

Multiscale vortical phonons and vorticity wave vortices as coherent excitations in two-dimensional dusty plasma crystals

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Counterintuitively, cooperative spatiotemporal motions and localized coherent entities (CEs) with distinct topological features can emerge in various extended systems exhibiting disordered multiscale fluctuations. Examples include multiscale vortices in nonlinear hydrodynamic turbulent flows and multiscale wave vortices in traveling acoustic wave turbulence. However, their fundamental nature and generic behaviors in stochastically driven, disordered multiscale linear waves remain elusive. Here, we utilize two-dimensional crystals supporting thermally excited, multiscale disordered vibrations, as a model system to address these issues. By combining two-dimensional dusty plasma crystal experiments with numerical simulations of Yukawa and Lennard-Jones crystals, we report the following observations at the discrete level: (a) multiscale vortical phonons (also named as vorticity waves, VWs), in the form of propagating vortices with alternating signs of vorticities and varying vortex shape, predominately contributed by the superposition of transverse waves, as basic cooperative acoustic vibrations; and (b) vorticity wave vortices (VWVs), where the VW crest surfaces helically wind around screw-dislocation filaments in the xyt space, as localized CEs in VWs. These VWVs exhibit stretched-exponential lifetime distribution and short-range spatial correlations.

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Introduction. Microscopically, the ordered structure of a crystal supports the coherent propagation of longitudinal as well as transverse phonons [1,2]. In contrast, thermal agitation serves as a multiscale disordered source stochastically exciting and deexciting phonons, and thereby disrupting their spatiotemporal coherence. The interplay of the above order and disorder makes the cold crystal a good candidate for uncovering the emergence of coherence from disorder. Beyond the previous studies of thermal phonons such as their power spectra, density of states, mode-mode coupling, and effects on heat transport [1–8], an intriguing and fundamental question remains relatively unexplored: What are the basic cooperative vibrations, especially their spatiotemporal waveforms, after decomposing thermal vibrations into different-scale modes? It is difficult to directly visualize for real crystals in nature due to the small atomic scale.

From a broader perspective of extended ordered and disordered systems, identifying and/or generating localized coherent entities (CEs), as well as uncovering their distinct topological and dynamical features and applications, has been fundamentally challenging. Examples include skyrmions and merons in magnetic materials and water surface waves [9–12], along with wave vortices (WVs) in optical, sound, elastic, water, plasma, quantum-fluid, and atom waves [11,13–26].

These CEs are universal, localized scalar or vector fields with quantized phase increments circulating around singular phase defects and thereby carry quantumlike angular momenta. However, CEs of acoustic waves in crystals down to the discrete level have never been revealed.

Past studies about CEs have been mainly limited to *single-scale* linear and nonlinear wave systems, through the linear superposition of single-scale waves from wave source arrays with programed phase delays [11,17,21,25], and modulation instability causing waveform modulation [13,15,20,27,28], respectively. However, CE excitations in *multiscale* linear and nonlinear *disordered* waves have received much less attention, aside from the recent experimental study demonstrating the spontaneous excitations of multiscale acoustic WVs in three-dimensional *nonlinear* traveling dust acoustic wave turbulence within gaseous dusty plasmas, through modulation-instability-induced wave crest breaking and reconnection [29]. These WVs can be used to characterize the disordered wave dynamics. Certainly, this raises another important question: What kinds of CEs exist in thermal phonons and how do they behave spatiotemporally?

In essence, in addition to the significance in condensed matter physics and applications, thermal phonons in cold crystals serve as a model system of linear disordered waves driven by multiscale random agitation for uncovering coherence emerging from disorder. In this work, the relatively unexplored issues about the basic cooperative acoustic vibrations and their spatiotemporal waveforms, and CEs of thermal phonons at different temporal scales, are unraveled. By combining experimental study of two-dimensional (2D) dusty plasma crystals through direct visualization, and numerical simulations of 2D Yukawa and Lennard-Jones crystals,

