

Global topological protection of entanglement in integrated photonic quantum walks

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Photon entanglement is a fundamental resource for quantum technologies, yet it is extremely fragile especially in large quantum circuitry, which is necessary for quantum information processing and computing. We propose an integrated scheme for direct topological protection of entanglement, enabling robust transport of entangled photonic states, even when the photon dynamically evolves during propagation. Our platform relies on a two-dimensional discrete-time quantum walks in synthetic dimensions, supporting topological edge states at the interface between distinct domains. Thus far, in all previous approaches for exploiting topology to protect entanglement, the protection was linked to the position of the photon in the actual structure. In contrast, our scheme relies on a synthetic temporal dimension (time bins) where the entangled state is always reproduced, despite fluctuations or variations that may emerge in the system. Thus, our scheme provides precise temporal control important for synchronization and timing in quantum computation. By suppressing phase-sensitive errors—the most critical noise source in photonic platforms—our approach can lower the physical error rate below fault-tolerance thresholds and reduce the overhead typically required for scalable quantum information processing. The robustness to errors arises from the fact that the entangled state is preserved through correlations in the synthetic (temporal) dimension. As a result, we ensure that at specific designated times, the emergent state faithfully reproduces the initial entangled state, despite the photon having taken any dynamically evolving paths. This approach establishes a robust framework for directly protecting entanglement.

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I. INTRODUCTION

Quantum topological photonics, an emerging field at the intersection of quantum optics and topological photonics, offers a powerful platform for realizing robust quantum devices [1–5]. Topological photonic systems support protected edge states that are immune to scattering from imperfections, enabling unidirectional transport and reduced sensitivity to fabrication defects [6–8]. These features are particularly advantageous for quantum optics, where even minor losses or fabrication errors can degrade entanglement and fidelity to below a threshold where errors can be corrected through quantum error corrections [9]. This issue is especially crucial in multiqubit systems [10]. By utilizing topological protection, quantum light sources can be stabilized, entanglement can be preserved over long distances [11], and quantum states can become more resilient to disorder—key requirements for quantum communication, computation, and sensing [12,13].

Apart from photonics, all other physical platforms for quantum information science suffer from decoherence (loss of coherence due to interaction with the environment). Although photons do not decohere in the conventional sense, they are still susceptible to loss. When two photons are entangled and one of them is lost, whether due to absorption or due to scattering away from its destination, the surviving photon acts as “noise,” significantly reducing the fidelity of quantum measurements. In quantum information processing, such fidelity degradation is particularly harmful: there exists a strict threshold for acceptable error rates, typically below 10^{-4} per operation—beyond which errors can no longer be corrected, rendering the computation unreliable [14,15]. Obviously, one cannot “protect” against loss of photons due to absorption, but in the current platforms for photonic information processing the underlying material (silicon or other materials) is essentially lossless. In fact, the critical reason for fidelity reduction in current quantum photonic circuitry is scattering loss, where even minor fabrication imperfections can give rise to scattering, and the fidelity reduction accumulates over many operations, making scalability in photonic quantum systems especially challenging. This vulnerability is pronounced in large-scale integrated photonic platforms, where dense circuit layouts and miniaturization amplify the impact of even slight imperfections. Without a robust mechanism to protect entanglement from scattering and disorder, this issue becomes

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a critical bottleneck—a fundamental obstacle for scalable quantum photonic technologies.

Addressing the need for protection of entanglement in photonic systems, it has been proposed that using concepts from topological photonics could offer a promising route forward, exploiting spatial modes that are inherently immune to various forms of disorder, including fabrication imperfections and scattering [1–3]. While such protection has been demonstrated for both quantum and classical light, its application for entangled quantum states has thus far required guiding the light in specific channels that form edge states in the photonic circuit. More specifically, previous approaches utilized topological protection of each mode of a bipartite state separately for clarity, and the protection of the entanglement was a by-product of the individually protected constituents [2,3,16–23]. These methods safeguard the photonic modes, but not the quantum property of entanglement itself. Although topological protection of individual photon modes can protect entanglement under such specific conditions [5,21,23], this approach relies on the assumption that the quantum correlations will remain intact as long as each mode stays confined to its respective edge state. However, entanglement is not a property of the individual modes—it is encoded in the joint quantum state, and therefore its preservation requires protecting the entire state as a whole, not just its components.

Let us give an example that shows the problem of protecting each mode individually rather than protecting as a whole. Consider a polarization-entangled photon pair, propagating in two polarization-dependent channels, A and B :

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle_A |V\rangle_B + |V\rangle_A |H\rangle_B), \quad (1)$$

Where H and V are horizontal and vertical polarizations, respectively, and A and B denote the two propagation channels. For the general case that channel A is protected up to a global phase for H polarization $|H\rangle \rightarrow e^{i\theta_H^A} |H\rangle$ and V polarization is protected up to a (potentially different) global phase $|V\rangle \rightarrow e^{i\theta_V^A} |V\rangle$, and channel B analogously applies global phases $e^{i\theta_H^B}$, $e^{i\theta_V^B}$, the state becomes

$$|\psi\rangle = \frac{1}{\sqrt{2}}(e^{i(\theta_H^A + \theta_V^B)} |H\rangle_A |V\rangle_B + e^{i(\theta_V^A + \theta_H^B)} |V\rangle_A |H\rangle_B), \quad (2)$$

Which gives a relative phase between the two components of Eq. (1):

$$\Delta = \theta_H^A + \theta_V^B - \theta_V^A - \theta_H^B. \quad (3)$$

Thus, because the topological protection is up to a global phase, the relative phase between the two components in Eq. (1) can change in this configuration, leading to degraded fidelity. For example, if these phases fluctuate differently for the two polarizations, the relative phase becomes effectively random. Note that, while such a global phase has no physical consequence for a single state, for entangled states involving internal degrees of freedom—the accumulated phase can depend on these degrees of freedom, introducing relative phase shifts between components of the superposition. These relative phases are physically meaningful and must be preserved, as their variation can degrade the entanglement even when all subsystems remain topologically confined. Therefore, topo-

logical protection of each subsystem individually does not always guarantee the robustness of entanglement in the presence of certain types of scattering—such as asymmetric loss, mode-dependent phase noise, or intermode coupling—all of which are common in integrated photonics. In fact, as shown in recent works [24–26], the quantum correlations degrade due to some overlap with unprotected bulk modes or differential phase shifts. These effects accumulate and directly reduce entanglement fidelity.

To truly ensure robustness, there is a clear need for an approach that encodes the entire entangled state into a topologically protected subspace within the joint Hilbert space, such that this subspace allows transport in different channels (e.g., waveguides), as necessary for quantum information applications. Such an approach could provide a more direct route to preserving the quantum correlations themselves, rather than relying on the indirect protection of the individual photon paths [27].

