3) is generally not trivial, but we obtain further understanding of the system's relaxation dynamics by deriving an analytical expression for the time-dependent second moment,

$$\langle \tilde{x}^2 \rangle^{\text{SR}}(\tilde{t}) = \frac{2S}{S+2} (1 - e^{-(S+2)\tilde{t}}). \quad (4)$$

Equation (4) shows that the relaxation of the second moment is characterized by the timescale $(S+2)^{-1}$. In the same dimensionless form, the relaxation timescale of the second moment in the absence of resetting is $1/2$. Figure 2(c) shows that the numerically measured speedup exactly follows the ratio of these two timescales $(S+2)/2$. Namely, the speedup in relaxation time of the entire distribution can be described via the ratio of the two second moments alone.

It is instructive to estimate how many resetting events are required to establish the steady state. Since resetting events

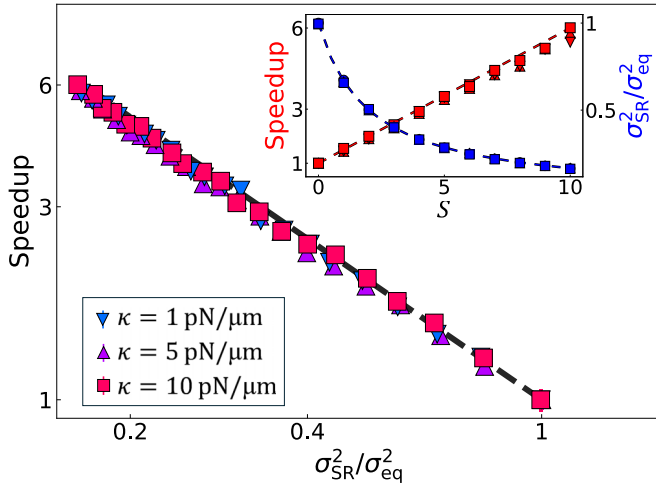


FIG. 3. Speedup vs spatial exploration shows a direct inverse relation, and black dashed line follows the analytical expression $y = x^{-1}$; here, $\text{Speedup} = (\sigma_{SR}^2/\sigma_{eq}^2)^{-1}$. Inset: red, speedup τ_{eq}/τ_{SR} ; blue, relative spatial exploration $\sigma_{SR}^2/\sigma_{eq}^2$. Dashed lines represent the corresponding analytical expressions; red, $(S+2)/2$; blue, $2/(S+2)$.

occur as a Poisson process with rate r , over the intrinsic relaxation timescale of the dominant moment, $\tau_{SR}^{(2)} = [(S+2)\omega_0]^{-1}$, the expected number of resetting events per trajectory until the ensemble reaches steady state as measured by its variance is

$$\langle N_r \rangle = r\tau_{SR}^{(2)} = \frac{r}{(S+2)\omega_0} = \frac{S}{S+2}. \quad (5)$$

This shows that the number of resetting events needed to relax the dominant moment remains of order unity across parameter regimes. We note that a few resetting events per trajectory (~ 1) are enough for the ensemble to converge faster to a steady state (see Appendix D).

Spatial exploration. As can be seen from Eq. (3) and in the inset of Fig. 1, S determines not only the relaxation but also the resulting NESS. Here, we examine how the increase in speedup with S affects the broadness of the NESS, as characterized by the second moment of the position. We define the relative *spatial exploration* through the ratio between the steady-state second moments, with and without resetting $\sigma_{SR}^2/\sigma_{eq}^2$ (where $\sigma^2 \equiv \langle x^2(t \rightarrow \infty) \rangle$). Equation (4) implies that the ratio of spatial exploration takes the form $\sigma_{SR}^2/\sigma_{eq}^2 = 2/(S+2)$, which is exactly the inverse of the ratio of relaxation durations. Namely, we obtain the exact relation

$$\frac{\tau_{eq}}{\tau_{SR}} = \left(\frac{\sigma_{SR}^2}{\sigma_{eq}^2} \right)^{-1}. \quad (6)$$

This demonstrates that relaxation acceleration and steady-state narrowing are not merely correlated, but are governed by a single parameter and obey a universal one-to-one scaling relation without adjustable coefficients (Fig. 3).

