

Near-ultimate quantum-enhanced sensitivity in dissipative critical sensing with partial access

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Quantum sensors are powerful devices that exploit quantum effects to detect minute quantities with extremely high precision. Two obstacles to harnessing the full capacity of quantum probes are the resource-intensive preparation of the probe and the need for sophisticated measurements that typically require full access to the entire probe. Here, we address these challenges by investigating the driven Jaynes-Cummings system undergoing a dissipative quantum phase transition as a quantum sensor. We show that detuning the system off resonance significantly improves sensing performance by adequately selecting a preferred bistable state in phase space. Our dissipative sensor, independent of the initial probe preparation, exhibits sensitivity that scales faster than linearly with respect to an effective thermodynamic resource—the strong-coupling regime ratio—for appropriate choices of parameters, and this enhanced scaling manifests in both the full system and the field subsystem. Hence, quantum-enhanced sensitivity persists even when only partial system accessibility is available. Remarkably, we show that a homodyne detection of the field state, combined with Bayesian estimation, nearly saturates the ultimate sensitivity limit of the entire system.

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I. INTRODUCTION

By exploiting distinctive quantum phenomena, quantum sensors have been shown to surpass the standard quantum limit of precision [1–7], enabling a wide range of high-precision field estimations [8–16] as well as the estimation of both Hamiltonian [17–28] and non-Hamiltonian parameters [29–40]. Among the diverse platforms and sensing strategies [41–49], critical quantum sensors [18,24,50–62] stand out as promising candidates for achieving quantum-enhanced sensing by efficiently using available resources [18,58]. Intuitively, a system near a phase transition can act as a quantum sensor, since infinitesimal changes in the control parameter induce divergent responses [5]. In particular, the ground state of a quantum many-body system at a quantum phase transition has been extensively explored for this purpose [17,18,56–59,63–69]. However, their applicability faces several challenges: (1) preparing the ground state of a many-body system is demanding; (2) exploiting the enhanced sensitivity often requires unfeasible measurements across the entire system;

and (3) restricting measurements to subsystems can significantly reduce the achievable sensing performance. Failure to meet any of these conditions usually nullifies any quantum advantage. It is therefore desirable to identify a system that achieves quantum-enhanced sensitivity with local, feasible measurements and without complex state preparation.

To overcome the accessibility challenge, certain probes have been shown to extract either part [8,9,22,70] or all [14,15,71] of the relevant information from only a reduced portion of the system. However, there is no universal sensing method that can reach or approach the full sensitivity of the entire system by measuring only part of it. To address the state-preparation challenge, probes undergoing dissipative quantum phase transitions [72–76], which occur due to the interplay between Hamiltonian dynamics and Lindbladian processes [74], have been explored as an alternative. In this approach, instead of preparing a specific initial state, the system evolves toward a steady state that exhibits critical properties suitable for sensing [77]. Indeed, dissipative quantum phase transitions, which have been extensively studied theoretically [53,78–98] and realized experimentally [81,99–112], have also been proposed as a resource for quantum sensing [77,85,113–119]. Even though dissipative probes solve the state-preparation challenge, it is still unknown whether they can reach quantum-enhanced sensitivity when only practical, partial-access measurements are allowed. In this work, we show that they can.

In this work, we study the driven-dissipative Jaynes-Cummings (JC) system as a quantum sensor undergoing a dissipative quantum phase transition [87] (see also

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Refs. [50,120]). This model is paradigmatic, capturing the essential physics of a broad class of light-matter interaction systems [121], and has been experimentally demonstrated across multiple distinct platforms [105,122,123]. Most notably, the dissipative quantum phase transition at the heart of our proposal has already been directly observed in experiments [107,124]. In the thermodynamic limit, this system exhibits quantum-critical behavior characterized by polarization of the two-level (qubit) system and bistability of the field mode. Remarkably, at the critical point, the steady state becomes weakly entangled in the thermodynamic limit, effectively encoding the unknown parameter in the field subsystem and suggesting that local probing can attain the system's ultimate sensitivity. We present three main results. First, we demonstrate that detuning the system off resonance favors one of the two bistable states, leading to a significant enhancement in sensing precision. Second, our sensor achieves quantum-enhanced sensitivity off resonance, under both full and partial probing. Third, a feasible homodyne field measurement combined with a Bayesian estimator nearly saturates the ultimate sensing precision of the whole quantum state. Even with this practical local measurement, the quantum-enhanced sensitivity is clearly demonstrated.

Our work is organized as follows. In Sec. II, we briefly introduce the tools of quantum parameter estimation. In Sec. III, we describe the open dynamics of the quantum probe under consideration. In Sec. IV, we analyze the quantum sensing limits at qubit-field resonance. In Sec. V, we study the scaling of the sensing performance of each subsystem with respect to the relevant sensing resource. In Sec. VI, we investigate the sensing precision limits in the off-resonant regime. In Sec. VII, we determine the corresponding scaling behavior for each subsystem off resonance. In Sec. VIII, we present a detailed analysis of an explicit estimation protocol based on Bayesian inference. In Sec. IX, we examine the robustness of the sensor near optimal operating parameters. In Sec. X, we discuss the experimental feasibility of our proposal. Finally, in Sec. XI, we summarize our conclusions.

II. QUANTUM ESTIMATION BACKGROUND

For a given measurement, the uncertainty in estimating an unknown parameter θ parametrized in a quantum state $\rho(\theta)$ is lower bounded by $\text{Var}[\tilde{\theta}] \geq [M\mathcal{F}(\theta)]^{-1}$ [125–127]. Here, $\text{Var}[\tilde{\theta}]$ is the variance of the unbiased local estimator $\tilde{\theta}$, M is the number of measurement trials, $\mathcal{F}(\theta) = \sum_k [p_k(\theta)]^{-1} (\partial p_k(\theta)/\partial \theta)^2$ is the classical Fisher information (CFI) [128], and $p_k = \text{Tr}[\Pi_k \rho(\theta)]$ is the probability distribution for a given positive operator-valued measure (POVM) Π_k with measurement outcome k . By maximizing over all POVMs, a more fundamental lower bound known as quantum Cramér-Rao theorem is derived [128–132]:

$$\text{Var}[\tilde{\theta}] \geq \frac{1}{M\mathcal{F}(\theta)} \geq \frac{1}{MQ(\theta)}, \quad (1)$$

where $Q(\theta) = \max_{\{\Pi_k\}} \mathcal{F}(\theta)$ is the quantum Fisher information (QFI), given by [128]

$$Q(\theta) = 2 \sum_{m,n} \frac{|\langle \psi_m | \partial_\theta \rho(\theta) | \psi_n \rangle|^2}{\lambda_m + \lambda_n}, \quad (2)$$

where the sum runs over all pairs (m, n) with $\lambda_m + \lambda_n > 0$. In the above, $\rho(\theta) = \sum_i \lambda_i |\psi_i\rangle \langle \psi_i|$ is expressed in spectral decomposition with eigenvalue λ_i and corresponding eigenvector $|\psi_i\rangle$. While the CFI determines the best possible precision for estimating an unknown parameter given a specific measurement, the QFI sets the ultimate precision limit for estimating a parameter encoded in the quantum state $\rho(\theta)$. A higher QFI corresponds to a lower estimation uncertainty.