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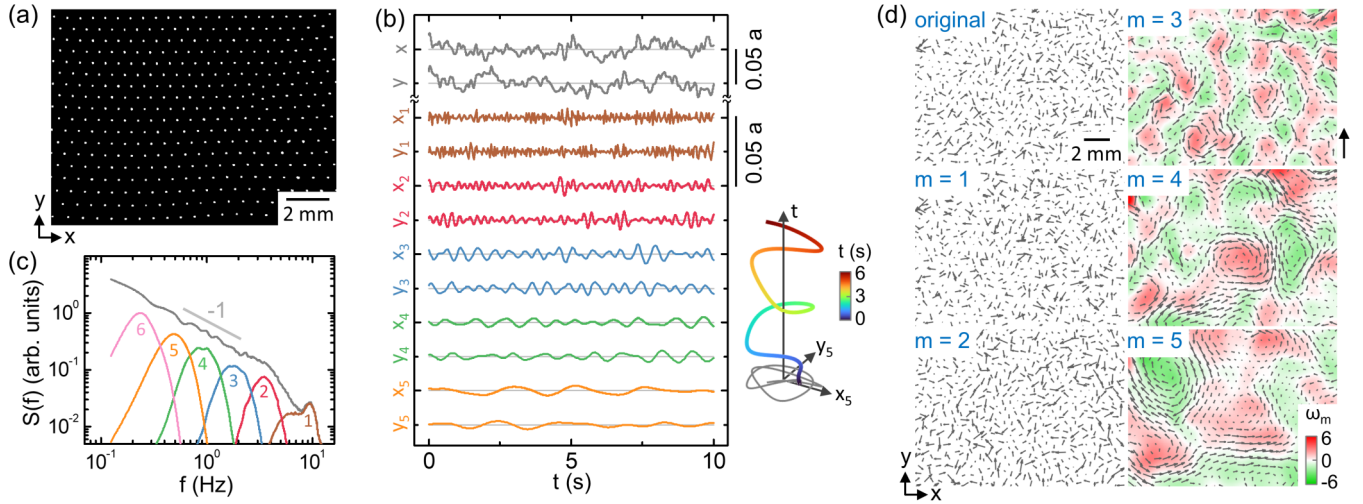


FIG. 1. (a) Typical image of the 2D monolayer dusty plasma crystal. (b) Left: example of the temporal fluctuations of $x(t)$ and $y(t)$, and the decomposed x_m and y_m for modes 1 to 5. Right: example of the irregular helical trajectory of mode 5 of a single particle around its equilibrium position in the x_5y_5t space, and its projection (gray trajectory) on the x_5y_5 plane. (c) Frequency power spectra of $x(t)$ and $x_m(t)$, averaged over the corresponding spectra of 600 particles. The number by the gray line corresponds to the scaling exponent of the power spectrum of $x(t)$. (d) Representative instantaneous configurations of the original and the decomposed particle velocities of modes 1 to 5 (represented by vectors with their ends located at particle positions), color coded by the instantaneous local vorticity ω_m for $m \geq 3$. The length of the vector on the right corresponds to the particle speed of each mode (normalized by the mean particle speeds of each mode) and equals 6.

we report the following discoveries: (a) multiscale vortical phonons (i.e., vorticity waves, VWs), in the form of propagating vortices with alternating signs of vorticities and varying vortex shapes, as basic cooperative acoustic vibration, and (b) vorticity wave vortices (VWVs) with VW crest surfaces helically winding around screw dislocations as localized CEs in VWs. Their generic dynamical behaviors are also characterized.

The dusty plasma can be formed by micrometer-size dust particles suspended in low-pressure radio-frequency discharge. Particles are negatively charged ($\sim 10^4$ e/particle) and interact through screened Coulomb interaction [30–33]. The submillimeter spatial scale and 100-millisecond temporal scales make it a good experimental platform to mimic and understand the microstructure, motion, and waves of solids and liquids through direct visualization [34–48]. Previous studies on phonons in dusty plasma crystals have predominantly focused on the dispersion relation, and mode coupling near melting [36,40,41,44,46].

