

Erratum: Z_2 Dirac points with topologically protected multihelicoid surface states **[Phys. Rev. Research 4, 033170 (2022)]**

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We have found that the definition of the Z_2 topological invariant $\mathcal{Q}$ given in Eq. (3) in our original paper is not correct because it does not actually fix the gauge. In the paper, the gauge is chosen to be continuous within the region $\mathcal{A}$ in Fig. 1(a) [also in Figs. 2(a) and 2(c) in our paper]. Meanwhile, this region contains the Dirac point, where the eigenstates become singular and the gauge choice becomes problematic. Thus, we need to circumvent this singularity by deforming the region $\mathcal{A}$ so that it does not pass through the Dirac point. It is accomplished, e.g., by removing a small disk around the Dirac point from the region $\mathcal{A}$ and replacing it by a hemisphere [Fig. 1(b)], resulting in the region $\mathcal{A}'$ [see Fig. 1(c)]. Thus the argument in our paper becomes correct by choosing the gauge to be continuous on the region $\mathcal{A}'$.

By this change, we have found that the argument on Ref. [1] (Ref. [21] in our paper) on the ill-definedness of the Z_2 topological invariant given in Sec. III A in the paper was not correct. In fact, if the gauge is chosen to be continuous on the hemisphere described above (as assumed in Ref. [1]), the Z_2 topological invariant given in Ref. [1] [see Eqs. (1) and (2) in our paper] is gauge independent, and well defined. Thus it correctly gives the Z_2 topological invariant. These changes do not alter the main conclusions of our paper.

Furthermore, in Sec. III, we wrote that the equality of the two Z_2 invariants, $\mathcal{Q} = \nu$ [Eq. (15) in our paper], holds both in spinless and spinful systems, but we have found that it holds only in the spinless cases. It is because in spinful systems, the $\tilde{\Theta} = \mathcal{TG}$ operation does not switch the glide- $\pm$ sectors on the ABA' and EDE' lines, which invalidates the discussion leading to Eq. (15). This does not affect the subsequent discussions since in Sec. IV we used this equation only for spinless systems.

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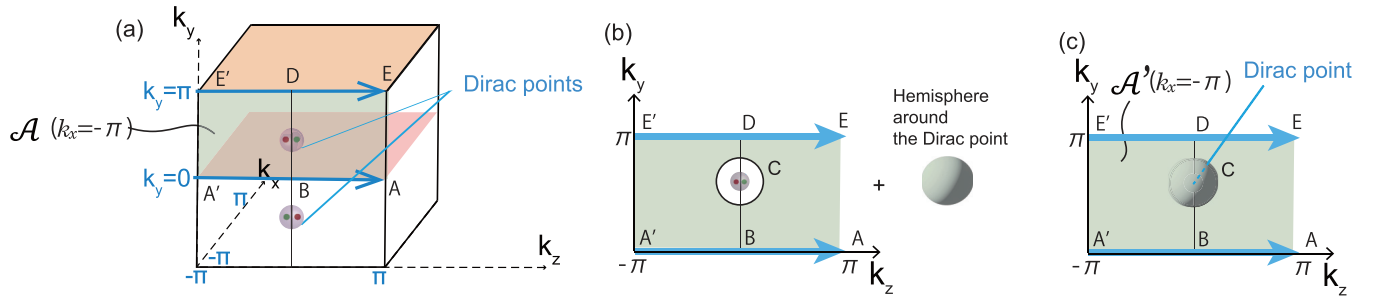


FIG. 1. Brillouin zone for the space group No. 7, which only has a glide symmetry. To define the $\tilde{\Theta}$ polarizations along the $k_y = 0$ and $k_y = \pi$ lines to calculate the Z_2 topological invariant, we need to fix the gauge. In our paper it was chosen to be continuous across the region $\mathcal{A}$ in (a). However, the wave functions are singular at the Dirac point located on the DB line, we need to circumvent this point. Thus we remove a small disk around the Dirac point from the region $\mathcal{A}$ and replace it by a small hemisphere around the Dirac point, as shown in (b). Thus, to define the Z_2 topological invariant, we fix the gauge to be continuous across the region $\mathcal{A}'$ in (c).

[1] C. Fang, L. Lu, J. Liu, and L. Fu, Topological semimetals with helicoid surface states, *Nat. Phys.* **12**, 936 (2016).