III. THE MODEL

We consider the dissipative quantum dynamics of a driven qubit-field system described by the master equation [87]

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H_{\text{int}}, \rho] + \kappa(2a\rho a^\dagger - \rho a^\dagger a - a^\dagger a \rho), \quad (3)$$

where a and $a^\dagger$ are the annihilation and creation operators of the field mode, κ is the decay rate, and H_{int} is the Hamiltonian in the frame rotating at the drive frequency Ω . It is obtained from the laboratory-frame Hamiltonian H_{DJC} by the time-dependent unitary transformation $T = \exp[i\Omega t(a^\dagger a + \sigma^+ \sigma^-)]$ as

$$H_{\text{int}} = T H_{\text{DJC}} T^\dagger + i\hbar(\partial_t T) T^\dagger \quad (4)$$

$$= T H_{\text{DJC}} T^\dagger - \hbar\Omega(a^\dagger a + \sigma^+ \sigma^-). \quad (5)$$

The laboratory-frame Hamiltonian H_{DJC} describes a driven Jaynes-Cummings interaction between a qubit and a field mode,

$$\begin{aligned} \frac{H_{\text{DJC}}}{\hbar} &= \omega_f a^\dagger a + \omega_q \sigma^+ \sigma^- + g(\sigma^+ a + \sigma^- a^\dagger) \\ &+ \mathcal{E}(ae^{i\Omega t} + a^\dagger e^{-i\Omega t}), \end{aligned} \quad (6)$$

where ω_f is the field resonance frequency, ω_q is the qubit transition frequency, g is the qubit-field coupling strength, $\mathcal{E}$ is the amplitude of the external drive, and σ^+ , σ^- are the qubit raising and lowering operators. Assuming $\omega_f = \omega_q = \omega$, the Hamiltonian simplifies to the time-independent form

$$H_{\text{int}} = -\hbar\Delta(a^\dagger a + \sigma^+ \sigma^-) + \hbar g(\sigma^+ a + \sigma^- a^\dagger) + \hbar\mathcal{E}(a + a^\dagger), \quad (7)$$

where $\Delta = \Omega - \omega$ is the detuning between the drive frequency Ω and the subsystem frequency ω . In this work, we focus on estimating $\mathcal{E}$ from the steady state ρ_{ss} of Eq. (3). Following Ref. [87], we take the ratio

$$N = \left(\frac{g}{2\kappa}\right)^2 \quad (8)$$

as a bona-fide sensing resource, which quantifies the competition between coherent interactions and incoherent losses (for a detailed discussion of the thermodynamic limit, see Ref. [87]). All results presented in this work are obtained through numerical simulations performed using the Python-based quantum simulation toolbox QuTiP [133].

Note that the drive amplitude $\mathcal{E}$ acts on the field mode in exactly the same way as a linear displacement of a harmonic oscillator. To make this explicit, consider the quadratic field Hamiltonian

$$H_{\text{osc}} = \hbar|\Delta|a^\dagger a + \hbar\mathcal{E}(a^\dagger + a), \quad (9)$$

which differs from the field sector of Eq. (7) only by an overall sign convention for Δ and an additive constant, with $|\Delta|$ setting the relevant frequency scale. Introducing the canonical quadratures

$$a = \sqrt{\frac{|\Delta|}{2\hbar}} \left(x + \frac{ip}{|\Delta|} \right), \quad a^\dagger = \sqrt{\frac{|\Delta|}{2\hbar}} \left(x - \frac{ip}{|\Delta|} \right) \quad (10)$$

implies

$$x = \sqrt{\frac{\hbar}{2|\Delta|}} (a + a^\dagger), \quad p = i\sqrt{\frac{\hbar|\Delta|}{2}} (a^\dagger - a), \quad (11)$$

which satisfy $[x, p] = i\hbar$. In terms of these quadratures, one finds

$$H_{\text{osc}} = \frac{p^2}{2} + \frac{|\Delta|^2}{2} (x + x_0)^2 + \text{const}, \quad (12)$$

with

$$x_0 = \mathcal{E} \sqrt{\frac{2\hbar}{|\Delta|^3}}. \quad (13)$$

Thus, the linear drive term $\hbar\mathcal{E}(a^\dagger + a)$ shifts the equilibrium point of the effective oscillator by x_0 . Equivalently, the potential can be written as

$$V(x) \sim \frac{|\Delta|^2}{2} x^2 - Fx, \quad (14)$$

where the effective constant force is

$$F = -|\Delta|^2 x_0 = -\sqrt{2\hbar|\Delta|}\mathcal{E}. \quad (15)$$

Once $|\Delta|$ is fixed, the displacement x_0 , the drive amplitude $\mathcal{E}$, and the magnitude $|F|$ of this effective force are in one-to-one correspondence—estimating any one of them is therefore equivalent for metrological purposes.

IV. ON-RESONANCE QUANTUM-ENHANCED SENSOR

We first focus on the sensing capabilities of the dissipative probe when it operates on resonance, $\Delta = 0$ in Eq. (7). In Figs. 1(a)–1(c), we plot the QFI for the entire probe $\mathcal{Q}_{\text{whole}}$ as well as for the field (qubit) subsystem $\mathcal{Q}_{\text{field}}$ ($\mathcal{Q}_{\text{qubit}}$) obtained from the reduced steady state $\rho_{\text{SS}}^{\text{field}} = \text{Tr}_{\text{qubit}}[\rho_{\text{SS}}]$ ($\rho_{\text{SS}}^{\text{qubit}} = \text{Tr}_{\text{field}}[\rho_{\text{SS}}]$) as a function of $\mathcal{E}$ for selected values of N . Several qualitative insights can be drawn from Figs. 1(a)–1(c). First, all QFI functions increase as the sensing resource ratio N becomes larger. This supports the choice of N as sensing resource.

Note that g is taken as the reference frequency throughout. Since the drive amplitude $\mathcal{E}$ has units of frequency, it is natural to introduce the (scaled by g) dimensionless parameter $\mathcal{E}/g$ and to consider the QFI with respect to $\mathcal{E}/g$. By the chain rule, $\mathcal{Q}(\mathcal{E}/g) = g^2 \mathcal{Q}(\mathcal{E})$, where $\mathcal{Q}(\mathcal{E}) \equiv \mathcal{Q}$ is the QFI with respect to $\mathcal{E}$. In practice, we therefore plot the dimensionless combination $g^2 \mathcal{Q}(\mathcal{E}) = g^2 \mathcal{Q}$, which fixes the units of the QFI and does not affect its dependence on the sensing parameter $N = (g/2\kappa)^2$.

Second, the QFI of the field subsystem closely follows the behavior and values of the entire system, whereas the qubit subsystem does not. Hence, the field encodes significantly more information about $\mathcal{E}$ than the qubit, which is

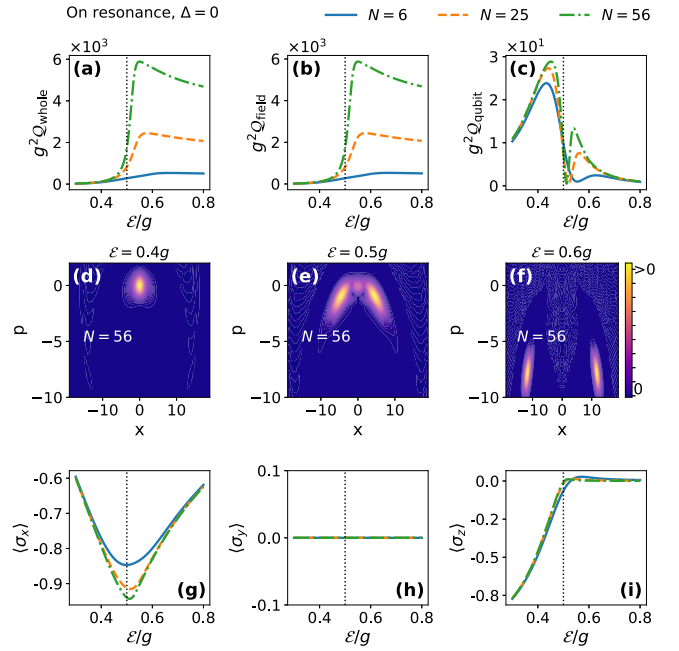


FIG. 1. On-resonance case, $\Delta = 0$: (a)–(c) QFI for the entire probe $\mathcal{Q}_{\text{whole}}$, the field subsystem $\mathcal{Q}_{\text{field}}$, and qubit subsystem $\mathcal{Q}_{\text{qubit}}$, as a function of $\mathcal{E}$ for some choices of N . (d)–(f) Wigner function of the field subsystem as a function of the phase-space variables x and p for $N = 56$ and for several values of $\mathcal{E}/g$. (g)–(i) Expectation values of the qubit components $\langle\sigma_\alpha\rangle$ ($\alpha = x, y, z$) as a function of $\mathcal{E}/g$ and various values of the ratio N .

