

Long-range quantum emitter interactions mediated by a nonlocal metasurface: Application to qubit-qubit entanglement

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Scalable quantum technologies demand long-range interactions between many distant quantum emitters (QEs). We introduce nonlocal metasurfaces supporting bound states in the continuum (BICs) as a promising platform to achieve this goal. We show that efficient QE interactions depend almost entirely on emitter-BIC coupling efficiencies (β factors), which in our system can exceed 80% even without additional mode engineering. These values rival those of one-dimensional waveguides but are achieved here in a geometry that naturally accommodates large two-dimensional QE arrays. Using this platform, we explore entanglement generation between two remote QEs, finding that it develops faster than in free space, is significantly amplified, and persists over separations spanning several emission wavelengths. Optimal inter-QE interactions require large β factors but only moderately small Purcell factors, both within experimentally achievable ranges. Our results establish nonlocal metasurfaces as a practical and scalable platform for leading-edge quantum nanophotonics.

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I. INTRODUCTION

Long-range interactions in quantum systems are highly sought after because they enable collective quantum phenomena and play a central role in advancing emerging quantum technologies [1]. In particular, they facilitate robust and scalable entanglement between spatially separated quantum emitters (QEs), a key requirement for quantum computing [2]. Such interactions also circumvent the need to fabricate subwavelength QE arrays [3]—a process that is experimentally demanding, and undesirable when individual optical, electrical, or magnetic control over each QE is required, since dense packing inevitably leads to crosstalk.

Photon-mediated interactions can be enhanced by tailoring structured photonic environments [4], which has inspired extensive theoretical [5–8] and experimental [9–11] studies. In particular, one-dimensional (1D) waveguides [12] have been proposed to enable long-range QE-QE coupling and entanglement creation [13–18], with a recent experiment

demonstrating enhanced coupling of quantum dots via a photonic crystal waveguide [19]. However, the intrinsic 1D geometry of such platforms poses a challenge to scalability, which motivates the exploration of two-dimensional (2D) architectures capable of supporting entanglement across planar QE arrays.

Metasurfaces, comprising planar arrays of engineered nanoparticles (“meta-atoms”), enable subwavelength control of light into the quantum regime [20,21], while also offering scalability, along with advantages in fabrication and integration [22–26]. A theoretical study by Jha *et al.* [27] demonstrated the potential of metasurfaces to effectively facilitate entanglement generation by considering atomic qubits in the metasurface *far field*. However, suspending the QEs in the far field requires additional systems, such as optical tweezers [28], and the metasurface design was limited to two atoms at specific positions, making it unsuitable for QE arrays. More recently, Zundel *et al.* [29] studied long-range interactions mediated by lattice resonances in the *near field* of a metallic metasurface, which are compatible with planar QE arrays. While near-field coupling to such modes offers a promising architecture for compact integration [30–32], it also introduces losses inherent to metallic systems.

Here, we propose instead to use *nonlocal* metasurfaces that support *bound states in the continuum* (BICs). BICs are spatially delocalized wave states spectrally embedded in the radiation continuum but decoupled from it, preventing energy leakage and yielding infinite lifetimes and quality factors (Q

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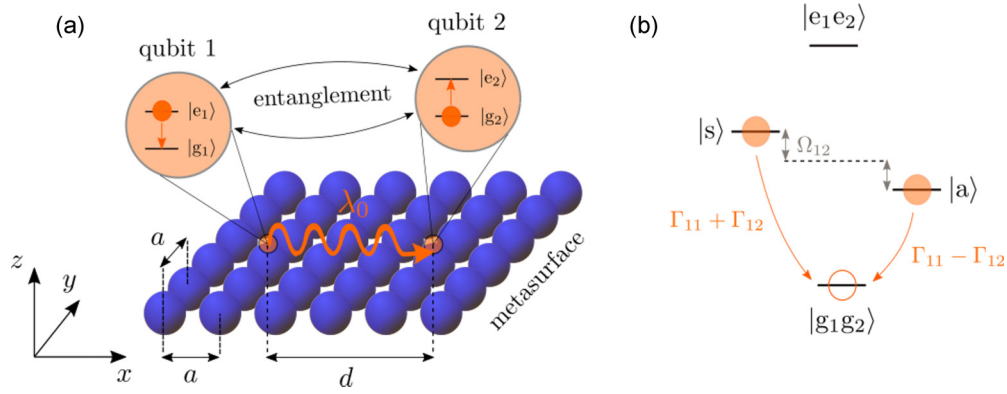


FIG. 1. (a) Schematic illustrating long-range metasurface-mediated interactions between two qubits. Initially uncorrelated QEs (qubits) spontaneously emit into delocalized BIC modes, resulting in entanglement over distances much larger than the wavelength of emitted photons. (b) Dicke states of the two emitter system are connected by decay channels with rates $\Gamma_{11} \pm \Gamma_{12}$. Initially QE1 is excited, such that the two-QE system is in an equal superposition of symmetric state $|s\rangle$ and the antisymmetric state $|a\rangle$.

factors) [33–37]. Although they cannot be excited via the far field, BICs can be efficiently accessed using localized near-field sources [38–41]. In practice, finite and imperfectly fabricated metasurfaces host quasi-BICs with very high yet finite Q factors.

We find that nonlocal metasurfaces supporting BICs can mediate long-range interactions and entanglement between QEs over separations exceeding several emission wavelengths, persisting indefinitely in the ideal case of an infinite metasurface with perfect spectral and spatial matching between the QEs and the BIC mode. QE-BIC coupling efficiencies, quantified by β factors [42–44], are found to be exceptionally high for a 2D platform, exceeding 80% and potentially improvable through further metasurface optimization. For configurations that maximize QE-BIC coupling, the resulting QE-QE entanglement depends almost entirely on the β factor, increasing monotonically with it. This is counterintuitive from a classical perspective, where one would expect that the source dipole field would spread across the metasurface plane such that only a small fraction reaches the target emitter, significantly reducing their coupling and entanglement. Moreover, the metasurface induces a notable Purcell effect that accelerates entanglement generation, a key advantage for fast quantum operations.