Here, we propose an integrated photonic scheme for topologically protected transport of entanglement, not relying on the protection of the individual photon paths. Specifically, we show topological edge states at the interface between distinct domains in the synthetic dimension of time bins. Our scheme allows dynamically evolving paths during propagation, yet ensures that the output state—retrieved at designated time steps—faithfully reproducing the original entangled input. Here, global means that, at the protected times, the entire Bell state is an eigenstate of a topological edge mode, not just each spatial mode separately. This approach lays the foundation for a scalable and robust framework for quantum information processing, where entanglement is preserved even in the presence of imperfections.

II. ENTANGLEMENT IN SYNTHETIC DIMENSIONS

Our scheme is based on entanglement in synthetic dimensions—a platform in which spatial coordinates are replaced (or extended) by internal or modal degrees of freedom, such as energy levels, time bins, waveguide modes, or orbital angular momentum [28–31]. These systems enable access to higher-dimensional physics and allow for nonlocal correlations and protected transport that go beyond conventional real-space lattices. A particularly flexible platform for exploring various phenomena in synthetic dimensions is the discrete-time quantum walk (DTQW) [32,33]. For example, in these systems, spin- or polarization-dependent shifts over discrete time steps induce effective topological phases [34,35]. DTQWs have been used to implement quantum gates, simulate quantum algorithms [36], generate high-dimensional entangled states [37,38], and have also been shown to enable universal quantum computation [39]. When implemented in photonic chips, DTQWs offer fine-grained control over spatial and temporal dynamics, enabling the design of protection mechanisms that act globally on the entangled state. By incorporating topology into the DTQW scheme, we can control the specific time bins in which the state remains protected—an important capability for quantum computing operations.

Crucially, our scheme of topologically protected entanglement in time bins is fully integrated, offering a compact and stable platform compatible with on-chip architectures—

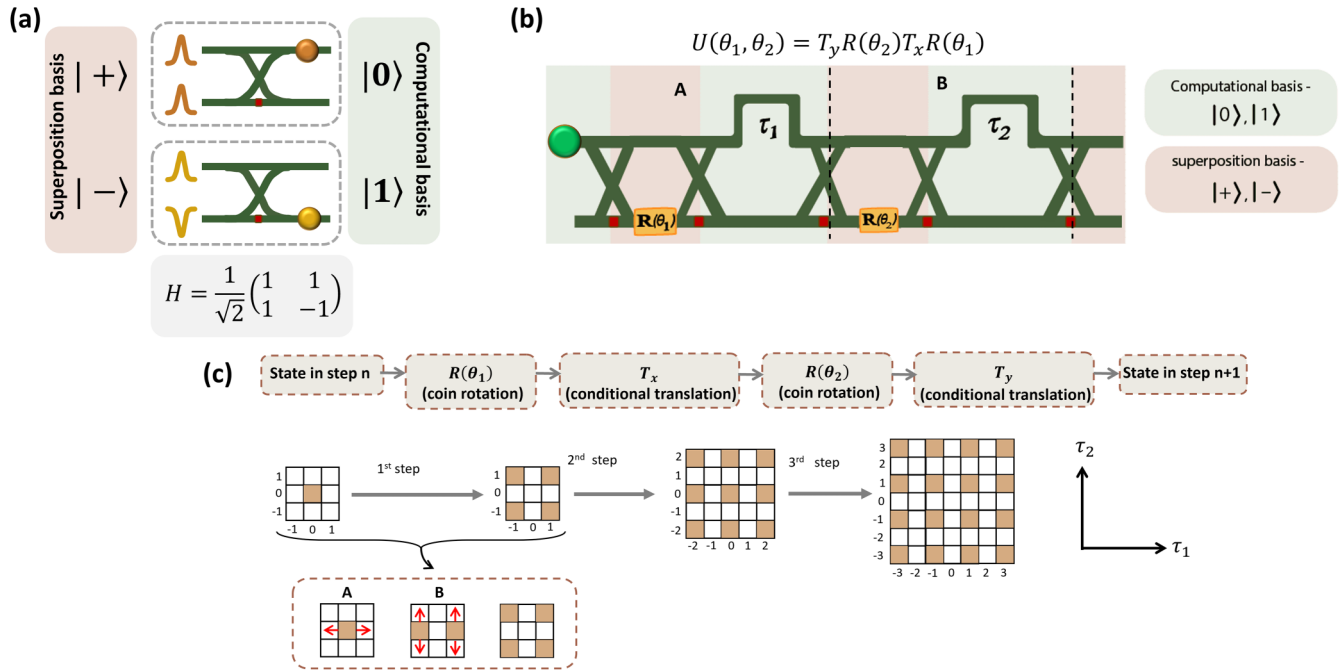


FIG. 1. Schematic illustration of the integrated platform for one qubit. (a) Implementation of a Hadamard gate: a photon injected into the upper waveguide evolves into a superposition over both waveguides with zero relative phase, while a photon injected into the lower waveguide evolves into a superposition with a relative phase of π . The process also works in reverse, mapping superposition states back to the computational basis. (b) An incident photon follows, depending on which of the waveguides it is injected, four different paths in the pair of connected waveguides. The four paths correspond to the four different directions a walker can take in a single step on a 2D time-bin lattice. Different path lengths in the system generate temporally encoded states, where different position states are represented by discrete time bins. (c) Mapping the evolution of the photon through the waveguides to the 2D synthetic lattice of time bins. Each lattice site (x, y) corresponds to a unique arrival time, defined by the two independent delays τ_1 and τ_2 .

an essential step toward large-scale quantum photonic technologies. Furthermore, our method opens the door to topologically encoded multiphoton states, where entanglement is distributed across synthetic dimensions or high-dimensional spaces, a direction not naturally accessible in standard topological schemes.

III. PHYSICAL MODEL

A natural setting in which to implement this approach is single-rail encoding, where a single photon occupies a superposition across multiple optical modes, and the quantum information is encoded in correlations between populated and vacuum states [40–42]. While this form of entanglement involves only a single excitation, it is nonetheless a genuinely nonclassical resource [43], used in quantum experiments and protocols [44–46].

The topological protection in our scheme is realized through a DTQW on a two-dimensional (2D) synthetic lattice [Fig. 1(c)]. Each site of this lattice corresponds to a distinct time bin, defined by two independent delays, τ_1 and τ_2 , which are the horizontal and vertical directions of the synthetic lattice, respectively. The photon is initially injected at the origin of the synthetic lattice $(x, y) = (0, 0)$. Each evolution step in the network introduces a temporal shift associated with the delays τ_1 and τ_2 and the difference between them. The coordinates (x, y) in the synthetic space therefore represent the net imbalance accumulated along the two delay

directions relative to the reference path. Positive and negative coordinates correspond to arrival times that are delayed or advanced relative to the reference time bin, rather than to positive or negative physical times. For example, if the reference path includes a delay τ_1 , a path that does not acquire this delay is mapped to $x = -1$, even though all physical propagation times remain positive. The propagation of photons in the synthetic 2D space is governed by alternating coin and shift operations, physically encoded by a sequence of interferometric directional couplers and delay lines. By encoding the photon evolution in time bins, we can perform postselection on the output time slots, allowing us to isolate and analyze specific quantum states that are robust.

