

Realizing a topological staggered Su-Schrieffer-Heeger ladder with atomic momentum lattice

Yi-Yi Mao^{1,2}, Chao Zeng^{1,2}, Fei-Fei Wu^{1,2}, Yan-Jun Xie^{1,2}, Tao Yuan^{1,2}, Yu-Ao Chen^{1,2,3,4,*},
Han-Ning Dai^{1,2,3,†} and Jian-Wei Pan^{1,2,3,‡}

¹Hefei National Research Center for Physical Sciences at the Microscale and School of Physical Sciences,
University of Science and Technology of China, Hefei 230026, China

²Shanghai Research Center for Quantum Sciences and CAS Center for Excellence in Quantum Information and Quantum Physics,
University of Science and Technology of China, Shanghai 201315, China

³Hefei National Laboratory, University of Science and Technology of China, Hefei 230088, China

⁴New Cornerstone Science Laboratory, School of Emergent Technology,
University of Science and Technology of China, Hefei 230026, China



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Quasi-one-dimensional or ladder models serve as a bridge between one-dimensional and two-dimensional models, providing a simplified framework for incorporating intricate physics. In this study, we extend the concept of the momentum-state lattice by introducing couplings between next-nearest-neighbor sites throughout the entire momentum chain to simulate the staggered Su-Schrieffer-Heeger (SSH) ladder model. This ladder model is a direct extension of the well-known SSH model, recognized as a fundamental progression in modeling systems. Through this model, we explore compelling physical phenomena and successfully replicate the topological phase diagram of the Kitaev-Majorana chain. Utilizing time-averaged mean displacements of wave packets as a metric, we ascertain the topological invariants of the model. By varying staggered dimerization and interchain hopping parameters, we map out the topological phase diagram of the staggered SSH ladder, elucidating the presence of three distinct phases classified by their winding numbers: $\mathcal{W} = +1, 0$, and -1 , respectively.

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Introduction. The topological nature of a Bloch band in a system is characterized by certain topological invariants, which are typically obtained by integrating a local function over a closed surface in the Brillouin zone [1–3]. Among various models of topological bands, the Su-Schrieffer-Heeger (SSH) model [4,5], initially proposed to describe the spinless electronic structure of polyacetylene, serves as the simplest model for capturing the quintessence of topological band insulators. In the SSH model, the winding number is the topological invariant that classifies two distinct topological phases. Additionally, the bulk-edge correspondence predicts the presence of localized edge states at the ends of a finite SSH chain in the topological nontrivial phase [6]. Over the years, one-dimensional (1D) SSH chain and its extended models have been extensively studied across various platforms, including atomic [7–15], photonic [16–18], and superconducting [19] systems. Meanwhile, the quasi-one-dimensional (Q1D) chains or ladders [20–25], which bridge 1D and two-dimensional systems, not only can reveal distinct physics but

are also more feasible in experiments. As an immediate extension of the SSH model, the SSH ladder [26–29], which consists of coupled SSH chains, promises to showcase more intricate physics and applications. The SSH ladder is characterized by two distinct dimerization patterns: columnar and staggered. The columnar pattern resembles the SSH chain, whereas the noninteracting staggered pattern corresponds to an O(2)-symmetric model [29] comprising two decoupled Kitaev-Majorana (KM) chains [30–32]. In mechanical metamaterial systems, Majorana-like bound states of the SSH ladder have been observed [33,34]. However, realizing the staggered SSH ladder in a controlled quantum system remains challenging due to obstacles in achieving the staggered dimerization pattern and interchain hopping.

The momentum-state lattice (MSL), which enables the manipulation of individual tunneling amplitudes and phases in an atomic momentum chain, along with site-resolved density distribution measurement, offers a promising physical implementation for both 1D and Q1D models, such as the SSH model [13,14,35], the Aubry-André model [36,37], the Aharonov-Bohm ring [38,39], etc. Nevertheless, the realization of various ladders in a 1D MSL chain necessitates the inclusion of next-nearest-neighbor (NNN) tunnelings throughout the entire chain. This requirement poses a challenge within the existing framework of MSL theory.