The article is organized as follows. We first introduce the composite QE-metasurface system, where the QEs are described using a Lindblad master equation. The metasurface-modified single and collective QE decay rates are expressed in terms of the local and cross densities of states (LDOS and CDOS) [45]. We compute these quantities for two distinct BICs—electric dipole (ED-BIC) and magnetic dipole (MD-BIC)—which exhibit different optimal near-field coupling conditions. A combined analytical and numerical analysis yields β factors that quantify QE-BIC coupling and determine the resulting dynamics. To quantify QE-QE entanglement generated through spontaneous emission [46], we employ concurrence and derive an approximate expression in terms of β and Purcell factors, which can serve as a design guide for metasurfaces interfaced with QE arrays.

II. RESULTS

A. The QE-metasurface system

We consider a pair of two-level QEs, labeled $\mu = 1, 2$ [Fig. 1(a)], with ground and excited states $|g_\mu\rangle$ and $|e_\mu\rangle$, respectively, and identical transition energy $\hbar\omega_0$, where $\hbar$ is the reduced Planck constant. The corresponding transition wavelength is $\lambda_0 = 2\pi c/\omega_0$, where c is the speed of light. We assume identical transition dipole moment magnitudes, $p_1 = p_2 = p$, but allow different orientations; $\hat{\mathbf{p}}_1 \neq \hat{\mathbf{p}}_2$ in general. Initially, QE1 is prepared in its excited state ($|e_1\rangle$), while QE2 is in its ground state ($|g_2\rangle$).

The metasurface under study is depicted in Fig. 1(a), comprising a square array of 21×21 silicon nanospheres in the xy plane, with lattice constant $a = 400$ nm, radius $R = 100$ nm, and refractive index $n = 3.5$ [47]. This finite size balances computational tractability and physical realism. The QEs are positioned at a vertical height $h = z - R = 4$ nm above the metasurface, enabling efficient near-field coupling to its optical resonances.

The two-QE system can be described in terms of four (Dicke) levels, with two decay channels of rates $\Gamma_{11} \pm \Gamma_{12}$, separated by energy Ω_{12} [15,27,46] [shown in Fig. 1(b)]. The coefficients $\Gamma_{\mu\nu}$ and $\Omega_{\mu\nu}$ are expressed in terms of the classical Green's tensor $\mathbf{G}$ of Maxwell's equations, evaluated at the QE positions $\mathbf{R}_\mu$ and $\mathbf{R}_\nu$, as [48]

$$\Gamma_{\mu\nu} = \frac{2p_i p_j \omega_0^2}{\hbar \varepsilon_0 c^2} \Im(G_{ij}(\mathbf{R}_\mu, \mathbf{R}_\nu, \omega_0)) \quad (1)$$

and

$$\Omega_{\mu \neq \nu} = -\frac{\omega_0^2}{\hbar \varepsilon_0 c^2} p_i p_j \Re(G_{ij}(\mathbf{R}_\mu, \mathbf{R}_\nu, \omega_0)), \quad (2)$$

where ε_0 is the vacuum permittivity and repeated Cartesian indices i, j are summed (Einstein convention). We denote Γ_{11} (Γ_{22}) as the *single-emitter decay rate* of QE 1 (QE 2), Γ_{12} as the *collective decay rate*, and Ω_{12} as the *collective coupling rate*. In the dissipative regime, ($\Omega_{12} \simeq 0$, justified later), the effective coupling between the two QEs is quantified by Γ_{12} .

