

Enhancing the performance of quantum switch refrigeration with an information pool of qubits

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Indefinite causal order has been shown to enable nonclassical refrigeration, with recent experimental verification using an ensemble of nuclear spins in the nuclear magnetic resonance system that boosted prior coefficient of performance (COP) through the approach of density matrix exponentiation (DME). In this paper, we demonstrate an approach to improve the COP of this setup by incorporating an information pool of qubits to absorb the entropic cost of refrigeration. If these qubits are reused until their resources are expended, an increase of the COP beyond the DME approach can be achieved in certain temperature ranges. This improvement is found to hold even for the case when the working body is hotter than the cold bath, and in the prethermalization regimes where further enhancements can be observed. Our work thus introduces a technique to enhance quantum refrigeration with indefinite causal order.

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I. INTRODUCTION

By placing multiple quantum channels in a controlled superposition of different orders, one can achieve quantum processes with indefinite causal orders that can perform quantum information and thermodynamical tasks not possible with fixed causal orders [1–12]. One of such processes known as the quantum switch, Φ^{sw} , places the applications of two channels, Φ_A and Φ_B , in a superposition of $\Phi_B \circ \Phi_A$ and $\Phi_A \circ \Phi_B$ [1]. If Φ_A and Φ_B are fully thermalizing channels that thermalize any state to the thermal state of a bath of some temperature, the quantum switch allows refrigeration of the bath even if the working body starts at a temperature that is greater than or equal to the bath, i.e., $T_{\text{working}} \geq T_{\text{bath}}$, up to a bound [12,13].

The case of $T_{\text{working}} = T_{\text{bath}}$ was verified experimentally in a nuclear magnetic resonance (NMR) system, and further enhanced using a density matrix exponentiation (DME) approach that features repeated interaction with the cold bath [14]. Moreover, the inclusion of an information reservoir in the form of an infinite tape of memory bits or qubits was shown to enhance classical [15,16] and quantum refrigeration tasks [17,18]. In these setups, the interaction with a heat bath is repeated with a fresh memory bit or qubit in each repetition, dumping entropy to them while enhancing heat extraction. While the memory tape is typically assumed to be of infinite length, a more realistic case of a finite memory tape such that the memory tape is reused multiple times was also shown to grant enhancements to classical thermodynamical tasks [19,20].

Motivated by these works, we investigate how the resources of coherence and purity can be utilized by the quantum switch to improve the coefficient of performance (COP) of refrigeration. Particularly, we show that the quantum switch operation left untapped resources that can be expended via repeated operations of the quantum switch in a single refrigeration cycle. Furthermore, by augmenting the quantum switch to function as a Maxwell's refrigerator similar to the finite-tape ratchet system [18–20], we show that the augmented quantum switch can also utilize the purity of a pool of qubits to further enhance the COP to even surpass the DME approach in certain temperature ranges.

We first review the quantum switch refrigeration cycle of Ref. [12] in an environmental representation [13,14,21,22] (Sec. II A), before demonstrating how there exists unexpended resources that can be expended via repeated operations to enhance its COP (Sec. II B). Next, we augment the quantum switch's operation to utilize resources from an additional memory qubit, before demonstrating the increased heat flow due to the presence of this resource (Sec. III A). Finally, by considering a finite memory tape of three qubits, we demonstrate the enhanced COP of the refrigeration cycle (Sec. III B). We will focus on the enhancement to COP for $T_{\text{working}} = T_{\text{bath}}$, while the extension to the $T_{\text{working}} > T_{\text{bath}}$ case follows simply with an alternate refrigeration cycle, which we demonstrate numerically in Appendix A.

II. CONVENTIONAL QUANTUM SWITCH Φ^{sw}

A. Refrigeration cycle

To achieve the quantum switch operation Φ^{sw} on a qubit of a main system ρ^Q , i.e., a superposition of $\Phi_B \circ \Phi_A(\rho^Q)$ and $\Phi_A \circ \Phi_B(\rho^Q)$, one can condition each operation order on each basis of a control qubit ρ^C and place it in a superposition of that basis, e.g., $|+\rangle^C = (|0\rangle^C + |1\rangle^C)/\sqrt{2}$ with $\rho^C = |+\rangle\langle+|^C$. Note that the superscript indicates the subsystem. Taking an environmental representation, Φ^{sw} can be

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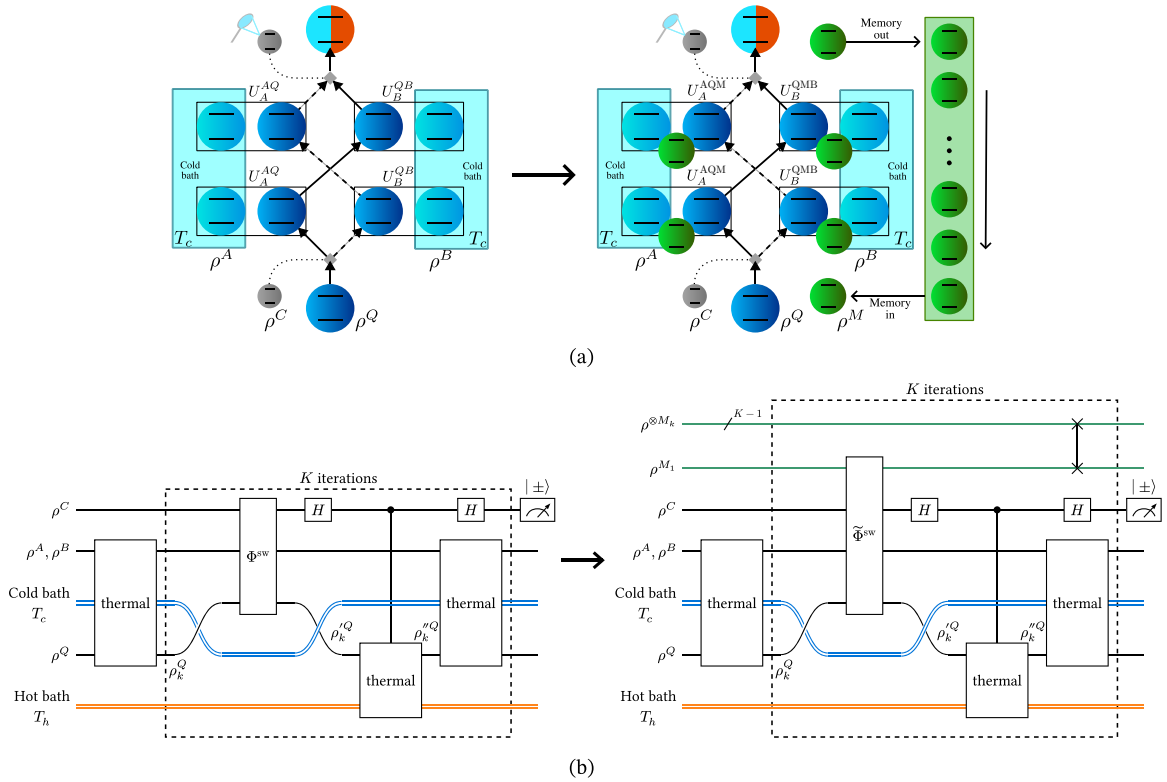


FIG. 1. (a) Conventional quantum switch operation Φ^{sw} and its augmentation $\tilde{\Phi}^{\text{sw}}$ to include a memory tape. In the standard quantum switch refrigeration, the control qubit ρ^C is measured in the $|\pm\rangle^C$ basis at the end of the operation. Depending on the measurement outcome, heat can be extracted out of the cold baths by ρ^Q , which can be passed to a hot bath to allow refrigeration. (b) The corresponding quantum circuits for the $T_{\text{working}} = T_{\text{bath}}$ refrigeration cycle, and its augmentation to include a memory tape, where the conditioning of the control qubit's measurement outcome to pass heat to a hot bath is shown as a controlled thermalization with the measurement deferred to the end of the cycle. A complete refrigeration cycle features K iterations of the operations in the dashed box, expending the resources in the control and memory qubits.

defined as [12–14,21,22]

$$\Phi^{\text{sw}}(\rho^C \otimes \rho^Q) = \text{Tr}_{A,B}[U_2 U_1(\rho^C \otimes \rho^A \otimes \rho^Q \otimes \rho^B) U_1^\dagger U_2^\dagger], \quad (1)$$

where, without loss of generality, the conditionings are performed by the controlled unitaries of

$$U_1 = |0\rangle\langle 0|^C \otimes U_A^{AQ} \otimes I^B + |1\rangle\langle 1|^C \otimes I^A \otimes U_B^{QB}, \quad (2)$$

$$U_2 = |0\rangle\langle 0|^C \otimes I^A \otimes U_B^{QB} + |1\rangle\langle 1|^C \otimes U_A^{AQ} \otimes I^B, \quad (3)$$

such that U_A^{AQ} and U_B^{QB} are the unitaries that implement $\Phi_A(\rho^Q) = \text{Tr}_A[U_A^{AQ}(\rho^A \otimes \rho^Q) U_A^{AQ\dagger}]$ and $\Phi_B(\rho^Q) = \text{Tr}_B[U_B^{QB}(\rho^Q \otimes \rho^B) U_B^{QB\dagger}]$, with subsystems A and B being the environmental subsystems, also known as the bath qubits. This is illustrated on the left side of Fig. 1(a).