In this work, we report on the realization of the staggered SSH ladder in an atomic momentum chain [as shown in Fig. 1(a)] and the measurement of the topological phase diagram of the model. By appropriately tuning the de-

*Contact author: yuaochen@ustc.edu.cn

†Contact author: daihan@ustc.edu.cn

‡Contact author: pan@ustc.edu.cn

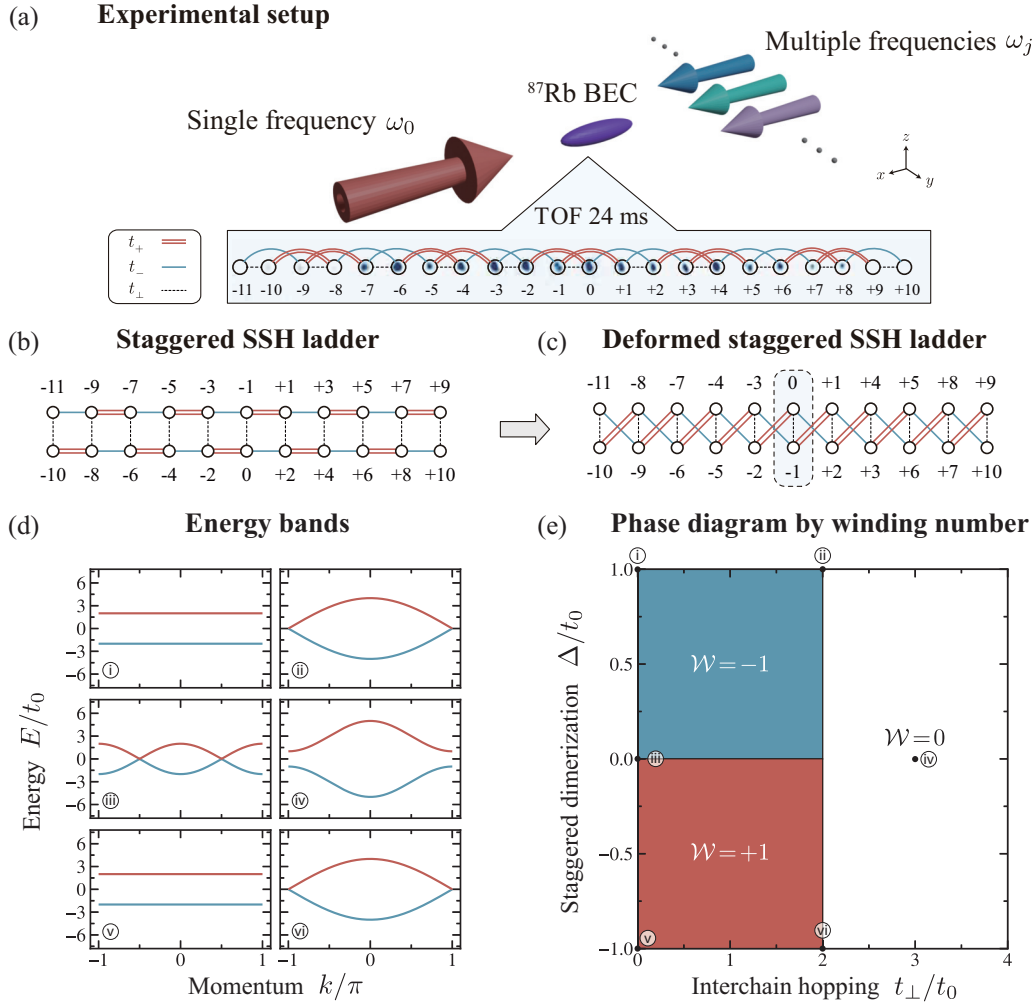


FIG. 1. (a) Schematic of a typical staggered SSH ladder constructed from an atomic momentum chain, with momentum states ranging from $n_i = -11$ to $n_f = +10$. BEC atoms, suffering from two propagating lasers, show a discrete chain structure after a time-of-flight of 24 ms. (b) The staggered SSH ladder model. The odd (even) tunneling amplitude in one SSH chain and the even (odd) tunneling amplitude in another SSH chain are both t_- (t_+). Furthermore, these two SSH chains are coupled together with an amplitude $t_\perp$. (c) The deformed staggered SSH ladder model. After a straight deformation, the staggered SSH ladder Hamiltonian could be rewritten in a new form. In this geometry, one could directly find that a unit cell contains two sites. (d) Energy bands under different settings of interchain hoppings and staggered dimerizations. ① to ⑥ correspond to parameters $(t_\perp/t_0, \Delta/t_0) = (0, 1), (2, 1), (0, 0), (3, 0), (0, -1),$ and $(2, -1)$. (e) Topological phase diagram by theoretically calculated winding number. Three distinct topological phases are classified by different winding numbers.

tuning configuration of Bragg processes, we introduce an extended MSL scheme containing NNN tunnelings over the entire chain. Leveraging the advanced MSL theory mentioned above, we experimentally fabricate a 22-site staggered SSH ladder with a Bose-Einstein condensate (BEC) of ^{87}Rb atoms along a series of momentum states. Subsequently, we delineate the topological phase diagram by assessing the winding number within a two-dimensional parameter space. Through this process, we pinpoint two discernible topological nontrivial phases, characterized by winding numbers $\mathcal{W} = \pm 1$, and a topological trivial phase, marked by a winding number of $\mathcal{W} = 0$.

