

Information dynamics and natural computing in two-skyrmion system

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The probabilistic information flow and natural computational capability of a system with two magnetic skyrmions at room temperature have been experimentally evaluated. Probabilistic behavior has attracted attention for its potential to enable unconventional computing paradigms that achieve ultralow-energy computation. However, information propagation and computation in such systems are more complex than in conventional computers, making their visualization essential.

In this study, a two-skyrmion system confined within a square potential well at thermal equilibrium was analyzed using information thermodynamics. Transfer entropy and the time derivative of mutual information were employed to investigate the information propagation speed, the absence of a Maxwell's demon in thermal equilibrium, and the system's non-Markovian properties. Furthermore, it was demonstrated that the system exhibits a small but finite computational capability for the nonlinear XOR operation, potentially linked to hidden information in the non-Markovian system.

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I. INTRODUCTION

How can we design a computing machine that operates with minimal energy [1]? To address this question, Bennett proposed the concept of Brownian computation [2], in which thermal energy drives the exploration process, and the computation proceeds once the correct solution is identified. To further develop practical implementations of such circuits, Peper and co-workers introduced a universal Brownian computation circuit based on token dynamics [3]. In this approach, a mechanism analogous to Maxwell's demon, called a Conservative-Join, is employed to determine the correct answer. Potential candidates for tokens include electrical charges in single-electron transistors and skyrmions in magnetic thin films. However, single-electron transistors can

only function at extremely low temperatures [4], limiting their practical applications.

Magnetic skyrmions are particlelike magnetic structures that emerge in continuous magnetic ultrathin films and are expected to find applications in various memory and computing devices, such as racetrack memory [5] and Brownian computers [6]. Racetrack memory utilizes the translational motion of skyrmions driven by an electric current [7] and their nonvolatility. In contrast, Brownian computers leverage the probabilistic behavior arising from the Brownian motion of skyrmions at room temperature [8–12] and their mutual repulsive interactions [13,14], enabling intelligent computations distinct from conventional computers. To realize the Brownian computer with ultralow energy consumption, the techniques for potential control using film structure and/or ion irradiation [6,15] and for electric field manipulation [16–18] have been developed. So far, applications to brainlike computers, including Brownian computers [2], reservoir computing [19–24], neuromorphic computing [25,26], and cellular automata [27], have been proposed, yielding preliminary results.

In these computing systems, information propagates in a complex manner, making it impossible to understand using the simple information propagation models employed in conventional computers. Therefore, it is critically important to

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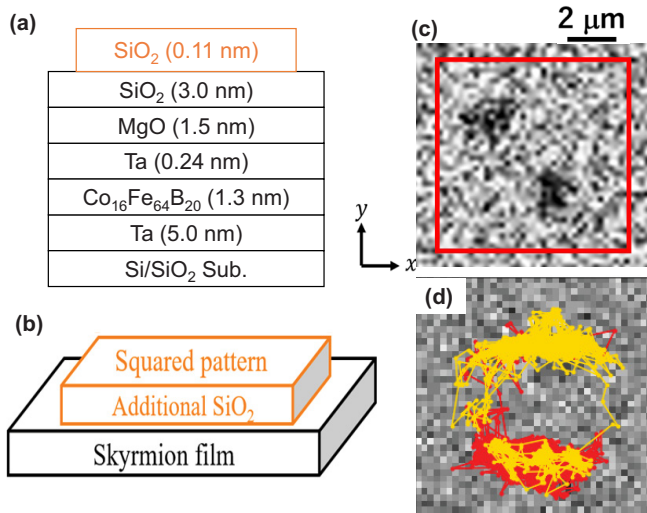


FIG. 1. (a) Schematic diagram of the sample structure. Skyrmions are formed in the $\text{Co}_{16}\text{Fe}_{64}\text{B}_{20}$ layer (atomic percentages are indicated in subscript). (b) By depositing an additional SiO_2 layer (shown in orange) on top of the existing SiO_2 , skyrmions are confined within that region, forming a potential box. (c) MOKE microscope image of skyrmions confined within the box. The red line represents the estimated edge of the box, determined based on skyrmion behavior. (d) The yellow and red solid lines indicate the trajectories of skyrmions A and B, respectively.

visualize how information propagates and to elucidate the computational capabilities available. Transfer entropy [28] is a useful physical quantity for clarifying the information dynamics in complex systems. Transfer entropy quantifies the causal flow of information between time-series data from the probability distribution of a system [29]. It is effective in systems that handle complex time-series data and has wide-ranging applications such as biology [28], neuroscience [30,31], physics [32], economics [33], food web [34], and social network service (SNS) [35].