Dependence on initial conditions. So far, we have considered the case where the resetting position X_r coincides with the minimum of the harmonic potential, $X_r = x_0 = 0$. We now turn to a more general setting in which the resetting and initial

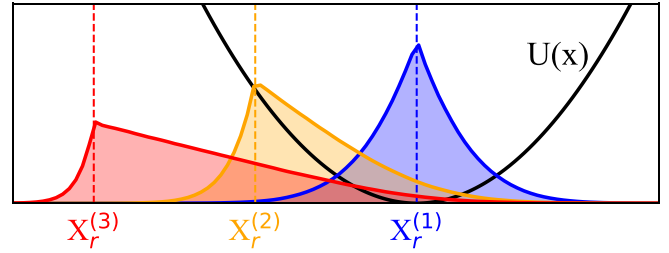


FIG. 4. The NESS $P_s(x)$ of an SR process for various resetting positions from right to left: centered process $X_r^{(1)} = 0$ (blue), $X_r^{(2)} \approx 2.25\sigma$ (yellow), and $X_r^{(3)} \approx 4.5\sigma$ (red). Increasing the distance between the resetting position and the center of the potential leads to skewed distributions, but also to larger spatial extension.

position are displaced from the minimum ($X_r = x_0 \neq 0$). This introduces a difference from thermal systems: In equilibrium, the final distribution is independent of the initial condition, whereas under stochastic resetting the NESS explicitly depends on the resetting position (see Fig. 4). Interestingly, even when $X_r \neq 0$, SR accelerates the relaxation compared to purely thermal dynamics, as shown in Fig. 5. Furthermore, two distinct regimes emerge. For small X_r , the speedup in relaxation grows with S as $(S+2)/2$, as observed for $x_0 = 0$, whereas for large X_r it scales as $S+1$. This transition can be understood by noting that, for small X_r , the first moment of the distribution remains close to zero and therefore the relaxation is governed primarily by the second moment. In contrast, for large X_r , the first moment becomes significant and, since it relaxes more slowly, dominates the relaxation process (see Appendix E). Consequently, applying SR for large X_r results in greater acceleration than for small X_r at the same resetting rate.

Conclusions. We have shown that stochastic resetting enhances relaxation toward an NESS for Brownian motion in

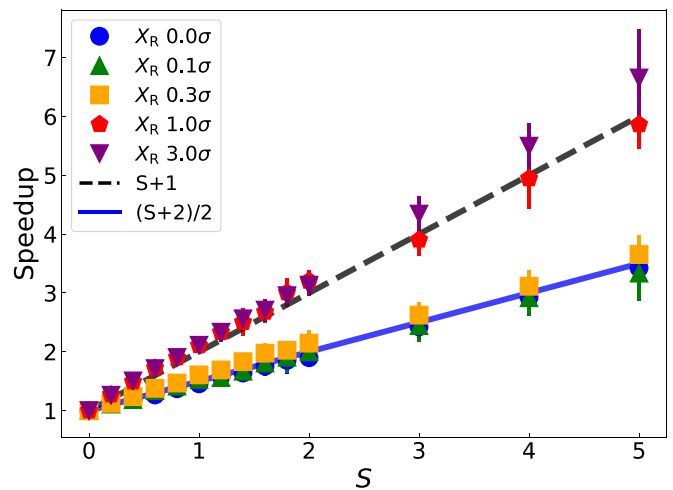


FIG. 5. The speedup is presented as a function of S for various initial positions. Dashed lines (dashed black and solid blue) represent the ratio of the first and second moments' timescales, respectively. The speedup follows either the first or second moment ratios, depending on initial conditions, with some transition between these two regimes.