Methods. Experimentally, a monolayer of a cold dusty plasma crystal composed of melamine resin (MF) particles (9.3 μm in diameter) suspended in a capacitively coupled radio-frequency argon discharge is used for investigation (see more details in the Supplemental Material, SM [49]). The motion of individual particles in the horizontal monolayer is optically tracked. The low viscous damping in the low-pressure argon background (1.3 Pa) enables underdamped particle vibrations.

Numerically, the large-scale atomic/molecular massively parallel simulator (LAMMPS) [50] is used for molecular dynamics simulations of 2D Yukawa and Lennard-Jones crystals with 16384 particles. The system is governed by the coupled Langevin equation of motion under periodic boundary conditions and low viscous damping (see more details in the SM [49]).

Similar to the previous studies on the acoustic wave turbulence in gaseous and liquid dusty plasmas, multidimensional complementary ensemble empirical mode decomposition (MCEEMD) is employed [29,45,47,51,52]. The temporal evolution $\mathbf{x}_i(t)$ of the displacement of particle i from its equilibrium position on the xy plane is adaptively decomposed (sequentially from the highest-frequency oscillation) into amplitude-frequency-modulated oscillatory intrinsic mode functions $\mathbf{x}_{im}(t)$ ($m = 1, 2, \dots, N$) with zero mean, until reaching the mean trend $\mathbf{r}_i(t)$. Namely, $\mathbf{x}_i(t) = \sum_{m=1}^N \mathbf{x}_{im}(t) + \mathbf{r}_i(t)$ and $\mathbf{x}_{im}(t) = \mathbf{a}_{im}(t) \cos \phi_{im}(t)$. After extending this nearly complete and orthogonal decomposition to all particle motions, the multiscale spatiotemporal waveforms, with the information of local amplitudes $[\mathbf{a}_{im}(t)]$ and phases $[\phi_{im}(t)]$ of mode m in the xyt space, can be revealed (see more details in the SM [49]).

Multiscale vortical phonons as cooperative vibrational excitations. Now we present our experimental findings. Figure 1(a) shows a typical image of the monolayer dusty plasma crystal with triangular lattice structure. The left panel of Fig. 1(b) shows an example of the temporal fluctuations of $x(t)$ and $y(t)$, the displacements along x and y axes of a typical particle from its equilibrium position, and its corresponding decomposed x_m and y_m for modes 1 to 5, respectively. Figure 1(c) shows the power spectra $S(f)$ of $x(t)$ and $x_m(t)$, averaged over the corresponding spectra of 600 particles (see Fig. S2 for the spatiotemporal spectra $S(k, f)$ for the transverse and longitudinal phonons in the SM [49]). Note that the averaged power spectra of $y(t)$ and $y_m(t)$ coincide with those of $x(t)$ and $x_m(t)$, respectively (not shown). All modes exhibit amplitude and frequency modulations.

From $\mathbf{x}_{im}(t)$, the temporal evolution of the instantaneous velocity $\mathbf{v}_{im}(t)$ of particle i can be extracted. Figure 1(d) presents the typical snapshots of the original particle velocity configuration and decomposed particle velocity