This paper experimentally investigates the information dynamics of thermal motion in a confined square region containing two skyrmions. Skyrmions exchange information through their repulsive interactions. The flow of information in thermal equilibrium was analyzed using transfer entropy and the subsystem-time derivative of mutual information [36]. The latter allows us to determine whether Maxwell's demon exists in this physical system. Furthermore, the non-Markovian nature of the system and its capability for natural computation were also examined.

II. METHODS

The sample structure is shown in Fig. 1(a). Ta (5.0 nm) $|\text{Co}_{16}\text{Fe}_{64}\text{B}_{20}$ (1.3 nm) $|\text{Ta}$ (0.24 nm) $|\text{MgO}$ (1.5 nm) $|\text{SiO}_2$ (3.0 nm) was deposited onto the Si $|\text{SiO}_2$ substrate using a magnetron sputtering system (Canon ANELVA, E-880SM in Osaka University). By additionally depositing 0.11 nm of SiO_2 on a portion of the surface of this film, skyrmions can be confined within that area [6,27]. We used photolithography to confine two skyrmions in an $8\text{ }\mu\text{m} \times 8\text{ }\mu\text{m}$ square cell [Fig. 1(b)]. Figure 1(c) is a MOKE microscope image of the sample, and the area surrounded by the red line represents the

area where SiO_2 was additionally deposited. Two skyrmions with about $1.5\text{ }\mu\text{m}$ in diameter are confined. Brownian motion of two interacting skyrmions was observed. Due to the skyrmion-skyrmion repulsive interaction within a cell, the two skyrmions prefer to be arranged at opposite corners of the cell as the top-right or bottom-left pairs or the top-left and bottom-right pairs [27]. Observations were taken in video for 10 s at a frame rate of 250 fps (time step $\Delta t = 4\text{ ms}$). The number of records was taken 75 times (in total 187 500 frames). During observation, a magnetic field was applied perpendicularly upward to the sample surface, and the sample temperature was controlled using a heater. The optimal conditions were at around 0.25 mT and around 51°C . At elevated temperature, skyrmions show a high diffusion coefficient as high as $34.4\text{ }\mu\text{m}^2/\text{s}$ at 51.5°C and around $100\text{ }\mu\text{m}^2/\text{s}$ at 57.5°C .

In order to investigate information dynamics in the skyrmion system, we created time-series data from the trajectory of skyrmions exhibiting Brownian motion inside the square cell. Figure 1(d) shows the trajectory of each skyrmion. The one with the larger y coordinate of the initial position is defined as skyrmion A, and the one with the smaller initial y coordinate is defined as skyrmion B. The solid line shown in yellow represents the trajectory of skyrmion A, and the solid line shown in red represents the trajectory of skyrmion B.

Here, A_n and B_n are the random variables of the state (position) of skyrmions A and B at discrete time n , respectively. We consider the observed x position (event) of a_n and b_n of skyrmions A and B, which are two time-series datasets $\{a_1, a_2, \dots, a_n, \dots, a_N\}$ and $\{b_1, b_2, \dots, b_n, \dots, b_N\}$, respectively. In our experiments, the time step is equal to 4 ms, and the position data are binarized. We set a threshold value for the x coordinate of each skyrmion and binarized it. Coordinates smaller than the threshold are defined as state “0” and coordinates larger than the threshold are defined as state “1”. The threshold value is determined so that the number of cases of “0” and “1” would be equal for all position coordinates of skyrmion A (B). In this analysis, this binary probability distribution was used.