a harmonic potential; however, it leads to a different steady state. The speedup is governed by a single dimensionless parameter S and there is a trade-off between the extent of spatial exploration and the speedup obtained. These insights could inform optimal control strategies [10,56,57], search processes [22–25], and trap-assisted experimental setups. Furthermore, in the control of mechanical systems, this combination of accelerated relaxation and smaller variance may lead to smoother control protocols, with possible applications in soft robotics [58] and active solid actuation [59]. The acceleration induced by stochastic resetting comes at a thermodynamic cost, which has been quantified at steady state using information-theoretic measures [53,60]. In applications such as Brownian heat engines, this additional dissipation directly affects performance and efficiency [50]. In contrast, in accelerated numerical sampling [31], no physical energetic cost is involved, rendering the associated thermodynamic dissipation irrelevant in practice. We focused on a harmonic potential, but the results are more broadly applicable, since any potential can be approximated by a harmonic trap near local minima. Future work will extend our analysis to more complex systems with different potentials, such as anharmonic, nonlinear, or disordered systems. We expect more significant speedups for these scenarios, which have a broader available parameter space, such as in computer simulations [31,32,61]. We will also explore different resetting protocols, such as space-dependent resetting, or different types of dynamics, such as underdamped or active.

Acknowledgments. Y.R. acknowledges support from the Israel Science Foundation (Grant No. 385/21). Y.R., R.G., and N.S. acknowledge support from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation program (Grant Agreement No. 101002392). R.G. acknowledges support from the Mark Ratner Institute for Single Molecule Chemistry at Tel Aviv University. B.H. acknowledges support by the Israel Science Foundation (Grants No. 1037/22 and No. 1312/22) and the Pazy Foundation of the IAEC-UPBC (Grant No. 415-2023).

Data availability. The data that support the findings of this article are openly available [62].

Appendix A: Steady state under resetting. The steady-state distribution for an O-U process characterized by a diffusion coefficient D , frequency ω_0 , and undergoing resetting at a rate r is known analytically [41,53] and is given by

$$P(x) = \sqrt{\frac{a}{\pi p^2}} \Gamma\left(\frac{1}{p}\right) e^{-\frac{ax^2}{2}} (ax^2)^{-\frac{1}{4}} W_{\frac{1}{4}-\frac{1}{p}, \frac{1}{4}}(ax^2), \quad (\text{A1})$$

where $W_{k,m}(x)$ is the Whittaker W function, $a = \omega_0/2D$, $b = 2\omega_0$, and $p = b/r$.

Appendix B: Determining relaxation metric and threshold. We define the full relaxation time of a process as the time at which the Kullback-Leibler divergence, $\mathcal{D}_{\text{KL}}$, crosses a specified threshold (see Fig. 1). This definition enables us to treat, on an equal footing, processes with different functional forms of relaxation and those exhibiting more than one characteristic relaxation time. As can be expected, the steady-state value of the $\mathcal{D}_{\text{KL}}$ depends on the potential stiffness κ and resetting rate r . Because of the variation in $\mathcal{D}_{\text{KL}}$, the threshold is determined adaptively relative to its value at steady state. To do so,

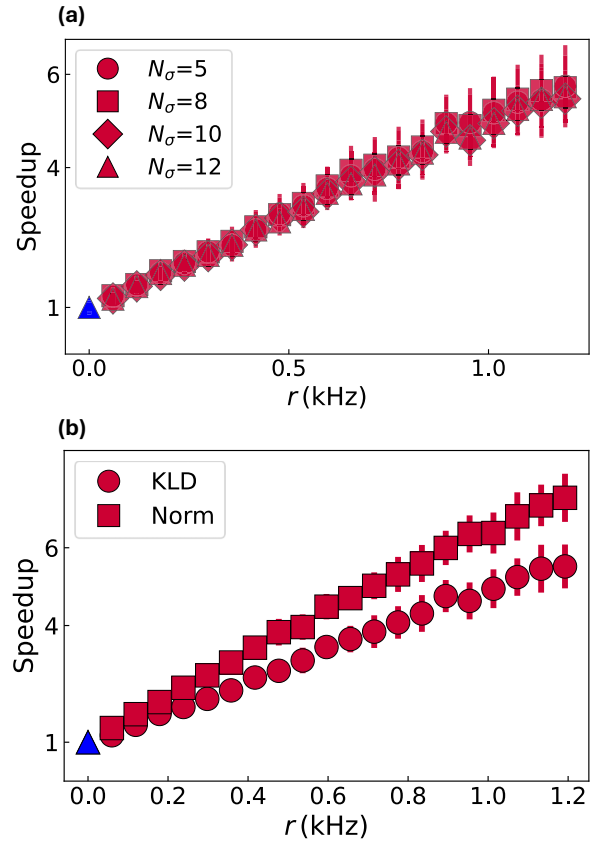


FIG. 6. (a) Speedup for different choices of threshold given by the number of standard deviations from the $\mathcal{D}_{\text{KL}}$ plateau values. (b) Speedup for different distance metrics, $\mathcal{D}_{\text{KL}}$ (circles) and L_2 norm (squares). The threshold is set to ten standard deviations from the plateau values ($\kappa = 1$ for all cases).