also evident in the magnitude of the QFIs. Third, there is an abrupt change across all QFI functions near $2\mathcal{E} \sim g$. Fourth, the QFI exhibits a plateau with a moderate drop of its maximum value for parameters $2\mathcal{E} \gtrsim g$. The above observed behavior can be explained by the fact that the system undergoes a well-established dissipative quantum phase transition at the critical point $2\mathcal{E} = g$, in the *thermodynamic limit*, where $N \gg 1$ [87]. Specifically, for $2\mathcal{E} < g$, the quasienergy spectrum of H_{int} is discrete with eigenvalues $E_0 = 0$ and doublets given by $E_{n,\pm} = \pm \sqrt{n\hbar g [1 - (\frac{2\mathcal{E}}{g})^2]^{3/4}}$, with $n \in \mathbb{Z}_+$. Remarkably, at the critical point $2\mathcal{E} = g$, all quasienergies collapse to zero. For $2\mathcal{E} > g$, the discrete spectrum disappears and becomes continuous. Thus, the point $2\mathcal{E} = g$ marks and organizes a dissipative quantum phase transition [87]. Note that Figs. 1(a)–1(c) address key challenges: First, they show a definitive sensing enhancement of the unknown drive amplitude $\mathcal{E}$, irrespective of the initial state. Second, they tackle the partial-access challenge by demonstrating that information about $\mathcal{E}$ is predominantly encoded in the field subsystem, implying that local probing of the field alone may suffice to reveal this enhancement. The latter can be intuitively explained since the unknown parameter $\mathcal{E}$ is locally imprinted onto the never-saturating infinite-dimensional field subsystem. Hence, the excitation cascade triggered by the photon blockade breakdown can always populate additional regions of the Hilbert space as N increases.

To gain deeper insight into the probe's dynamics, in Figs. 1(d)–1(f) we plot the Wigner quasiprobability function of the field subsystem, defined as

$W(x, p) = \frac{1}{\pi\hbar} \int_{-\infty}^{\infty} \langle x+y | \rho_{\text{field}} | x-y \rangle e^{-2ipy/\hbar} dy$, as a function of the phase-space variables x and p for (an arbitrary choice of) $N = 56$ and various $\mathcal{E}/g$. As shown in Figs. 1(d)–1(f), $W(x, p)$ remains strictly positive for all values of $\mathcal{E}/g$, indicating the absence of any Wigner-function negativity. This rules out one common signature of strong nonclassicality, even though more refined notions of nonclassicality (e.g., based on the Glauber-Sudarshan P function or photon statistics) may still be present. More importantly, the field subsystem undergoes an abrupt transition from a vacuumlike state to a quasiprobability distribution characterized by two well-separated peaks. This bistable behavior emerges around $2\mathcal{E} \sim g$, with the critical point $2\mathcal{E} = g$ in the thermodynamic limit $N \rightarrow \infty$ marking the sudden departure from the vacuumlike to the bistable state [87]. Note that the vacuumlike state originates from the photon blockade effect, while the emergence of two well-separated peaks results from the semiclassical steady-state solutions of the field amplitude α , which admit two stable values of equal magnitude and opposite phase [87]. In Figs. 1(g)–1(i), we plot the expectation values of the qubit components, denoted as $\langle \sigma_\alpha \rangle$ for $\alpha = x, y, z$, as a function of $\mathcal{E}/g$ and several values of the ratio N .

As shown in Figs. 1(g)–1(i), $\langle \sigma_y \rangle$ is always zero— independent of the ratio $\mathcal{E}/g$, whereas the expectation value along the z -axis transitions from nonzero when $2\mathcal{E} < g$ to approximately zero when $2\mathcal{E} > g$. Along the x axis, the qubit subsystem acquires a nonzero expectation value across all values of $\mathcal{E}/g$. Notably, it approaches $\langle \sigma_x \rangle \rightarrow -1$ as $N \gg 1$, signaling near-complete polarization in the $|- \rangle$ eigenstate of σ_x . In the thermodynamic limit, the dissipative critical point is associated with spontaneous breaking of a symmetry between the two dressed states $|\pm\rangle = (|e\rangle \pm |g\rangle)/\sqrt{2}$ [87], so that the steady state becomes only weakly entangled. Note that there is no guarantee that such a combination of weak entanglement and partial purity in the dressed state will generally occur in dissipative quantum phase transitions. In the parameter regime explored here, our numerics select the $|- \rangle$ branch, and the steady state is well approximated by

$$\rho_{\text{ss}} \approx |- \rangle \langle - | \otimes \rho_{\text{field}}(\mathcal{E}), \quad (16)$$

with $\rho_{\text{field}}(\mathcal{E})$ the bistable field state. This factorization clearly demonstrates that the strongest dependence on $\mathcal{E}$ is encoded in the field subsystem ρ_{field} , thus making enhanced sensitivity achievable via local field probing alone. See the Appendix for entanglement and purity analysis.

V. ON-RESONANCE SCALING ANALYSIS

We now analyze how the precision scales with the sensing resource N , considering both the entire system and its subsystems. To this end, we define the driving strength that yields the maximum QFI as

$$\left(\frac{\mathcal{E}}{g}\right)_j^* = \arg \max_{\left\{\left(\frac{\mathcal{E}}{g}\right)\right\}} \mathcal{Q}_j(\mathcal{E}) \Big|_{\Delta=0}, \quad (17)$$

where j accounts for the whole, qubit, and field parties. In Fig. 2(a), we show how the optimal driving amplitude $(\mathcal{E}/g)^*$ varies with the sensing resource N for the entire system and its subsystems. The optimal values $(\mathcal{E}/g)^*$ for the entire system

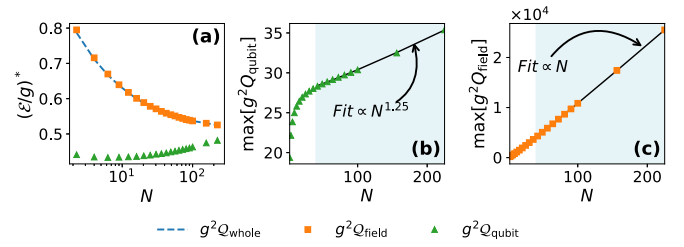


FIG. 2. (a) Optimal driving amplitude $(\mathcal{E}/g)^*$ as a function of N . (b) Maximum QFI for the qubit subsystem as a function of N . (c) Maximum QFI as a function of the sensing resource N for both the entire system and the field subsystem. In panels (b) and (c), the exponent B obtained from fitting the data is explicitly shown in each plot.

and the field subsystem nearly coincide across the full range of N , whereas the qubit subsystem exhibits a different trend. Notably, as N increases, the optimal drive amplitude $(\mathcal{E}/g)^*$ for all three cases converges toward the value $1/2$, consistent with the critical point expected at resonance. In Fig. 2(b), we plot the maximum QFI for the qubit subsystem as a function of N . To capture this behavior, we fit the data using a function of the form $AN^B + C$. To exclude finite-size effects, we fit the curve using only data points with $N > 20$, highlighted by the light blue shaded region. Beyond this point, the data follow a polynomial trend with an exponent $B \approx 1.25$, i.e., a superlinear growth in the effective system-size parameter N . Within Carmichael’s thermodynamic-limit construction, where $N = (g/2\kappa)^2$ plays the role of an effective “system size” [87], this indicates that the sensitivity improves faster than linearly as the strong-coupling parameter is increased. In Fig. 2(c), we plot the maximum QFI as a function of the sensing resource N for both the entire system and the field subsystem. The plot reveals a similar trend in both cases, characterized by clear polynomial growth. A fitting analysis yields a linear $B \approx 1$ exponent, as shown in the main plot.