Generally, the transfer entropy from skyrmion A to B between discrete times n and $n + 1$ is defined by Eq. (1) [25]:

$$I_{A \rightarrow B}^{tr} = S(B_{n+1}|B_1, \dots, B_n) - S(B_{n+1}|A_1, \dots, A_n, B_1, \dots, B_n). \quad (1)$$

Here, $S(B_{n+1}|A_1, \dots, A_n, B_1, \dots, B_n)$ is the conditional Shannon entropy [37,38]. The Shannon entropy $S(B_{n+1})$ of the random variable of B at discrete time $n + 1$ is expressed by $S(B_{n+1}) = -k_B \sum_{b_{n+1}} p(b_{n+1}) \ln p(b_{n+1})$, where $p(b_{n+1})$ is the probability of finding skyrmion B in position b_{n+1} . Conditional Shannon entropy is the Shannon entropy for conditional probabilities, i.e., $S(B_{n+1}|B_1, B_2, \dots, B_n) \equiv -k_B \sum_{b_1, b_2, \dots, b_{n+1}} p(b_1, b_2, \dots, b_{n+1}) \ln p(b_{n+1}|b_1, b_2, \dots, b_n)$. The transfer entropy is a reduction in skyrmion B's conditional entropy by knowing the past positions of skyrmion A and expressing a causal information flow (IF) from skyrmion A to B. To discuss information dynamics, we extend the above definition to longer time separation between cause and result

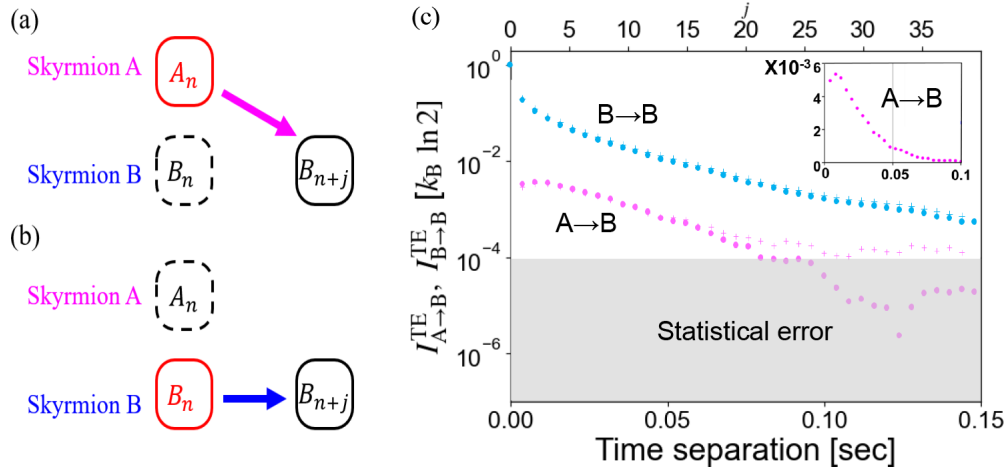


FIG. 2. (a) Schematic diagram of the transfer entropy $I_{A→B}^{TE}(j)$ from the current skyrmion A to the future skyrmion B, given that the current skyrmion B is known. The nodes shown as dotted lines represent already known nodes, while the nodes shown in red represent newly learned nodes. (b) Schematic diagram of the transfer entropy $I_{B→B}^{TE}(j)$ from the current skyrmion B to the future skyrmion B, given that the current skyrmion A is known. (c) Experimentally obtained transfer entropies as functions of time separation. The results shown as pink and blue closed circles represent $I_{A→B}^{TE}(j)$ and $I_{B→B}^{TE}(j)$, respectively. Statistical error is indicated by gray hatching.

as follows:

$$I_{A→B}^{TE}(j) = S(B_{n+j}|B_n) - S(B_{n+j}|A_n, B_n),$$

$$I_{B→B}^{TE}(j) = S(B_{n+j}|A_n) - S(B_{n+j}|A_n, B_n). \quad (2)$$

Here, time separation is changed from one time step to j time steps. In addition, information from the past, i.e., $n - 1, \dots, 1$, is neglected. Later treatment facilitates numerical analysis using a limited number of datasets and does not affect the results if the system is Markovian. The first line uses the extended definition of the transfer entropy (TE) from A to B. The transfer entropy from B to B is also defined for comparison purposes in the second line.