Theoretical model. The SSH ladder model, advancing from the celebrated SSH model and consisting of two interconnected SSH chains, exhibits rich physics. This makes it a promising intermediate stage for further exploration of two-dimensional models. The noninteracting Hamiltonian for the

staggered SSH ladder is given by

$$\mathbf{H}_0 = - \sum_{n,\sigma} \left[t_0 + \frac{1}{2} \Delta_\sigma (-1)^n \right] (\mathbf{c}_{n,\sigma}^\dagger \mathbf{c}_{n+1,\sigma} + \text{H.c.}) - t_\perp \sum_{n,\sigma} \mathbf{c}_{n,\sigma}^\dagger \mathbf{c}_{n,-\sigma}, \quad (1)$$

which incorporates intrachain hopping amplitude t_0 , staggered dimerization $\Delta_+ = -\Delta_- = \Delta$, and interchain hopping amplitude $t_\perp$. Here, $\mathbf{c}_{n,\sigma}^\dagger$ and $\mathbf{c}_{n,\sigma}$ denote second-quantized creation and annihilation operators for spinless bosons at the n th site of the chain in the upper (for $\sigma = +1$) and lower (for $\sigma = -1$) ladder sections.

Following a straightforward deformation, as shown in Figs. 1(b) and 1(c), the model can be reexpressed with a unit cell that includes two sites. Under the assumption of a periodic

boundary condition, the Hamiltonian in momentum space is

$$\mathbf{h}_0(k) = \begin{pmatrix} 0 & t_+ e^{ik} + t_- e^{-ik} + t_\perp \\ t_+ e^{-ik} + t_- e^{ik} + t_\perp & 0 \end{pmatrix}, \quad (2)$$

where $t_+ = t_0 + \Delta/2$ and $t_- = t_0 - \Delta/2$. The energy spectrum derived from this Hamiltonian reads

$$E^\pm(k) = \pm \sqrt{[t_\perp + \epsilon(k)]^2 + \Delta(k)^2}, \quad (3)$$

with $\epsilon(k) = 2t_0 \cos(k)$ and $\Delta(k) = \Delta \sin(k)$. The gap between two energy bands closes at the following conditions shown in Fig. 1(d): (i) $t_\perp = -2t_0$, $k = 0$; (ii) $t_\perp = 2t_0$, $k = \pi$; and (iii) $\Delta = 0$, $|t_\perp/t_0| < 2$, and $\cos(k) = -t_\perp/(2t_0)$. These conditions correspond to the topological phase boundary, as indicated by the winding number in Fig. 1(e).