VI. OFF-RESONANCE QUANTUM-ENHANCED SENSOR

Although the dissipative probe at resonance is robust and highly precise, it is natural to ask whether it is indeed the optimal probe. To address this, we now consider $\Delta \neq 0$ in Eq. (7). Specifically, we analyze the sensing precision, for estimating $\mathcal{E}$, at the optimal detuning $(\frac{\Delta}{g})_j^*$ and optimal driving amplitude $(\frac{\mathcal{E}}{g})_j^*$, which maximizes the QFI $\mathcal{Q}_j$ for each $j = \text{whole, field, qubit system}$. These optimal values are defined as

$$\left[\left(\frac{\Delta}{g}\right)_j^*, \left(\frac{\mathcal{E}}{g}\right)_j^*\right] = \arg \max_{\left\{\left(\frac{\Delta}{g}, \frac{\mathcal{E}}{g}\right)\right\}} \mathcal{Q}_j(\mathcal{E}, \Delta). \quad (18)$$

For the purpose of this section, we focus on the detuning that maximizes $\mathcal{Q}_{\text{field}}$ and refer to it simply as $\Delta \neq 0$. All explicit values of $(\Delta/g)_j^*$ and $(\mathcal{E}/g)_j^*$ as shown in Eq. (18) will be examined in later sections.

In Figs. 3(a)–3(c), we plot the QFI for the entire probe $\mathcal{Q}_{\text{whole}}$, the field subsystem $\mathcal{Q}_{\text{field}}$, and the qubit subsystem $\mathcal{Q}_{\text{qubit}}$ as a function of $\mathcal{E}/g$ for some choices of N . As shown in Figs. 3(a)–3(c), the dissipative probe achieves higher

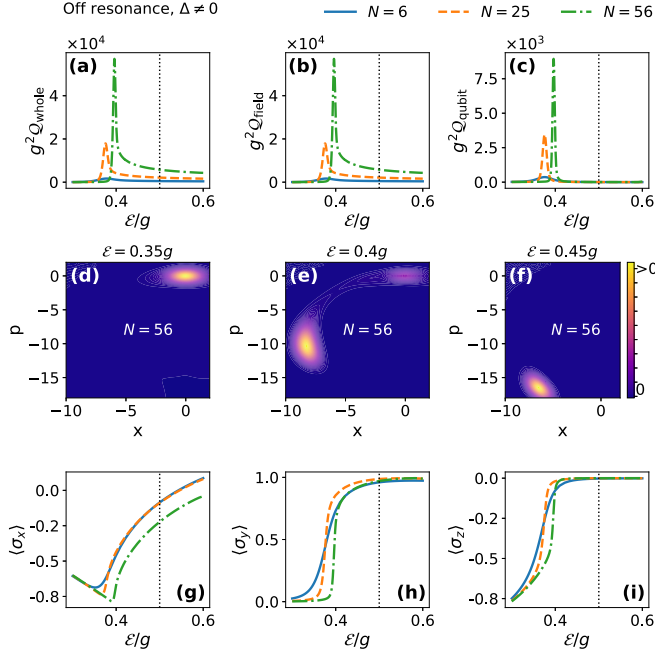


FIG. 3. Off-resonance case, $\Delta \neq 0$: QFI for the entire probe Q_{whole} , the field subsystem Q_{field} , and the qubit subsystem Q_{qubit} as a function of $\mathcal{E}$ for some choices of N . (d)–(f) Wigner quasiprobability function of the field subsystem in phase space for $N = 56$ for several values of $\mathcal{E}/g$. (g)–(i) Expectation values of the qubit components $\langle \sigma_\alpha \rangle$ ($\alpha = x, y, z$) as a function of $\mathcal{E}/g$ for several choices of N .

sensitivity as N increases, similar to the behavior observed in the on-resonance case. However, in contrast to the on-resonance results shown in Figs. 1(a)–1(c) and 3(a)–3(c) reveal two stark differences: (1) a very sharp peak for all QFI functions emerges around $\mathcal{E}/g \sim 0.4$ and (2) the QFI reaches significantly higher values—indicating improved precision—compared to the on-resonance case as $N \gg 1$.

To better understand these differences, in Figs. 3(d)–3(f) we show the Wigner quasiprobability distribution of the field subsystem in phase space for $N = 56$ for driving values around $\mathcal{E}/g \sim 0.4$. As seen from panels (d)–(f), the field transitions from a vacuumlike state for $\mathcal{E}/g < 0.4$ to a strongly displaced state for $\mathcal{E}/g > 0.4$. No signs of bistability, as seen in the on-resonance scenario, are observed. This indicates that, in the off-resonance case, the field exhibits a preferred localization in phase space. Note that choosing $\Delta < 0$ preferentially selects the other bistable peak. To further analyze the system, Figs. 3(g)–3(i) show the expectation values of the qubit components $\langle \sigma_\alpha \rangle$ ($\alpha = x, y, z$) as a function of $\mathcal{E}/g$ for several values of the sensing resource N . As shown in Figs. 3(g)–3(i), increasing the sensing resource N causes the qubit subsystem to become abruptly polarized in the XY plane, along a superposition of the $|-\rangle$ and $|+i\rangle$ directions— $| \pm i \rangle = (|e\rangle \pm i|g\rangle)/\sqrt{2}$. The emergence of a nonzero component along the y axis, due to $\Delta \neq 0$, breaks the bistability and leads to the preferred localization of the field in phase space—choosing $\Delta < 0$ (i.e., the right bistable peak) causes the qubit’s y component to acquire a phase, resulting in the state $| -i \rangle$ [134]. See the Appendix for entanglement and purity details for this case.

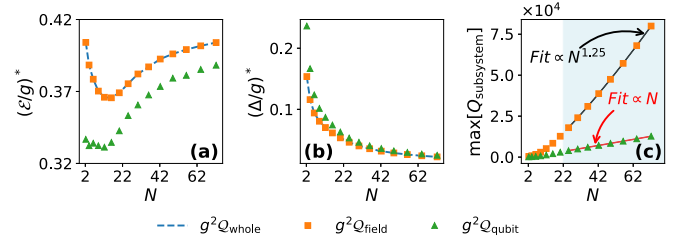


FIG. 4. (a) Optimal driving amplitude $(\mathcal{E}/g)^*$ as a function of N . (b) Optimal detuning $(\Delta/g)^*$ as a function of the sensing resource N . (c) Fit of the optimized maximum QFI as a function of N .

VII. OFF-RESONANCE SCALING ANALYSIS

In Fig. 4(a), we show how the optimal driving amplitude $(\mathcal{E}/g)^*$ varies with N . We consider $N \gtrsim 20$ to exclude finite-size effects. As seen in the figure, a clear convergence toward $\mathcal{E}/g \sim 0.4$ is observed across subsystems. In Fig. 4(b), we plot the optimal detuning $(\Delta/g)^*$ as a function of the sensing resource N for each subsystem. As N increases, the differences between optimal detunings across subsystems become smaller. In Fig. 4(c), we show a fit of the maximum QFI—optimized over $(\Delta/g)^*$ and $(\mathcal{E}/g)^*$ —using a polynomial of the form $AN^B + C$ as a function of N . We avoid finite-size effects by fitting the curve from $N > 20$. As seen in the figure, the exponent B can be either linear or superlinear, depending on the specific subsystem. Importantly, the off-resonance sensing performance significantly exceeds that of the on-resonance case, making it the optimal choice for quantum-enhanced sensing.