III. RESULTS

Figure 2(a) shows the schematic diagram (three-node diagram) of the transfer entropy $I_{A→B}^{TE}(j)$ from the state of the current skyrmion A_n to the state of future skyrmion B_{n+j} in a situation where the state of current skyrmion B_n is known. This evaluates the transfer entropy flowing between two different skyrmions. In the range where j is large, the correlation between B_{n+j} and (A_n, B_n) reduces; thus, $I_{A→B}^{TE}(j)$ is expected to approach 0. Figure 2(b) is the schematic diagram of the transfer entropy $I_{B→B}^{TE}(j)$ from the state of the current skyrmion B_n to the state of future skyrmion B_{n+j} in a situation where the state of current skyrmion A_n is known. This represents the transfer entropy of the same skyrmion from its own past to the future.

Figure 2(c) shows the time-separation dependence of the transfer entropies in the two-skyrmion systems evaluated from experimental data using three-node analysis [Eq. (2)]. The vertical axis of the figure is the value of transfer entropy normalized by one-bit Shannon entropy. The larger the value, the more information is flowing, and the state is completely determined when the value is 1.0. The horizontal axis represents time separation, and the larger j indicates the transfer of entropy to the distant future. Plotting was performed at 38 points from $j = 0$ to 37, and was converted to the actual time

separation. Since the transfer entropy $I_{A→B}^{TE}(j)$ is not defined for $j = 0$, it is not evaluated. Results represented by pink dots indicate $I_{A→B}^{TE}(j)$, and results represented by blue dots indicate $I_{B→B}^{TE}(j)$. In the figure, statistical error is indicated by gray hatching. In a range with values comparable to this statistical error, the results cannot be determined to be significant. Since both $I_{A→B}^{TE}(j)$ and $I_{B→B}^{TE}(j)$ have values larger than the statistical error of the analysis in the range where the time separation is less than 0.1 s, it can be seen that significant results were obtained.

$I_{A→B}^{TE}(j)$ increases in value from $j = 1$ and reaches its maximum value at the time separation between $j = 2$ and 3 [approximately 10 ms of the time separation; see the inset of Fig. 2(c)]. This indicates that skyrmion B receives the largest amount of information from skyrmion A at this time. Afterward, it attenuates as j increases. The skyrmion diffusion length over 10 ms is about 0.8 μm . It is close to the skyrmion radius (0.8 μm). Therefore, the peak structure in the transfer entropy graph is attributed to the fact that information transfer requires a transient transition of skyrmion B's position from 0 to 1 or from 1 to 0, necessitating a finite time corresponding to the diffusion time needed to switch the skyrmion position [39]. On the other hand, $I_{B→B}^{TE}(j)$ is equal to 1 at $j = 0$, decreases rapidly at first, and then decays exponentially with a smaller slope as a function of j .

In addition, the attenuation time τ was evaluated from the fitting of the logarithmic attenuation region in the graphs, using the fitting function of $\alpha \exp[-j / f\tau]$. Here, f is the frame rate of 250 fps and α is a coefficient. As a result, it was found that the attenuation time of $I_{A→B}^{TE}(j)$ is about 21 ms, and that of $I_{B→B}^{TE}(j)$ is about 26 ms, indicating that skyrmions retain propagated information for more than 20 ms. The initial rapid decrease in $I_{B→B}^{TE}(j)$ is thought to correspond to the dissipation of information during the diffusion process of a single skyrmion. On the other hand, the subsequent decrease in transfer entropies, which occurs at nearly the same rate for both entropies but with a longer decay time, suggests that this process corresponds to the slower dissipation of