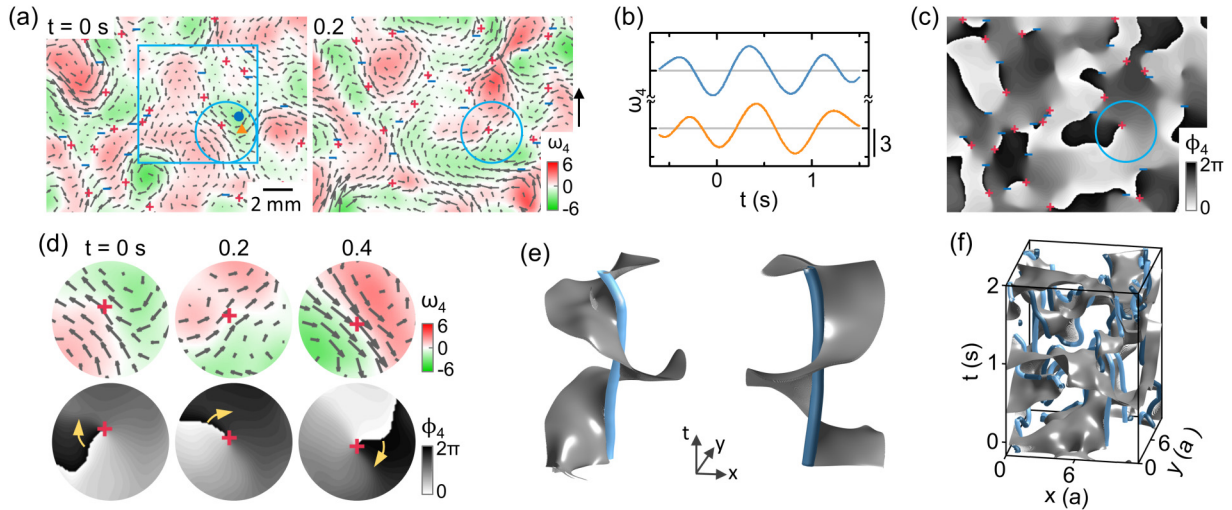


FIG. 2. (a) Snapshots of velocity fields of mode 4, color coded by ω_4 , at $t = 0$ and 0.2 s. (b) Temporal evolutions of the oscillating ω_4 with phase delay at two neighboring sites indicated by the dot and the triangle in (a). The gray lines represent the zero level of ω_4 . (c) Phase field $\phi_4(x, y)$ of ω_4 at $t = 0$ for the same regime of (a). Screw dislocations with $+1$ and -1 topological charges are denoted by $+$ and $-$, respectively, as singular points in (a) and (c), where the phases are undefined. (d) Sequential snapshots of the velocity fields color coded by ω_4 (left) and the corresponding phase fields (right) from the circled region in (a) and (c). (e) Left: Plot of the left-handed VW crest ($\phi_4 = 0$) surface helically winding around the SDF in the xyt space from the same region as (d). Right: example of the right-handed helical VW crest winding around another SDF. (f) Similar plot to that of (e) from the box region of (a). SDFs with $+1$ and -1 topological charges are color coded by brighter and darker blues, respectively, in the xyt space.

configurations of modes 1 to 5, represented by vectors with their ends located at particles. The slower modes 3 to 5 in the right column are color coded by ω_m , the instantaneous local vorticity Ω_m normalized by the spatiotemporal average of the absolute value of Ω_m , for $m \geq 3$ (see the SM for obtaining Ω_m from $\mathbf{v}_{im}$ [49]).

Basically, the velocity configurations of the left column of Fig. 1(d) exhibit disordered fluctuations. However, for the slower modes with $m \geq 3$, the instantaneous velocity configuration of each mode is composed of irregular vortices with similar scales, and spatially alternating signs of vorticities (labeled by red and olive green for positive and negative vorticities) and varying shapes. The vortex scale increases with increasing mode number.

Beyond the above spatial behaviors, what is the generic behavior of the temporal evolutions of our decomposed velocity and associated vorticity fields? The varying phase delays and ratio of the amplitudes of x_m to y_m , the example of a single-particle trajectory of mode 5 in Fig. 1(b), and the decomposed local velocity evolution in video S1 in the SM [49], reveal the rotational particle motion around their equilibrium positions with uncertain elliptical trajectories in the xy space, manifesting the *spin* nature of particle velocity. As presented in the sequential images of the velocity field color coded by ω_4 in Fig. 2(a) and video S1 in the SM [49], the spatiotemporally fluctuating swirls associated with alternating signs of vorticity, which changes their shapes and locations with time, reveal that the cooperative vibrational excitations appear in the form of propagating *vortical phonons*. Figure 2(b) demonstrates that the ω_4 at the two different sites, indicated by the dot and the triangle in the circled region of Fig. 2(a), exhibit temporal oscillations with uncertain amplitude and frequency modulations, and uncertain finite phase difference between the two curves. It mani-

festes that these propagating vortical phonons are *vorticity waves* (VWs).