Note that, in general, U_A^{AQ} and U_B^{QB} can be taken to be the unitary representation of the generalized amplitude damping channel, also known as the one-qubit thermal operation [23,24]. These unitaries are generated from an energy-conserving interaction Hamiltonian of

$$H_{\text{int}}^{AQ} = J_A(e^{i\varphi} \sigma_-^A \sigma_+^Q + e^{-i\varphi} \sigma_+^A \sigma_-^Q), \quad (4)$$

$$H_{\text{int}}^{QB} = J_B(e^{i\varphi} \sigma_-^B \sigma_+^Q + e^{-i\varphi} \sigma_+^B \sigma_-^Q), \quad (5)$$

respectively, where J_A and J_B are the coupling strengths, $\sigma_\pm$ are the raising and lowering operators for their corresponding subsystems, and φ is an arbitrary phase that we take to be $3\pi/2$ [23,24]. This will be useful when we augment the quantum switch later (Sec. III A).

The values of J_A , J_B , and time t then control the amount of thermalization, reducing to the fully thermalizing operation if $\cos J_A t = \cos J_B t = 0$. For simplicity, we take $x_A = \cos J_A t$ and $x_B = \cos J_B t$, such that $x_A, x_B \in [-1, 1]$ determines the amount of thermalization. Thus, the full thermalization case is achieved for $x_A = x_B = 0$, where U_1 and U_2 are composed of controlled SWAP gates. Experimentally, these are each decomposed as three Toffoli gates, which in turn can be further decomposed into single- and two-qubit gates to be implemented as pulse sequences in an NMR system [14]. As for $x_A, x_B \neq 0$, we have partial or prethermalization where the system does not fully thermalize to the thermal state of the ancillary environments A and B (we demonstrate numerically in Appendix B that our results continue to hold in such cases).

By initializing $\rho^C = |+\rangle\langle +|^C$, $\rho^A \otimes \rho^Q \otimes \rho^B = \rho_{T_c} \otimes \rho_{T_c} \otimes \rho_{T_c}$ such that the working system Q and environments A and B are in a thermal state ρ_{T_c} of a cold bath of temperature T_c , i.e., $T_{\text{working}} = T_{\text{bath}} = T_c$, and by taking $x_A = x_B = 0$ such that the resulting Φ_A and Φ_B are fully thermalizing channels at that temperature, it was shown in Refs. [12,14] that heat can flow into subsystem Q from the cold baths after the operation

of Φ^{sw} if ρ^C is measured to be $|-\rangle^C = (|0\rangle^C - |1\rangle^C)/\sqrt{2}$. A refrigeration cycle can then be constructed by passing this gained heat to a hot bath of temperature T_h with $T_h > T_c$, where after the Φ^{sw} operation, ρ^Q thermalizes with the hot bath conditioned on the $|-\rangle\langle -|^C$ measurement outcome, before resetting ρ^Q , ρ^A , and ρ^B back to ρ_{T_c} , completing the cycle [Fig. 1(b)].

Such a system can be initialized with pseudopure states in the NMR system before undergoing single-qubit rotations to the desired polarization, followed by gradient pulses to decohere them into thermal states. Likewise, the conditioned thermalization can again be implemented as a controlled SWAP gate [14].

Since the quantum switch conserves energy [12], after the quantum switch operation Φ^{sw} , we have

$$\begin{aligned} \Delta E(\rho^Q) &= -\Delta E(\rho^A) - \Delta E(\rho^B), \\ E(\rho'^Q) - E(\rho_{T_c}^Q) &= E(\rho_{T_c}^A) - E(\rho'^A) + E(\rho_{T_c}^B) - E(\rho'^B), \end{aligned} \quad (6)$$

or

$$E(\rho'^Q) = E(\rho_{T_c}^A) - E(\rho'^A) + E(\rho_{T_c}^B) - E(\rho'^B) + E(\rho_{T_c}^Q), \quad (7)$$

where we defined ρ'^Q , ρ'^A , and ρ'^B as the states after the quantum switch operation Φ^{sw} , and $E(\rho) = \text{Tr}[H\rho]$ is the energy of the state ρ with Hamiltonian H . Note that since the measurement-feedback control scheme of the quantum switch refrigerator is taken as a controlled quantum operation here, the expectation values of the energy of the system after the controlled operations are already weighted by the success probability of the measurement outcome.

The heat flow into the hot bath in the controlled thermal operation is

$$Q^h = E(\rho'^Q) - E(\rho''^Q), \quad (8)$$

where we defined ρ''^Q as the state of Q after the controlled thermal operation. Finally, at the end of the cycle, the heat flow out of the cold bath is simply the change of energy of each system Q , A , and B , as they thermalize back to the thermal state ρ_{T_c} , i.e.,

$$\begin{aligned} -Q^c &= E(\rho_{T_c}^A) - E(\rho'^A) + E(\rho_{T_c}^B) - E(\rho'^B) \\ &\quad + E(\rho_{T_c}^Q) - E(\rho''^Q) \\ &= E(\rho'^Q) - E(\rho''^Q) = Q^h, \end{aligned} \quad (9)$$

where we used Eq. (7) in the second equality.

The COP of the refrigeration cycle is thus

$$\eta = -Q^c/W_r = Q^h/W_r, \quad (10)$$

where $W_r = H(p_{\pm})T_r$ is the work expended to erase the memory register of the measurement of ρ^C at the end of the cycle due to Landauer's principle [12,25], $H(p_{\pm})$ is the Shannon entropy of the measurement probabilities $p_{\pm} = \text{Tr}[|\pm\rangle\langle\pm|^C \rho^C]$ at the end of the cycle, and T_r is the temperature of the resetting bath.

For simplicity, we take the self Hamiltonian of ρ^Q to be $H^Q = \omega|1\rangle\langle 1|^Q$, whereas ρ^C has no energy, and we set the

arbitrary $T_r = \omega/k_B$ for comparison with Ref. [14]. Depending on the physical system, ρ^A and ρ^B can have different energy gaps from ρ^Q , e.g., their self Hamiltonians are $H^A = \omega_A|1\rangle\langle 1|^A$ and $H^B = \omega_B|1\rangle\langle 1|^B$, respectively, but they can be arbitrary as long as they are initialized and reset to a fixed thermal state ρ_{T_c} of say subsystem Q [14].

Note that the idealized case of $T_h = T_c$ is assumed in Ref. [14], such that all the heat extracted by ρ^Q is transferred into the hot bath. This allows the probing of the heat extraction capabilities of Φ^{sw} without T_h being a factor. We will also assume this case for a fair comparison with Ref. [14].

B. Repeated operation of the quantum switch

Next, we show that the heat extracted Q^h is dependent on the initial coherence of the control qubit ρ^C , and that the quantum switch operation Φ^{sw} leaves unexpended coherence that can be expended by repeated iteration of the dashed box in Fig. 1(b) in a single refrigeration cycle.

To be clear, while the resetting of ρ^Q , ρ^A , and ρ^B occurs in each repeated iteration, a single refrigeration or thermodynamic cycle is only complete after the control qubit ρ^C is measured and has its memory register erased. Therefore, for K iterations in a single refrigeration cycle, we have

$$Q^h = \sum_k^K Q_k^h = \sum_k^K E(\rho_k'^Q) - E(\rho_k''^Q), \quad (11)$$

where $\rho_k'^Q$ and $\rho_k''^Q$ are the states of the working system ρ^Q before and after the controlled thermalization with the hot bath in the k th iteration as shown in Fig. 1(b).

Since only ρ^C survives across iterations and is not reset until it is measured at the end of all the iterations, we consider an arbitrary

$$\rho_k^C = \frac{1}{2}(I + r_x\sigma_x + r_y\sigma_y + r_z\sigma_z) \quad (12)$$

for each iteration k , where $\{\sigma_i\}$ are the Pauli matrices, and $r_i \in \mathbb{R}$. Taking $\rho_{T_c} = \begin{pmatrix} 1-N & 0 \\ 0 & N \end{pmatrix}$ where $N = 1/(1 + e^{\omega/T_c})$, we can compute the heat flow into the hot bath in the k th iteration as

$$Q_k^h = E(\rho_k'^Q) - E(\rho_k''^Q) = \frac{\omega}{2}(N - 3N^2 + 2N^3)r_x, \quad (13)$$

which depends on the real part of the coherence r_x (in the energy eigenbasis) of ρ^C in that iteration, as well as T_c as parametrized by N .