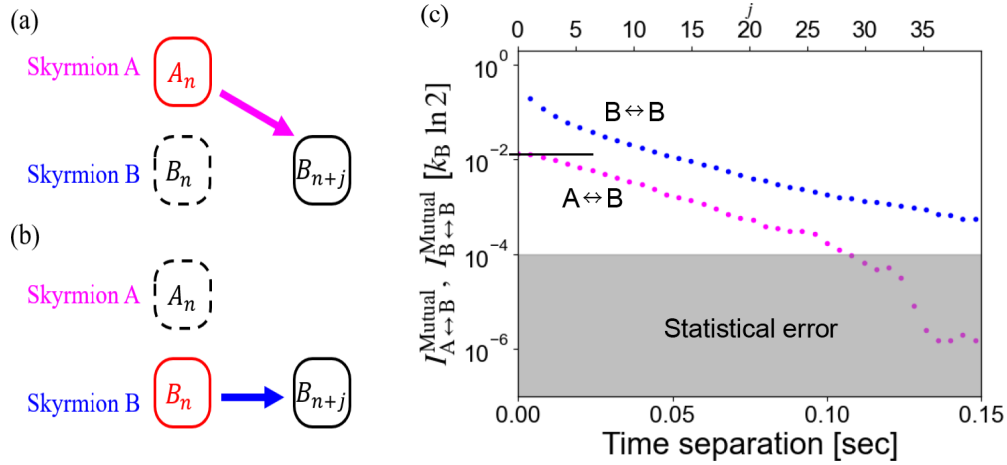


FIG. 3. (a) Schematic diagram of the mutual information $I_{A \leftrightarrow B}^{Mutual}(j)$ between the current skyrmion A and the future skyrmion B. (b) Schematic diagram of the mutual information $I_{B \leftrightarrow B}^{Mutual}(j)$ between the current skyrmion B and the future skyrmion B. (c) Experimentally obtained mutual entropies as functions of the time separation. Results shown in pink and blue indicate $I_{A \leftrightarrow B}^{Mutual}(j)$ and $I_{B \leftrightarrow B}^{Mutual}(j)$, respectively. Statistical error is indicated by gray hatching.

information due to the sharing of information between two skyrmions.

To validate the Markov approximation, we performed an analysis of transfer entropy considering the contribution of the previous node using the following equations:

$$I_{A \rightarrow B}^{TE(2)}(j) = S(B_{n+j}|B_n, B_{n-1}) - S(B_{n+j}|A_n, A_{n-1}, B_n, B_{n-1}),$$

$$I_{B \rightarrow B}^{TE(2)}(j) = S(B_{n+j}|A_n, A_{n-1}) - S(B_{n+j}|A_n, A_{n-1}, B_n, B_{n-1}). \quad (3)$$

The results are shown with plus signs in Fig. 2(c). There is a slight difference between the analysis assuming a Markov process and the analysis considering the contribution of the previous node, but the general time dependence remains unchanged.

It was pointed out that the subsystem-time derivative of the mutual entropy expresses a directional information flow (4) [36]:

$$I_{A \rightarrow B}^{IF} \equiv \lim_{\Delta t \rightarrow 0} (I_{A \leftrightarrow B}^{Mutual}(1) - I_{A \leftrightarrow B}^{Mutual}(0)) / \Delta t. \quad (4)$$

In the above expression, the mutual information $I_{B \leftrightarrow A}^{Mutual}(j)$ is defined by first line of the following expressions:

$$I_{A \leftrightarrow B}^{Mutual}(j) \equiv S(B_{n+j}) - S(B_{n+j}|A_n),$$

$$I_{B \leftrightarrow B}^{Mutual}(j) \equiv S(B_{n+j}) - S(B_{n+j}|B_n). \quad (5)$$

As in the above definition, the mutual information in the first and second lines is equivalent to the nonlocal and local active information storages, respectively [40]. The above mutual information (active information storage) is plotted in Fig. 3 to observe the time derivative as a slope of a graph.

In Figs. 3(a) and 3(b), the schematic diagram (two-node diagram) of the above-defined mutual entropy is shown. Figure 3(c) shows the time-separation dependence of the mutual

information evaluated from experimental data using two-node analysis [Eq. (5)]. The vertical axis of the figure is the value of the mutual information. The horizontal axis is the time separation, j . Plotting was performed at 38 points from $j = 0$ to 37. Results represented by pink dots indicate $I_{A \leftrightarrow B}^{Mutual}(j)$, and results represented by blue dots indicate $I_{B \leftrightarrow B}^{Mutual}(j)$. It can be seen that below 0.12 s, significant results were obtained.

Both mutual information are monotonically decreasing as the time separation increases. At the beginning ($j = 0$ to 1), however, the behaviors are very different between two graphs. The slope of $I_{A \leftrightarrow B}^{Mutual}(j)$ is almost zero at $j = 0$, and that of $I_{B \leftrightarrow B}^{Mutual}(j)$ shows the largest slope there. The initial slope of the $I_{A \leftrightarrow B}^{Mutual}(j)$ corresponds to $I_{A \rightarrow B}^{IF}$. Here, it should be mentioned that $I_{B \leftrightarrow B}^{Mutual}(j)$ is almost equivalent to $I_{B \rightarrow B}^{TE}(j)$. This means that the information flows from A to B are minor compared with B-to-B flow. It is clear from the fact that $I_{A \rightarrow B}^{TE}(j)$ is about 1/15 of $I_{B \rightarrow B}^{TE}(j)$.