we quantify the mean and standard deviation of $\mathcal{D}_{\text{KL}}$ at full relaxation and define a threshold as N_σ standard deviations above the mean. As can be seen in Fig. 6(a), our results remain the same regardless of threshold choice. In the main text, we set the threshold at ten standard deviations from the mean.

In Fig. 6(b), we compare the L_2 norm with the $\mathcal{D}_{\text{KL}}$. Using a different metric does not change the qualitative results at all and only slightly modifies the quantitative results. In the main text, we use the $\mathcal{D}_{\text{KL}}$ metric.

Appendix C: Resetting to a Gaussian distribution. We consider resetting to a Gaussian distribution with a finite width. This scenario is experimentally relevant for systems such as optical tweezers, where resetting to an exact position is impractical, and particles are instead reset to a narrow distribution around the target point, x_0 . We use Gaussian distributions whose widths are defined as a fraction of the equilibrium distribution width. We focus on the regime $N_\sigma \leq 1$, which is the most experimentally relevant. The results (Fig. 7) show that the relaxation speedup toward the steady state is largely unaffected by the broadening of the resetting distribution. Remarkably, the acceleration persists even when the variance of the resetting distribution approaches that of the equilibrium Boltzmann distribution in the harmonic potential.

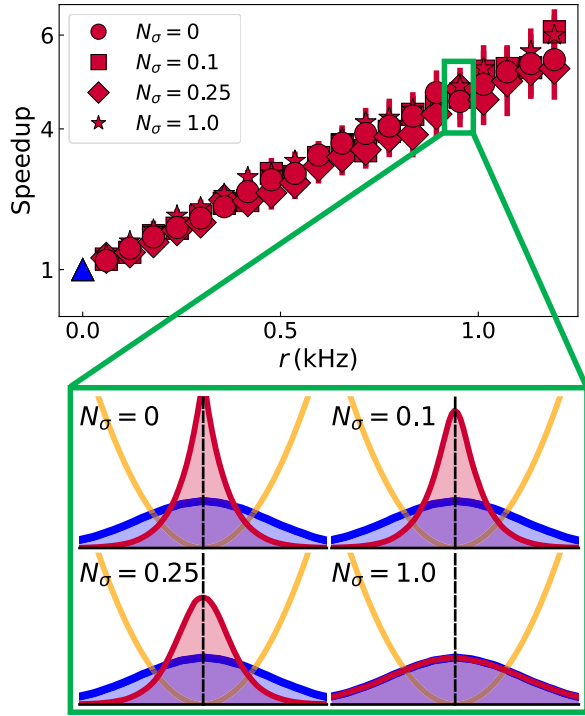


FIG. 7. The speedup is plotted for the process of resetting to a Gaussian distribution where N_σ is the fraction of the steady-state distribution width $\sigma_{ss} = \sqrt{kT/\kappa}$. Here, $\kappa = 1$, and for the plotted histograms, $r \approx 1$.

This indicates that the acceleration mechanism does not rely on confinement to a very narrow nonequilibrium steady state. In the limiting case $N_\sigma = 1$, the resetting distribution coincides with the equilibrium distribution itself and acceleration still persists. Importantly, the dependence of the speedup on the resetting rate r remains universal and independent of the width of the resetting distribution.

Appendix D: Mean number of resetting events before relaxation. We numerically verify Eq. (5) characterizing the mean number of resetting events per trajectory before the ensemble reaches steady state. The numerically obtained $\langle N_r \rangle$ agrees with Eq. (5) up to a scaling constant that relates the full relaxation time $\mathcal{T}_{KLD}$ of $\mathcal{D}_{KL}$ to the timescale of the second moment τ_{var} [Fig. 8(b)].

