

Characterization and comparison of energy relaxation in fluxonium qubits

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Fluxonium superconducting qubits have demonstrated long coherence times and high single- and two-qubit gate fidelities, making them a favorable building block for superconducting quantum processors. We investigate the dominant limitations to fluxonium qubit energy relaxation time T_1 using a set of eight planar, aluminum-on-silicon qubits. We find that a circuit-based model for capacitive dielectric loss best captures the frequency dependence of T_1 , which we analyze within both a two-level and a six-level energy relaxation model. We convert the measured T_1 into an effective capacitive quality factor Q_C^{eff} to compare qubits on equal footing, accounting for independently estimated contributions from $1/f$ flux noise and radiative loss to the control and readout circuitry. We apply this methodology to compare qubits from two fabrication processes: a baseline process and one that applies a fluorine-based wet treatment prior to Josephson junction deposition. We resolve a small improvement of $13.8\% \pm 8.4\%$ in the process mean Q_C^{eff} , indicating that the fluorine treatment may have reduced loss from the metal-substrate interface, but did not address the primary source of loss in these fluxonium qubits.

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I. INTRODUCTION

Superconducting qubits are a promising platform for future quantum computing applications [1,2]. Most recent advances toward utility-scale superconducting processors have been based on the transmon qubit [3–6], a weakly anharmonic LC oscillator realized by a Josephson junction (JJ) inductor. Though demonstrations with transmon qubits have been promising, alternative circuits with intrinsic resilience to noise and higher fidelity of operations are also being pursued. The fluxonium qubit [7] is one such alternative, a circuit comprising a capacitor, JJ, and linear inductor in parallel. Fluxonium-based architectures have been used to realize millisecond coherence times [8–10], fast single-qubit gates with fidelities exceeding 0.9999 [10–12], and two-qubit gates with fidelities above 0.999 [10,13,14].

Identifying dominant decoherence channels is key to improving superconducting qubit coherence and is a clear avenue toward higher gate fidelities. Over the past decade, studies have indicated that fluxonium coherence times are limited by a variety of mechanisms, including dielectric loss, likely from charge-coupled environmental two-level systems (TLSs) [15–20], $1/f$ flux noise [17,19], and quasiparticles [8], depending on device parameters and the materials used to construct the qubits. In particular, the materials and processing associated with JJ fabrication [21–26] are expected to play a strong role in the performance of fluxoniums with JJ-array-based superinductors [27,28]. Leveraging data from multiple qubits in conjunction with modifications to materials processing can validate process improvements and elucidate process reproducibility, for which consistency will be key for building utility-scale quantum processors [15,29]. Qubit-design and materials-process studies have primarily used transmon qubits [26,30–33], with fewer examples utilizing multiple fluxonium qubits [15,34,35].

In this article, we investigate the frequency dependence of the energy relaxation time T_1 of eight fluxonium qubits. To identify the dominant source of loss, we model the effects of various energy relaxation mechanisms, accommodating for population-transfer dynamics outside of the computational basis of the qubit [19]. Over a majority of the tunable frequency range, we find the trend in T_1 to be well described by a circuit model for capacitive dielectric loss. Accounting for the contributions from $1/f$ flux noise and radiative loss to the microwave environment, we transform T_1 into an