In a steady state, the total time derivative of the mutual information is zero. Therefore, $I_{A \rightarrow B}^{IF} = -I_{B \rightarrow A}^{IF}$. This means that if $I_{A \rightarrow B}^{IF} > 0$, skyrmion B acquires information about skyrmion A, thereby imparting negative entropy to skyrmion A, and skyrmion B can be regarded as a Maxwell's demon for $I_{A \rightarrow B}^{IF} > 0$. In our experiment, the information flow between subsystems was zero, and no Maxwell's demon was observed. This is a natural and reasonable result for a system in thermal equilibrium.

As shown in Fig. 2(c), the analysis results of transfer entropy, assuming a Markov process, were slightly modified by taking into account the contribution of the preceding nodes. This indicates that the system comprising the two skyrmions under consideration exhibits a non-Markovian response. To investigate the non-Markovian property of our system, we compared the following two mutual information:

$$I_{Present \leftrightarrow Future}^{Mutual}(j) = S(B_{n+j}|A_{n-1}, B_{n-1}) - S(B_{n+j}|A_{n-1}, B_{n-1}, A_n, B_n),$$

$$I_{Past \leftrightarrow Future}^{Mutual}(j) = S(B_{n+j}|A_n, B_n) - S(B_{n+j}|A_n, B_n, A_{n-1}, B_{n-1}). \quad (6)$$

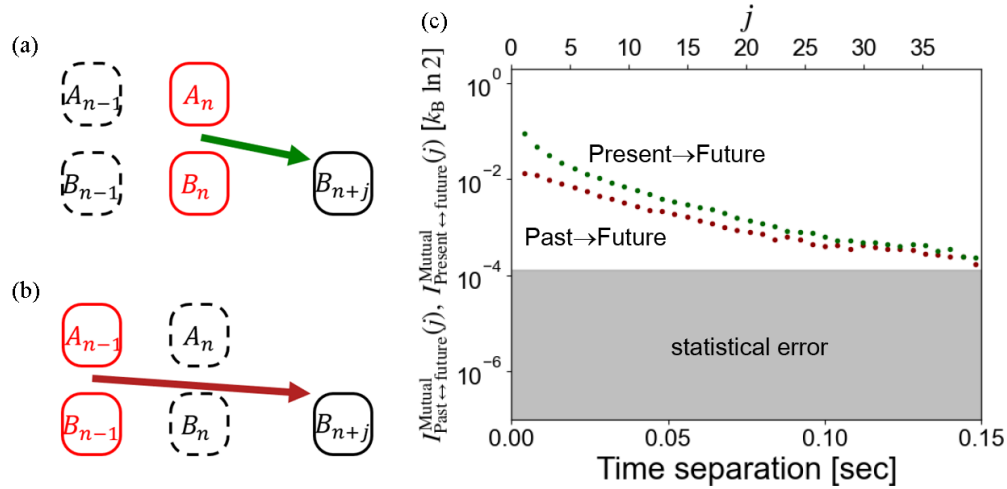


FIG. 4. (a) Schematic diagram of the mutual information $I_{\text{Present} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ between (A_n, B_n) and B_{n+j} when the past configuration (A_{n-1}, B_{n-1}) is known. (b) Schematic diagram of the mutual information $I_{\text{Past} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ between (A_{n-1}, B_{n-1}) and B_{n+j} when the present configuration (A_n, B_n) is known. (c) Experimentally obtained mutual information as a function of the time separation. Dark green and red dots represent $I_{\text{Present} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ and $I_{\text{Past} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$, respectively. Statistical error is indicated by gray hatching.

The first equation, $I_{\text{Present} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$, represents the mutual information between the current skyrmion (B_n, A_n) and the future skyrmion (B_{n+j}) , given the past position (B_{n-1}, A_{n-1}) . Since the system continually generates new information, this value is expected to be finite. The second equation, $I_{\text{Past} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ represents the mutual information between the past skyrmion (B_{n-1}, A_{n-1}) and the future skyrmion (B_{n+j}) , given the current position (B_n, A_n) . If the system is Markovian, this value should be zero. In Figs. 4(a) and 4(b), diagrammatic expressions (three-nodes) of $I_{\text{Present} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ and $I_{\text{Past} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ are shown, respectively.