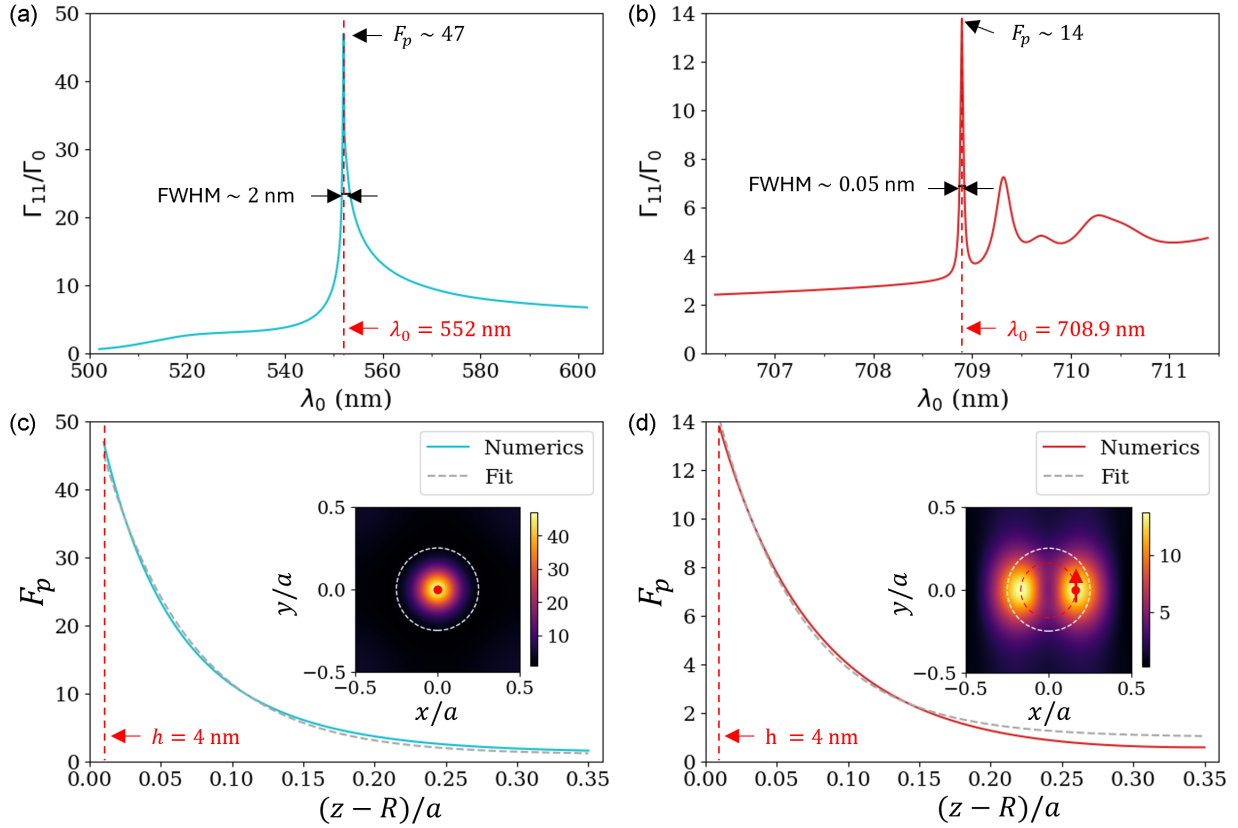


FIG. 2. Normalized single-emitter decay rates Γ_{11}/Γ_0 (LDOS) as a function of emitter wavelength λ_0 for the ED-BIC [panel (a)] and MD-BIC [panel (b)] at emitter height $h = z - R = 4$ nm above the metasurface, and Purcell factor F_p as a function of z for the ED-BIC at $\lambda_{\text{BIC}} = 552.0$ nm [panel (c)] and for MD-BIC at $\lambda_{\text{BIC}} = 708.9$ nm [panel (d)]. In panels (a) and (b), the emitter's position and orientation are chosen to maximize the coupling (see main text). Dashed gray lines are exponential fits of the form $1 + Ae^{-B(z-R)/a}$ with fitting parameters $A = 51.80$ and $B = 16.05$ [panel (c)], and $A = 15.47$ and $B = 16.97$ [panel (d)]. Insets in panels (c) and (d): Purcell factors as a function of lateral position in the xy plane at $h = 4$ nm for an emitter with orientation along the z axis (out-of-plane) [panel (c)] and orientation along the y axis (in-plane) [panel (d)]. Red dots and arrows show the lateral positions and orientations of maximum coupling considered in panels (a) and (b). Dashed white circles show a nanosphere cross section. The dashed red circle in panel (d) shows the positions where maximum coupling occurs.

More generally, when normalized by the free space dipole decay rate $\Gamma_0 = \omega_0^3 p^2 / (3\pi \epsilon_0 \hbar c^3)$, $\Gamma_{\mu\mu}$ and Γ_{12} depend only on the electromagnetic environment. They are related, respectively, to the LDOS and CDOS, which are proportional to the imaginary part of $\mathbf{G}$. Unlike the LDOS, the CDOS can take negative values, though the inequality $|\Gamma_{12}| \leq \sqrt{\Gamma_{11}} \sqrt{\Gamma_{22}}$, stemming from properties of the Green's tensor, is always satisfied [45].

In free space, significant dipole-dipole interactions only occur for separations $d < \lambda_0$ [49], since radiation from one dipole spreads isotropically and only a small fraction reaches the target. In contrast, we will show that QE-metasurface platforms can be engineered to enhance emission through resonant modes, enabling dipole-dipole interactions over macroscopic separations $d > \lambda_0$. As mentioned earlier, we focus here on two types of optical resonance: the electric-dipole BIC (ED-BIC) and the magnetic-dipole BIC (MD-BIC) [47]. In each case, the LDOS and the CDOS are computed numerically using the Python package SMUTHI (see Sec. I of the Supplemental Material [50]).

To model the two-QE system dynamics, we use the reduced density matrix $\rho(t)$, which satisfies a Lindblad master

equation of the same form as in free space [51], but with coefficients given by Eqs. (1) and (2) to account for the metasurface environment (details are provided in Sec. II of the Supplemental Material [50]). This equation is valid in the weak-coupling regime. We have verified that at $h = 4$ nm the QEs remain weakly coupled to the metasurface provided dipole moment magnitudes do not exceed ~ 90 D (see Sec. III of the Supplemental Material [50]). Consequently, in solving the master equation with the Python package QuTIP, we restrict to $p \sim 3$ D (see Sec. I of the Supplemental Material [50]).

B. Metasurface local density of states

Figures 2(a) and 2(b) show the normalized single-emitter decay rates Γ_{11}/Γ_0 , also referred to as the LDOS, as a function of λ_0 near the resonance wavelengths of the ED-BIC and MD-BIC, respectively. The maximum value, achieved when λ_0 matches the BIC resonance, corresponds to the Purcell factor F_p . This is plotted in Figs. 2(c) (ED-BIC) and 2(d) (MD-BIC) as a function of the QE vertical position above the metasurface. At $h = 4$ nm, $F_p = 46.9$ for the ED-BIC,