Multiscale vorticity wave vortices as localized coherent entities. Do wave vortices exist as coherent entities in the above vortical phonons? From the oscillating $\omega_m(\mathbf{x}, t)$, its local instantaneous phase $\phi_m(\mathbf{x}, t)$ can be extracted using Hilbert transform [53]. Figure 2(c) depicts the corresponding ϕ_4 field of Fig. 2(a) at $t = 0$ s (left panel). It clearly demonstrates that there are many screw-dislocation defects with their locations denoted by $+$ and $-$, as singular points, where phases are undefined. Namely, each defect has $+1$ or -1 topological charge because of the 2π or -2π phase change after circulating one cycle counterclockwise around each defect.

As shown in Fig. 2(d), sequential images of the velocity field, color coded by ω_4 and ϕ_4 , enlarged from the circled region of Fig. 2(a) (mode 4), illustrate more clearly how the VW propagates spatially around the defect. The red vortex with positive vorticity trailed by the adjacent olive green vortex with negative vorticity, circles clockwise around the $+1$ defect, as indicated by the yellow arrows (also see video S2 in the SM [49]). This propagation causes a temporal oscillation of the local vorticity with phase delay between two separated points, as demonstrated in Fig. 2(b).

The left panel of Fig. 2(e) again manifests that in the xyt space, the above $+1$ defect appears in the form of a screw-dislocation filament (SDF), wound around by the left-handed helical VW crest ($\phi_4 = 0$) surface of vorticity oscillation. The right panel of Fig. 2(e) shows another example of the -1 SDF filament wound around by the right-handed helical VW crest surface. These examples evidence that they are the cores of VWVs serving as coherent entities of thermally excited vibrations. Figure 2(f) shows more SDFs and the wave crest surface for mode 4 in the xyt space, from the boxed region of Fig. 2(a).