Experimental setup. The experiments commence with a BEC of approximately 6×10^4 atoms that are optically trapped with dipole trap frequencies of $(\omega_x, \omega_y, \omega_z) = 2\pi \times (40, 100, 100)$ Hz. As shown in Fig. 1(a), the BEC atoms are driven by a pair of counterpropagating lasers with wavelength $\lambda = 1064$ nm and wave number $k = 2\pi/\lambda$. One of the lasers has a single frequency component, while the opposite laser consists of multiple frequency components. These counterpropagating laser fields provide two-photon and four-photon Bragg processes to couple synthetic NN and NNN momentum sites, respectively. More detailed information on our MSL setup can be found in Refs. [35,40,41].

By employing a series of two-photon Bragg processes, the couplings between adjacent discrete momentum states of free particles can be meticulously managed [42]. This approach, relying on Bragg dynamics, crafts a synthetic momentum chain, enabling the simulation of 1D lattice models that incorporate only NN couplings. Nevertheless, the simulation of certain Q1D lattice models, such as two-leg ladder models [25,43–45], demands detailed control of NNN couplings over the whole chain.

In addressing NNN couplings within the two-photon-Bragg style synthetic 1D chain, introducing four-photon Bragg processes emerges as a natural step. These processes occur between a momentum site and its second adjacent site. However, careful selection of the detuning is crucial. The detuning between the virtual level of the four-photon Bragg processes and the intermediate energy level between two NNN levels must be meticulously chosen. Otherwise, the desired couplings will be disrupted by interference from the optical electric field.

Based on the MSL theory [42,46], the ground-state MSL Hamiltonian can be derived by performing adiabatic elimination of a series of significantly higher-energy excited states,

$$\mathbf{H}(t) = \sum_n [K_n(t)|n+1\rangle\langle n| + \text{H.c.}], \quad (4)$$

where $|n\rangle = |n \cdot 2\hbar k\rangle$ represents the n th momentum state. $K_n(t)$ encompasses all time-dependent coupling terms arising from two-photon Raman processes $\{u\}$ and virtual two-photon Raman transitions $\{v_+\}$ and $\{v_-\}$ in four-photon Bragg processes:

$$K_n(t) = \sum_{j,s} \frac{\hbar \Omega_{js}}{2} e^{-i\delta_{js,n}t}. \quad (5)$$

Here, Ω_{js} represents the effective two-photon Rabi frequency of the j th two-photon group, where $s \in \{u, v_+, v_-\}$. $\delta_{js,n} = \omega_{js} - \omega_{n+1,n}$ denotes the two-photon detuning of the s component within the j th group of laser fields for the transition $|n\rangle \leftrightarrow |n+1\rangle$. $\hbar\omega_{n+1,n} = E_{n+1} - E_n$ is the energy difference between $|n+1\rangle$ and $|n\rangle$, where $E_n = 4n^2 E_r$ is the energy of the n th momentum state, and E_r is the recoil energy. Specifically, $\hbar\omega_{ju} = E_{j+1} - E_j$ and $\hbar\omega_{jv_\pm} = \frac{1}{2}(E_{j+2} - E_j) \pm \delta_j$. The Hamiltonian is already transformed into the interaction picture and treated using the rotating-wave approximation.