As only ρ_k^C survives across iterations, we are interested in its transformation in each iteration, i.e., $r_i \rightarrow r'_i = \text{Tr}[\rho_{k+1}^C \sigma_i]$, where ρ_{k+1}^C is the final state at the end of each k th iteration, which gives us

$$r'_x = (1 - 3N - 3N^2)r_x, \quad (14)$$

$$r'_y = (1 - 4N + 6N^2 - 4N^3 + 2N^4)r_y, \quad (15)$$

$$r'_z = (1 - 2N + 2N^2)r_z, \quad (16)$$

where their magnitudes become smaller, i.e., $|r'_i| \leq |r_i|$ for all $N \in [0, 0.5]$, such that ρ^C approaches the maximally mixed

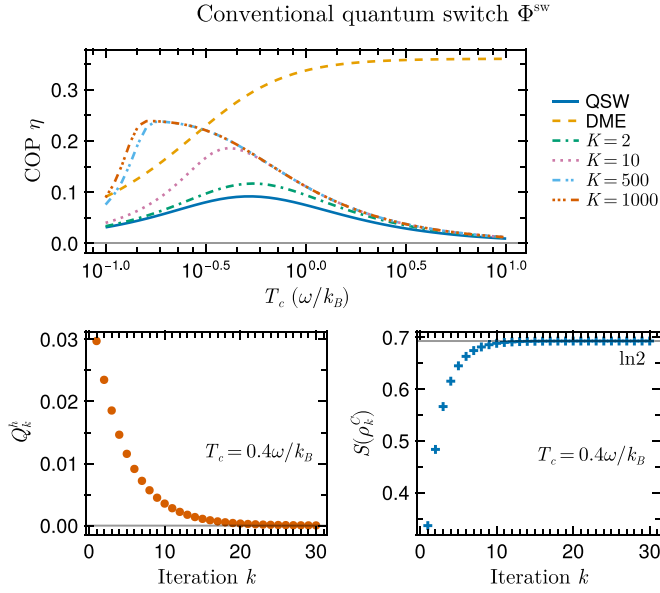


FIG. 2. Saturation of enhanced COP, Q_k^h , and $S(\rho_k^C)$ for repeated operations of Φ^{sw} with initial $\rho^C = |+\rangle\langle+|^C$. The label “QSW” refers to the conventional quantum switch with $K = 1$. The COP can be enhanced for repeated iterations, i.e., $K > 1$, in a single refrigeration cycle, but will saturate as $S(\rho_k^C)$ approaches maximum entropy where heat extraction is no longer possible. An arbitrary temperature of $T_c = 0.4\omega/k_B$ is chosen as an example for the evolution of Q_k^h and $S(\rho_k^C)$.

state in each iteration, until eventually refrigeration is no longer possible with $Q_k^h = 0$ as $r_x \rightarrow 0$.

This is not surprising as the entropy production in each k th iteration is

$$\begin{aligned} \Sigma_k &= \Delta S(\rho_k^C) + \frac{Q_k^h}{T_h} + \frac{Q_k^c}{T_c} \\ &= \Delta S(\rho_k^C) + \left(\frac{1}{T_h} - \frac{1}{T_c} \right) Q_k^h \geq 0, \end{aligned} \quad (17)$$

where $\Delta S(\cdot)$ is the change in von Neumann entropy. Since $T_h > T_c$, the second term is negative in the case of refrigeration, and thus, refrigeration is only possible if $\Delta S(\rho_k^C) \geq (1/T_c - 1/T_h)Q_k^h > 0$. This means that the entropic cost of refrigeration is paid for by increasing $S(\rho_k^C)$, expending the resource of coherence, such that Q^h and thus the COP saturate as $K \rightarrow \infty$. This cost is manifested as the work required to erase the memory register after the measurement of ρ^C at the end of the refrigeration cycle after K iterations.

Taking the best initial ρ^C of $|+\rangle\langle+|^C$ for maximal heat extraction according to Eq. (13), the COP for K iterations for different values of T_c , and Q_k^h and $S(\rho_k^C)$ for an arbitrarily chosen $T_c = 0.4\omega/k_B$ are shown in Fig. 2. We have also reproduced the COP for the DME approach from Ref. [14] for comparison. We see that while the COP can be increased for repeated iterations of Φ^{sw} , it will saturate as ρ^C eventually runs out of coherence with $S(\rho_k^C)$ saturating to maximum entropy, and Q_k^h saturating to 0.

We note that similar to the repeated iteration of Φ^{sw} , the DME approach also features repeated interaction that consumes the coherence of the control qubit, and its plotted

COP is the theoretical maximum as K approaches infinity. However, instead of a controlled SWAP operation like in the quantum switch, the DME approach performs a controlled $e^{iS\Delta\theta}$ operation, where S is the SWAP operation and $\Delta\theta$ is a parameter that is optimized to achieve the best COP in each iteration [14]. This optimization of $\Delta\theta$ in each iteration might explain the supremacy of the DME approach’s COP as compared to the conventional quantum switch that has the unoptimized $\Delta\theta = \pi/2$.

One other difference is that each controlled $e^{iS\Delta\theta}$ operation is performed on a fresh environmental subsystem in the DME approach. The quantum switch, on the other hand, shares the two environmental subsystems A and B between the four controlled SWAP operations that make up the quantum switch. This allows non-Markovian effects to manifest in the quantum switch, which was shown to enhance the COP [13]. This might explain the presence of a small temperature regime ($\sim 10^{-1}$ to $10^{-0.5}\omega/k_B$) where the conventional quantum switch’s COP is able to outperform the DME approach’s despite having no optimization of $\Delta\theta$.

III. AUGMENTED QUANTUM SWITCH $\tilde{\Phi}^{\text{sw}}$

Next, we show that in addition to the resource of coherence in the control qubit, the quantum switch operation Φ^{sw} can be augmented to leverage additional resources in the form of an information pool of memory qubits to enhance heat extraction. Particularly, we demonstrate that heat extraction can be enhanced by dumping entropy into this pool of memory qubits in accordance with the information processing second law [15–18].

In this context, we first consider how the augmented quantum switch can leverage the resource of purity in the memory qubits for heat extraction. Afterward, we consider an example of three memory qubits that can be reused and expended for multiple iterations, until they are reset along with the control qubit at the end of the refrigeration cycle.

A. Enhanced heat extraction with the inclusion of a memory qubit

We define the augmented quantum switch operation $\tilde{\Phi}^{\text{sw}}$ where it now includes a memory qubit ρ^M with

$$\begin{aligned} \tilde{\Phi}^{\text{sw}}(\rho^C \otimes \rho^Q) &= \text{Tr}_{A,B}[U_2 U_1(\rho^C \otimes \rho^A \otimes \rho^Q \otimes \rho^M \otimes \rho^B)U_1^\dagger U_2^\dagger], \end{aligned} \quad (18)$$

$$U_1 = |0\rangle\langle 0|^C \otimes U_A^{\text{AQM}} \otimes I^B + |1\rangle\langle 1|^C \otimes I^A \otimes U_B^{\text{QMB}}, \quad (19)$$

$$U_2 = |0\rangle\langle 0|^C \otimes I^A \otimes U_B^{\text{QMB}} + |1\rangle\langle 1|^C \otimes U_A^{\text{AQM}} \otimes I^B, \quad (20)$$

where we now have U_A^{AQM} and U_B^{QMB} that are to be generated from Maxwell’s demonlike interaction. For simplicity, we consider a simple interaction where the memory qubit acts as a Maxwell’s demon that keeps a record on whether heat is extracted or not by the working body ρ^Q with the energy-conserving interaction Hamiltonians of [18]

$$H_{\text{int}}^{\text{AQM}} = J_A(e^{i\varphi}\sigma_-^A\sigma_+^Q\sigma_+^M + e^{-i\varphi}\sigma_+^A\sigma_-^Q\sigma_-^M), \quad (21)$$

$$H_{\text{int}}^{\text{QMB}} = J_B(e^{i\varphi}\sigma_+^Q\sigma_+^M\sigma_-^B + e^{-i\varphi}\sigma_-^Q\sigma_-^M\sigma_+^B). \quad (22)$$

Note that again we are able to take $x_A = \cos J_A t$ and $x_B = \cos J_B t$, such that $x_A = x_B = 0$ and $x_A, x_B \neq 0$ indicate full thermalization and prethermalization, respectively.

While this recording or dumping of entropy into an information reservoir is in general different from the measurement-feedback control scheme typical of Maxwell's demon system, where information gained about the system is leveraged for enhanced refrigeration, it is still able to describe certain measurement-feedback control schemes [26], and thus can have thermodynamic consequence of traditional Maxwellian refrigerators.