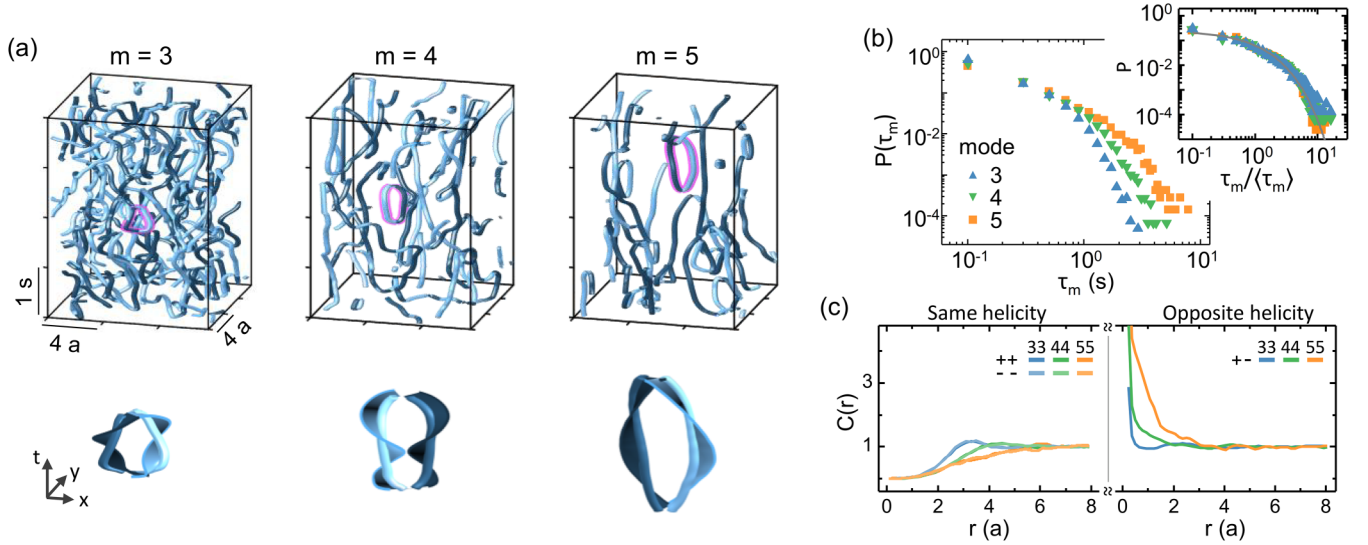


FIG. 3. (a) Top row: examples of irregular SDFs in the xyt space for modes 3 to 5, with $+1$ and -1 SDFs topological charges, labeled by brighter and darker blues, respectively. Bottom row: SDF loops and the VW crest surfaces magnified from SDF loops labeled by pink edges in the top row. (b) $P(\tau_m)$, the probability distribution functions of SDF lifetimes τ of different modes, showing the increasing mean lifetime $\langle\tau_m\rangle$ of modes 3 to 5. Inset: collapsing of those curves after rescaling by $\langle\tau_m\rangle$ of each mode. The gray line is the curve for the best fit of the corresponding collapsed curves from the simulation of Yukawa crystals (Fig. S5 of the SM [49]), exhibiting stretched exponential decay: $P(\tau/\langle\tau\rangle) \propto \exp^{-1.5(\tau/\langle\tau\rangle)^{0.75}}$. (c) Radial pair-correlation function $C(r)$, the probabilities of finding two SDFs with separation r , under different combinations of helicity.

Other slow modes also exhibit the similar behavior. For example, video S1 in the SM [49] shows the evolutions of the velocity, vorticity, and phase fields labeled by defect locations (and charges) of modes 3 to 5. Figure 3(a) further presents the SDFs of several typical modes in the xyt space over a longer time, with the same color coding as those in Fig. 2(f). Namely, the excitation of each mode can be viewed as a collection of irregular VWVs winding around wormlike SDF cores with alternating helicities.

The dynamics of irregular SDFs is not completely disordered. In the xyt space, SDFs with opposite charges can be pairwise generated, propagate, and be pairwise annihilated with their twin sisters, as shown in Figs. 3(a), or be pairwise annihilated with other conjugate SDFs from other SDF pair generations, as shown in videos S1 and S3 in the SM [49], under the conservation of topological charges. This leads to the finite lifetime of each SDF and the formation of SDF loops in the xyt space.

Figure 3(b) shows that $P(\tau)$, the probability distribution functions of SDF lifetimes of different modes, basically all follow stretched exponential decays. Their mean lifetimes $\langle\tau_m\rangle$ increase with m . As shown in the inset, those curves nearly collapse after rescaling by $\langle\tau_m\rangle$ (the gray line is the best fitted curve $[P(\tau/\langle\tau\rangle) \propto \exp^{-1.5(\tau/\langle\tau\rangle)^{0.75}}]$). It indicates their self-similar nature over different modes.

How are screw dislocations with the same/opposite charge correlated spatially? Figure 3(c) shows the radial pair-correlation function $C(r)$, the probabilities of finding two screw dislocations separated by distance r of modes 3 to 5, under different combinations of topological charges. For the same (opposite) charge, the strong negative (positive) deviation of $C(r)$ from 1 in the small- r regime implies a

repulsive (attractive) like nature leading to the smaller (larger) likelihood of the appearance of an SDF nearby another SDF with the same (opposite) charge. The slower increase (decay) of $C(r)$ curves with increasing mode number indicates the increasing averaged inter-screw-dislocation separation scale.

Numerical results. To test the generality of our above experimental findings, we further conduct numerical simulation of 2D Yukawa and Lennard-Jones crystals. They also demonstrate the multiscale vortical phonons and VWVs exhibiting the similar generic spatiotemporal behaviors (see details of the simulation method and results in the SM [49]).

Discussion. In our 2D lattice, thermal agitations act as spatiotemporally uncorrelated stochastic sources for exciting and deexciting multiscale transverse and longitudinal phonons. For the slow modes with $m \geq 3$, the average frequency of each mode is less than 2.5 Hz (see Fig. 1(c) here and Fig. S2 in the SM [49]). At a fixed frequency, the higher compression modulus compared to the shear modulus causes longitudinal waves to propagate faster and are harder to excite. Consequently, longitudinal waves exhibit longer wavelengths and lower intensities than transverse waves in the power spectra $S(k, f)$ of Fig. S2 in the SM [49]. Namely, each low-frequency mode is contributed predominantly by transverse acoustic waves with uncertain amplitudes, phases, and propagation directions within its frequency band. How the superposition of these waves leads to the emergence of observed VWs and VWVs with irregular SDF cores in each mode is an interesting issue and needs further theoretical investigation. For the first two fast modes, the corresponding short wavelength scales and additional contribution from stronger longitudinal modes (see Fig. S2 in the SM [49]) cause the disordered patterns of the velocity fields shown in Fig. 1(d).