The effective Hamiltonian can be derived through a second-order perturbative approximation of the MSL Hamiltonian (see the Supplemental Material [47]),

$$\mathbf{H}_{\text{eff}} = \sum_n [t_n^u \mathbf{c}_{n+1}^\dagger \mathbf{c}_n + t_n^v \mathbf{c}_{n+2}^\dagger \mathbf{c}_n + \text{H.c.}]. \quad (6)$$

Here, the tunneling amplitudes t_n^u are derived solely from Rabi frequencies Ω_{ju} in two-photon Bragg processes; the amplitudes t_n^v arise from all Rabi frequencies Ω_{ju} , Ω_{jv_+} , and Ω_{jv_-} . One might attempt to align the parameters of this effective Hamiltonian with those of a staggered SSH ladder. However, achieving such a correspondence proves impossible if the detunings $\delta_{jv_+,n}$ and $\delta_{jv_-,n}$ are set without considering the appropriate configuration of δ_j .

We define two distinct detuning configurations: one, following a straightforward approach, assumes $\delta_j = 0$ and is known as the constant detuning configuration; the other employs a stepwise distribution, setting $\delta_j = E_r$ if $(j - n_i) \bmod 4 = 0, 1$ and $\delta_j = 0$ if $(j - n_i) \bmod 4 = 2, 3$, referred to as the stepped detuning configuration. Here, $|n_i\rangle$ is the initial momentum state. By comparing numerical simulation results of wave packet expansion dynamics for both the ideal Hamiltonian and the MSL Hamiltonian with experimental data, we benchmark the effective NNN tunneling amplitudes under these configurations [47]. Experimentally, we implement a few-site sample with only t_{-1}^v and t_{+1}^v activated and a complete-site sample with all NNN tunnelings opened, as depicted in Fig. 2. We find good agreement between the experimental results and the simulation results for both the ideal Hamiltonian and the MSL Hamiltonian under the stepped detuning configuration. However, the simulation results for the MSL Hamiltonian under the constant detuning configuration show unexpected behaviors.

In the case of the few-site sample shown in Fig. 2(a), atoms are initially located at site $|0\rangle$ and are expected to remain localized at the original site due to the absence of tunneling amplitude between $|0\rangle$ and any other site. However, simulation results for the MSL Hamiltonian under the constant configuration reveal that atoms could tunnel into site $|+2\rangle$. This indicates a nonvanishing tunneling amplitude t_0^v , an unexpected behavior arising from the combined effects of Ω_{-1v_+} and Ω_{+1v_+} , which meet the resonance condition for the transition $|0\rangle \leftrightarrow |+2\rangle$. Similarly, for the complete-site sample illustrated in Fig. 2(b), the absence of all tunnelings in the simulation results for the MSL Hamiltonian under the constant detuning configuration stems from the undesired path interference among various two-photon transitions. These interferences disappear under the stepped configuration due to detuning mismatch. With the MSL Hamiltonian under the