With this $\tilde{\Phi}^{\text{sw}}$, we can iterate through a tape of memory qubits such that in each iteration, one of the memory qubit is sent to $\tilde{\Phi}^{\text{sw}}$ for interaction before swapping out with another memory qubit in the same tape at the end of the iteration. This is illustrated on the right side of Fig. 1(a), and the corresponding quantum circuit to implement it is shown in the right circuit of Fig. 1(b), which simply includes the memory qubits into the cycle.

Similar to the case of the conventional quantum switch, since only ρ^{CM} survives between each iteration, we consider the arbitrary two-qubit state of

$$\rho_k^{\text{CM}} = \frac{1}{4} \left[I \otimes I + \vec{r} \cdot \vec{\sigma} \otimes I + I \otimes \vec{s} \cdot \vec{\sigma} + \sum_{i,j} t_{ij} \sigma_i \otimes \sigma_j \right], \quad (23)$$

where $r_i, s_i, t_{ij} \in \mathbb{R}$, and compute the heat flow into the hot bath in each cycle k as

$$Q_k^h = \frac{\omega}{4} [(N - 3N^2 + 2N^3)r_x + 2(N - N^2)s_z - (N + N^2 - 4N^3 + 2N^4)t_{xz}], \quad (24)$$

where we see that it depends on the real part of the coherence of ρ^C , r_x , the purity of ρ^M , s_z , and the term t_{xz} that captures their correlations.

Again, we are interested in their transformation in each iteration, i.e., $r_x \rightarrow r'_x = \text{Tr}[\rho_{k+1}^C \sigma_x]$, $s_z \rightarrow s'_z = \text{Tr}[\rho_{k+1}^M \sigma_z]$, and $t_{xz} \rightarrow t'_{xz} = \text{Tr}[\rho_{k+1}^{\text{CM}} \sigma_x \otimes \sigma_z]$. These are computed explicitly, yielding

$$r'_x = (1 - 1.5N + 1.5N^2)r_x - (0.5N - 1.5N^2 + N^3)t_{xz}, \quad (25)$$

$$t'_{xz} = (1 - 1.5N + 1.5N^2)t_{xz} - (0.5N - 1.5N^2 + N^3)r_x, \quad (26)$$

$$s'_z = (1 - 2N + 2N^2)s_z, \quad (27)$$

where it is straightforward to see that $|s'_z| \leq |s_z|$ for all $N \in [0, 0.5]$, which eventually approaches 0 for $k \gg 1$. As for r'_x and t'_{xz} , first we note that

$$r'_x = \delta r_x - \gamma t_{xz}, \quad (28)$$

$$t'_{xz} = \delta t_{xz} - \gamma r_x, \quad (29)$$

$$r'_x - t'_{xz} = (\delta + \gamma)(r_x - t_{xz}), \quad (30)$$

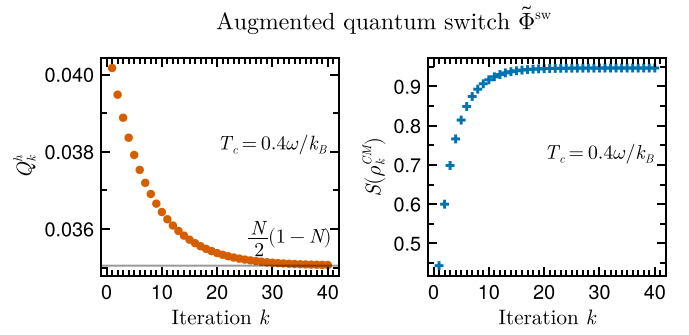


FIG. 3. Q_k^h and $S(\rho_k^{\text{CM}})$ do not saturate to 0 and maximum entropy of $2 \ln 2$, respectively, for repeated operations of $\tilde{\Phi}^{\text{sw}}$ with an initially uncorrelated memory tape of qubits. An arbitrary temperature of $T_c = 0.4\omega/k_B$ is chosen as an example.

where we rewrote the temperature-dependent terms in parentheses as δ and γ , and that $0 \leq \delta + \gamma \leq 1$ for all $N \in [0, 0.5]$, implying that $t'_{xz} = r'_x$ as $k \gg 1$, i.e.,

$$r'_x = \delta r_x - \gamma r_x = (\delta - \gamma)r_x \quad (31)$$

or

$$t'_{xz} = \delta t_{xz} - \gamma t_{xz} = (\delta - \gamma)t_{xz}, \quad (32)$$

which approaches 0 since $0 \leq \delta - \gamma \leq 1$ for all $N \in [0, 0.5]$, i.e., $|r'_x| \leq |r_x|$ and $|s'_z| \leq |s_z|$.

Therefore, the augmented quantum switch $\tilde{\Phi}^{\text{sw}}$ is able to extract heat by expending the coherence of the control qubit ρ^C and the purity of the memory qubit ρ^M , though the presence of correlations between them will result in a negative contribution. Note that in addition to r'_x , t'_{xz} , and s'_z , it can be shown that we also have $|r'_i| \leq |r_i|$, $|s'_i| \leq |s_i|$, and $|t'_{ij}| \leq |t_{ij}|$ for all $N \in [0, 0.5]$, such that ρ^C and ρ^M approach the maximally mixed state after every iteration k .

In the case where we have fresh memory qubits that are initially uncorrelated with the system, we have $\rho_k^{\text{CM}} = \rho_k^C \otimes \rho_k^M$ or

$$\begin{aligned} \rho_k^{\text{CM}} &= \frac{1}{4} (I + \vec{r} \cdot \vec{\sigma}) \otimes (I + \vec{s} \cdot \vec{\sigma}) \\ &= \frac{1}{4} \left[I \otimes I + \vec{r} \cdot \vec{\sigma} \otimes I + I \otimes \vec{s} \cdot \vec{\sigma} + \sum_{i,j} r_i s_j \sigma_i \otimes \sigma_j \right], \end{aligned} \quad (33)$$

then there remains a nonzero amount of heat extraction even when the control qubit ρ^C becomes the maximally mixed state with $r_x \rightarrow 0$ (Fig. 3), i.e.,

$$\begin{aligned} Q_k^h &= \frac{\omega}{4} [(N - 3N^2 + 2N^3)r_x + 2(N - N^2)s_z - (N + N^2 - 4N^3 + 2N^4)r_x s_z] \\ &= \frac{\omega}{2} N(1 - N)s_z, \end{aligned} \quad (34)$$

which demonstrates that this resource can be independent of the quantum switch operation.

Likewise, the entropy production in each iteration k is

$$\begin{aligned}\Sigma_k &= \Delta S(\rho_k^{CM}) + \frac{Q_k^h}{T_c} + \frac{Q_k^c}{T_c} \\ &= \Delta S(\rho_k^{CM}) + \left(\frac{1}{T_h} - \frac{1}{T_c} \right) Q_k^h \geq 0,\end{aligned}\quad (35)$$

where the condition for refrigeration is $\Delta S(\rho_k^{CM}) > (1/T_c - 1/T_h)Q_k^h > 0$. It should be clear that as long as there is a fresh supply of memory qubits that are not maximally mixed, i.e., $S(\rho_k^{CM}) \neq 2 \ln 2$, we can always have $\Delta S(\rho_k^{CM}) > 0$, as $S(\rho_k^{CM})$ can be greater than $S(\rho_k^C)$.

B. Enhanced COP with a three-qubit memory tape

Similar to the case of the control qubit in the conventional quantum switch, we expect the memory qubit to have unexpended resources after the $\tilde{\Phi}^{\text{sw}}$ operation. Therefore, we can reuse these additional memory qubits over multiple iterations. This is akin to treating the memory qubits as a one-dimensional memory tape that is “circular” or with periodic boundary conditions such that it is “scanned” repeatedly, similar to a finite-tape Maxwell’s demon system [18–20]. This also allows us to complete the thermodynamic cycle where the memory qubits can be reset after K iterations.

The COP now has to account for the costs of resetting the memory qubits in addition to the erasure of the memory register for the control qubit’s measurement. As an example, we consider a three-qubit memory tape where the joint memory qubit state $\rho^{\tilde{M}} = \rho^{M_1 M_2 M_3}$ is to be reset at the end of the refrigeration cycle. The COP is thus

$$\eta = \frac{Q^h}{W_r + T_r [S(\rho_K^{\tilde{M}}) - S(\rho_0^{\tilde{M}})]}, \quad (36)$$

where $T_r = \omega/k_B$ is the temperature of the resetting bath, and $\rho_0^{\tilde{M}}$ and $\rho_K^{\tilde{M}}$ are the states of the joint memory qubit state before and after K iterations. Experimentally, the resetting of the memory qubits can be achieved by contact with a bath of $T_r \ll \omega_M$, where ω_M is the energy gap of the memory qubit.