The behaviors of SDFs as cores of VWVs, such as pair generation/annihilation, stretched exponential lifetime distribution, and short-range spatial correlation, are similar to those of the SDFs as the cores of the acoustic vortices in the xyt space of the 3D *nonlinear* traveling longitudinal dust acoustic (density) wave turbulence in the *gaseous* dusty plasma [18,27], spontaneously generated through nonlinear waveform undulation instability. It manifests the robust universal features and dynamical behaviors of those topological defects in disordered wave systems, regardless of their different generation mechanisms in different types of waves.

Note that in the incompressible hydrodynamic turbulence which only allows transverse relative motion, multiscale vortical (swirl) flows with varying shapes and spatially alternating signs of vorticities are the basic coherent excitations. From this perspective, predominated by transverse relative motions, similar vortical velocity field patterns can be expected for the slow modes in solids. However, unlike vortices in hydrodynamic flows which persist over much longer times, the accumulated shear strain in the solid due to the relative transverse (sheared) displacements in the saddle region of the vortical pattern generates a rebounding force, which is the oscillatory origin for the multiscale vortical phonon excitations.

Also note that swirls of the instantaneous displacement field of low-frequency normal modes have been demonstrated in the recent studies on the particle vibrations in ordered crystals, and amorphous solids and glasses, using dynamical matrix or Hessian matrix methods [54–57]. It agrees with our finding of swirls of slow modes contributed by transverse vibrations. However, the spatiotemporal evolutions of swirls have not been addressed to demonstrate that they are propagating vortical phonons. Also, VWVs have not been demonstrated. In particular, in the colloidal systems commonly used for the corresponding experimental studies [54,55], the highly viscous background liquid overdamps particle vibrations.

Conclusion. Using empirical mode decomposition to separate disordered particle vibrations into multiscale modes, we have demonstrated the following key findings, experimentally in 2D dusty plasma crystals and numerically in 2D Yukawa and Lennard-Jones crystals under low viscous damping: (a)

multiscale vortical phonons (i.e., vorticity waves) as cooperative acoustic excitations, and (b) multiscale vorticity wave vortices as localized coherent entities.

We find that each particle undergoes irregular elliptical motion around its equilibrium position. For slow modes with $m \geq 3$, particle vibrations collectively manifest as vortical phonons—coherent patterns of traveling vortices with varying shapes and alternating signs of vorticities—leading to what we term vorticity waves, with spatiotemporal oscillation of local vorticity. The typical vortex scale increases with m . In the xyt space, the multiscale vortical phonons can be represented by a collection of multiscale vorticity wave vortices, serving as localized coherent entities, where the vorticity wave crests helically wind around irregular screw-dislocation filaments. These screw-dislocation filaments with opposite helicities can be pairwise generated, propagate, and be pairwise annihilated with their twin sisters or other screw-dislocation filaments in the xyt space, under topological charge conservations. They exhibit stretched-exponential lifetime distribution and short-range spatial correlations. The superposition of stochastically excited transverse phonons, each with uncertain amplitude, phase, and propagation direction in each low-frequency mode, is the key for the emergence of vortical phonons and vorticity wave vortices.

Our study lays the groundwork for future studies using VWVs as coherent entities to characterize the disordered dynamics of thermally excited phonons, and to correlate them with structural heterogeneity and rearrangements, and melting processes, in various 2D and 3D crystals, amorphous solids, glasses, and cold liquids. Furthermore, it opens avenues for identifying WVs and exploring whether similar generic dynamical behaviors of WVs are also followed in other stochastically driven multiscale disordered wave media.

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Data availability. The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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