We begin by describing the basic two-waveguide configuration and its role in implementing the quantum walk (Fig. 1), then extend the discussion to the complete four-waveguide scheme (Fig. 2), and finally explain the creation of a topological interface.

The encoding in our system relies on two complementary bases. The first is the computational basis: a single photon can occupy either of the two waveguides. The second is the superposition (Hadamard) basis, where the photon is in a superposition of occupying both waveguides. In this basis, the information is encoded in the relative phase between the two occupation probabilities. A relative phase of zero corresponds to the state: $|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$, while a relative phase of π corresponds to $|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$. These two complementary bases are illustrated in Fig. 1(a).

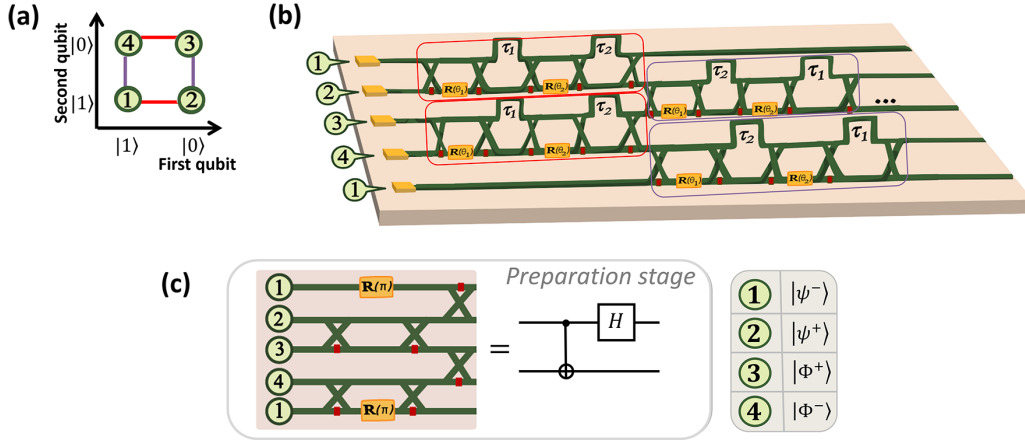


FIG. 2. Schematic illustration of the integrated platform for an entangled state. (a) Encoding two qubits in the four-waveguide configuration: waveguides 1–4 are arranged in a square, where the horizontal axis corresponds to the first qubit and the vertical axis to the second qubit. Purple nodes indicate the first-qubit states, while red nodes indicate the second-qubit states. (b) Evolution scheme of the four waveguides: in the first step, coupling occurs within the pairs (1, 2) and (3, 4); in the next step, coupling occurs within the pairs (2, 3) and (1, 4). This sequence then alternates repeatedly (red and purple boxes mark the two alternating coupling configurations). (c) Preparation stage, where each waveguide is associated with a distinct Bell state prior to the evolution in panel (b). After this stage, the state evolves according to the alternating coupling pattern.

The unitary evolution of each DTQW step is given by

$$U = T_y R(\theta_2) T_x R(\theta_1), \quad (4)$$

Where $R(\theta)$ denotes a spin rotation (in our case, a relative phase between the probabilities for the photon to occupy the two waveguides), and T_x , T_y describe conditional translations of the photon along the horizontal and vertical synthetic directions, determined by whether the photon acquires the corresponding delay. These operators are defined as $T_x = \sum_x |x+1\rangle\langle x| \otimes |\uparrow\rangle\langle\uparrow| + |x-1\rangle\langle x| \otimes |\downarrow\rangle\langle\downarrow|$, with an analogous expression for T_y . The corresponding Floquet Hamiltonian $\mathcal{H}(\theta_1, \theta_2)$ is extracted from the unitary operation via $U(\theta_1, \theta_2) = e^{-i\mathcal{H}(\theta_1, \theta_2)t/\hbar}$, where t is the time duration of one unitary operation (we set $t/\hbar \equiv 1$). The band structure derived from $\mathcal{H}(\theta_1, \theta_2)$ determines the topological properties of the system, characterized by a winding number [34,47].

Note that, to unambiguously resolve the path propagation of the photon through time-bin analysis, the time delays τ_1 and τ_2 must be incommensurate (i.e., their ratio must be irrational), preventing path ambiguity due to temporal overlap. This modular setup is fully compatible with integrated photonic platforms and can serve as a building block for topologically robust quantum circuits.

IV. EVOLUTION IN THE TIME-BIN SYSTEM

We initiate the unitary evolution presented in Eq. (4) by launching a photon into one of the waveguides [Fig. 1(b)]. We then introduce an interferometric coupler: an integrated interferometer that functions as a Hadamard gate, switching between the computational and superposition bases [Fig. 1(a)]. This coupler splits the light guided in each waveguide into two arms: one continues straight, while the other is coupled to the adjacent waveguide. The two arms are designed with a path length difference such that—when the photon is launched into the upper waveguide—the resulting superposi-

tion across the two waveguides is in phase (relative phase 0). Conversely, when the photon is launched into the lower waveguide, the resulting superposition across the waveguides acquires a relative phase of π (See the Appendix A for more details).

Next, we apply a phase rotation $R(\theta_1)$ using a controlled phase delay. This can be implemented, for instance, by a heater (or by using applied field acting through the electro-optic effect) on the upper waveguide. This operation mixes the $|+\rangle$ and $|-\rangle$ states, since the relative phase between the photon's occupation of the two waveguides becomes neither 0 nor π . The interferometric coupler is then applied again to return the system to the computational basis. That is, the photon, initially in a superposition state across the two waveguides, now emerges in one of them. The interferometric coupler acts as follows: for the $|+\rangle$ state (equal-phase superposition), constructive interference occurs in the upper waveguide and destructive interference in the lower, whereas for the $|-\rangle$ state (superposition with a relative phase of π), the interference pattern is reversed, yielding destructive interference in the upper waveguide and constructive interference in the lower.

At this stage, a temporal delay τ_1 is introduced, by extending one of the waveguides, producing horizontal translation along the synthetic lattice [corresponding to the operator T_x , see the mapping in Fig. 1(c)]. Applying the temporal delay τ_1 is equivalent to a translation operation in time domain (T_x).

This sequence is then repeated: we apply another interferometric coupler to switch to the superposition basis (states $|+\rangle$ and $|-\rangle$) and perform a second phase rotation $R(\theta_2)$ in the lower waveguide (which again mixes the $|+\rangle$ and $|-\rangle$ states). Then, we apply the interferometric coupler once more to return to the computational basis and introduce a second temporal delay τ_2 by again using a path length difference between the waveguides (which is mapped to a vertical translation in the synthetic lattice— T_y). This sequence completes

a full DTQW step [Fig. 1(b)], with Fig. 1(c) illustrating the corresponding mapping onto the synthetic time-bin lattice. We note that the rotations $R(\theta_1)$ and $R(\theta_2)$ are implemented in the Hadamard basis, while the translations T_x and T_y are implemented in the computational basis.