Note that we have assumed the ideal case where the memory qubits can be reset to a pure state, and that its work cost is simply due to the Landauer’s principle, though in realistic finite-time scenarios, there can be additional work costs due to corrections that are inversely proportional to the operational time [27,28]. Alternatively, the reset can also be performed by a projective measurement on the memory qubits similar to the resetting of the control qubit, or simulated by the reparation of pseudopure states in the case of the NMR system [14].

Taking the best initial $\rho^C = |-\rangle\langle -|^C$ and $\rho^M = |0\rangle\langle 0|^M$ for maximal heat extraction as deduced from Eq. (24), we plot the COP, Q_k^h , and $S(\rho_k^{CM})$, for the case of a three-qubit memory tape in Fig. 4 where the same three memory qubits are reused for multiple iterations. As the three memory qubits are reused in each iteration, not only is the coherence of the control qubit expended with $r_x \rightarrow 0$, the purity of the three memory qubits will also become expended with $s_z \rightarrow 0$, such that $Q_k^h \rightarrow 0$ as $S(\rho_k^{CM}) \rightarrow 2 \ln 2$ as ρ_k^{CM} approaches the maximally mixed state.

Despite this, the enhancement to COP, in addition to surpassing the conventional quantum switch’s, can surpass

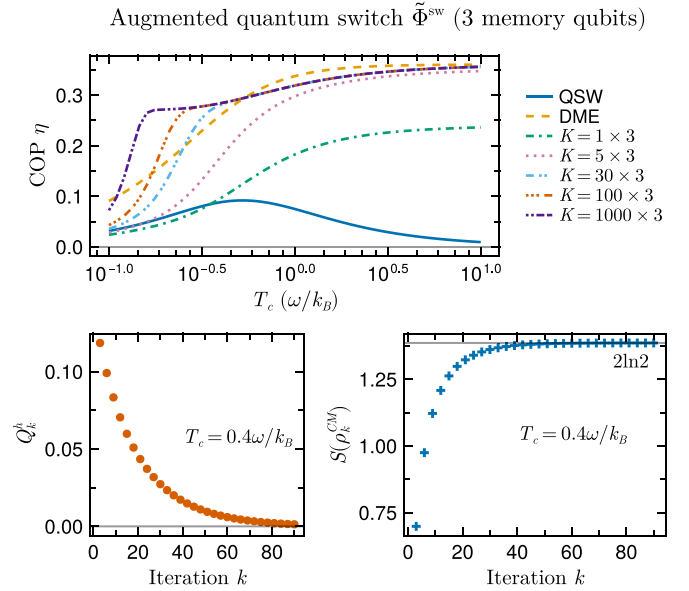


FIG. 4. Saturation of enhanced COP, Q_k^h , and $S(\rho_k^{CM})$ for repeated operations of $\tilde{\Phi}^{\text{sw}}$ with initial $\rho^C = |-\rangle\langle -|^C$ and three memory qubits that are reused multiple times. The label “QSW” refers to the conventional quantum switch with $K = 1$. The COP can be enhanced for repeated iterations, i.e., $K > 1$, in a single refrigeration cycle and can surpass the DME approach in certain temperature ranges, but will saturate as $S(\rho_k^{CM})$ approaches maximum entropy where heat extraction is no longer possible. An arbitrary temperature of $T_c = 0.4\omega/k_B$ is chosen as an example for the evolution of Q_k^h and $S(\rho_k^C)$.

DME’s for certain temperature ranges. Though, we note that this is only true for a large number of iterations K , where the purity of the three memory qubits can be largely expended. A small K can even result in a decreased COP as compared to the conventional quantum switch at low temperature ranges due to the increased work costs. Furthermore, even though the enhanced COP can surpass DME’s, the maximum COP achievable by the DME approach is still higher, and can only be approached as the number of memory qubits increase. The same holds for the minimum COP, albeit the approach is slower unless K is large (Fig. 5).

This means that the choice of using one protocol over the other is highly dependent on the temperature regimes. For example, at low temperatures, the conventional quantum switch Φ^{sw} with repeated operations might suffice as its COP is higher than the DME approach’s and almost on par with the augmented quantum switch $\tilde{\Phi}^{\text{sw}}$, but at low to mid-temperatures, the augmented quantum switch $\tilde{\Phi}^{\text{sw}}$ outperforms the others, and finally at high temperatures, the DME approach grants a higher COP instead.

IV. CONCLUSION

Our results showed that a single use of the quantum switch operation still leaves untapped resources that can be expended by repeated use of the quantum switch. We demonstrated this by showing that the COP can be enhanced as the coherence of the control qubit is largely expended after multiple iterations in a single thermodynamic cycle. Furthermore, by

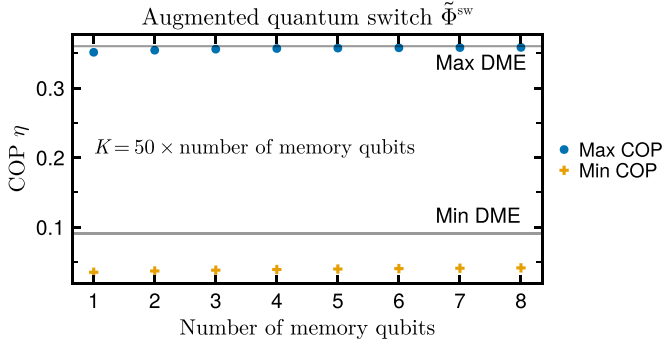


FIG. 5. The minimum and maximum COP are dependent on the number of memory qubits, and approach the minimum and maximum COP of the DME approach as it increases. The case of $K = 50 \times$ number of memory qubits, i.e., each memory qubit is reused 50 times, is chosen as an example.

augmenting the quantum switch operation to include an additional resource of memory qubits that serves as a Maxwell's demon memory, the quantum switch functions as a Maxwell's refrigerator that further enhances heat extraction.

Heat extraction is possible even when the coherence of the control qubit is expended as the purity of the memory qubit contributes to the amount of heat extracted. However, as the augmented quantum switch also leaves unexpended purity in the memory qubits, we showed that reusing the memory qubits until they are approaching maximum entropy is viable to enhance the COP that can surpass the DME approach in certain temperature ranges, and approaching it in others. Specifically, we demonstrated this with three memory qubits.

As this work extends on the experimental work of Ref. [14], an experimental implementation of the refrigeration cycle with a small number of memory qubits might be possible with one challenge being the increased number of required qubits, which might increase errors due to crosstalk. Though we note that seven qubits might be achievable in certain NMR systems, e.g., in dichlorocyclobutanone NMR systems [29], or in the perfluorobutadienyl iron complex [30], such that the three memory qubits and the four qubits required for the quantum switch operation can be implemented.

Also note that our theoretical formulation considers ideal cases where pure states such as the control and memory qubits can be prepared. The presence of coherent or pure states in the (augmented) quantum switch operation differs from traditional refrigeration cycle where there is only access to passive or thermal states, such as the Otto cycle. Instead of being driven by external work or a thermal gradient, the resources of coherence and purity common in quantum information processing can lead to refrigeration cycles that operate in nonclassical regimes such as the case of $T_{\text{working}} = T_{\text{bath}}$ as demonstrated in this work and Ref. [12].

Furthermore, while the main text focused on the case of $T_{\text{working}} = T_{\text{bath}}$, these results also extend to the case of $T_{\text{working}} > T_{\text{bath}}$ of Ref. [13] that simply features a different refrigeration cycle (Appendix A), and that more heat extraction and thus a higher COP is possible by generalizing to the prethermalizing regimes, i.e., U_A^{AQ} , U_B^{QB} , U_A^{AQM} , and U_B^{QMB} that are not SWAP operations (Appendix B).

Finally, an interesting question is if there are better quantum processes that can consume the resources from the control and memory qubits more efficiently, such that a lesser number of iterations are required to achieve a comparable COP, while still maintaining the quantum switch's ability to perform refrigeration at the nonclassical temperatures of $T_{\text{working}} \geq T_{\text{bath}}$; e.g., the case of a superposition of independent channel [13,31] is also able to perform refrigeration at $T_{\text{working}} = T_{\text{bath}}$. Future works can compare or augment these processes to better consume the resources in the system.

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DATA AVAILABILITY

The data supporting this study's findings are available within the article.