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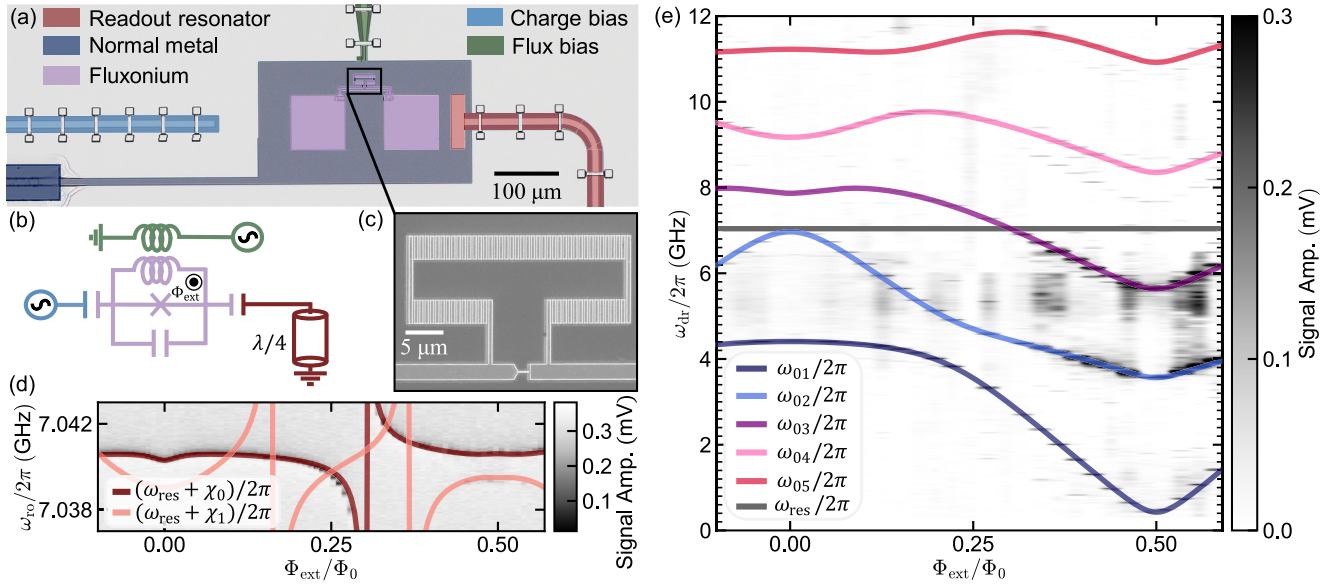


FIG. 1. Device design summary. (a) Optical micrograph of a representative fluxonium circuit used in this study, with key elements highlighted in color. (b) Circuit schematic with colors corresponding to those in panel (a). (c) Scanning electron micrograph of the qubit JJs from panel (b). (d) Dressed resonator frequency vs applied magnetic flux bias for qubit *B1*. The dressed readout resonator frequency for the qubit in ground $\omega_{\text{res}} + \chi_0$ and first excited state $\omega_{\text{res}} + \chi_1$ are overlaid. (e) Qubit drive frequency $\omega_{\text{dr}}/2\pi$ vs applied magnetic flux bias for qubit *B1*. Overlaid transitions out of the qubit ground state are highlighted in color, with bare resonator frequency $\omega_{\text{res}}/2\pi$ in dark gray.

effective capacitive dielectric quality factor Q_C^{eff} . We observe frequency-dependent variation of Q_C^{eff} consistent with limitation from TLS defects, and with our experimental protocol determining the mean Q_C^{eff} to within approximately $\pm 20\%$. We compare the mean Q_C^{eff} of qubits fabricated with two distinct fabrication processes (here, *A* and *B*), used for transmon qubits in Ref. [26] [there, “POR” (process of record) and “E”]. Process *B* introduces a pad etch substrate cleaning step (described in detail in Sec. II), shown in Ref. [26] to improve the metal-substrate interface underneath the JJs of transmons resulting in a $2\times$ improvement in their quality factor. Here, we resolve a statistically significant difference in the Q_C^{eff} means of $13.8\% \pm 8.4\%$, indicating that the metal-substrate interface was improved, but was not the predominant interface contributing to the energy relaxation rate in these fluxoniums. The methodology and analysis used here are extensible to comparisons of any superconducting qubit designs or fabrication processes, the optimization of which is critical for the development of more performant superconducting quantum processors.

II. DEVICE DESIGN AND FABRICATION

The fluxonium circuit [7] is modeled by the Hamiltonian

$$\hat{H} = 4E_C \hat{n}^2 - E_J \cos \hat{\varphi} + \frac{1}{2} E_L \left(\hat{\varphi} - 2\pi \frac{\Phi_{\text{ext}}}{\Phi_0} \right)^2, \quad (1)$$

which contains an energy term for each of the elements in the circuit. Here, $E_J = I_C \Phi_0 / (2\pi)$ is the Josephson energy of the JJ, where $\Phi_0 = h/(2e)$ is the superconducting magnetic flux quantum and I_C is the JJ critical current, which depends on its size. $E_C = e^2/(2C_\Sigma)$ is the single-electron charging energy, where C_Σ is the total capacitance shunting the JJ. Finally, $E_L = \Phi_0^2/(4\pi^2 L_A)$ is the inductive energy, where in