Next, we describe the four-waveguide configuration used to protect the entangled states, as shown in Fig. 2. Each is a single-mode waveguide. To establish the two-qubit entangled state, we launch a single photon in a superposition state into the system of four waveguides. The two qubits are encoded in the spatial position of the photon in the waveguides, labeled from 1 to 4 [Fig. 2(a)], which are arranged in a cyclic (ringlike) geometry. The mapping is defined as follows: the horizontal axis corresponds to the logical state of the first qubit, while the vertical axis corresponds to the second qubit. Specifically, a photon in waveguides 1 or 4 encodes the logical state $|1\rangle$ of the first qubit, while a photon in waveguides 2 or 3 encodes $|0\rangle$. For the second qubit, a photon in waveguides 1 or 2 encodes $|1\rangle$, and a photon in waveguides 3 or 4 encodes $|0\rangle$, as shown in Fig. 2(a).

With this mapping, each physical waveguide corresponds to a unique logical two-qubit basis state:

$$\begin{aligned} \text{waveguide 1} &\rightarrow |1, 1\rangle, \\ \text{waveguide 2} &\rightarrow |0, 1\rangle, \\ \text{waveguide 3} &\rightarrow |0, 0\rangle, \\ \text{waveguide 4} &\rightarrow |1, 0\rangle. \end{aligned} \quad (5)$$

Thus, for example, injecting a photon in a superposition between waveguides 2 and 4 realizes the Bell state $\frac{1}{\sqrt{2}}(|1, 0\rangle + |0, 1\rangle)$.

To proceed, we change the encoding such that each individual waveguide is associated with a different Bell state. This requires a preparation stage, illustrated in Fig. 2(c), together with the corresponding quantum circuit describing the preparation steps. After this stage, the four waveguides encode the Bell states according to the following mapping:

$$\begin{aligned} \text{waveguide 1} &\rightarrow |\psi^-\rangle = \frac{1}{\sqrt{2}}(|1, 0\rangle - |0, 1\rangle), \\ \text{waveguide 2} &\rightarrow |\psi^+\rangle = \frac{1}{\sqrt{2}}(|1, 0\rangle + |0, 1\rangle), \\ \text{waveguide 3} &\rightarrow |\phi^+\rangle = \frac{1}{\sqrt{2}}(|0, 0\rangle + |1, 1\rangle), \\ \text{waveguide 4} &\rightarrow |\phi^-\rangle = \frac{1}{\sqrt{2}}(|0, 0\rangle - |1, 1\rangle). \end{aligned} \quad (6)$$

Subsequently, the unitary evolution defined in Eq. (4) is implemented in a stepwise manner: in the first step, coupling occurs within the pairs of waveguides (1, 2) and (3, 4) [red frame in Fig. 2(b)], followed by coupling within the pairs (2, 3) and (1, 4) in the next step [purple frame in Fig. 2(b)], and so forth in an alternating sequence.

This quantum-walk evolution is repeated iteratively, resulting in multiple steps in the synthetic space. As we show below, using this DTQW scheme, it is possible to identify and utilize specific time instants at which the system is protected from noise. Physically, this means that the two-qubit entangled state appears at certain predesigned time, despite variations

in the rotation angles $R(\theta_1)$ and $R(\theta_2)$ and in the lengths of the waveguides in the system. This robustness to variations (whether random or systematic) allows single-photon detectors to sample only at time points that are topologically protected by the DTQW evolution, enhancing the robustness and fidelity of quantum information processing. This is in fact one of the advantages of our topological structure: the topological interface can be engineered such that the protected entangled state appears at predesigned (arbitrary) times. For example, it can be designed to reappear periodically in time, such that it is synchronized with a clock. Technically, the change in the rotation angle from θ_{2-} to θ_{2+} can be implemented with, for example, an electro-optic modulator, which can be triggered on demand to define the topological interface in the synthetic time-bin lattice.

V. RESULTS—TOPOLOGICAL EDGE STATES

The evolution of the photon in our system is mapped onto a 2D synthetic lattice of time bins, defined by two different time delays: the horizontal axis corresponds to a delay line with time shift τ_1 , and the vertical axis corresponds to a delay line with time shift τ_2 . Each time bin thus represents a lattice site (x, y) [48,49]. The spacing between neighboring sites in the x (y) direction is determined by the discrete time delays τ_1 (τ_2). As shown in Fig. 1(c), a single step of the DTQW, as defined in Eq. (4), is realized by the evolution of the photon through the four coupled waveguides.

The purpose of this article is to provide topological protection for the quantum information in this integrated photonic system. The topological protection is on the topological edge states (or interface); hence, we first need to explain how to introduce an interface within this synthetic lattice.

In the unitary transformation described in Eq. (4), each choice of the angles θ_1 and θ_2 determines a specific effective Hamiltonian and its associated winding number [34]. To create a topological interface, we introduce an effective transition time t_0 in the synthetic lattice, defined as

$$t_0 = \tau_1 n_1 + \tau_2 n_2, \quad (7)$$

Where τ_1 , τ_2 are delay times corresponding to the two delays in the waveguide, and n_1 , n_2 denote the number of steps the photon has accumulated along each delay line. The condition $t = t_0$ defines a diagonal boundary in the synthetic lattice, shown in Fig. 3. Across this boundary, the rotation angle used in the operation $R(\theta_2)$ is changed from θ_{2-} to θ_{2+} . Specifically, lattice sites satisfying $t < t_0$ (lower half plane in Fig. 3) evolve with θ_{2-} , whereas sites satisfying $t > t_0$ (upper half plane in Fig. 3) evolve with θ_{2+} . These yield two different effective Hamiltonians: $\mathcal{H}(\theta_1, \theta_{2-})$ for $t < t_0$ and $\mathcal{H}(\theta_1, \theta_{2+})$ for $t > t_0$. When the corresponding winding numbers differ, a topological interface emerges along the diagonal boundary $t = t_0$. In the topological interface case [Figs. 3(a)–3(c)], for $t < t_0$, the parameters are chosen to yield a winding number $W = +1$, with $\theta_1 = \pi/2$, $\theta_{2-} = 0$, while for $t > t_0$, the system evolves with a winding number $W = -1$, with the parameters $\theta_1 = \pi/2$, $\theta_{2+} = 0.55\pi$ (see the Appendix for more details) [50]. As a result, the probability distribution, initially localized at the center of the synthetic lattice, continues to