APPENDIX A: REFRIGERATION CYCLE

WITH $T_{\text{working}} > T_{\text{bath}}$

In the main text, we focused on the case of $T_{\text{working}} = T_{\text{bath}}$ of Refs. [12,14], where the temperature of the working body is initialized at the same temperature as the baths to be cooled, i.e., $\rho^A \otimes \rho^Q \otimes \rho^B = \rho_{T_c} \otimes \rho_{T_c} \otimes \rho_{T_c}$. However, it was shown in Ref. [13] that refrigeration is still possible even with $T_{\text{working}} > T_{\text{bath}}$, where the temperature of the working body is higher than the baths to be cooled, i.e., $\rho^A \otimes \rho^Q \otimes \rho^B = \rho_{T_c} \otimes \rho_{T_h} \otimes \rho_{T_c}$, where $T_h > T_c$. The case of $T_{\text{working}} = T_{\text{bath}}$ demonstrates the novelty of quantum switch refrigeration as one would expect no net heat flow. Likewise, the case of $T_{\text{working}} > T_{\text{bath}}$ is also novel as one would expect heat flow from the hotter working body to the colder bath, rather than refrigerating it, presenting opportunities for refrigeration applications.

Also note that we do not compare the case of $T_{\text{working}} > T_{\text{bath}}$ to the DME approach of Ref. [14] as the DME approach considers only the case of $T_{\text{working}} = T_{\text{bath}}$. Qualitatively, one would expect the COP for the case of $T_{\text{working}} > T_{\text{bath}}$ to be smaller than the case of $T_{\text{working}} = T_{\text{bath}}$, with the difference being greater as the temperature difference increases.

An alternate refrigeration cycle for this case, as well as its augmentation to include a memory tape, is shown in Fig. 6, where one difference with the $T_{\text{working}} = T_{\text{bath}}$ case in the main text is that ρ^Q is initialized and reset with the hot bath instead of the cold bath.

Similar to the case of $T_{\text{working}} = T_{\text{bath}}$, since the quantum switch conserves energy, we have

$$\Delta E(\rho^Q) = -\Delta E(\rho^A) - \Delta E(\rho^B),$$

$$E(\rho^Q) - E(\rho_{T_h}^Q) = E(\rho_{T_c}^A) - E(\rho'^A) + E(\rho_{T_c}^B) - E(\rho'^B), \quad (\text{A1})$$

or

$$-E(\rho_{T_h}^Q) = E(\rho_{T_c}^A) - E(\rho'^A) + E(\rho_{T_c}^B) - E(\rho'^B) - E(\rho^Q), \quad (\text{A2})$$

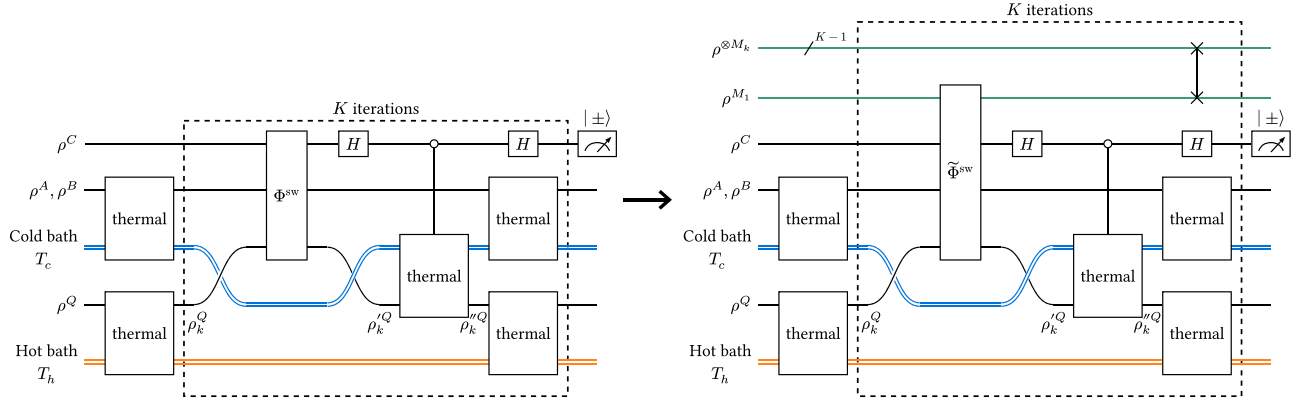


FIG. 6. Refrigeration cycle for $T_{\text{working}} > T_{\text{bath}}$ in quantum circuit representation, and its augmentation to include a memory tape in its operation. A complete refrigeration cycle features K iterations of the operations in the dashed box, expending the resources in the control and memory qubits.

after the quantum switch operation Φ^{sw} . Note that we have omitted the subscript k . The heat flow out of the cold bath consists of the controlled thermal operation and the resetting of ρ^A and ρ^B at the end of the cycle. They are, respectively,

$$-Q_1^c = E(\rho''^Q) - E(\rho'^Q) \quad (\text{A3})$$

and

$$-Q_2^c = E(\rho_{T_c}^A) - E(\rho'^A) + E(\rho_{T_c}^B) - E(\rho'^B). \quad (\text{A4})$$

Thus, the total heat flow out of the cold bath is

$$\begin{aligned} -Q^c &= -Q_1^c - Q_2^c = E(\rho''^Q) - E(\rho'^Q) \\ &\quad + E(\rho_{T_c}^A) - E(\rho'^A) + E(\rho_{T_c}^B) - E(\rho'^B) \\ &= E(\rho''^Q) - E(\rho_{T_h}^Q) = Q^h, \end{aligned} \quad (\text{A5})$$

where we substituted in Eq. (A2) in the third equality, and we note that the heat flow into the hot bath Q^h is simply due to the thermalization of system Q at the end of the cycle, i.e., $Q^h = E(\rho''^Q) - E(\rho_{T_h}^Q)$. Therefore, similar to the case of $T_{\text{working}} = T_{\text{bath}}$, to compute the COP, it will suffice to compute the heat flow into the hot bath.

As an example, we take $T_{\text{working}} = 1.05T_{\text{bath}}$ and plot the COP, Q_k^h , and $S(\rho_k^C)$ for the conventional quantum switch Φ^{sw} in Fig. 7, where an enhancement can be seen from the repeated operation of the quantum switch. However, since the resource is limited, we observe diminishing returns as the number of iterations increase, as Q_k^h falls to negative values as the control qubit approaches the maximally mixed state where refrigeration is no longer possible and the thermal gradient dominates.

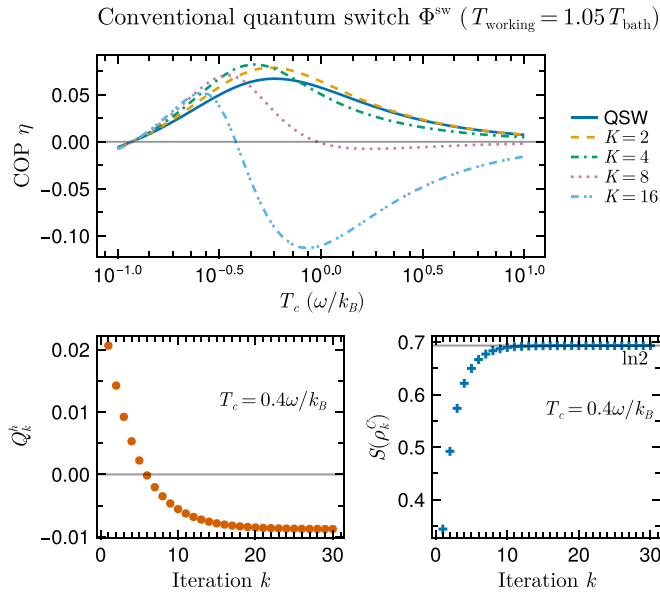


FIG. 7. COP, Q_k^h , and $S(\rho_k^C)$ for repeated use of Φ^{sw} with initial $\rho^C = |++\rangle|C^C$. The label “QSW” refers to the conventional quantum switch with $K = 1$. The COP can be slightly enhanced for repeated iterations, i.e., $K > 1$, but will result in diminishing returns for large K as Q_k^h and $S(\rho_k^C)$ saturates to negative values and maximum entropy, respectively, where heat extraction is no longer possible and the thermal gradient dominates. An arbitrary temperature of $T_c = 0.4\omega/k_B$ is chosen as an example for the evolution of Q_k^h and $S(\rho_k^C)$.

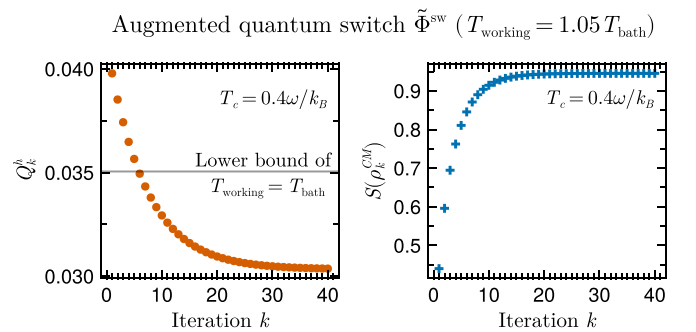


FIG. 8. Q_k^h and $S(\rho_k^{CM})$ do not saturate to 0 and maximum entropy of $2 \ln 2$, respectively, for repeated operations of Φ^{sw} with an initially uncorrelated memory tape of qubits. However, Q_k^h saturates to a lower value than the case of $T_{\text{working}} = T_{\text{bath}}$ due to the thermal gradient. An arbitrary temperature of $T_c = 0.4\omega/k_B$ is chosen as an example.