our devices L_A is the inductance of the JJ array that we use to construct this element. The operator $\hat{\varphi}$ represents the phase drop across JJ and is canonically conjugate to the reduced charge $\hat{n}$. By threading an externally applied magnetic flux Φ_{ext} through the loop formed by the JJ and linear inductor, the eigenenergies, eigenstates, and transition matrix elements of the circuit are varied.

In order to differentiate qubit designs and the two fabrication processes, we label each qubit with a letter and a number (e.g., “A1”). The letter in the fluxonium labels denotes one of two different fabrication processes. All qubits in this work are fabricated using an aluminum-based process on 50 mm intrinsic silicon wafers. In both process versions, an Al film is deposited on a precleaned wafer and patterned to form all device components: microwave control lines, readout resonators, and qubit shunt capacitors. The qubit JJs are deposited via shadow evaporation, using a Ge-based mask patterned for Dolan-bridge style Al-AlOx-Al JJs [36]. We fabricate the single JJ branch and an array of 151 series JJs [Fig. 1(c)] this way, where the series JJs are used as the fluxonium inductor, which we approximate as a linear inductance.

Our fabrication processes were introduced in Ref. [26], which in part motivated this experiment. In process *A*, referred to as the POR in Ref. [26], the surface of the exposed Si and patterned Al base metallization is cleaned before JJ deposition with an Ar ion mill to remove surface oxides and organic contaminants. This recipe was shown to leave Ge-based dry-etch byproducts on the Si surface under the JJ bridge, which were interpreted to be limiting the performance of transmon qubits made using this process. In process *B* (labeled as “E” in Ref. [26]), an additional fluorine-based wet etch (commercially available Transene Silox Vapox III) was added before the milling step to clean the substrate without removing the Al base metallization. This process was shown to reduce the

TABLE I. Summary of measured device parameters. Hamiltonian energies, the bare resonator frequency $\omega_{\text{res}}/2\pi$, and qubit-resonator coupling $g/2\pi$ are extracted from measurements of single- and two-tone spectroscopy as a function of Φ_{ext} [Figs. 1(d) and 1(e)]. The linewidth of the readout resonator $\kappa/2\pi$, the relative dispersive shift $\chi_{01}/2\pi$ of the qubit on the resonator, qubit frequency $\omega_{01}/2\pi$, as well as the mean, maximum, and minimum of five repeated measurements of T_1 , T_2^R , and T_2^E are reported at $\Phi_{\text{ext}}/\Phi_0 = 0.5$. Values are reported as $\text{mean}_{\text{min}}^{\text{max}}$. $\sqrt{A_\Phi}$ is the extracted flux noise amplitude at 1 Hz.

Process	Design	E_J/h (GHz)	E_C/h (GHz)	E_L/h (GHz)	$\omega_{\text{res}}/2\pi$ (GHz)	$g/2\pi$ (MHz)	$\kappa/2\pi$ (MHz)	$\chi_{01}/2\pi$ (MHz)	$\omega_{01}/2\pi$ (GHz)	$\sqrt{A_\Phi}$ ($\mu\Phi_0/\sqrt{\text{Hz}}$)	T_1 (μs)	T_2^R (μs)	T_2^E (μs)
A	1	3.54	1.05	0.53	7.090	124	0.25	1.44	0.362	10.4	124 ¹⁵⁰ ₉₅	46 ⁵⁵ ₃₃	112 ¹³³ ₈₇
A	2	3.94	1.04	0.53	7.182	124	0.17	1.63	0.279	5.3	157 ²²⁶ ₁₁₇	77 ⁹¹ ₆₅	144 ¹⁴⁹ ₁₂₉
A	3	4.38	1.02	0.53	7.273	123	0.34	1.80	0.214	4.3	138 ¹⁸⁸ ₉₂	76 ⁸⁶ ₆₉	97 ¹¹⁷ ₈₇
A	4	5.41	1.00	0.53	7.297	124	0.27	2.29	0.115	6.4	404 ⁴⁴⁶ ₃₄₃	107 ¹⁵³ ₈₃	154 ¹⁹⁷ ₁₃₆
A	5	7.11	0.95	0.53	7.500	120	0.60	2.20	0.042	2.4	129 ¹⁵² ₁₁₃	43 ⁶² ₁₈	66 ⁸⁶ ₃₉
B	1	3.15	1.04	0.50	7.039	118	0.29	1.11	0.427	5.2	85 ³⁰⁹ ₂₄	54 ⁹² ₄₀	86 ¹⁵⁴ ₆₃
B	2	3.52	1.04	0.51	7.126	120	0.30	1.30	0.343	5.7	101 ¹¹⁸ ₈₇	98 ¹¹¹ ₇₅	103 ¹²⁴ ₈₉
B	3	3.81	1.03	0.50	7.229	120	0.35	1.39	0.284	4.3	325 ³⁷⁰ ₂₉₅	120 ¹³³ ₉₅	137 ¹⁴² ₁₂₈