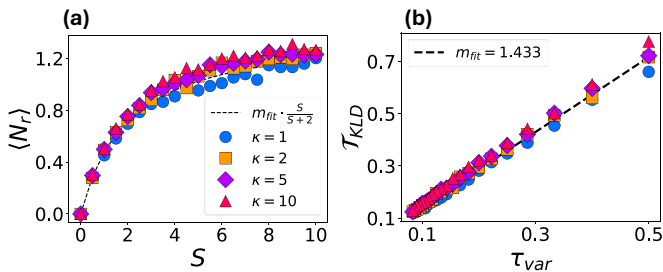


FIG. 8. (a) $\langle N_r \rangle$ for various potential stiffnesses and plotted against S . (b) The scaling between the relaxation of the distribution (vertical) and the relaxation of the leading moment (horizontal) is numerically measured by linear fit.

Appendix E: Derivation of the time-dependent moments.

We derive the moments of the probability distribution for a centered ($\langle x \rangle = 0$) Ornstein-Uhlenbeck process under Poisson stochastic resetting with rate r and an arbitrary resetting point X_r . The renewal equation states that

$$P(x, t|x_0) = G_0(x, t|x_0)e^{-rt} + r \int_0^t G_0(x, t|X_r)e^{-r\tau} d\tau, \quad (E1)$$

where $G_0(x, t|x_0)$ is the centered O-U propagator with an initial condition $G_0(x, 0) = \delta(x - x_0)$ and, here, we enforce $X_r = x_0$ i.e., the resetting point coincides with the initial position. The complete propagator is given in Eq. (2). Our aim is not to solve the last renewal equation for the general time-dependent probability distribution under resetting. We will instead calculate the time-dependent first and second moments. To do so, we multiply both sides by x^n and integrate with respect to x . This gives us a time-dependent expression for the n th moment of the distribution under resetting, denoted by $\langle x^n \rangle^{SR}(t)$.

Left-hand side:

$$\int_{-\infty}^{\infty} x^n P(x, t|x_0) dx = \langle x^n \rangle^{SR}(t). \quad (E2)$$

Right-hand Side:

$$\begin{aligned} & \left[\int_{-\infty}^{\infty} x^n G_0(x, t|x_0) dx \right] e^{-rt} \\ & + r \int_0^t e^{-r\tau} \int_{-\infty}^{\infty} x^n G_0(x, t|X_r) dx d\tau \\ & = \langle x^n \rangle^{eq}(t) e^{-rt} + r \int_0^t e^{-r\tau} \langle x^n \rangle^{eq}(\tau) d\tau, \end{aligned} \quad (E3)$$

where $\langle x^n \rangle^{eq}(t)$ is the time-dependent moment of the corresponding thermal system ($r = 0$). Equating the two terms, we arrive at an equation that allows us to compute a general time-dependent moment

$$\langle x^n \rangle^{SR}(t) = \langle x^n \rangle^{eq}(t) e^{-rt} + r \int_0^t e^{-r\tau} \langle x^n \rangle^{eq}(\tau) d\tau. \quad (E4)$$

Using Eq. (E4), we derive the following expressions for the moments, $\omega_0 = \kappa/\gamma$:

$$\begin{aligned} \langle x \rangle^{SR}(t) &= x_0 e^{-(r+\omega_0)t} + \frac{x_0 r}{r + \omega_0} (1 - e^{-(r+\omega_0)t}), \\ \langle x^2 \rangle^{SR}(t) &= \frac{2D + rx_0^2}{r + 2\omega_0} + \frac{2\omega_0 x_0^2 - 2D}{r + 2\omega_0} e^{-(r+2\omega_0)t}. \end{aligned} \quad (E5)$$

In terms of the dimensionless parameters presented in the main text, S , $\tilde{x}$, $\tilde{t}$, and $\tilde{x}_0$, which is the dimensionless starting position x_0/α , we obtain

$$\begin{aligned} \langle \tilde{x} \rangle^{SR}(\tilde{t}) &= \tilde{x}_0 e^{-(S+1)\tilde{t}} + \frac{\tilde{x}_0 S}{S+1} (1 - e^{-(S+1)\tilde{t}}), \\ \langle \tilde{x}^2 \rangle^{SR}(\tilde{t}) &= \frac{2\tilde{x}_0^2 + 2S}{S+2} + \frac{2\tilde{x}_0^2 - 2S}{S+2} e^{-(S+2)\tilde{t}}. \end{aligned} \quad (E6)$$

The timescales of the first and second moments are $(S+1)^{-1}$ and $(S+2)^{-1}$, respectively. However, for the thermal system,

in this dimensionless form the timescales for the moments are 1 and $1/2$, respectively. Thus, the ratios of the moments'

timescales are $(S + 1)/1$ for the mean and $(S + 2)/2$ for the second moment.