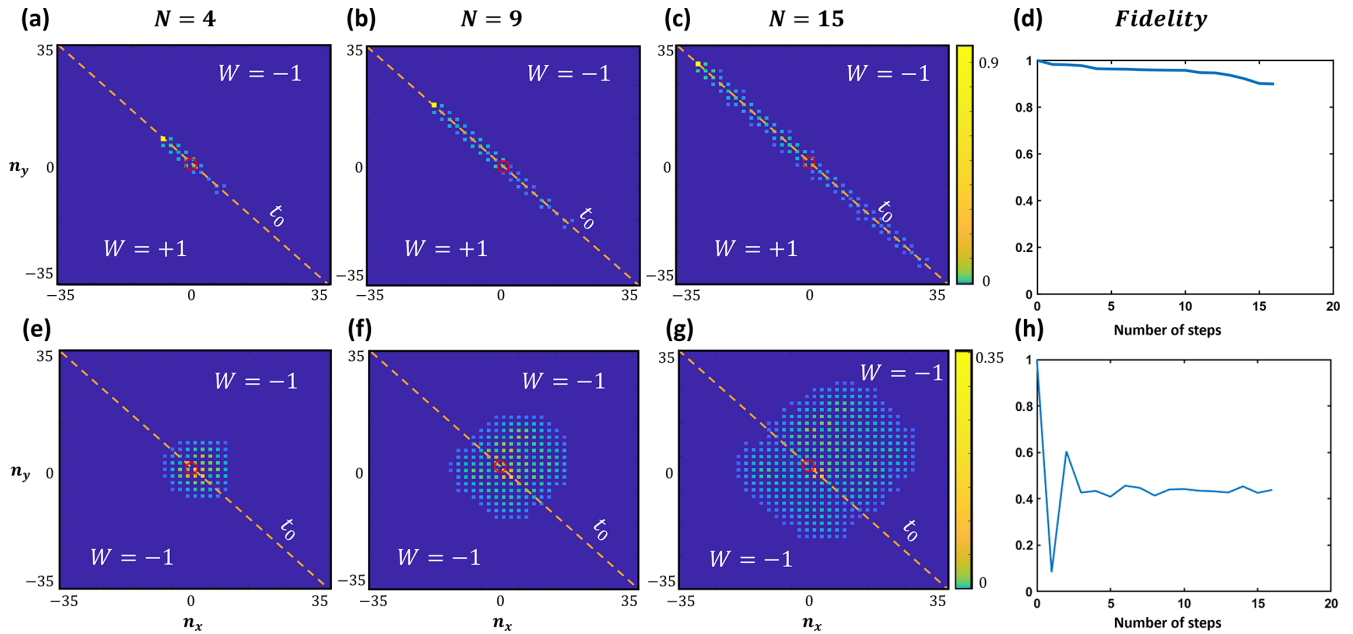


FIG. 3. Evolution of an entangled state in a synthetic time-bin lattice. (a)–(c) Evolution of the photon probability distribution at three propagation steps ($N = 4, 9$, and 15) while evolving on the topological interface case. (e)–(g) Corresponding evolution in the nontopological case, where the winding numbers are identical on both sides of the interface. In all panels, the initial photon position is marked by a red circle and the dashed line indicates the time interface t_0 . The color scale shown on the right-hand side of each row represents the occupation probability associated with each synthetic lattice site (time bin) and is identical for all panels within that row. (d), (h) Fidelity of detecting the initial state $\frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle)$ as a function of propagation step for the topological and nontopological cases, respectively.

propagate along the interface despite the presence of disorder in the rotation angles θ_{2-} and θ_{2+} . We also introduce disorder in the waveguide lengths, representing fabrication errors that are likely to occur in practice.

The dynamics of the 2D quantum walk are traced by the evolving probability distribution, shown in Fig. 3, which follows the diagonal interface defined by Eq. (7). After state preparation, a photon is injected into waveguide 2, corresponding to the Bell state $\frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle)$. Figures 3(a)–3(c) show the case of a nontrivial topological interface (different winding numbers across the boundary), while Figs. 3(e)–3(g) show a trivial interface (same winding number, $W = -1$, on both sides of the interface). The trivial case is implemented with parameters $\theta_1 = \pi/13$, $\theta_{2-} = 0.84\pi$, and $\theta_{2+} = 0.34\pi$. In both scenarios, we introduce random fluctuations of 5% in the rotation angles and 10% in the waveguide lengths.

As shown in Fig. 3, the topological case exhibits robust edge-state propagation confined to the interface, manifested as photon detections at specific arrival times forming a diagonal line in the synthetic lattice. The color scale represents the occupation probability associated with each time bin (synthetic lattice site), with brighter regions corresponding to a higher probability of detecting the photon. In contrast, in the trivial configuration, the probability distribution spreads into the bulk and loses confinement to the interface. Figures 3(d) and 3(h) quantify this behavior by showing the fidelity $\text{Tr}(\rho_f |\psi_i\rangle\langle\psi_i|) = \text{Tr}(\rho_f \rho_i)$, measuring the overlap of the evolved state with the initial Bell state in the boundary region as a function of the number of steps N . The decay of fidelity in the trivial case reflects spreading into other

waveguides, whereas in the topological case the state remains localized at the interface. Thus, this setting, for suitable choices of θ_1 , θ_{2-} , and θ_{2+} , provides topological protection of entangled states, by using only linear optical elements. The underlying topology is two dimensional, effectively forming a 2D topological insulator in synthetic space, where the protected (entangled) state is transported in time. This formulation ensures that the initial state is reproduced at specific time steps, even in the presence of considerable disorder.

Last, we compare our four-waveguide configuration, where the Bell state is encoded across a single photon occupying one of the four waveguides, with a scheme based on two separate pairs of coupled waveguides [Fig. 4(a)]. This comparison is designed to directly contrast global topological protection of an entangled state with local topological protection applied to each qubit. In the two-pair configuration, each pair encodes a single qubit: launching a photon to each of the pairs into the upper (lower) waveguide represents logical state “|0⟩” (“|1⟩”), and an entangled state is created by distributing two photons across the two pairs, for example, using spontaneous parametric down conversion followed by a polarizing beam splitter. In this setting, each qubit is individually protected by its own topological structure, but the relative phase between the qubits is not globally protected.

As shown in Fig. 4, we apply the same parameters and disorder strength to both configurations. We find that the two-photon state in the two-pair configuration exhibits lower fidelity compared to the single-photon Bell state encoded in the four-waveguide configuration [Fig. 4(b)]. Furthermore, we compare the fidelity of the initial Bell state under three scenarios: (1) a topological interface with different winding