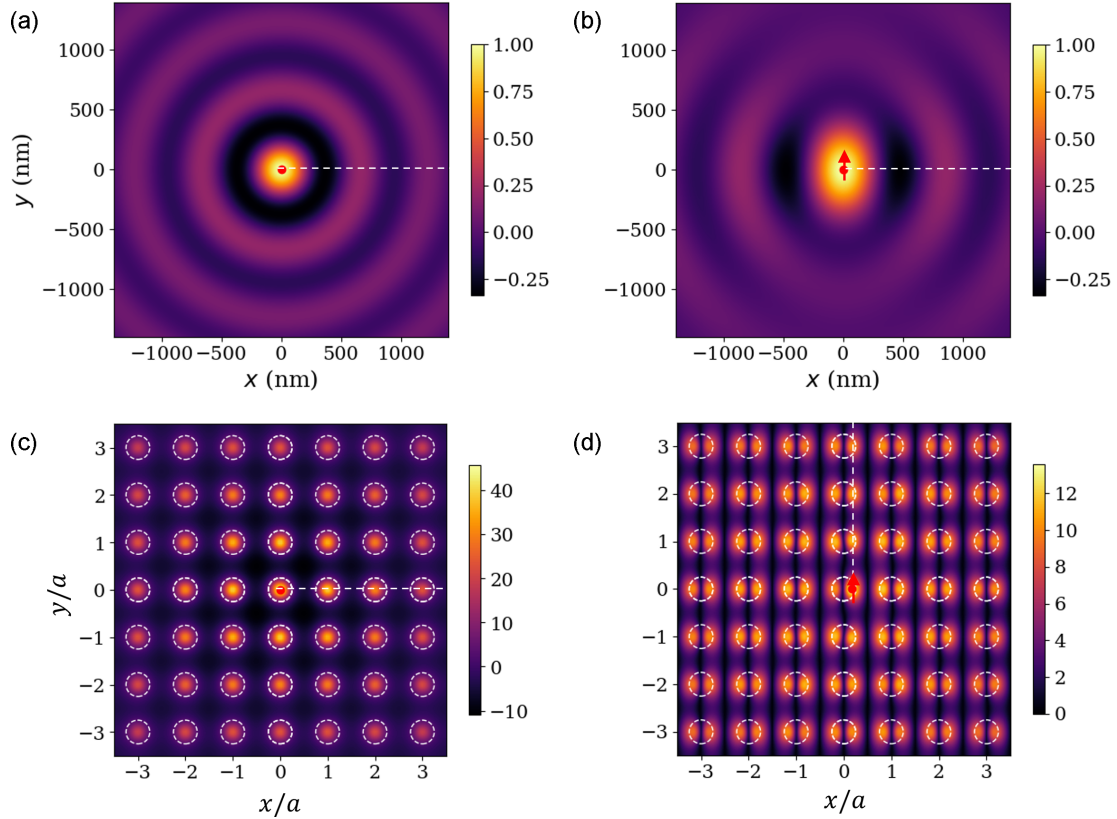


FIG. 3. (a), (b) Normalized collective decay rate Γ_{12}/Γ_0 (CDOS) in free space, with the first emitter fixed at $(x = 0, y = 0)$ (red dot) and the second emitter scanned laterally across the xy plane. In panel (a), both dipoles are oriented along z , and in panel (b) they are oriented along y . (c) Γ_{12}/Γ_0 in the presence of the metasurface for the ED-BIC, with the first emitter again at $(0, 0)$. (d) Collective decay rate for the MD-BIC, plotted as the absolute value $|\Gamma_{12}|/\Gamma_0$ to enhance visual contrast (the full signed distribution is given in Sec. V of the Supplemental Material [50]). Here, the first emitter is located at $(x = 0.163a, y = 0)$. In the left (resp. right) column, both emitters are oriented along the z axis (out-of-plane) [resp. y axis (in-plane)], with transition wavelength $\lambda_0 = 552.0$ nm (resp. $\lambda_0 = 708.9$ nm). In the metasurface cases [panels (c) and (d)], both emitters are positioned at a height $h = 4$ nm above the array. The spatial regions shown for free space [panels (a) and (b)] match those for the metasurface [panels (c) and (d)], covering 7×7 nanospheres centered on the central element, for direct comparison. Dashed white circles indicate nanosphere cross sections. Dashed white lines mark the cuts used in later analysis: along x at $y = 0$ in panels (a)–(c) and along y at $x = 0.163a$ in panel (d).

occurring at resonance wavelength $\lambda_0 = 552$ nm, and $F_p = 13.7$ for the MD-BIC, occurring at $\lambda_0 = 708.9$ nm. Although the MD-BIC possesses a smaller Purcell factor, it exhibits a much narrower linewidth (smaller full width at half maximum), corresponding to a larger Q factor, which becomes relevant in the strong-coupling regime (see Sec. III of the Supplemental Material [50]). Insets in Figs. 2(c) and 2(d) show the lateral dependence of F_p at fixed emitter orientations.

The emitter orientation and lateral position are chosen to maximize coupling. For the ED-BIC, the emitter must be placed at the center of a nanosphere with an out-of-plane orientation (z direction), as illustrated in Fig. 2(c). The MD-BIC allows greater flexibility: The emitter can be positioned anywhere on a circle of radius $r = 0.163a$ centered on a nanosphere, with an in-plane orientation tangential to this circle [Fig. 2(d)]. Note that small positional changes will not affect the coupling drastically; for example, the Purcell factor decreases by less than 10% for an emitter within a circle of diameter 30 nm around the optimum position for the ED-BIC (40 nm for the MD-BIC). Such positioning accuracy may be well within reach experimentally [52,53].

For both BICs, the dependence of QE-BIC coupling (Purcell factor) on vertical position z is essentially exponential and well described by the function $1 + Ae^{-B(z-R)/a}$, where A and B are dimensionless fitting parameters [dashed gray lines in Figs. 2(c) and 2(d)]. This behavior is characteristic of near-field coupling to a surface-bound mode and indicates that varying z provides predictable control over the coupling strength. It also allows prediction of the variation of the coupling with vertical position changes. Contrary to the lateral position, the Purcell factor is much more sensitive to variation in height: It decreases by 10% when the height is increased by ~ 2 nm for the ED-BIC (~ 3 nm for the MD-BIC) from the value $h = 4$ nm considered here. However, the height can be controlled very precisely with certain materials, such as 2D materials [31].

C. Metasurface cross density of states

We now consider the coupling between two QEs. Beginning with free space, Figs. 3(a) and 3(b) show the spatial dependence of the normalized collective decay rate Γ_{12}/Γ_0