-
- [1] D. Guéry-Odelin, A. Ruschhaupt, A. Kiely, E. Torrontegui, S. Martínez-Garaot, and J. Muga, Shortcuts to adiabaticity: Concepts, methods, and applications, *Rev. Mod. Phys.* **91**, 045001 (2019).
 - [2] I. A. Martínez, A. Petrosyan, D. Guéry-Odelin, E. Trizac, and S. Ciliberto, Engineered swift equilibration of a Brownian particle, *Nat. Phys.* **12**, 843 (2016).
 - [3] M. Baldovin, D. Guéry-Odelin, and E. Trizac, Control of active Brownian particles: An exact solution, *Phys. Rev. Lett.* **131**, 118302 (2023).
 - [4] D. Raynal, T. de Guillebon, D. Guéry-Odelin, E. Trizac, J.-S. Lauret, and L. Rondin, Shortcuts to equilibrium with a levitated particle in the underdamped regime, *Phys. Rev. Lett.* **131**, 087101 (2023).
 - [5] T. Schmiedl and U. Seifert, Optimal finite-time processes in stochastic thermodynamics, *Phys. Rev. Lett.* **98**, 108301 (2007).
 - [6] S. A. Loos, S. Monter, F. Ginot, and C. Bechinger, Universal symmetry of optimal control at the microscale, *Phys. Rev. X* **14**, 021032 (2024).
 - [7] E. Aurell, C. Mejía-Monasterio, and P. Muratore-Ginanneschi, Optimal protocols and optimal transport in stochastic thermodynamics, *Phys. Rev. Lett.* **106**, 250601 (2011).
 - [8] K. Proesmans, J. Ehrich, and J. Bechhoefer, Finite-time Landauer principle, *Phys. Rev. Lett.* **125**, 100602 (2020).
 - [9] K. Proesmans, J. Ehrich, and J. Bechhoefer, Optimal finite-time bit erasure under full control, *Phys. Rev. E* **102**, 032105 (2020).
 - [10] R. Goerlich, T. D. Keidar, and Y. Roichman, Resetting as a swift equilibration protocol in an anharmonic potential, *Phys. Rev. Res.* **6**, 033162 (2024).
 - [11] M. R. Evans and S. N. Majumdar, Diffusion with stochastic resetting, *Phys. Rev. Lett.* **106**, 160601 (2011).
 - [12] M. R. Evans, S. N. Majumdar, and G. Schehr, Stochastic resetting and applications *J. Phys. A: Math. Theor.* **53**, 193001 (2020).
 - [13] S. C. Manrubia and D. H. Zanette, Stochastic multiplicative processes with reset events, *Phys. Rev. E* **59**, 4945 (1999).
 - [14] I. Eliazar, T. Koren, and J. Klafter, Searching circular DNA strands, *J. Phys.: Condens. Matter* **19**, 065140 (2007).
 - [15] M. R. Evans and S. N. Majumdar, Diffusion with optimal resetting, *J. Phys. A: Math. Theor.* **44**, 435001 (2011).
 - [16] M. R. Evans and S. N. Majumdar, Diffusion with resetting in arbitrary spatial dimension, *J. Phys. A: Math. Theor.* **47**, 285001 (2014).
 - [17] L. Kusmierz, S. N. Majumdar, S. Sabhapandit, and G. Schehr, First order transition for the optimal search time of Lévy flights with resetting, *Phys. Rev. Lett.* **113**, 220602 (2014).
 - [18] Ł. Kuśmierz and E. Gudowska-Nowak, Optimal first-arrival times in Lévy flights with resetting, *Phys. Rev. E* **92**, 052127 (2015).
 - [19] U. Bhat, C. De Bacco, and S. Redner, Stochastic search with Poisson and deterministic resetting, *J. Stat. Mech.* (2016) 083401.
 - [20] A. Pal and S. Reuveni, First passage under restart, *Phys. Rev. Lett.* **118**, 030603 (2017).