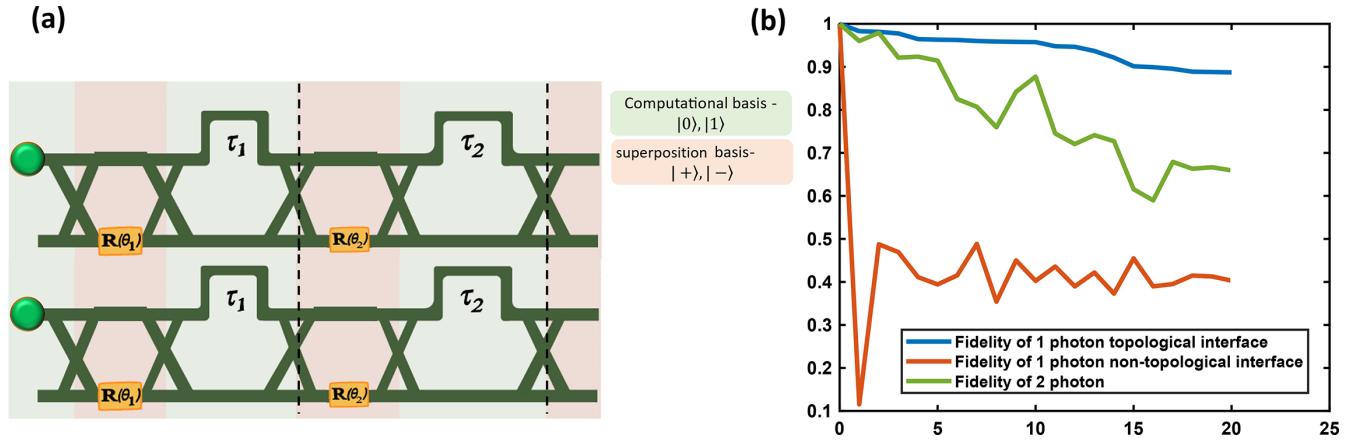


FIG. 4. Comparing the topological protection of a Bell state to the topological protection of a pair of single qubits. (a) Topological protection of two separate qubits, implemented by two separate pairs of coupled waveguides. A single photon is injected into each pair and allowed to evolve. (b) Fidelity in three scenarios: blue curve—fidelity of the initial Bell state in the four-waveguide topological interface; red curve—same scheme with a nontopological interface; and green curve—topological interface protecting two separate qubits, corresponding to the configuration in panel (a).

numbers [blue curve in Fig. 4(b)], (2) a trivial interface with identical winding numbers [red curve in Fig. 4(b)], and (3) the two-pair configuration, where each qubit is locally protected by its own topological structure [green curve in Fig. 4(b)]. The results show that global topological protection of the entangled state in the coupled four-waveguide configuration is considerably superior to protecting the two qubits separately. This highlights the importance of preserving the relative phase structure of entanglement, rather than only stabilizing individual qubits.

VI. CONCLUSIONS AND DISCUSSION

To conclude, we presented a scheme for topologically protected transport of entangled photonic states in a synthetic time-bin lattice. Unlike previous approaches that rely on protecting each photon mode individually, our method directly encodes the entire entangled state into a globally protected subspace. This ensures that the quantum correlations themselves remain robust during evolution despite significant disorder and fabrication imperfections, even when individual waveguides are affected considerably. Using a discrete-time quantum walk framework, we showed that entangled states propagate along a topological interface and reemerge faithfully at specific time steps, despite the presence of disorder in the rotation angles and the lengths of the waveguides. Such a scheme is compatible with contemporary integrated photonic implementations based on waveguide delay lines, directional couplers, and tunable phase shifters, which are already being used for quantum information processing; a discussion of possible experimental realizations and practical considerations is provided in Appendix E. Our comparison between the four-waveguide single-photon encoding and the two-pair encoding highlights the importance of global protection over protecting each qubit individually: while local protection of individual qubits can sometimes preserve entanglement, embedding the full entangled state in a topologically nontrivial subspace ensures robust fidelity across a broader range of conditions.

Importantly, this concept of topological protection of entanglement is general and not restricted to photonic systems. For instance, in a cold atomic ensemble, a photonic time-bin lattice could be realized by mapping each photon time bin onto a separate collective atomic spin excitation (spin wave), with read-out pulses reproducing the same lattice structure [51]. As such, the topological protection of entanglement studied here can be implemented in other physical systems that are relevant for quantum information.

Scalability. Beyond generality across physical platforms, another important consideration for practical quantum information processing is how the protected subspace scales when encoding for a large number of qubits. Representing an N -qubit state in our scheme corresponds to 2^N waveguides, which naturally raises the question of scalability. A key consideration when extending our approach to multiqubit encodings is how such scaling can be made practical. We address this by analyzing two standard error-correction frameworks—the surface code and concatenated codes—and show how the number of required physical qubits can be reduced with our approach.

In fault-tolerant quantum information processing, the central figure of merit is the physical error rate. Our approach suppresses the dominant phase-sensitive errors in photonic circuits, leading to two complementary advantages. First, quantum computing fundamentally requires quantum error correction, which has a sharp threshold: above it, increasing the number of physical qubits cannot eliminate accumulated errors. Thus, from the outset, our approach can reduce the physical error rate to below the threshold. By lowering the physical error rate below this threshold, the additional waveguides become an enabling resource rather than an overhead. Second, even below threshold, fault-tolerant schemes often require a large number of physical qubits. Consequently, even below the error threshold, further suppressing the error rate directly reduces this resource overhead. Thus, although enlarging our scheme with N qubits scales with 2^N waveguides, the resulting error suppression can lead to a significant net reduction in the number of physical qubits required for

fault-tolerant operation. A detailed discussion of these error-correction frameworks is presented in Appendix D.

While the discussion above focuses on reducing resource overhead through improved fault tolerance, an alternative route toward scalability is to reduce the number of physical modes required for encoding. In this context, the proposed topological encoding could be combined with schemes that encode multiple qubits within a single photon using several degrees of freedom. Such approaches effectively increase the accessible Hilbert space without requiring a proportional increase in the number of physical photons. Related multiplexing and multi-degree-of-freedom encoding strategies have recently been proposed in Ref. [52]. Combining these approaches with the present architecture may provide a pathway toward protecting more complex multipartite entangled states while mitigating some of the exponential resource requirements associated with direct waveguide-based encodings.

Looking ahead, this work highlights the importance of protecting the relative phases between the components of entangled states, rather than only preserving individual photonic modes. By embedding quantum information in synthetic dimensions, our approach provides a route to such global phase protection within an encoded Hilbert space. While we present a scheme focusing on a single-photon encoding, the underlying principle is not restricted to this setting and can, in principle, be extended to qubits encoded across multiple photons. This perspective opens the door to more complex architectures, including multiphoton and high-dimensional entangled states, and suggests avenues for robust quantum information processing on integrated photonic platforms.

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The authors declare no conflicts of interest.

DATA AVAILABILITY

The data that support the findings of this article will be available upon request.

APPENDIX A: IMPLEMENTATION OF THE INTERFEROMETRIC COUPLER

In this Appendix, we explain how to implement the interferometric coupler introduced in Fig. 1(a), which in our scheme serves as a Hadamard gate. The coupler is realized by splitting the incoming photon into two arms of nominal

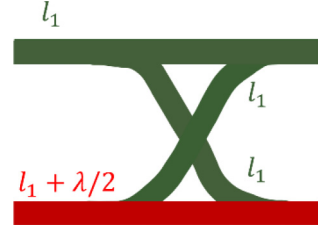


FIG. 5. Schematic of the interferometric coupler, illustrating the path-length difference that produce the required relative phase between the two waveguides.

length l_1 . To generate the required relative phase between the two paths, one of the arms is extended by an additional optical length of $\lambda/2$. This path length difference introduces a relative phase shift of π , which ensures that the photon launched into the upper waveguide results in a symmetric superposition with relative phase 0, while photon launched into the lower waveguide results in a superposition with relative phase π .