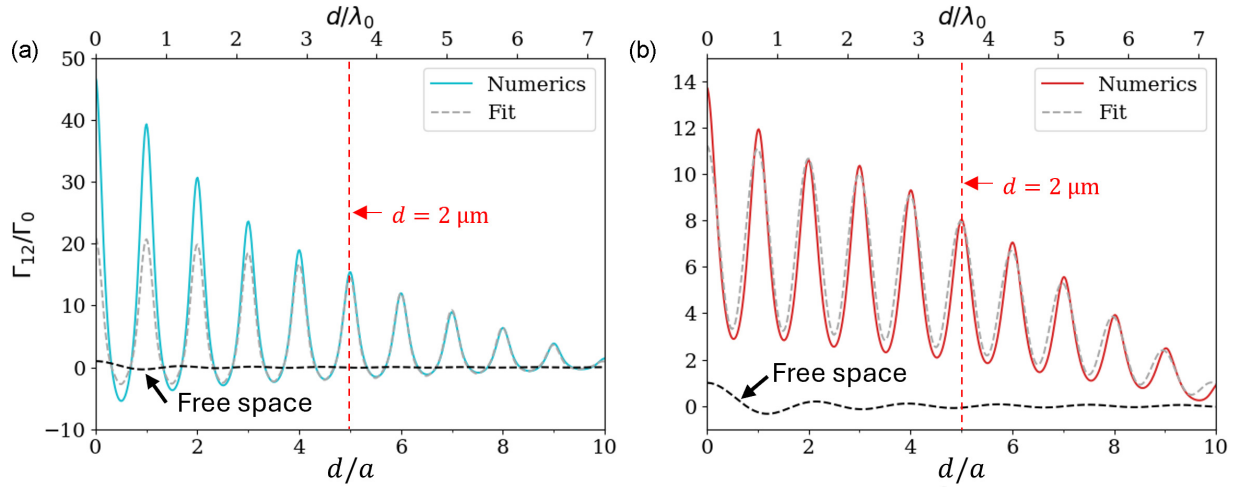


FIG. 4. Normalized collective decay rate Γ_{12}/Γ_0 (CDOS) as a function of emitter separation d for the ED-BIC [panel (a)] and for the MD-BIC [panel (b)], for cross sections indicated by the white dashed lines in Figs. 3(c) and 3(d), respectively. The dashed gray curves are fits to the numerical data using Eq. (3) with fitting parameters $\beta = 44.80\%$, $k_p = 0.581 \text{ rad}/\mu\text{m}$ for the ED-BIC and $\beta = 81.79\%$, $k_p = 0.562 \text{ rad}/\mu\text{m}$ for the MD-BIC. The fits are made over the range $d \geq 5a$ for the ED-BIC and $d \geq 2a$ for the MD-BIC, for which Γ_{12} is well approximated by Eq. (3) (see Sec. VI of the Supplemental Material [50]). The dashed black curves correspond to the free space case, for cross sections indicated by the white dashed lines in Figs. 3(a) and 3(b).

(i.e., the CDOS), when QE1 is fixed (red dot) and QE2 is scanned across the surface. In Fig. 3(a), both dipoles are oriented along z , while in Fig. 3(b) they are oriented along y . In order to directly compare with the CDOS in the presence of the metasurface, the transition wavelength is chosen to match the ED-BIC ($\lambda_0 = 552.0 \text{ nm}$) in Fig. 3(a) and the MD-BIC ($\lambda_0 = 708.9 \text{ nm}$) in Fig. 3(b). The CDOS in free space is calculated analytically from Eq. (1) using the free-space Green's tensor (see Sec. IV of the Supplemental Material [50]). As expected, in three dimensions (3D), coupling between two point emitters is limited to close distances ($d < \lambda_0$) because of the isotropic spread of the radiated energy into free space.

In stark contrast, with the introduction of the metasurface, spatial interactions are completely redefined, as shown in Figs. 3(c) and 3(d). Here, the position and orientation of QE1 are chosen to maximize coupling to the BIC, as in Figs. 2(a) and 2(b). Remarkably, there is a significant enhancement of the CDOS whenever QE2 is located at nanosphere positions equivalent to that of QE1 within the unit cell. Coupling through the BIC modes yields large CDOS values over separations beyond the transition wavelength, $d > \lambda_0$, even exceeding several wavelengths, as we shall see later. This result is rather counterintuitive, since the BIC system corresponds in principle to an isotropic 2D system; hence, the radiation should be spread more or less uniformly, but in a 2D plane instead of 3D space. We will discuss later why the BIC system behaves effectively like a 1D system.

While in these plots QE2 has the same orientation as QE1, interestingly in the MD-BIC case two QEs with orthogonal in-plane orientations (e.g., QE2 aligned along x , while QE1 is aligned along y) can still couple strongly (see Sec. V of the Supplemental Material [50]). In fact, for the MD-BIC, the CDOS associated with any in-plane orientation of QE2 can be generated by summing dipole moment magnitude weighted heatmaps of the x - and y -oriented dipoles. This provides greater flexibility than the ED-BIC, where orthogonal ori-

entations do not couple. Note that for the MD-BIC, we plot the absolute value of the collective decay rate to improve visual contrast; the full signed Γ_{12} distribution is shown in Sec. V of the Supplemental Material [50]. For the cross section considered in later analysis (along the y axis at $x = 0.163a$), Γ_{12} is positive, so taking the absolute value does not alter subsequent results.

D. QE-BIC β factors

To further evidence the peculiar nature of the 2D BIC system, we show in Figs. 4(a) and 4(b) cross sections of the CDOS for the ED-BIC and MD-BIC, taken along the white dashed lines in Figs. 3(c) and 3(d), respectively. In the ED-BIC case, the separation is along x , while for the MD-BIC it is along y . As the emitter separation increases, the CDOS exhibits oscillations with gradually decaying envelopes, which are different in the two cases. Interestingly, the decay is not due to a spread of the energy radiated by QE1 in 2D (as it would be in a classical picture), but is primarily a finite-size effect. This is confirmed by comparison with the *infinite* metasurface case (see Sec. VI of the Supplemental Material [50]), where the coupling between the two point emitters does not decay with distance, in a similar way as for a (lossless) 1D system such as a waveguide [54]. Therefore, the BIC system behaves effectively as a 1D system when mediating interactions between distant emitters, but presents the inherent advantages of a 2D system, such as being compatible with 2D arrays of emitters. For reference, the corresponding free space results (in 3D) are also shown (dashed black curves).