presence of Ge byproducts and other surface residues. This process was found to provide a $2\times$ improvement in the transmon qubit quality factor compared to the process A.

The number in the qubit label denotes a distinct single JJ size used to set the qubit E_J . The size and number of the array JJs used for the linear inductance, the design of coplanar shunt capacitor pads, charge and flux control lines, and the geometry of the ground plane were identical for all qubits. As such, all qubits were designed to have nominally the same E_C and E_L (Table I), though E_C decreases slightly with increasing E_J (corresponding to an increased JJ size and therefore increased parallel-plate capacitance of the JJ). Each qubit is capacitively coupled to its own readout resonator [Figs. 1(a) and 1(b)], which in turn is inductively coupled to a shared microwave feedline.

The qubits characterized in this study are from three unique chips, two fabricated on the same wafer with process A (one chip with A1, A2, and A3, the other with A4 and A5) and one fabricated on a separate wafer with process B. The design of qubit A5 was identical to the device reported in Ref. [19], and all qubits share similarities with the qubit designs in Ref. [37]. Both references utilize devices fabricated using process A, albeit drawing from different wafers and fabrication runs.

III. QUBIT CHARACTERIZATIONS

We utilize standard microwave control and dispersive-readout techniques (see Appendix A for details). For all measurements, we apply flux bias using a global coil as opposed to the on-chip line, which we cap with a $50\ \Omega$ termination at the mixing chamber stage of the dilution refrigerator (Appendix A). All devices include a TiPt patch labeled as “Normal metal” in Fig. 1(a), which we use to prevent screening currents in the ground plane induced by the magnetic field bias.

We obtain the resonator and qubit parameters by fitting single- and two-tone spectroscopy [Figs. 1(d) and 1(e)] measurements versus applied magnetic flux bias to extract the bare resonator frequency $\omega_{\text{res}}/2\pi$, qubit-resonator coupling rate g , and qubit energy scales E_J , E_C , and E_L (Table I and

Appendix B). We also report the resonator linewidth $\kappa/2\pi$, relative dispersive shift of the qubit on the resonator $\chi_{01}/2\pi$, and qubit transition frequency $\omega_{01}/2\pi$ at $\Phi_{\text{ext}}/\Phi_0 = 0.5$ (Table I). We characterize T_1 as a function of Φ_{ext} for each qubit (Fig. 2 and Appendix B 2). Around $\Phi_{\text{ext}}/\Phi_0 = 0.5$, we performed interleaved measurements of T_1 , the Ramsey dephasing time T_2^R , and the echo dephasing time T_2^E (Appendices B 2 and B 3). The average and standard deviation