Figure 5 shows the path length difference required for the implementation of the Hadamard gate.

In practice, this phase shift can be implemented not only by static design but also dynamically, using a thermo-optic (or electro-optic) phase shifter. A heater (or an electrode) deposited on one arm of the interferometer allows precise control over the relative phase: applying the correct voltage changes the local refractive index, effectively tuning the optical path length. This makes the interferometric coupler reconfigurable and tolerant to fabrication variations, while still realizing the desired Hadamard operation.

APPENDIX B: PREPARATION STAGE: ENCODING A BELL STATE ON EACH OF THE WAVEGUIDES

In this Appendix, we show explicitly how the four Bell states are encoded into the spatial modes of the four-waveguide system, following the encoding scheme introduced in Fig. 2(a). Using the preparation circuit shown in Fig. 2(c), each Bell state is mapped to a specific waveguide. For example, launching a photon in a superposition to occupy waveguides 2 and 4 is equivalent, according to our encoding, to starting from the Bell state $\frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B)$. After applying a cnot gate, we obtain $\frac{1}{\sqrt{2}}(|1\rangle_A|1\rangle_B + |0\rangle_A|1\rangle_B) = |+\rangle_A|1\rangle_B$. Applying a Hadamard on qubit A then gives $|0\rangle_A|1\rangle_B$, which corresponds to waveguide 2.

Below, we describe how each Bell state is encoded into a different waveguide:

$$\frac{1}{\sqrt{2}}(|1\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B) \xrightarrow{\text{CNOT}} \frac{1}{\sqrt{2}}(|1\rangle_A|1\rangle_B + |0\rangle_A|1\rangle_B) = \frac{1}{\sqrt{2}}(|1\rangle_A + |0\rangle_A)|1\rangle_B = |+\rangle_A|1\rangle_B \xrightarrow{H_1} |0\rangle_A|1\rangle_B \rightarrow \text{waveguide 2}, \quad (\text{B1})$$

$$\frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B) \xrightarrow{\text{CNOT}} \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|0\rangle_B) = \frac{1}{\sqrt{2}}(|0\rangle_A + |1\rangle_A)|0\rangle_B = |+\rangle_A|0\rangle_B \xrightarrow{H_1} |0\rangle_A|0\rangle_B \rightarrow \text{waveguide 3}, \quad (\text{B2})$$

$$\frac{1}{\sqrt{2}}(|1\rangle_A |0\rangle_B - |0\rangle_A |1\rangle_B) \xrightarrow{\text{CNOT}} \frac{1}{\sqrt{2}}(|1\rangle_A |1\rangle_B - |0\rangle_A |1\rangle_B) = \frac{1}{\sqrt{2}}(|1\rangle_A - |0\rangle_A)|1\rangle_B = |-\rangle_A |1\rangle_B \xrightarrow{H_1} |1\rangle_A |1\rangle_B \rightarrow \text{waveguide 1}, \quad (\text{B3})$$

$$\frac{1}{\sqrt{2}}(|0\rangle_A |0\rangle_B - |1\rangle_A |1\rangle_B) \xrightarrow{\text{CNOT}} \frac{1}{\sqrt{2}}(|0\rangle_A |0\rangle_B - |1\rangle_A |0\rangle_B) = \frac{1}{\sqrt{2}}(|0\rangle_A - |1\rangle_A)|0\rangle_B = |-\rangle_A |0\rangle_B \xrightarrow{H_1} |1\rangle_A |0\rangle_B \rightarrow \text{waveguide 4}. \quad (\text{B4})$$

Thus, each of the four Bell states is mapped to a distinct mode in the four-waveguide system.

Next, we recall that the CNOT gate can be decomposed as

$$\text{CNOT} = (\mathbf{I} \otimes H) \text{CZ} (\mathbf{I} \otimes H), \quad (\text{B5})$$

where the controlled-Z applies a Z operation on the target when the control is $|1\rangle$. Since $HZH = X$, conjugating cz with Hadamards on the target transforms it into CNOT.

In our implementation, this $H\text{-CZ-}H$ sequence is realized in the four-waveguide platform. The interferometric coupler [Fig. 1(a)] performs a Hadamard operation. We first apply a Hadamard on the target qubit, encoded in waveguides (2, 3) and (1, 4). Then, we apply a phase rotation $R(\pi)$ on waveguide 1, which introduces a phase of π whenever the control qubit is in state $|1\rangle$. This is followed by another Hadamard on the target qubit, and finally a Hadamard on the control qubit, implemented by couplers between waveguides (1, 2) and (3, 4).

APPENDIX C: SIMULATION RESULTS

To complement the results presented in the main text, we also simulated the propagation in the absence of an interface, i.e., without changing the rotation angle from θ_{2-} to θ_{2+} . In this case, the lattice is homogeneous throughout the evolution, and no edge-localized transport is observed. The corresponding probability distribution is shown in Fig. 6.

For this simulation, we chose the rotation angles $\theta_1 = 0.3\pi$, $\theta_2 = 1.8\pi$. The absence of an interface leads to spreading characteristic of a bulk quantum walk, in contrast to the dynamics confined along the edge in the topological case.

It is important to note that in the simulations we presented here and in the main text, the colormap was stretched to

emphasize the evolution, as the overall probability of photon occupation decreases with time.

In addition, we provide three videos in the Supplemental Material [50] that visualize the full evolution of the quantum walk under the different conditions:

(1) Supplemental Video A: Evolution in the topological interface configuration, corresponding to the setup in Figs. 3(a)–3(c), for $N = 17$ steps.

(2) Supplemental Video B: Evolution in the nontopological interface, as introduced in Figs. 3(e)–3(g), for $N = 17$ steps.

(3) Supplemental Video C: Evolution without any interface (no change from θ_{2-} to θ_{2+}), for $N = 17$ steps.

APPENDIX D: SCALABILITY THROUGH ERROR-CORRECTION CODES

As discussed in the main text, we note that extending our approach to an N -qubit state requires 2^N waveguides, raising the question of scalability. To examine this question quantitatively, we analyze two widely used error-correction frameworks—the surface code and concatenated codes—and show how the reduction in physical error rate provided by our scheme can directly decrease the number of physical qubits required for fault-tolerant operation.