To gain further insight, we analyze an infinite metasurface, where closed-form expressions for Γ_{12} can be derived by expressing the CDOS in terms of an integral over the first Brillouin zone. Using a single-mode approximation for the metasurface, we obtain (see Sec. VI of the Supplemental

TABLE I. Coefficients c_n of the $(N + 1)$ -term Fourier cosine series used in Eq. (3). For the ED-BIC, $N = 4$, and for the MD-BIC, $N = 2$.

	c_0	c_1	c_2	c_3	c_4
ED-BIC	0.273	0.516	0.160	0.048	0.001
MD-BIC	0.642	0.351	0.005	—	—

Material [50])

$$\Gamma_{12}(d) = \beta \Gamma_{11} \text{osc}(d) \text{env}(d) \quad (3)$$

with oscillatory part

$$\text{osc}(d) = \sum_{n=0}^N c_n \cos\left(2n\pi \frac{d}{a}\right), \quad (4)$$

and envelope defined in terms of a Bessel function of the first kind:

$$\text{env}(d) = J_0(k_{\parallel}^{\text{res}}|d|), \quad (5)$$

where Γ_{11} is the single-emitter decay rate and $\beta \in [0, 1]$ is the β factor, defined as the ratio of the emission rate into the BIC mode to Γ_{11} [42–44]. The oscillatory part is an $(N + 1)$ -term Fourier cosine series, whose expansion coefficients are given in Table I. Note that these coefficients are specific to the metasurface parameters and QE configuration considered, and would need to be recalculated for different parameters and configurations. Within the envelope, the parameter $k_{\parallel}^{\text{res}}$ quantifies the detuning between the QE transition ω_0 and the BIC resonance, vanishing for perfect frequency matching ($k_{\parallel}^{\text{res}} = 0$), which then yields $\text{env}(d) = 1$. As shown in Sec. VI of the Supplemental Material [50], very good agreement is found between numerical simulations and Eq. (3) for the infinite metasurface [55]. In this case, the fitted β factors are $\beta = 75.18\%$ for the ED-BIC and $\beta = 82.43\%$ for the MD-BIC, which are remarkably high for a 2D platform.

In a real (finite) metasurface, the BIC resonance broadens into a quasi-BIC, effectively introducing detuning from λ_0 . This can be accounted for within Eq. (3) by allowing a nonzero value for $k_{\parallel}^{\text{res}}$ to quantify the decay of Γ_{12} with distance, which is treated as a fitting parameter. In general, the larger (smaller) the metasurface, the smaller (larger) the value of $k_{\parallel}^{\text{res}}$. Note also that if weak material absorption were present, it could similarly be accounted for by a nonzero value of $k_{\parallel}^{\text{res}}$. For the fits shown in Figs. 4(a) and 4(b) (21×21 nanoparticles), we obtain $\beta = 44.80\%$ with $k_{\parallel}^{\text{res}} = 0.581 \text{ rad}/\mu\text{m}$ for the ED-BIC and $\beta = 81.79\%$ with $k_{\parallel}^{\text{res}} = 0.562 \text{ rad}/\mu\text{m}$ for the MD-BIC. Agreement between numerical results and Eq. (3) is good for both BIC types, especially for larger separations ($d \geq 5a$ for the ED-BIC and $d \geq 2a$ for the MD-BIC), where the BIC mode dominates and the single-mode approximation is accurate. At shorter distances, nonresonant modes contribute more strongly, reducing the approximation's accuracy. The nonresonant contribution is shown in Sec. VI of the Supplemental Material [50].

Finite-size effects thus reduce β for the ED-BIC, but it remains remarkably high for the MD-BIC, comparable to values calculated for photonic crystal waveguides [42,43]. Importantly, however, BIC modes are delocalized in 2D, making metasurfaces much more readily integrable with large ensembles of QEs in 2D arrays. Moreover, while for comparison with existing literature we have chosen metasurface parameters identical to those in Ref. [47], optimization of the metasurface geometry and meta-atom type (e.g., nanodisks, rings) could significantly increase β values.

Although finite size also introduces decay of the long-range interaction ($k_{\parallel}^{\text{res}} \neq 0$), coupling persists across several micrometers for both BIC types, representing a substantial enhancement compared to free space. While the ED-BIC yields higher LDOS and CDOS, its β factor is smaller. As we show in the next section, the β factor is critical in determining entanglement generation over remote distances, which is made possible by BIC modes.

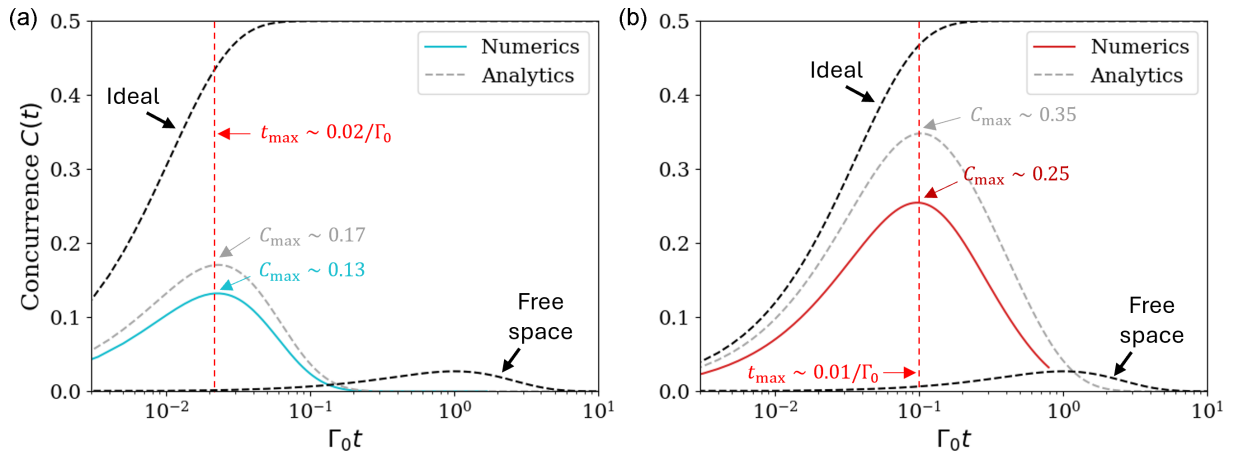


FIG. 5. Concurrence $C(t)$ as a function of dimensionless time $\Gamma_0 t$ for a QE separation of $d = 2 \mu\text{m}$ in the case of the ED-BIC [panel (a)] and the MD-BIC [panel (b)]. Solid curves are found numerically, while the dashed gray curves are obtained from Eq. (7) with $\beta = 44.80\%$, $F_p = 46.9$ for the ED-BIC and $\beta = 81.79\%$, $F_p = 13.7$ for the MD-BIC. Ideal metasurface coupling cases ($\beta = 1$) and the concurrence in free space for similar separations [$d = 1.79 \mu\text{m}$ in panel (a) and $d = 2.30 \mu\text{m}$ in panel (b), chosen to match CDOS free space maxima] are also shown.