VIII. ACTUAL ESTIMATION

The precision bounds discussed above demonstrate that the majority of the information about $\mathcal{E}$ is encoded in the field’s degree of freedom. Thus, we focus on extracting the information about $\mathcal{E}$ from the field subsystem. To this end, we propose a feasible homodyne detection scheme combined with a Bayesian estimator. Both are experimentally routine. In essence, the homodyne scheme combines the unknown signal with a strong local oscillator of known and tunable phase φ on a 50:50 beam splitter. Subsequently, the outputs are subtracted, from which one computes the probability distributions $p(X_\varphi|\mathcal{E}) = \langle X_\varphi | \rho_{\text{SS}}^{\text{field}} | X_\varphi \rangle$, where $\rho_{\text{SS}}^{\text{field}} = \text{Tr}_{\text{qubit}}[\rho_{\text{SS}}]$ is the reduced density matrix of the field subsystem, and $|X_\varphi\rangle$ is the eigenstate of the field quadrature operator $\hat{X}_\varphi = \frac{1}{\sqrt{2}}(\hat{a}e^{-i\varphi} + \hat{a}^\dagger e^{i\varphi})$, rotated by the homodyne (local oscillator) phase angle φ [107,135,136]. After optimizing $p(X_\varphi|\mathcal{E})$ over φ , i.e., after selecting the optimal homodyne strategy for a given $\mathcal{E}$, Bayesian inference directly utilizes the probability distribution $p(X_\varphi|\mathcal{E})$ to build the likelihood function, straightforwardly constructed from experimentally acquired measurement data. The full technical details of the homodyne (and heterodyne) measurement, classical Fisher information, and estimation protocol are provided in the Appendix.

In Fig. 5, we compare the estimator variance for the dimensionless parameter $\mathcal{E}/g$, denoted as $\text{Var}[\tilde{\mathcal{E}}]$, with the inverse of the global QFI of the whole system, evaluated at the optimal point $(\mathcal{E}/g)^*$, i.e., with $[Mg^2Q_{\text{whole}}(\mathcal{E}^*)]^{-1}$. As discussed

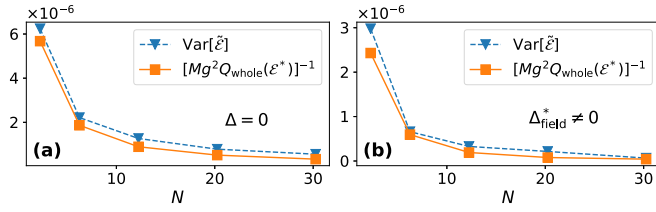


FIG. 5. Estimator variance $\text{Var}[\tilde{\epsilon}]$ and inverse of the QFI $[Mg^2Q_{\text{whole}}(\epsilon^*)]^{-1}$ as a function of N . (a) On-resonance case. (b) Off-resonance case.

above, the combination $g^2Q_{\text{whole}}(\mathcal{E})$ is exactly the QFI with respect to $\mathcal{E}/g$; hence, $[Mg^2Q_{\text{whole}}]^{-1}$ is the corresponding quantum Cramér-Rao bound for estimating $\mathcal{E}/g$. Notably, as clearly seen from Figs. 5(a)–5(b), homodyne detection combined with Bayesian estimation nearly saturates this ultimate precision limit when only partial access to the system is available. This near-ultimate performance can be attributed to two key factors: (1) the nontrivial qubit-field disentangling mechanisms at criticality, which effectively transfer most of the information to the field degree of freedom, and (2) the fact that the field’s phase-space quasiprobability distribution (see middle panels of Figs. 1 and 3) remains positive and approximately Gaussian shaped, making homodyne detection particularly effective in this scenario.

IX. ROBUSTNESS

Quantum probes tuned at the critical point of a quantum phase transition are known to achieve quantum-enhanced sensitivity [5,18,53,58,62]. However, a common drawback of critical sensors with a single critical point is that their QFI peak is narrow, and thus small deviations from optimal conditions rapidly degrade the quantum advantage [10,137]. This has been rigorously demonstrated through the establishment of the universal scaling behavior of the fidelity susceptibility at, and in the vicinity of, the critical point of a second-order quantum phase transition [137]. Our driven-dissipative JC sensor is no exception. Hence, to fully exploit its quantum enhancement, the parameters must be calibrated precisely. This can be achieved via adaptive [138], global [10,13], or sequential calibration protocols [139,140]. Note that the experimental parameters Δ , $\mathcal{E}$, and g are independently tunable in practice (see the above section). In Fig. 6(a), we show the QFI for the off-resonant case as a function of the sensing

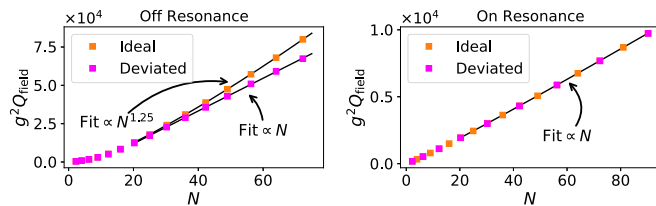


FIG. 6. Robustness of QFI against parameter miscalibration. (a) Off-resonance QFI for the reduced field state as a function of the sensing resource N . The ideally tuned case is compared to a scenario with a small 1% deviation from the optimal detuning $(\frac{\Delta}{g})^*$. (b) On-resonance case, same comparison as in panel (a).

resource N , comparing ideal tuning with a 1% error in $(\frac{\Delta}{g})^*$. Even such a small deviation fully eliminates the enhancement. Conversely, Fig. 6(b) shows the on-resonance case, where the QFI remains robust and maintains an approximately linear scaling with N (the analog of the standard quantum limit for this effective resource), even under the same 1% detuning error. This robustness arises from the broad plateau in the on-resonance response [Fig. 1(c)], which provides increased tolerance to parameter miscalibrations. Note that despite losing the scaling advantage in the off-resonance case under small detuning deviations, the absolute value of the QFI remains considerably larger compared to the on-resonance scenario. Therefore, even when scaling enhancement is absent, the off-resonance scenario continues to offer higher absolute sensitivity.

X. EXPERIMENTAL FEASIBILITY

The coherently driven-dissipative Jaynes-Cummings system has been firmly realized across optical, microwave, and solid-state platforms. In optical cavity QED with single neutral atoms, signatures of driven Jaynes-Cummings physics include photon blockade with antibunched transmission under drive [122,141], direct spectroscopic access to two-photon JC resonances [142], correlated two-photon emission [143], and strong-drive phenomena such as spontaneous dressed-state polarization and few-photon remnants of semiclassical bistability captured by driven-dissipative Jaynes-Cummings theory [144,145]. The above regimes of parameters and signatures of driven Jaynes-Cummings physics in optical cavity QED strongly support the feasibility of our theoretical findings. In circuit QED, the driven Jaynes-Cummings ladder has been resolved spectroscopically, demonstrating the characteristic $\sqrt{n}$ nonlinearity [146], the vacuum-Rabi doublet, and higher-order multiphoton peaks under stronger drives [147]. Furthermore, the nonlinear steady-state response of the strongly driven dispersive Jaynes-Cummings oscillator, including bistability, has been experimentally mapped and modeled [148]. This strongly supports the feasibility of our bistability-based driven-dissipative quantum sensor in circuit QED. Direct measurements of the second-order autocorrelation function $g^{(2)}(\tau)$ have established resonant photon blockade at microwave frequencies under continuous drive [149]. Notably, as discussed in the main text, at higher drive strengths, the driven-dissipative JC model undergoes a dissipative quantum phase transition, which has been experimentally observed [124], exactly matching our model and thus enabling our dissipative quantum sensor scheme. Interestingly, analogous physics has been extended to many-mode lattices, where a nonequilibrium phase transition has been experimentally observed [107]. Such realizations clearly indicate a promising direction for future work toward extending our quantum sensing proposal to many-mode lattice systems. Finally, solid-state photonic-crystal microcavities with embedded quantum dots reproduce the same driven-dissipative Jaynes-Cummings phenomenology on-chip, displaying photon blockade and tunneling effects in reflection measurements [150]. These experiments robustly establish that the parameter regime $g \gtrsim \kappa$ under coherent drive is readily accessible, fully supporting our theoretical

proposal to exploit the driven-dissipative Jaynes-Cummings system as a quantum sensor.