In the surface code, a widely used empirical relation for the logical error per cycle is

$$p_L \approx A \left(\frac{p}{p_{\text{th}}} \right)^{(d+1)/2},$$

where p denotes the physical error rate, p_{th} the threshold, A is a small implementation-dependent constant, and d is the code distance—a knob that sets how robust the code is. Since the total number of physical qubits scales approximately as

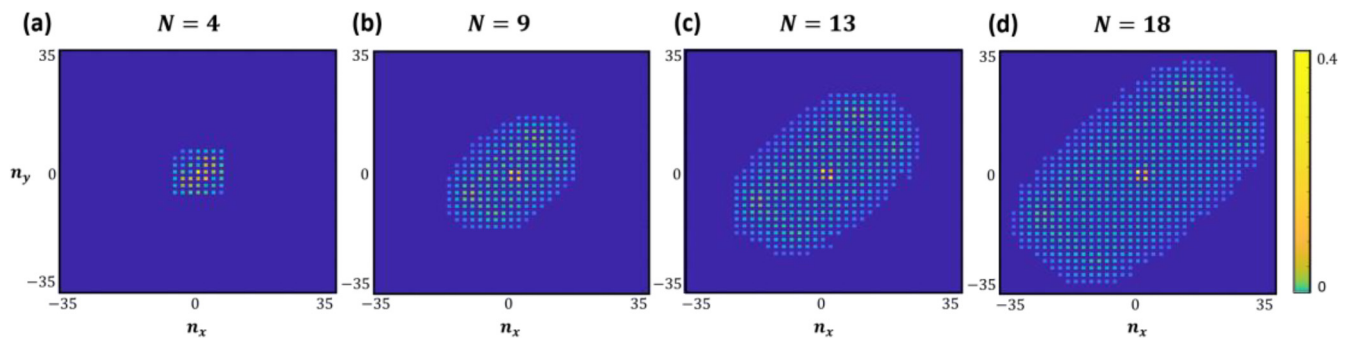


FIG. 6. Evolution of the photon probability distribution at four propagation steps ($N = 4, 9, 13$, and 18), without changing the rotation angle θ_2 (i.e., no transition from θ_{2-} to θ_{2+}). The photon, initially located at $(0, 0)$, exhibits spreading over time, characteristic of bulk evolution.

$Q \propto d^2$, reducing the required distance leads to a quadratic reduction in qubit overhead. Defining $r = \frac{p_{\text{th}}}{p} > 1$ as a measure of how far below threshold the system operates, one finds that improving the physical error rate p allows a smaller d for the same target logical error p_L . As a rule of thumb, when operating just below threshold ($r \approx 2$), improving p by an order of magnitude ($p \rightarrow \frac{p}{10}$) can reduce the required distance by about a factor of 4. This, in turn, corresponds to a decrease in the physical-qubit count by roughly a factor of 16–20, depending on the precise fit and the rounding of d to an odd integer.

Concatenated codes, in contrast, achieve error protection through hierarchical encoding rather than spatial scaling. In this approach, each logical qubit is first encoded into several physical qubits using a small “base code,” and then each of those physical qubits is reencoded recursively in additional layers. The notation $[[n, k, d]]$ describes a code that encodes k logical qubits into n physical qubits with distance d , meaning that it can correct up to $(d-1)/2$ errors. For example, the $[[5, 1, 3]]$ five-qubit code, the $[[7, 1, 3]]$ Steane code, and the $[[15, 1, 3]]$ Reed-Muller code, each encodes one logical qubit ($k = 1$) into $n = 5, 7$, or 15 physical qubits and can correct a single error ($d = 3$).

A similar argument applies to these concatenated codes: for distance-3, base codes such as those mentioned above, the logical error rate at successive concatenation levels can be approximated by

$$p_{l+1} \approx C p_l^2,$$

where C is a small constant capturing code-specific details. After L levels of concatenation, the logical error scales as $p_L \sim C^{2^L-1} p^{2^L}$, while the number of physical qubits grows as $Q \propto n^L$, where n is the block size (5, 7, or 15 for the respective codes). An improvement in the physical error rate that enables a reduction in the number of concatenation levels by ΔL results in a qubit-count reduction by a factor of $n^{\Delta L}$. Eliminating just one level of concatenation can therefore save between $5\times$ and $15\times$ in physical qubits, demonstrating that the saving is exponential in the number of levels one can remove.

In both frameworks, the parameters p and p_{th} represent the actual and threshold noise levels, while A encapsulates small implementation-specific constants. The quantities d and L serve as tunable parameters that trade hardware overhead for logical reliability. Thus, improving the physical error rate p , as we aim to do here, enables smaller d in surface codes, leading to large polynomial savings, or fewer concatenation levels L in concatenated codes, yielding exponential savings.

Overall, our proposed topological protection scheme can represent a hardware-level error-suppression mechanism that can effectively push the system below the fault-tolerance threshold, enabling scalable quantum information processing even at the cost of moderate increases in physical modes.

APPENDIX E: DESIGN CONSIDERATIONS FOR AN INTEGRATED PHOTONIC IMPLEMENTATION

Our proposed scheme can be readily implemented on the integrated photonic platform for quantum information processing [53], such as silicon photonics. In such an implementation, the delay lines associated with the synthetic lattice directions, τ_1 and τ_2 , can be realized using waveguides of different lengths, while the phase rotations can be implemented using thermo-optic phase shifters. By locally modifying the refractive index, these phase shifters enable the realization of the required coin operations $[R(\theta_1)]$ and $[R(\theta_2)]$. Moreover, actively driven heaters or electro-optic phase modulators could be used to dynamically switch the value of θ_2 , thereby creating different topological interfaces between distinct phases of the walk. Recent studies have also explored the implementation of coined DTQW as resource-efficient photonic quantum processors on integrated photonic platforms [54], further supporting the feasibility of realizing DTQW-based architectures using existing integrated photonic technologies.

The directional couplers required for the Hadamard operations can be implemented using low-loss integrated couplers, which have been extensively demonstrated in silicon and other integrated photonic platforms. The cyclic connectivity between waveguides $1 \leftrightarrow 2 \leftrightarrow 3 \leftrightarrow 4 \leftrightarrow 1$ can be realized through a sequence of coupling sections, where the physical ordering of the modes is rearranged along the chip. In particular, the coupling between waveguides 1 and 4 can be achieved by routing the corresponding waveguides into proximity using low-loss bends or waveguide crossings, allowing the effective cyclic graph to be implemented without requiring simultaneous coupling between all four modes. In addition, test ports distributed throughout the circuit may be incorporated to enable characterization and calibration of the implemented operations using weak classical light prior to operation in the quantum regime.

The dominant imperfections in such integrated photonic circuits typically arise from fabrication-induced disorder, including deviations in coupling strengths, phase shifts, and waveguide lengths. These imperfections lead to errors in the effective arrival times of the photon and in the implemented quantum walk dynamics. As discussed in the article, the proposed topological scheme in our work is specifically designed to provide robustness against moderate local perturbations of this type. In particular, small variations in the implemented phases, coupling coefficients, or delay lengths primarily perturb the evolution of the photon. Although a detailed analysis of fabrication tolerances is beyond the scope of the present work, these considerations suggest that the proposed architecture is compatible with realistic integrated photonic implementations, where fabrication-induced imperfections constitute one of the dominant sources of error.

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