TABLE II. Master equation coefficients used in the numerical computation of the concurrence in Figs. 5(a) and 5(b). The rates Γ_{11} are extracted from Figs. 2(a) and 2(b) at $\lambda_0 = 552.0$ nm and $\lambda_0 = 708.9$ nm, respectively, and Γ_{12} from Figs. 4(a) and 4(b) at $d = 5a$ (2 μ m). Γ_{22} and Ω_{12} are extracted from Fig. S4 of the Supplemental Material [50] at $d = 5a$ (2 μ m).

	Γ_{11}/Γ_0	Γ_{22}/Γ_0	Γ_{12}/Γ_0	Ω_{12}/Γ_0
ED-BIC	46.9	42.4	15.3	-2.6
MD-BIC	13.7	8.8	7.9	-0.2

E. Concurrence

As an important application of the above results, we now demonstrate that metasurfaces can generate long-range QE-QE entanglement. To quantify entanglement, we use the concurrence [56,57]

$$C(t) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \quad (6)$$

where λ_i are the eigenvalues (ordered by decreasing magnitude) of the non-Hermitian matrix $R = \rho \tilde{\rho}$, with $\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$, in which conjugation is taken in the Pauli-Z basis. The concurrence satisfies $0 \leq C(t) \leq 1$, with $C = 0$ for separable states and $C = 1$ for maximally entangled states.

We consider an initially uncorrelated QE state $|e_1 g_2\rangle$ in the electromagnetic vacuum. In this case, a closed-form expression for $C(t)$ in terms of the β factor can be derived under the following assumptions: (1) The QEs interact solely via the BIC mode, such that $\Gamma_{12}(d)$ is given by Eq. (3) (single-mode approximation); (2) the two QEs have the same LDOS at positions separated by integer multiples of the lattice constant, $d = na$ with n an integer, so that $\Gamma_{22} = \Gamma_{11} = F_p \Gamma_0$ (which is exact for QEs interacting with an infinite metasurface); and (3) the interaction is purely dissipative, that is, $\Omega_{12} = 0$.

Upon imposing approximations (1)–(3), and at separations $d = na$, the concurrence becomes (see Sec. II of the Supplemental Material [50])

$$C(t) \simeq \sinh(\beta F_p \Gamma_0 t) e^{-F_p \Gamma_0 t}. \quad (7)$$

An important consequence is that the concurrence becomes independent of the QE separation: As long as the emitters occupy equivalent positions with respect to the unit cell, the degree of entanglement is the same, regardless of d .

Figures 5(a) and 5(b) compare the analytical predictions of Eq. (7) (dashed gray curves) with numerical results (solid curves) for the concurrence evolution at $d = 5a = 2 \mu$ m. The numerical results are obtained by solving the two-QE master equation for a 21×21 metasurface (see Sec. I of the Supplemental Material [50]). We see that entanglement develops for both BIC types and that the agreement between the numerical and analytical curves is qualitatively good, though finite-size effects mean approximation (2) is not exactly satisfied (see Table II). The discrepancy is larger for the MD-BIC, where $\Gamma_{11} \neq \Gamma_{22}$ is more pronounced. Note that approximation (3) is relatively well satisfied (see Table II), and this is generally the case for $d > \lambda_0$ (see Sec. VII of the Supplemental Material [50]). For comparison, the free-space case is also shown, where $\Gamma_{11} = \Gamma_{22} = \Gamma_0$ and $\Omega_{12} \rightarrow 0$ for the separations chosen, such that Eq. (7) becomes exact [58]. Entanglement in free space develops at least an order of magnitude later and with much smaller magnitude than in the metasurface platform.

At long times, Eq. (7) converges to the population of the asymmetric Dicke state $\rho_{aa}(t)$ (see Sec. II of the Supplemental Material [50]), and the entanglement eventually decays exponentially as

$$C(t) \simeq \frac{1}{2} e^{-(1-\beta)F_p \Gamma_0 t}. \quad (8)$$

Thus, smaller Purcell factors F_p correspond to longer-lived entanglement, which explains why the MD-BIC ($F_p \sim 14$) sustains entanglement longer than the ED-BIC ($F_p \sim 47$). Note that the ED-BIC and MD-BIC also possess different β . As $\beta \rightarrow 1$, the exponential decay is increasingly suppressed and in the ideal case $C(t) \rightarrow 0.5$ for large t . We plot this for both BICs (corresponding to a metasurface with $\beta = 1$, keeping F_p fixed) in Fig. 5.