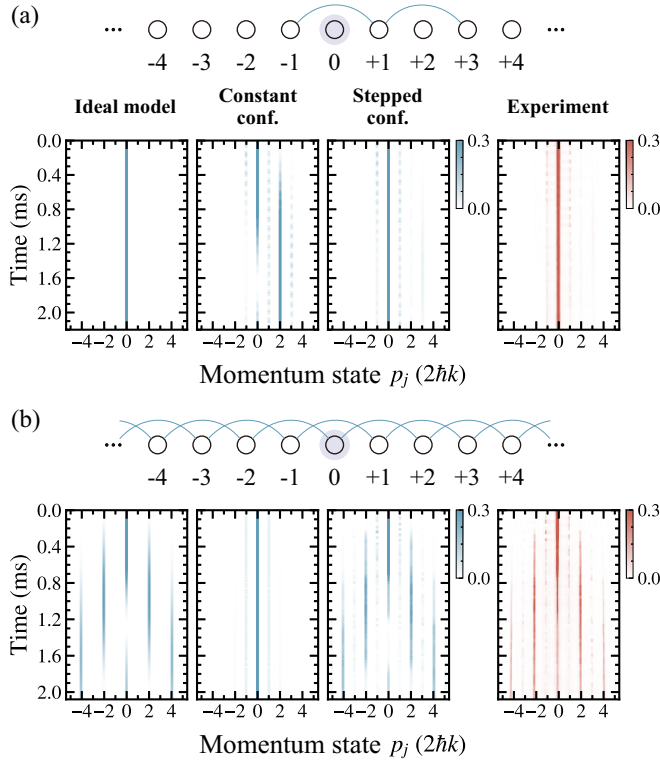


FIG. 2. Few-site and complete-site sample models under different detuning configurations. The upper part of each subfigure illustrates schematic lattices: Panel (a) represents the few-site sample, while panel (b) corresponds to the complete-site sample. In panel (a), only tunnelings between $|-1\rangle$ and $|+1\rangle$ sites and $|+1\rangle$ and $|+3\rangle$ sites are employed, with an amplitude of $2\pi\hbar \times 0.25$ kHz. In panel (b), all NNN tunnelings are opened, with an amplitude of $2\pi\hbar \times 0.15$ kHz. The numerical simulation results, depicting the wave packet evolution governed by the ideal Hamiltonian, the MSL Hamiltonian under the constant detuning configuration, and the MSL Hamiltonian in the stepped detuning configuration, are presented sequentially from left to right with blue color bars. In the experiment, a Hamiltonian in the stepped detuning configuration is employed. The experimental results are represented using orange color bars, demonstrating comparability with the theoretical predictions.

stepped configuration, one can ultimately achieve complete control over every NNN tunneling in a momentum-state chain.

Realization of the staggered SSH ladder. Considering the coherent time of momentum states and the perturbative condition, we set the intrachain tunneling amplitude $t_0 = 2\pi\hbar \times 0.15$ kHz. This setting results in a characteristic unit time of $\hbar/t_0 \approx 1.06$ ms. Following the quench dynamics, all laser fields are shut down and atoms after evolving in the synthetic momentum lattice thereby undergo a 24 ms time-of-flight (TOF) process. The atomic population distribution can be directly extracted from the TOF images. As illustrated in Fig. 3, we monitor the wave packet evolution dynamics under typical parameters corresponding to three different topological phases by scanning the evolution time sequence. The mean displacement of the wave packet can be calculated: $\langle m(t) \rangle = \sum_m m(A_m(t) + B_m(t))$, where $A_m(t)$ and $B_m(t)$ represent the occupation of sublattices A and B in the m th cell at time t , respectively. While the ladder is in the topological