 - [21] A. Pal and V. V. Prasad, First passage under stochastic resetting in an interval, *Phys. Rev. E* **99**, 032123 (2019).
 - [22] P. C. Bressloff, Search processes with stochastic resetting and multiple targets, *Phys. Rev. E* **102**, 022115 (2020).
 - [23] R. K. Singh, T. Sandev, A. Iomin, and R. Metzler, Backbone diffusion and first-passage dynamics in a comb structure with confining branches under stochastic resetting, *J. Phys. A: Math. Theor.* **54**, 404006 (2021).
 - [24] O. Tal-Friedman, A. Pal, A. Sekhon, S. Reuveni, and Y. Roichman, Experimental realization of diffusion with stochastic resetting, *J. Phys. Chem. Lett.* **11**, 7350 (2020).
 - [25] A. Pal, V. Stojkoski, and T. Sandev, Random resetting in search problems, in *Target Search Problems*, edited by D. Grebenkov, R. Metzler, and G. Oshanin (Springer Nature Switzerland, Cham, 2024), pp. 323–355.
 - [26] M. R. Evans, S. N. Majumdar, and G. Schehr, Stochastic resetting and applications, *J. Phys. A: Math. Theor.* **46**, 185001 (2013).
 - [27] P. C. Bressloff, Queueing theory of search processes with stochastic resetting, *Phys. Rev. E* **102**, 032109 (2020).
 - [28] A. Pal, S. Kostinski, and S. Reuveni, The inspection paradox in stochastic resetting, *J. Phys. A: Math. Theor.* **55**, 021001 (2022).
 - [29] O. L. Bonomo, A. Pal, and S. Reuveni, Mitigating long queues and waiting times with service resetting, *PNAS Nexus* **1**, pgac070 (2022).
 - [30] R. Roy, A. Biswas, and A. Pal, Queues with resetting: A perspective, *J. Phys. Complex.* **5**, 021001 (2024).
 - [31] O. Blumer, S. Reuveni, and B. Hirshberg, Combining stochastic resetting with metadynamics to speed-up molecular dynamics simulations, *Nat. Commun.* **15**, 240 (2024).
 - [32] O. Blumer, S. Reuveni, and B. Hirshberg, Stochastic resetting for enhanced sampling, *J. Phys. Chem. Lett.* **13**, 11230 (2022).
 - [33] O. Blumer and B. Hirshberg, Have you tried turning it off and on again? Stochastic resetting for enhanced sampling, *Wires Comput. Molecular Sci.* **15**, e70038 (2025).
 - [34] S. Reuveni, M. Urbakh, and J. Klafter, Role of substrate unbinding in Michaelis–Menten enzymatic reactions, *Biophys. Comput. Biol.* **111**, 4391 (2014).
 - [35] Y. Scher and S. Reuveni, Unified approach to gated reactions on networks, *Phys. Rev. Lett.* **127**, 018301 (2021).
 - [36] P. C. Bressloff, Diffusion-mediated surface reactions and stochastic resetting, *J. Phys. A: Math. Theor.* **55**, 275002 (2022).
 - [37] A. Biswas, A. Pal, D. Mondal, and S. Ray, Rate enhancement of gated drift-diffusion process by optimal resetting, *J. Chem. Phys.* **159**, 054111 (2023).
 - [38] S. Gupta, S. N. Majumdar, and G. Schehr, Fluctuating interfaces subject to stochastic resetting, *Phys. Rev. Lett.* **112**, 220601 (2014).
 - [39] X. Durang, M. Henkel, and H. Park, The statistical mechanics of the coagulation–diffusion process with a stochastic reset, *J. Phys. A: Math. Theor.* **47**, 045002 (2014).