Regarding the tunability of the ratio $\mathcal{E}/g$, input-output formalism establishes that $\mathcal{E} = \sqrt{\kappa_{\text{ext}}}\alpha_{\text{in}}$ [151], where κ_{ext} is the external coupling rate and $\alpha_{\text{in}} = \sqrt{P_{\text{in}}/\hbar\omega_f}$ is fixed by the incident power at the input port. Consequently, the essential control parameter $\mathcal{E}/g \propto \sqrt{P_{\text{in}}}/g$ can be continuously tuned over many orders of magnitude simply by varying P_{in} , without any need to alter the underlying Hamiltonian parameters [152]. Systematic sweeps of $\mathcal{E}/g$ have been extensively performed to explore the Jaynes-Cummings ladder beyond linear response, as explicitly demonstrated in both circuit QED [146–148] and optical cavity QED [122,141–145] experiments. While the coupling strength g typically depends on geometry and material parameters, it can also be engineered and even tuned *in situ* across multiple platforms. For example, in circuit QED, an rf-SQUID coupler allows continuous adjustment of the effective qubit-resonator coupling from zero up to approximately $2\pi \times 100$ MHz on-chip [123]. Similarly, in optical cavity QED, the precise positioning of individual atoms, for instance via optical tweezer techniques, within the standing-wave mode provides deterministic control over the coupling strength $g(\mathbf{r}) \leq g$ [153,154]. Thus, experimental calibration and systematic scanning of the ratio $\mathcal{E}/g$ is straightforward to achieve in practice.

XI. CONCLUSIONS

In this work, we address key challenges in quantum sensing, specifically: Complex state initialization, demanding measurement requirements, and achieving quantum-enhanced sensitivity using only local access. To demonstrate our main findings, we investigate the precision limits of the archetypical driven-dissipative Jaynes-Cummings model undergoing a dissipative quantum phase transition. Indeed, the driven-dissipative JC model is a paradigmatic system that accurately represents a wide class of light-matter interaction models, with several key experimental demonstrations validating its relevance. Most notably, the dissipative quantum phase transition—central to our sensing protocol—has already been clearly realized and verified experimentally [107,124]. By defining the sensing resource as the interplay between coherent and incoherent processes, we report three main findings: First, detuning the dissipative probe off resonance significantly enhances sensing performance compared to the on-resonance case, achieving quantum-enhanced sensitivity for both the whole probe and the field subsystem. Our probe circumvents demanding state-preparation requirements—such as those needed in ground-state metrology—by harnessing dissipative mechanisms. Second, analyzing each subsystem individually reveals that the field subsystem encodes nearly all the information about the unknown parameter. This overcomes the challenging requirement of accessing the entire system and instead allows us to probe only a part of the system. Finally, we show that homodyne detection combined with Bayesian estimation nearly saturates the ultimate precision bound set by the QFI of the full system, despite accessing only part of it. This overcomes the difficulty of performing challenging measurements by relying instead on a straightforward measurement.

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DATA AVAILABILITY

The data supporting the findings are either directly obtained from the code [155] or available upon reasonable request.

APPENDIX A: QUANTUM ENTANGLEMENT AND PURITY ANALYSIS

In the middle and bottom panels of Fig. 1 (on-resonance case), we observe that the qubit's steady state of the qubit-field system abruptly polarizes along a single axis, while the field subsystem bifurcates into two bistable locations in phase space. Similarly, for the off-resonance scenario (middle and bottom panels of Fig. 3), the steady state exhibits analogous behavior. The key difference is that, by appropriately choosing a detuning $\Delta \neq 0$, one can select a preferred location in phase space.

This behavior indicates, as discussed in the main text, that the system becomes weakly entangled and, due to the spontaneous polarization of the qubit, the steady state also becomes purified. To quantify this, we evaluate the entanglement of the steady state using the logarithmic negativity—a measure of entanglement valid for mixed states—defined as

$$E_N(\rho_{\text{SS}}) = \log_2 \|\rho_{\text{SS}}^{T_A}\|_1, \quad (\text{A1})$$

where $\rho_{\text{SS}}^{T_A}$ denotes the partial transpose of the steady-state density matrix with respect to one subsystem and $\|\cdot\|_1$ is the trace norm.

In addition, we compute the purity of the steady state as

$$\mathcal{P}(\rho_{\text{SS}}) = \text{Tr}[\rho_{\text{SS}}^2]. \quad (\text{A2})$$

In the left column of Fig. 7, we show the logarithmic negativity for both the on-resonance and off-resonance cases. In Fig. 7(a), the logarithmic negativity is plotted as a function of the driving amplitude $\mathcal{E}/g$ for different sensing resources N . As the figure illustrates, the steady-state entanglement drops sharply both as N increases and near the critical point, indicating that the qubit and field become only weakly entangled.

A similar trend is observed in the off-resonance case, shown in Fig. 7(c), where we plot the logarithmic negativity as a function of N at the optimal parameters $[(\Delta/g)^*, (\mathcal{E}/g)^*]$. Here, the logarithmic negativity decreases as the sensing resource N grows, signaling a partial disentanglement of the steady state in the thermodynamic limit.

In Fig. 7(b), we show the purity for the on-resonance case as a function of the driving amplitude $\mathcal{E}/g$ for several values of N . As the figure illustrates, consistent with the abrupt spontaneous polarization of the qubit, the purity exhibits a dip around the critical point. For larger driving amplitudes, the system becomes increasingly mixed.

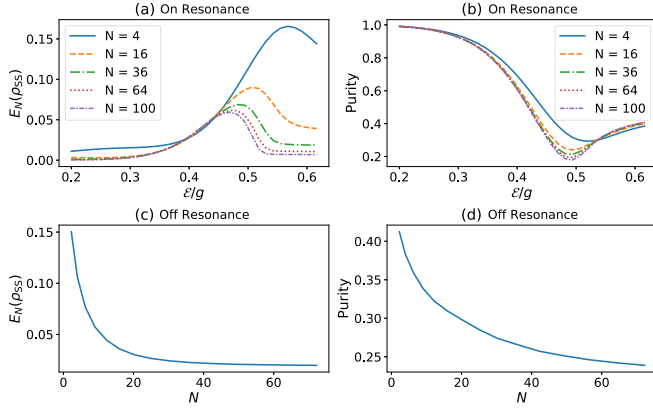


FIG. 7. On resonant: (a) Logarithmic negativity $E_N(\rho_{SS})$ plotted as a function of the driving amplitude $\mathcal{E}/g$ for different sensing resources N . (b) Purity as a function of the driving amplitude $\mathcal{E}/g$ for several values of N . Off resonant: (c) Logarithmic negativity as a function of N at the optimal parameters $[(\Delta/g)^*, (\mathcal{E}/g)^*]$. (d) Purity as a function of N at the optimal parameters $[(\Delta/g)^*, (\mathcal{E}/g)^*]$.

For the off-resonance scenario, Fig. 7(d) shows the purity as a function of N at the optimal parameters $[(\Delta/g)^*, (\mathcal{E}/g)^*]$. As the figure clearly demonstrates, the system becomes progressively more pure as N increases. As stated in the main text, the combination of weak entanglement and increased purification allows the field subsystem, when measured locally via a homodyne scheme and analyzed with a Bayesian estimator, to nearly saturate the ultimate sensing precision of the entire probe.