In Fig. 4(c), the experimentally obtained values of $I_{\text{Present} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ and $I_{\text{Past} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ are shown as red and green dots, respectively. As expected, $I_{\text{Present} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ exhibits a finite value. Notably, $I_{\text{Past} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ also has a finite value. This result indicates that the system of our two skyrmions is non-Markovian. However, the information flow from the past is small, with the ratio of $I_{\text{Past} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ to $I_{\text{Present} \leftrightarrow \text{Future}}^{\text{Mutual}}(j)$ being approximately 6.5 at $j = 1$. This suggests that the transfer entropy, which ignores the past as defined in Eq. (2), is approximately valid. The correction to Eq. (2) from the immediate past, represented as $I_{\text{Past} \leftrightarrow \text{Future}}^{\text{Mutual}}(j) - (S(B_{n+j}|B_n) - S(B_{n+j}|B_n, B_{n-1}))$, is estimated to be on the order of 10^{-4} and has only a minor contribution.

The reason why the skyrmion system in this study exhibits non-Markovian behavior is believed to lie in the coarse graining (binarization) of the skyrmion positions (see Sec. S1 of the Supplemental Material [41]). Figure 5 presents a heat map showing the spatial distribution of the integrated skyrmion's staying time. The brightness is proportional to the logarithm of the staying time at each pixel. Considering that the staying time in thermal equilibrium follows an exponential relationship with the potential depth, the brightness reflects the depth of the skyrmion's trapping potential. Approximately 15–20 trapping potentials can be observed in the figure. This means that each binary value corresponds to about 7–10 trapping positions, and in the binarized data sequence used for analysis, information about which specific trap the skyrmion

is captured in is lost (Effect of the trapping potential on the analysis is discussed in Sec. S2 of the Supplemental Material [41]). Systems with such hidden degrees of dynamical freedom generally exhibit non-Markovian behavior. Furthermore, in the field of machine learning, such dynamical systems are referred to as hidden Markov systems, which are known to possess computational capability and are applied to tasks such as filtering and maximum likelihood estimation [42].

The coupling between the hidden degrees of freedom in the skyrmion system studied in this paper was not designed for computational purposes, but it may still be performing some form of computation. To investigate this, we checked whether skyrmions were performing XOR computations. XOR computation, as expressed by Eq. (7), is a nonlinear operation that requires a certain level of computational capability:

$$x_n = \frac{1}{2} - 2(a_n - \frac{1}{2})(b_n - \frac{1}{2}). \quad (7)$$

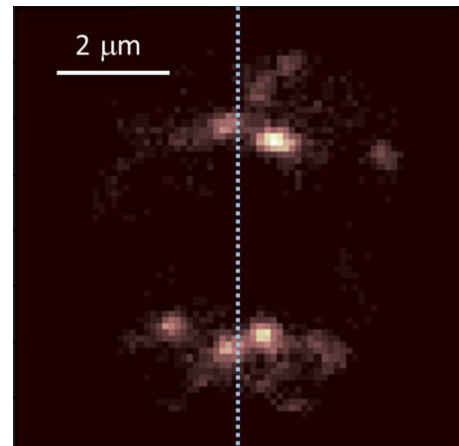


FIG. 5. Trapping potential distribution within the potential box. Brighter color indicates deeper potential energy. The blue dashed line indicates the threshold for the binarization. The potential was estimated based on the skyrmion's residence time distribution.