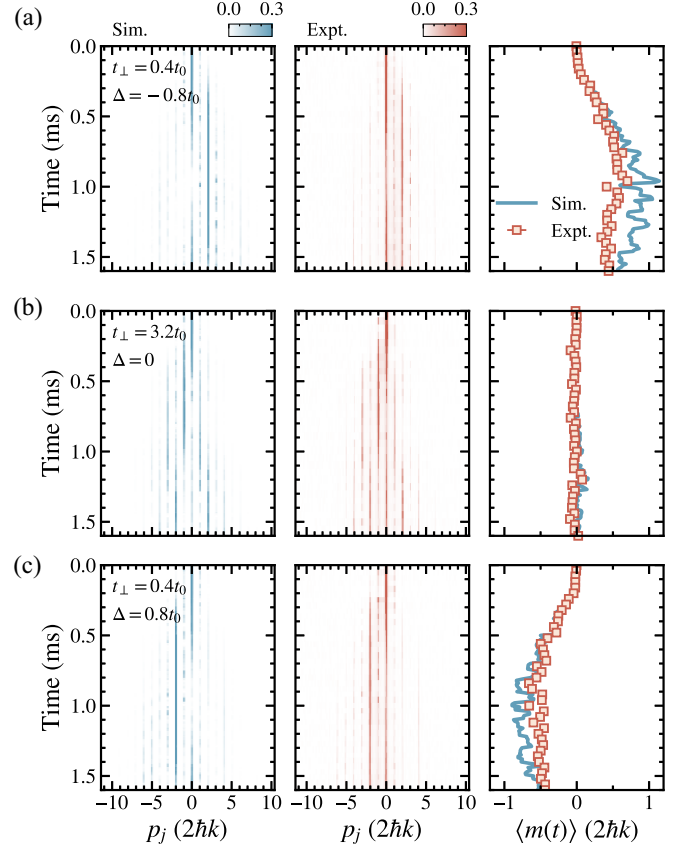


FIG. 3. Wave packet quench dynamics in a 22-site staggered SSH ladder. We monitor the evolution of the wave packet initially injected into the zero-momentum site for specific parameters $(t_\perp/t_0, \Delta/t_0)$: (a) $(0.4, -0.8)$, (b) $(3.2, 0.0)$, and (c) $(0.4, 0.8)$. From left to right are the numerical results of wave packet evolution, the experimental results of wave packet evolution, and the mean displacement of the wave packet $\langle m(t) \rangle$. All the data are averaged over four individual measurements, and the error bars represent the standard error.

nontrivial phase with $\mathcal{W} = +1$ ($\mathcal{W} = -1$), $\langle m(t) \rangle$ indicates a positive (negative) displacement of the center of mass. In contrast, when the ladder enters the topological trivial phase, $\langle m(t) \rangle$ remains near zero throughout the evolution. We find the experimental results match well with the simulation results of the MSL Hamiltonian.

In a two-band Hamiltonian of a 1D system, the winding number can be linked to the time-averaged mean displacement of a wave packet initially localized at a single central site, provided the Hamiltonian in momentum space lacks the σ_z Pauli matrix component [48–51]. The staggered SSH ladder model, being a two-band model with inversion symmetry, allows experimental classification according to the topological invariant. We determine the topology information of momentum ladders from the bulk dynamics response following a sudden quench. Specifically, we calculate the topological winding number by extracting the time average of the wave packet mean displacement, a method recently employed in various experimental setups.