APPENDIX B: CONTINUOUS-VARIABLE BAYESIAN ESTIMATION

This section clarifies the Bayesian estimation procedure illustrated in Fig. 5.

In a nutshell, Bayesian analysis is a powerful parameter estimation method in which observed measurement outcomes are used to update a probability distribution for an unknown parameter, based on a given statistical model. This approach relies on three key ingredients: measurement outcomes, a statistical model (i.e., modeled probability distributions), and an update rule. The more likely a measurement outcome is under a specific model, the higher the likelihood assigned to the corresponding parameter value. Bayesian estimation also uses prior knowledge (or beliefs) about the parameter in question. This prior information is updated step by step using extra data. At each step, the updated probability distribution (called the posterior) represents the best guess about the parameter given all the information available so far.

Mathematically, this update is governed by Bayes's rule:

$$P(\mathcal{E}|\text{data}) = \frac{P(\text{data}|\mathcal{E})P(\mathcal{E})}{P(\text{data})}, \quad (\text{B1})$$

where $\mathcal{E}$ is the unknown parameter we aim to estimate, and data represents the observed outcomes. Here, $P(\mathcal{E})$ is the *prior* distribution (the initial belief about $\mathcal{E}$), and $P(\text{data}|\mathcal{E})$ is the *likelihood* function—i.e., the probability of observing the data given the parameter $\mathcal{E}$; this constitutes the statistical model and can typically be computed or numerically simu-

lated. The *posterior* distribution $P(\mathcal{E}|\text{data})$ reflects the updated belief about $\mathcal{E}$ after incorporating the observed data. Finally, $P(\text{data})$, known as the *evidence*, ensures normalization so that the posterior remains a valid probability distribution.

1. Interlude I: Homodyne measurement

As discussed above, three key ingredients are required for Bayesian estimation: a statistical model, measurement data, and an update rule. We have already introduced the update rule—Bayes's rule—and the measurement data will be (simulated) experimentally obtained. What remains is to specify the statistical model and its corresponding probability distributions. To construct this, we must define a measurement basis.

From the main text, we know that the subsystem offering the highest sensitivity across all values of $\mathcal{E}$ is the field degree of freedom. Therefore, we focus exclusively on measurements performed on the field state. Rather than pursuing optimal measurement bases—which are often experimentally infeasible—we restrict our attention to a readily available continuous-variable field measurements: homodyne detection. This is because, as shown in the main text, the bistable states and their preferred localization in phase space yield a positive Wigner quasiprobability distribution with an approximately Gaussian shape. This makes homodyne detection an excellent candidate for nearly saturating the ultimate precision bound in our sensing task.

Let us now construct the corresponding probability distributions from

$$p(X_\varphi|\mathcal{E}) = \langle X_\varphi | \rho_{SS}^{\text{field}} | X_\varphi \rangle, \quad (\text{B2})$$

where $\rho_{SS}^{\text{field}} = \text{Tr}_{\text{qubit}}[\rho_{SS}]$ is the reduced density matrix of the field subsystem and $|X_\varphi\rangle$ is the eigenstate of the rotated field quadrature

$$\hat{X}_\varphi = \frac{1}{\sqrt{2}}(\hat{a}e^{-i\varphi} + \hat{a}^\dagger e^{i\varphi}), \quad (\text{B3})$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators of the field and φ is the homodyne phase angle.

Hence, as a very first step in our estimation procedure, we optimize the measurement basis by identifying the value of φ that yields the highest sensing performance within the chosen homodyne detection scheme. This optimal value of φ is determined by maximizing the sensing performance for our specific homodyne scheme, namely, by maximizing the classical Fisher information. As discussed in the main text, the CFI quantifies the sensitivity with respect to $\mathcal{E}$ for a given measurement. For clarity, we recall the expression for the CFI here as

$$\mathcal{F}(\varphi|\mathcal{E}) = \int_{-\infty}^{\infty} dX_\varphi \frac{1}{p(X_\varphi|\mathcal{E})} \left[\frac{\partial p(X_\varphi|\mathcal{E})}{\partial \mathcal{E}} \right]^2, \quad (\text{B4})$$

where $p(X_\varphi|\mathcal{E})$ accounts for the conditional probability of obtaining the value X_φ assuming the value $\mathcal{E}$. To determine the optimal measurement basis, we maximize the homodyne angle φ^* for a given set of parameters $(\frac{\mathcal{E}}{g})_j^*$ and $(\frac{\Delta}{g})_j^*$ and a given sensing resource N :

$$\varphi^* = \arg \max_{\varphi} \mathcal{F}(\varphi|\mathcal{E}). \quad (\text{B5})$$

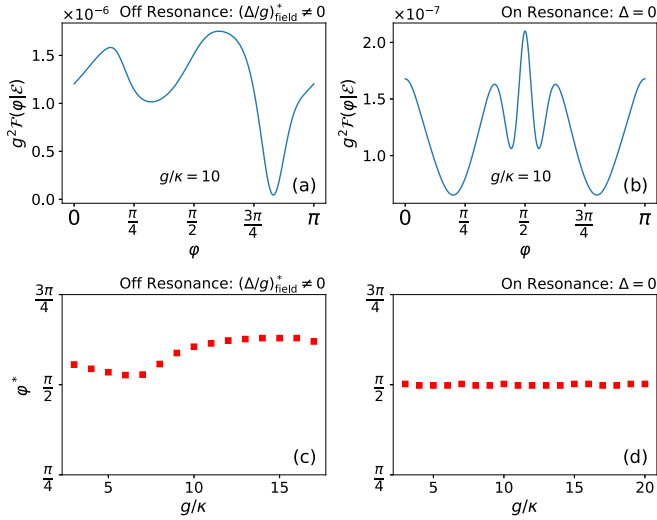


FIG. 8. (a), (b) CFI as a function of the homodyne angle φ for a fixed value of $g/\kappa = 10$ for the off-resonance and on-resonance cases, respectively. (c), (d) Optimized angle φ^* as a function of g/κ for the off-resonance and on-resonance scenarios, respectively.

In Figs. 8(a) and 8(b), we plot the CFI as a function of the homodyne angle φ for a fixed value of $g/\kappa = 10$ for the off-resonance and on-resonance cases, respectively. From these plots, one can directly identify the optimal angle φ^* that maximizes the CFI. In Figs. 8(c) and 8(d), we show how the optimized angle φ^* varies with g/κ for the off-resonance and on-resonance scenarios, respectively. As observed from the figures, in the off-resonance case, the optimal homodyne angle φ^* oscillates within the range $\pi/2$ to $3\pi/4$. In contrast, for the on-resonance case, the optimal angle remains nearly constant around $\varphi^* \approx \pi/2$ across the entire range of g/κ values considered.

2. Interlude II: Heterodyne measurement

Heterodyne measurement is another powerful experimental technique for probing the field state. In the main text, we argued that the homodyne scheme provides superior sensing performance. To quantify this claim, we define

$$\text{performance} = \frac{\mathcal{F}_{\text{field}}(\mathcal{E})}{\mathcal{Q}_{\text{whole}}(\mathcal{E})}, \quad (\text{B6})$$

where $\mathcal{F}_{\text{field}}(\mathcal{E})$ is the classical Fisher information, as described in Sec. II. It is given by Eq. (B4) for the homodyne case, and by

$$\mathcal{F}_{\text{field}}(\mathcal{E}) = \int d^2\Upsilon \frac{1}{p(\Upsilon|\mathcal{E})} \left(\frac{\partial p(\Upsilon|\mathcal{E})}{\partial \mathcal{E}} \right)^2 \quad (\text{B7})$$

for the heterodyne case, where the probability distribution is

$$p(\Upsilon|\mathcal{E}) = \frac{1}{\pi} \text{Tr}[\Upsilon \langle \Upsilon | \rho \rangle], \quad (\text{B8})$$

and $|\Upsilon\rangle$ is a coherent state.