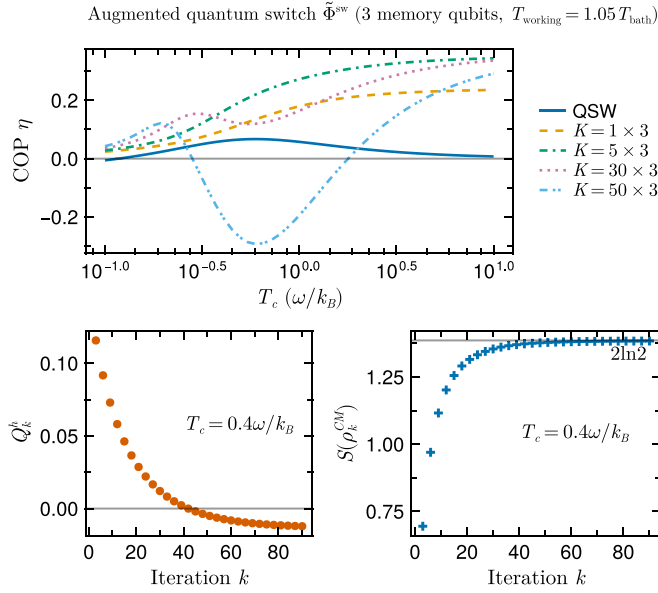


FIG. 9. COP, Q_k^h , and $S(\rho_k^{\text{CM}})$ for repeated use of $\tilde{\Phi}^{\text{sw}}$ with initial $\rho^C = |-\rangle\langle-|^C$ and three memory qubits that are reused multiple times. The label “QSW” refers to the conventional quantum switch with $K = 1$. The COP can be enhanced for repeated iterations, i.e., $K > 1$, but will result in diminishing returns for large K as Q_k^h and $S(\rho_k^{\text{CM}})$ saturates to negative values and maximum entropy, respectively, where heat extraction is no longer possible and the thermal gradient dominates. An arbitrary temperature of $T_c = 0.4\omega/k_B$ is chosen as an example for the evolution of Q_k^h and $S(\rho_k^{\text{CM}})$.

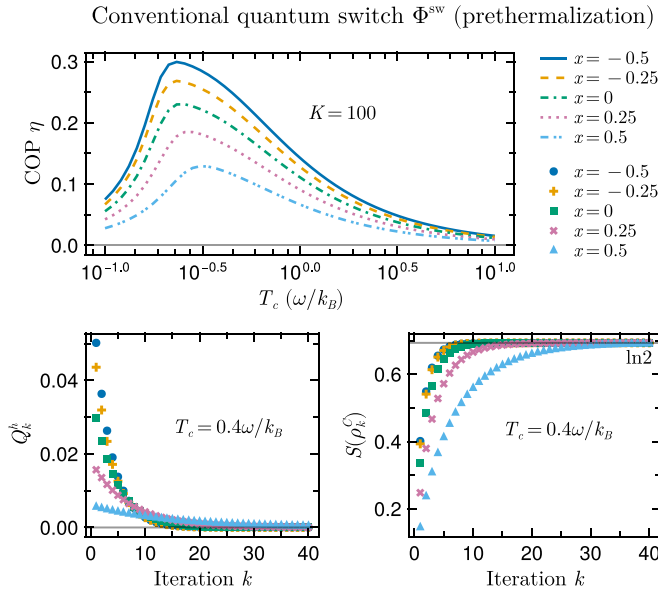


FIG. 10. COP, Q_k^h , and $S(\rho_k^C)$ for repeated operations of Φ^{sw} with initial $\rho^C = |+\rangle\langle+|^C$, for different values of x . Prethermalization ($x \neq 0$) can result in a greater COP than full thermalization ($x = 0$). For COP, the number of iterations is fixed arbitrarily at $K = 100$, while an arbitrary temperature of $T_c = 0.4\omega/k_B$ is chosen as an example for the evolution of Q_k^h and $S(\rho_k^C)$.

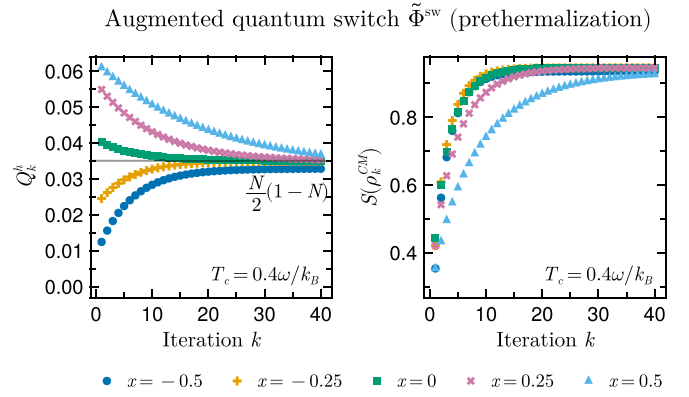


FIG. 11. Q_k^h and $S(\rho_k^{\text{CM}})$ do not saturate to 0 and maximum entropy of $2\ln 2$, respectively, for repeated operations of $\tilde{\Phi}^{\text{sw}}$ with an initially uncorrelated memory tape of qubits. An arbitrary temperature of $T_c = 0.4\omega/k_B$ is chosen as an example.

As for the augmented quantum switch $\tilde{\Phi}^{\text{sw}}$ where we have fresh memory qubits that are initially uncorrelated with the system, similar to the case of $T_{\text{working}} = T_{\text{bath}}$ in Fig. 3, Fig. 8 shows that the memory qubit can still grant positive heat extraction even when the control qubit runs out of resource, demonstrating that this resource from the memory qubits is independent of the quantum switch operation. Though in this case, the COP saturates at a value lower than the case of $T_{\text{working}} = T_{\text{bath}}$ due to the thermal gradient.

Finally, for the augmented quantum switch where the same three memory qubits are reused for multiple iterations in

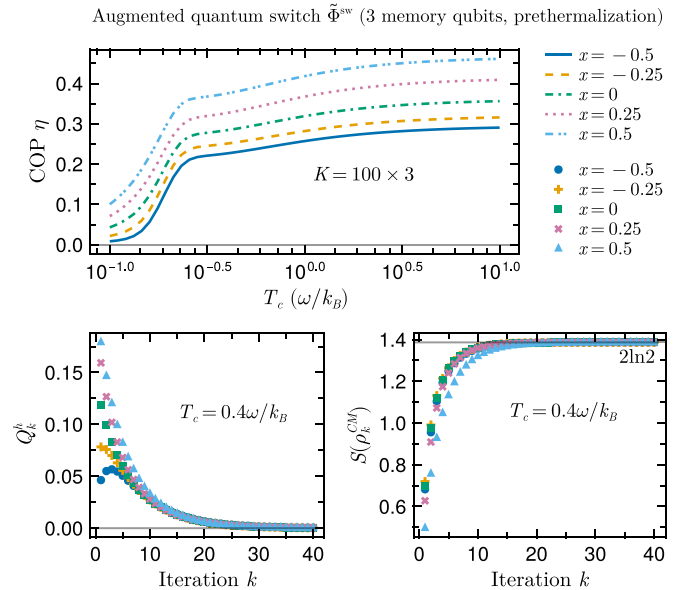


FIG. 12. COP, Q_k^h , and $S(\rho_k^{\text{CM}})$ for repeated operations of $\tilde{\Phi}^{\text{sw}}$ with initial $\rho^C = |-\rangle\langle-|^C$ and three memory qubits that are reused multiple times, for different values of x . Prethermalization ($x \neq 0$) can result in a greater COP than full thermalization ($x = 0$). For COP, the number of iterations is fixed arbitrarily at $K = 100 \times 3$, i.e., the three memory qubits are reused 100 times, while an arbitrary temperature of $T_c = 0.4\omega/k_B$ is chosen as an example for the evolution of Q_k^h and $S(\rho_k^C)$.

Fig. 9, similar to the conventional quantum switch, enhancement is possible but diminishing returns will occur as both the control and memory qubits run out of resource and the thermal gradient dominates.