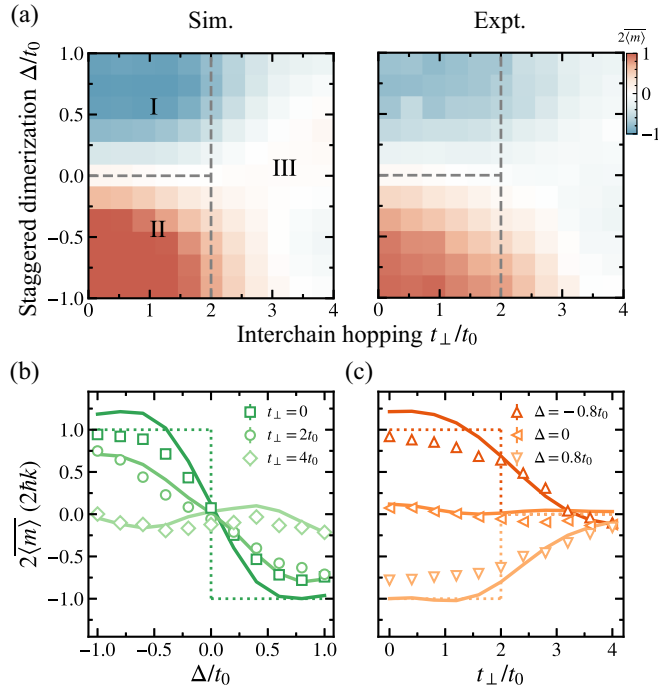


FIG. 4. Observation of topological phases of the staggered SSH ladder. (a) The phase diagrams, depicting numerical simulation result of the MSL Hamiltonian (left panel) and experimental result (right panel), respectively, are obtained from twice the time average of the wave packet mean displacement $2\langle m \rangle$. The gray dashed lines are delineated based on the ideal phase diagram in Fig. 1(e), indicating the theoretical phase boundary. (b) Twice the time-averaged mean displacement $2\langle m \rangle$ as a function of staggered dimerization Δ/t_0 at interchain hopping $t_\perp/t_0 = 0$, $t_\perp/t_0 = 2$ and $t_\perp/t_0 = 4$ (green, from dark to light). The data (filled squares, circles, and diamonds) are obtained from four individual experiments and averaged in time from 0 to $1.5\hbar/t_0$. The error bars denote the standard error. The solid lines are numerical results of the MSL Hamiltonian. (c) Twice the time-averaged mean displacement $2\langle m \rangle$ as a function of interchain hopping $t_\perp/t_0$ at staggered dimerization $\Delta/t_0 = -0.8$, $\Delta/t_0 = 0$, and $\Delta/t_0 = 0.8$ (orange, from dark to light). All the numerical simulation results use the same parameters as the experimental data.

The time-averaged mean displacement of the wave packet at time T is given by

$$\langle m(T) \rangle = \frac{1}{T} \int_0^T dt \langle m(t) \rangle. \quad (7)$$

With the inversion symmetry maintained in the Hamiltonian, the Zak phase becomes quantized, leading to $\mathcal{W} = \lim_{T \rightarrow \infty} 2\langle m(T) \rangle$.

In Fig. 4(a), we present the topological phase diagram of the SSH ladder within a $t_\perp - \Delta$ parametric space, identifying three distinct topological phases with winding numbers $W = +1, 0$, and -1 , respectively. As a result of connection between the SSH ladder and two decoupled KM chains, the phase boundaries in the phase diagram of the staggered SSH ladder show consistency with those in the phase diagram of the KM chain. As shown in Fig. 4(b), we present three specific lines $t_\perp = 0$, $2t_0$, and $4t_0$ in the diagram and

chart twice the time-averaged mean displacement at the final time as a function of dimerization Δ . A sharp variation is observed at $t_\perp = 0$, a slightly gentler slope at $t_\perp = 2t_0$, and an almost flat curve at $t_\perp = 4t_0$. By varying dimerization along the path $t_\perp = 0$, the ladder undergoes a phase transition from topological nontrivial phase II to topological nontrivial phase I. The slope becomes gentler as it approaches the theoretical boundary $t_\perp = 2t_0$ between topological nontrivial and trivial phases. This effect primarily arises from the finite size of the ladders in the experiment and imperfections of the ladder formed by the MSL chain containing NNN couplings. Additionally, we observe phase transitions from nontrivial phase II and I to trivial phase III along the path $\Delta/t_0 = -0.8$ and $\Delta/t_0 = 0.8$, respectively, as depicted in Fig. 4(c). It is also noted that the absolute values of the experimental data points are slightly lower than those in the simulation results, attributed to decoherence among atoms in different momentum states.

Conclusion. In this study, we have introduced an extended MSL scheme that includes both NN and NNN couplings over the entire momentum chain. Utilizing the extended MSL scheme, we have experimentally realized the staggered SSH ladder within a 22-site MSL chain. We further explored the topological two-dimensional parametric phase diagram by measuring the winding number derived from bulk dynamics. While our discussion here focused on tunnelings devoid of hopping phases, incorporating arbitrary phase terms into each tunneling process is straightforward. Introducing nonzero hopping phases could alter energy gaps and modify the symmetry class of systems such as the staggered SSH ladder or the bowtie ladder [25]. Additionally, the atomic interaction will lead to a nonlinear interaction term in a momentum chain [36,37]. By combining these elements within the extended MSL scheme, one may study the interplay between artificial gauge fields and interactions in the future [52]. Interactions in momentum space are intrinsically long ranged and state dependent, making it difficult to design on-site terms analogous to Hubbard-type interactions [53–55]. Nevertheless, such momentum-space interactions may also enable richer physics [36,37,56,57]. Our findings offer avenues for the physical implementation of quantum ladders and pave the way for realizing quantum ladder simulators with ultracold atoms [40,41].

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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