In Fig. 9(a), we show the performance ratio $\frac{\mathcal{F}_{\text{field}}(\mathcal{E})}{\mathcal{Q}_{\text{whole}}(\mathcal{E})}$ for the probe on and off resonance in the homodyne case as a function of N . As seen in the figure, off-resonance homodyne detection nearly saturates the ultimate sensing capability of

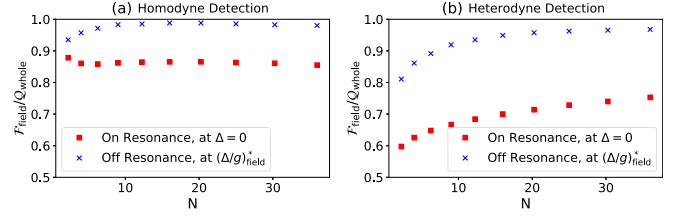


FIG. 9. (a) Homodyne performance ratio $\frac{\mathcal{F}_{\text{field}}(\mathcal{E})}{\mathcal{Q}_{\text{whole}}(\mathcal{E})}$ for the probe on and off resonance as a function of N . (b) Heterodyne performance ratio $\frac{\mathcal{F}_{\text{field}}(\mathcal{E})}{\mathcal{Q}_{\text{whole}}(\mathcal{E})}$ for the probe on and off resonance as a function of N .

the probe. This observation supports our Bayesian analysis, in which homodyne detection combined with Bayesian estimation approaches the ultimate sensitivity of the full probe.

In Fig. 9(b), we show the same performance ratio for heterodyne detection. As the figure demonstrates, homodyne detection outperforms heterodyne detection across all values of N . Interestingly, as N increases, heterodyne detection becomes comparable to homodyne detection. This is noteworthy because increasing N makes numerical simulations for homodyne detection challenging due to truncation issues, whereas the heterodyne scheme remains more reliable and feasible.

3. Likelihood function

Armed with the optimized homodyne angle φ^* , we now construct the likelihood function $P(\text{data}|\mathcal{E})$ for the continuous-variable case. To this end, we discretize the continuous probability density function $p(X_{\varphi^*}|\mathcal{E})$ into a set of m probabilities:

$$\mathcal{P}_m = \int_{y_m}^{y_{m+1}} p(X_{\varphi^*}|\mathcal{E}) dX, \quad (\text{B9})$$

where the integration limits satisfy

$$y_{m+1} = y_m + W, \quad (\text{B10})$$

with W being a fixed bin width along the quadrature axis X_{φ^*} . W is determined by the resolution of the measurement device. The resulting likelihood function, assuming independent and identically distributed measurements, is then given by

$$P(\text{data}|\mathcal{E}) \sim \prod_m \mathcal{P}_m^{C_m}, \quad (\text{B11})$$

where the exponent C_m denotes the number of measurement outcomes falling within the m th bin. Recall that the probabilities $\mathcal{P}_m$ correspond to the theoretical outcome probabilities predicted by the model. In our case, they are fully determined by the steady state of the system and can be computed directly from it. In contrast, the counts C_m represent the observed measurement data. These are simulated by drawing $M = \sum_m C_m$ independent random samples according to the distribution defined by the theoretical probabilities $\mathcal{P}_m$.

In Fig. 10, we illustrate the full procedure used to construct the observed counts C_m and, consequently, the likelihood function. In Fig. 10(a), we plot the modeled probability distribution $p(X_{\varphi^*}|\mathcal{E})$ for a fixed value of $\mathcal{E}$ and sensing resource N as a function of the eigenvalue of the optimized quadrature X_{φ^*} . Figure 10(b) shows the discretization of this continuous distribution, where the probabilities for each bin are computed

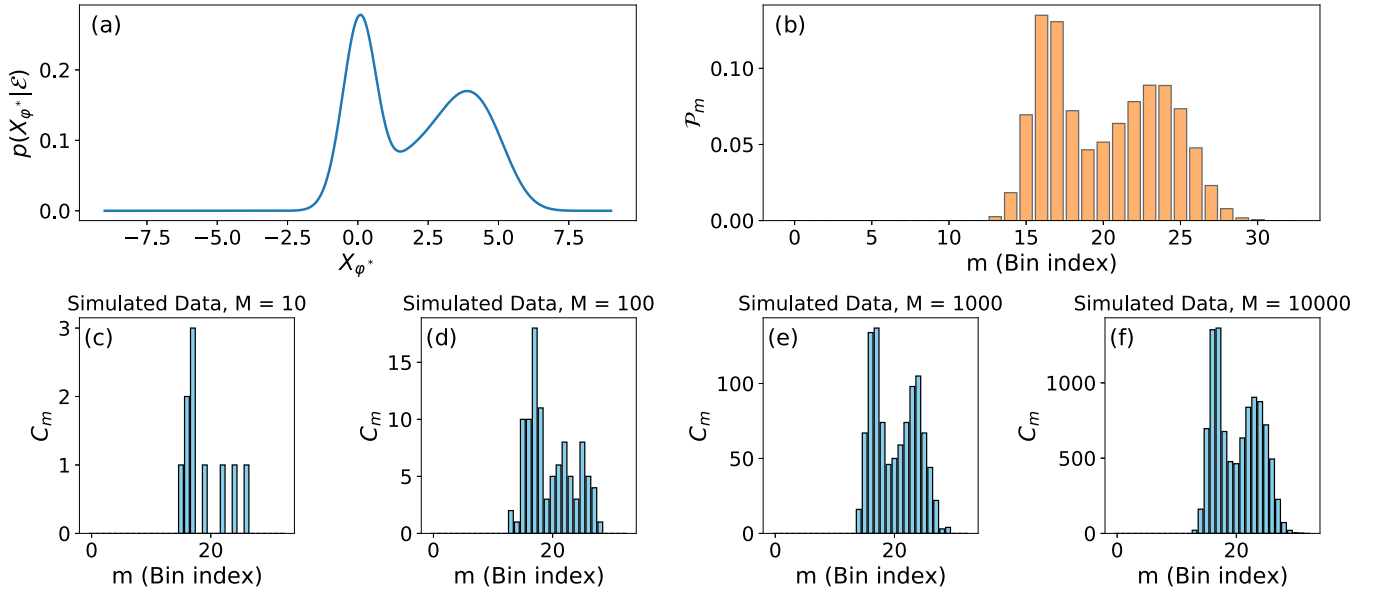


FIG. 10. (a) Probability distribution $p(X_\varphi^*|\mathcal{E})$ for a fixed value of $\mathcal{E}$ and sensing resource N as a function of the eigenvalue of the optimized quadrature X_φ^* . (b) Discretization of $p(X_\varphi^*|\mathcal{E})$. (c)–(f) Random counts C_m for increasing values of M .

using Eq. (B9). As seen in the figure, the chosen number of bins fully captures the values of X_φ^* . Since any continuous probability density function can be discretized, we can use this to simulate observed data. Specifically, we generate random samples by drawing M independent measurements according to the discretized distribution. Figs. 10(c)–10(f) show examples of the resulting random counts C_m for increasing values of M . As M increases, the sampled counts more accurately reflect the discretized distribution from Fig. 10(b), making the evaluation of the likelihood function straightforward.

4. Data simulation

The simulated statistics are as follows: We perform 100 independent experiments. In each experiment, we simulate $M = 1000$ measurements [we obtain a curve from simulated data as shown in Fig. 10(e)], which are used to compute a single estimate of $\tilde{\mathcal{E}}$ by finding the value of $\mathcal{E}$ that maximizes the posterior function. This results in 100 independent estimates of $\tilde{\mathcal{E}}$. We then calculate the variance across these 100 values to obtain the estimator variance $\text{Var}[\tilde{\mathcal{E}}]$. This procedure allows us to generate the Bayesian curves in Fig. 5.

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