- [40] R. K. Singh, K. Gorska, and T. Sandev, General approach to stochastic resetting, *Phys. Rev. E* **105**, 064133 (2022).
- [41] A. Pal, Diffusion in a potential landscape with stochastic resetting, *Phys. Rev. E* **91**, 012113 (2015).
- [42] A. Pal, A. Kundu, and M. R. Evans, Diffusion under time-dependent resetting, *J. Phys. A: Math. Theor.*, **49** 225001 (2016).
- [43] M. R. Evans and S. N. Majumdar, Run and tumble particle under resetting: A renewal approach, *J. Phys. A: Math. Theor.* **51**, 475003 (2018).
- [44] S. N. Majumdar, S. Sabhapandit, and G. Schehr, Dynamical transition in the temporal relaxation of stochastic processes under resetting, *Phys. Rev. E* **91**, 052131 (2015).
- [45] D. Gupta, Stochastic resetting in underdamped Brownian motion, *J. Stat. Mech.* (2019) 033212.
- [46] R. K. Singh, R. Metzler, and T. Sandev, Resetting dynamics in a confining potential, *J. Phys. A: Math. Theor.* **53**, 505003 (2020).
- [47] G. García-Valladares, D. Gupta, A. Prados, and C. A. Plata, Stochastic resetting with refractory periods: Pathway formulation and exact results *Phys. Scr.* **99**, 045234 (2024).
- [48] P. Trajanovski, P. Jolakoski, K. Zelenkovski, A. Iomin, L. Kocarev, and T. Sandev, Ornstein-Uhlenbeck process and generalizations: Particle dynamics under comb constraints and stochastic resetting, *Phys. Rev. E* **107**, 054129 (2023).
- [49] I. A. Martínez, É. Roldán, L. Dinis, and R. A. Rica, Colloidal heat engines: A review, *Soft Matter* **13**, 22 (2017).
- [50] S. Lahiri and S. Gupta, Efficiency of a microscopic heat engine subjected to stochastic resetting, *Phys. Rev. E* **109**, 014129 (2024).
- [51] A. Le Cunuder, I. A. Martínez, A. Petrosyan, D. Guéry-Odelin, E. Trizac, and S. Ciliberto, Fast equilibrium switch of a micro mechanical oscillator, *Appl. Phys. Lett.* **109**, 113502 (2016).
- [52] H. Risken, The Fokker-Planck equation methods of solution and applications, in *The Fokker-Planck Equation* (Springer, 1989), Chap. 5, pp. 96–125.
- [53] R. Goerlich, M. Li, L. B. Pires, P.-A. Hervieux, G. Manfredi, and C. Genet, Taming a Maxwell’s demon for experimental stochastic resetting, *Phys. Rev. E* **112**, 064116 (2025).
- [54] G. Volpe and G. Volpe, Simulation of a Brownian particle in an optical trap, *Am. J. Phys.* **81**, 224 (2013).
- [55] B. Besga, A. Bovon, A. Petrosyan, S. N. Majumdar, and S. Ciliberto, Optimal mean first-passage time for a Brownian searcher subjected to resetting: Experimental and theoretical results, *Phys. Rev. Res.* **2**, 032029(R) (2020).
- [56] R. Goerlich, K. S. Olsen, H. Löwen, and Y. Roichman, Time-energy tradeoff in stochastic resetting using optimal control, *Phys. Rev. E*, **113**, 014103 (2026).
- [57] B. De Bruyne and F. Mori, Resetting in stochastic optimal control, *Phys. Rev. Res.* **5**, 013122 (2023).
- [58] D. A. Haggerty, M. J. Banks, E. Kamenar, A. B. Cao, P. C. Curtis, I. Mezić, and E. W. Hawkes, Control of soft robots with inertial dynamics, *Sci. Rob.* **8**, (2023).
- [59] C. Hernández-López, P. Baconnier, C. Coulais, O. Dauchot, and G. Düring, Model of active solids: Rigid body motion and shape-changing mechanisms, *Phys. Rev. Lett.* **132**, 238303 (2024).
- [60] J. Fuchs, S. Goldt, and U. Seifert, Stochastic thermodynamics of resetting, *Europhys. Lett.* **113**, 60009 (2016).
- [61] J. R. Church, O. Blumer, T. D. Keidar, L. Ploutno, S. Reuveni, and B. Hirshberg, Accelerating molecular dynamics through informed resetting, *J. Chem. Theory Comput.* **21**, 605 (2025).
- [62] N. Sherf, nirs199820/FasterRelaxationDifferentDestination: v0.1 (v0.1), Zenodo (2025), <https://dx.doi.org/10.5281/zenodo.17710563>.