We see that for the ED-BIC, the maximum concurrence is smaller, occurs sooner, and decays faster than for the

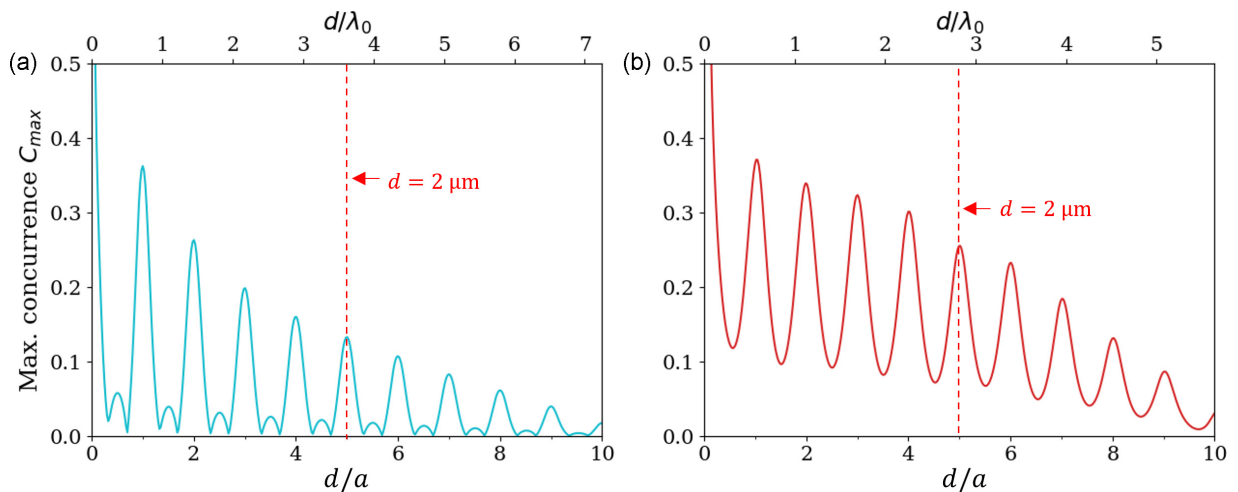


FIG. 6. Maximum concurrence $C_{\max}$ plotted with emitter separation d for the ED-BIC [panel (a)] and MD-BIC [panel (b)]. Other parameters are as in Fig. 5.

MD-BIC. This can be understood by noting that the maximum occurs at

$$t_{\max} = \frac{1}{2F_p\Gamma_0\beta} \ln\left(\frac{1+\beta}{1-\beta}\right), \quad (9)$$

with

$$C_{\max} = \frac{\beta}{\sqrt{1-\beta^2}} \left(\frac{1+\beta}{1-\beta}\right)^{-\frac{1}{2\beta}}. \quad (10)$$

This shows importantly that $C_{\max}$ is a monotonically increasing function of β alone, which is why it is larger in the case of the MD-BIC. Conversely, the time to reach maximum entanglement is set by both β and F_p , so the ED-BIC develops entanglement sooner, while the MD-BIC sustains it longer.

Finally, Fig. 6 shows the numerically computed $C_{\max}$ as a function of emitter separation d . The envelope follows the decay of the collective rates (Fig. 4), with oscillations that would thus persist indefinitely in the infinite-metasurface limit (see Sec. VI of the Supplemental Material [50]). At short separations, additional nonresonant modes enhance entanglement in the ED-BIC, while in the MD-BIC case significant enhancement appears for subwavelength separations $d < \lambda_0$, where dipole-dipole interactions become significant.

III. CONCLUSIONS

We have shown that quasi-BICs in nonlocal (finite) metasurfaces efficiently mediate long-range interactions between QEs placed in their near field (here, 4 nm above the metasurface). The ED-BIC couples to out-of-plane dipoles, while the MD-BIC couples to in-plane dipoles. In the weak-coupling and dissipative regimes, their interactions are governed by the Purcell factor F_p and the coupling efficiency β .

While F_p can be obtained numerically, we extracted β factors from fits of CDOS simulations to an analytical expression derived for infinite metasurfaces [Eq. (3)]. For the ED-BIC, we found $F_p \sim 47$ and $\beta \sim 0.45$, while for the MD-BIC we obtained $F_p \sim 14$ and $\beta \sim 0.82$. In the latter case, the β factor is remarkably high and comparable to those reported for one-dimensional waveguides [42,43].

As a key application, we studied entanglement generation between two QEs initially prepared in the state $|e_1, g_2\rangle$ (in the electromagnetic vacuum). We showed that the maximum concurrence $C_{\max}$ depends primarily on β , increasing monotonically with it. Since the MD-BIC possesses a higher β factor, it yields stronger entanglement ($C_{\max} \sim 0.25$) than the ED-BIC ($C_{\max} \sim 0.13$) at a QE separation of $2\ \mu\text{m}$ ($\simeq 3.6\lambda_0$ for the ED-BIC and $\simeq 2.8\lambda_0$ for the MD-BIC). Larger metasurfaces would further enhance performance, approaching the infinite limit predicted by Eqs. (7) and (10). This demonstrates that nonlocal metasurfaces can generate entanglement through

purely dissipative interactions, even across distances of several wavelengths.

While these values of concurrence are not necessarily larger than those computed in other systems [15,27], they can be improved by optimizing the metasurface design to engineer higher β factors, for example, by tailoring nanoparticle shapes to better isolate BIC resonances over a broader spectral range. Crucially, while the entanglement reported here spans separations of several wavelengths, it could in principle persist over arbitrarily larger distances in larger metasurfaces, since according to Eqs. (7) and (10) it is independent of the separation d , provided $d \ll c/\Gamma_{11}$ (beyond which retardation effects become significant [59]).

We also found that entanglement develops faster for the ED-BIC but decays more rapidly, since the time of maximum entanglement scales inversely with F_p , while its long-time decay rate scales with $F_p(1-\beta)$ [Eqs. (8) and (9), respectively]. In the ideal limit $\beta = 1$, the concurrence plateaus at $C_{\max} = 0.5$.

Although we focused here on two emitters, the same principles extend to larger ensembles. The framework naturally generalizes to many-body systems ($N > 2$ emitters) with long-range, all-to-all interactions mediated by metasurfaces [60,61]. Similarly, while we modeled QEs as ideal two-level systems, the formalism can be readily adapted to multilevel emitters and to include dephasing.

Looking forward, nonlocal metasurfaces offer a practical route to coupling distant qubits, as they naturally accommodate large 2D arrays of QEs due to their delocalized BIC modes. Their planar geometry and resulting integrability with QE arrays make them especially well suited for realizing scalable quantum technologies. For example, the architecture explored here could enable quantum simulation of spin models with nearest-neighbor or long-range interactions, offering a versatile platform for future quantum nanophotonics.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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