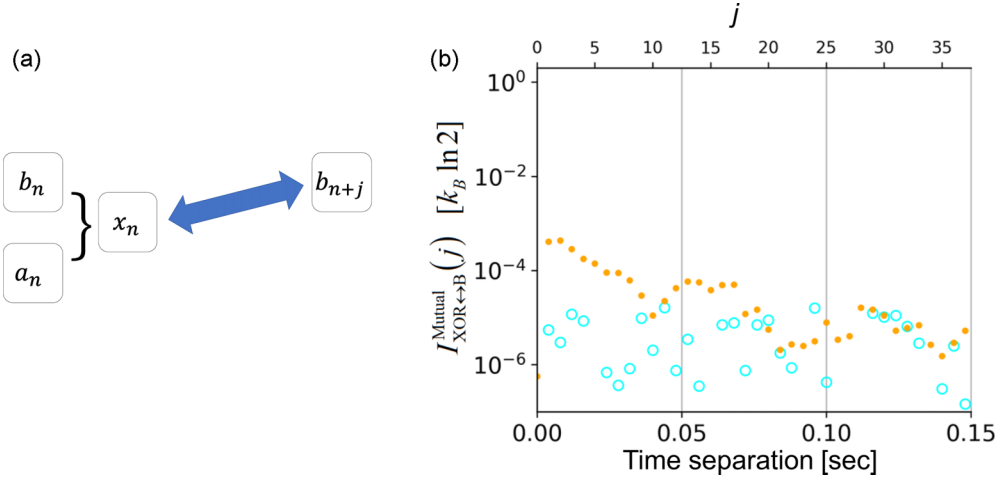


FIG. 6. (a) Schematic diagram of the mutual information $I(X_n : B_{n+j})$, where X_n represents the XOR operation between B_n and A_n . (b) Mutual information $I(X_n : B_{n+j})$ (shown in orange). Light blue markers indicate the statistical error of the analysis.

Figure 6(a) shows a diagram of the evaluated mutual information. First, we performed an XOR computation on the positional data of skyrmions A and B at the current time, treating the result as the first node. Then, we calculated the mutual information between this node and the future position of skyrmion B. If the mutual information is $k_B \ln 2$, it implies that the result of the XOR computation on the current positions of skyrmions A and B can be predicted with 100% accuracy from the future position of skyrmion B. This would indicate the presence of XOR computational capability:

$$I_{XOR \leftrightarrow B}^{Mutual}(j) = S(X_n) - S(X_n | B_{n+j}). \quad (8)$$

Figure 6(b) shows the time interval dependence of the mutual information as orange dots, as described in Eq. (8). Light blue dots represent statistical errors. The case $j = 0$ is not shown because the threshold is defined so that the probability of A and B taking the value 1 is 1/2, yielding $I_{XOR \leftrightarrow B}^{Mutual}(0) = 0$. Furthermore, in the limit $j \rightarrow \infty$, the variables become uncorrelated, and thus $I_{XOR \leftrightarrow B}^{Mutual}(\infty) = 0$. It can be seen that significant results are obtained for time separation below 0.04 s ($1 \leq j \leq 8$). From the value of mutual information, it is evident that the skyrmion system performs XOR computation at a rate of approximately 0.04% for $j = 1$. Although the value is small, it is statistically significant, and the fact that a very simple two-skyrmion system performs nonlinear natural computation in thermal equilibrium is intriguing. Here, we used relatively large skyrmions (1.5 μm) to enable observation with an optical microscope. However, skyrmions can be scaled down to approximately 10 nm, which would allow for an expected operation speed of near 1 ns. Additionally, in smaller skyrmions, a stronger repulsive force arises between skyrmions due to exchange interactions, making higher-temperature operation feasible.

IV. CONCLUSION

In this study, we conducted an information thermodynamic analysis of the thermal equilibrium state of two relatively large skyrmions confined within a box-shaped potential. Due to the repulsive force between the skyrmions, information transfer

occurs. Analyzing transfer entropy revealed that information transmission between the skyrmions takes approximately 10 ms.

An analysis of information flow, obtained from the time derivative of mutual information, confirmed that no information feedback mechanism resembling Maxwell's demon exists in this system. Furthermore, an analysis of the system's Markovian properties demonstrated that it exhibits non-Markovian behavior. Additionally, it was confirmed that the system can perform XOR computations, albeit at a minimal level. This can be regarded as a form of natural computation driven by thermal equilibrium fluctuations.

Skyrmions represent a rare system in which Brownian motion occurs in a solid at room temperature without involving material transport. The present data were obtained owing to the robustness of skyrmions against external perturbations provided by topological protection, which enabled nearly 2×10^5 sampling events. As models of information thermodynamic systems at room temperature, skyrmions hold potential for basic interest, such as violation of detailed balance [11,12,39], and future applications, such as the development of autonomous Maxwell's demons and other information thermodynamic engines, including ultralow-power computation circuits.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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