APPENDIX B: ENHANCEMENT TO HEAT EXTRACTION WITH PRETHERMALIZATION

In the main text, we considered only full thermalizations for a fair comparison with the standard quantum switch refrigeration of Ref. [12]. However, it was shown in Ref. [13] that the conventional quantum switch Φ^{sw} has non-Markovian backflow of information in its operation that the COP is dependent on, and that these backflows are greatest in the prethermalization regimes.

Therefore, here we demonstrate numerically that enhancements to the COP via repeated operation of the quantum switch can also extend to the prethermalization regimes. Particularly, we will replicate the results in the main text, but for the cases of $x = -0.5, -0.25, 0, 0.25, 0.5$, noting that $x = 0$ is simply the fully thermalizing case for comparison. Also note that we have taken $x_A = x_B = x$ for simplicity.

Particularly, we fix a large $K = 100$ for the conventional quantum switch Φ^{sw} , demonstrating the enhancement to COP for certain $x \neq 0$ (Fig. 10). Note that different amount of prethermalization results in different rates and amount of heat extraction, as well as different consumption of resource. While their corresponding Q_k^h and $S(\rho_k^C)$ will saturate at the

same values, their transient behaviors suffice to lead to different COP values.

As for the augmented quantum switch $\tilde{\Phi}^{\text{sw}}$, we note that similar to the fully thermalizing case, the memory qubit still grants positive heat extraction even when the control qubit is expended, saturating at the same value of $\omega N(1 - N)/2$ (Fig. 11). Finally, for the case where three memory qubits are reused for a hundred times, i.e., $K = 100 \times 3$ (Fig. 12), we see that again different amount of prethermalization results in different rates and amount of heat extraction and consumption of resource.

Note that in the case of Fig. 10 where the initial control qubit is set to be $|+\rangle\langle+|^C$, the presence of the negative phase of prethermalization, e.g., $x = -0.5$, leads to increased heat flow when the measurement outcome is $|-\rangle\langle-|^C$ (conversely, we will have decreased heat flow for the $|+\rangle\langle+|^C$ measurement outcome as the average heat flow is constant). This is why in Fig. 10 the best COP is achieved for $x = -0.5$, which decreases as x approaches $x = 0.5$.

On the other hand, in the case of Fig. 12, the initial control qubit is flipped to be $|-\rangle\langle-|^C$ instead. This flipping of the control qubit is equivalent to flipping the conditioned measurement outcome to the $|+\rangle\langle+|^C$ measurement outcome instead. This means that the COP now depends on the amount of heat flow when the system collapsed to the $|+\rangle\langle+|^C$ measurement outcome. Since the presence of the negative phase of prethermalization, e.g., $x = -0.5$, leads to decreased heat flow when the measurement outcome is $|+\rangle\langle+|^C$, we see in Fig. 12 that $x = -0.5$ now grants the lowest COP, which increases as x approaches $x = 0.5$.

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- [1] G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, Quantum computations without definite causal structure, *Phys. Rev. A* **88**, 022318 (2013).
 - [2] N. Loizeau and A. Grinbaum, Channel capacity enhancement with indefinite causal order, *Phys. Rev. A* **101**, 012340 (2020).
 - [3] A. Feix, M. Araújo, and Č. Brukner, Quantum superposition of the order of parties as a communication resource, *Phys. Rev. A* **92**, 052326 (2015).
 - [4] P. A. Guérin, A. Feix, M. Araújo, and Č. Brukner, Exponential communication complexity advantage from quantum superposition of the direction of communication, *Phys. Rev. Lett.* **117**, 100502 (2016).
 - [5] D. Ebler, S. Salek, and G. Chiribella, Enhanced communication with the assistance of indefinite causal order, *Phys. Rev. Lett.* **120**, 120502 (2018).
 - [6] G. Chiribella, M. Banik, S. S. Bhattacharya, T. Guha, M. Alimuddin, A. Roy, S. Saha, S. Agrawal, and G. Kar, Indefinite causal order enables perfect quantum communication with zero capacity channels, *New J. Phys.* **23**, 033039 (2021).
 - [7] M. Araújo, F. Costa, and Č. Brukner, Computational advantage from quantum-controlled ordering of gates, *Phys. Rev. Lett.* **113**, 250402 (2014).
 - [8] M. J. Renner and Č. Brukner, Computational advantage from a quantum superposition of qubit gate orders, *Phys. Rev. Lett.* **128**, 230503 (2022).
 - [9] F. Chapeau-Blondeau, Indefinite causal order for quantum metrology with quantum thermal noise, *Phys. Lett. A* **447**, 128300 (2022).
 - [10] K. Simonov, G. Francica, G. Guarnieri, and M. Paternostro, Work extraction from coherently activated maps via quantum switch, *Phys. Rev. A* **105**, 032217 (2022).
 - [11] G. Francica, Causal games of work extraction with indefinite causal order, *Phys. Rev. A* **106**, 042214 (2022).
 - [12] D. Felce and V. Vedral, Quantum refrigeration with indefinite causal order, *Phys. Rev. Lett.* **125**, 070603 (2020).
 - [13] J. W. Cheong, A. Pradana, and L. Y. Chew, Non-Markovian refrigeration and heat flow in the quantum switch, *Phys. Rev. A* **110**, 022220 (2024).
 - [14] X. Nie, X. Zhu, K. Huang, K. Tang, X. Long, Z. Lin, Y. Tian, C. Qiu, C. Xi, X. Yang, J. Li, Y. Dong, T. Xin, and D. Lu, Experimental realization of a quantum refrigerator driven by indefinite causal orders, *Phys. Rev. Lett.* **129**, 100603 (2022).
 - [15] D. Mandal, H. T. Quan, and C. Jarzynski, Maxwell's refrigerator: An exactly solvable model, *Phys. Rev. Lett.* **111**, 030602 (2013).
 - [16] A. B. Boyd, D. Mandal, and J. P. Crutchfield, Identifying functional thermodynamics in autonomous Maxwellian ratchets, *New J. Phys.* **18**, 023049 (2016).
 - [17] S. Deffner, Information-driven current in a quantum Maxwell demon, *Phys. Rev. E* **88**, 062128 (2013).

- [18] A. Chapman and A. Miyake, How an autonomous quantum Maxwell demon can harness correlated information, *Phys. Rev. E* **92**, 062125 (2015).
- [19] L. He, A. Pradana, J. W. Cheong, and L. Y. Chew, Information processing second law for an information ratchet with finite tape, *Phys. Rev. E* **105**, 054131 (2022).
- [20] L. He, J. W. Cheong, A. Pradana, and L. Y. Chew, Effects of correlation in an information ratchet with finite tape, *Phys. Rev. E* **107**, 024130 (2023).
- [21] J. W. Cheong, A. Pradana, and L. Y. Chew, Communication advantage of quantum compositions of channels from non-Markovianity, *Phys. Rev. A* **106**, 052410 (2022).
- [22] J. W. Cheong, A. Pradana, and L. Y. Chew, Effects of non-Markovianity on daemonic ergotropy in the quantum switch, *Phys. Rev. A* **108**, 012201 (2023).
- [23] M. Rosati, A. Mari, and V. Giovannetti, Narrow bounds for the quantum capacity of thermal attenuators, *Nat. Commun.* **9**, 4339 (2018).
- [24] S. Khatri, K. Sharma, and M. M. Wilde, Information-theoretic aspects of the generalized amplitude-damping channel, *Phys. Rev. A* **102**, 012401 (2020).
- [25] R. Landauer, Irreversibility and heat generation in the computing process, *IBM J. Res. Dev.* **5**, 183 (1961).
- [26] A. Barato and U. Seifert, Unifying three perspectives on information processing in stochastic thermodynamics, *Phys. Rev. Lett.* **112**, 090601 (2014).
- [27] T. Van Vu and K. Saito, Finite-time quantum Landauer principle and quantum coherence, *Phys. Rev. Lett.* **128**, 010602 (2022).
- [28] A. Rolandi and M. Perarnau-Llobet, Finite-time Landauer principle beyond weak coupling, *Quantum* **7**, 1161 (2023).
- [29] D. Lu, H. Li, D.-A. Trottier, J. Li, A. Brodutch, A. P. Krismanich, A. Ghavami, G. I. Dmitrienko, G. Long, J. Baugh, and R. Laflamme, Experimental estimation of average fidelity of a Clifford gate on a 7-qubit quantum processor, *Phys. Rev. Lett.* **114**, 140505 (2015).
- [30] L. M. K. Vandersypen, M. Steffen, G. Breyta, C. S. Yannoni, M. H. Sherwood, and I. L. Chuang, Experimental realization of Shor's quantum factoring algorithm using nuclear magnetic resonance, *Nature (London)* **414**, 883 (2001).
- [31] H. Nie, T. Feng, S. Longden, and V. Vedral, Quantum cooling activated by coherently-controlled thermalisation, [arXiv:2201.06954](https://arxiv.org/abs/2201.06954).