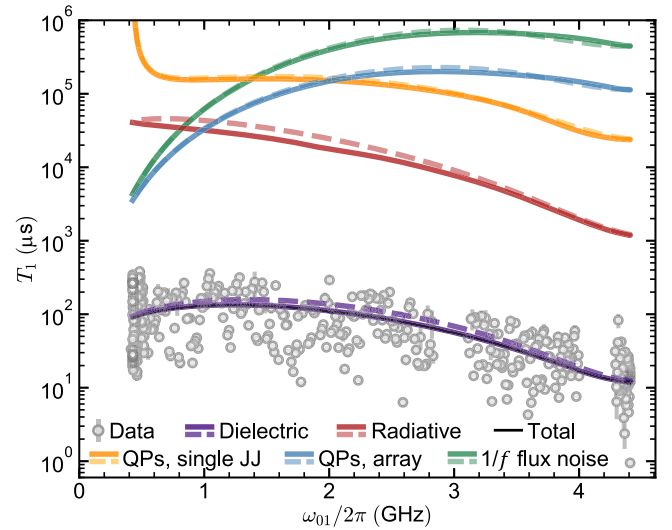


FIG. 2. Measured T_1 (gray points) vs $\omega_{01}/2\pi$ for qubit B1, with predicted T_1 due to individual noise models (see Fig. 12 for all qubits). Solid lines show predictions for a six-level description of the loss model. Dashed lines show the predictions for a two-level description of the loss model. $1/f$ flux noise and radiative losses are calculated using the measured qubit parameters (Table I). The dielectric loss model assumes Q_C^{eff} of 3.11×10^5 , with $\epsilon = 0.25$. The quasiparticle models assume x_{QP} of 1×10^{-9} . The total line represents the six-level description of T_1 due to all included noise sources. All models assume an effective qubit temperature of 40 mK and an effective resonator temperature of 65 mK. Gaps in the data near 3, 4.2 GHz are due to insufficient readout contrast (Appendix B).

of all coherence measurements taken at $\Phi_{\text{ext}}/\Phi_0 = 0.5$ are summarized in Table I. We use the measurements of T_1 and T_2^E taken away from $\Phi_{\text{ext}}/\Phi_0 = 0.5$, where we assume the qubit dephasing to be limited by $1/f$ flux noise, to extract the flux noise amplitude $\sqrt{A_\Phi}$ (Table I). We solve the echo pure dephasing rate $1/T_\phi^E = |\partial\omega_{01}/\partial\Phi_{\text{ext}}|\sqrt{A_\Phi\ln(2)}$ for $\sqrt{A_\Phi}$, where $|\partial\omega_{01}/\partial\Phi_{\text{ext}}|$ is the slope of the qubit transition frequency as a function of applied magnetic flux bias [38]. We observe a wide range of $\sqrt{A_\Phi}$ for these nominally identical designs, which we are able to partially attribute to noise from our instrumentation (Appendix B 4).

IV. MODELING ENERGY RELAXATION

To identify the loss limiting the measured T_1 , we consider various noise sources modeled perturbatively using Fermi's golden rule and compare to our data. The relaxation rate $1/T_1$ is $\sum_\lambda \Gamma_1^\lambda$, with λ indexing the noise sources (Appendix C). We consider an extension of usual models, allowing for population dynamics to exist beyond the first two-qubit energy levels, a consideration recently shown to be relevant for fluxonium [19] (Appendix D). We solve for the time-dependent qubit population vector $\vec{p}(\tau)$ under the differential equation $\frac{d}{d\tau}\vec{p}(\tau) = \mathbf{B}\vec{p}(\tau)$, where the transition matrix is

$$\mathbf{B} = \begin{pmatrix} -\sum_{i \neq 1}^N \Gamma_{0 \rightarrow i} & \Gamma_{1 \rightarrow 0} & \cdots & \Gamma_{N \rightarrow 0} \\ \Gamma_{0 \rightarrow 1} & -\sum_{i \neq 1}^N \Gamma_{1 \rightarrow i} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \Gamma_{N \rightarrow N-1} \\ \Gamma_{0 \rightarrow N} & \Gamma_{1 \rightarrow N} & \cdots & -\sum_{i \neq N}^N \Gamma_{N \rightarrow i} \end{pmatrix}, \quad (2)$$

and elements $\Gamma_{i \rightarrow j}$ are the transition rates from state $|i\rangle$ to state $|j\rangle$. The solution to the differential equation is $\vec{p}(\tau) = \exp(\mathbf{B}\tau)\vec{p}(0)$. We fit the decay of the population within the first six levels of the fluxonium back to thermal equilibrium to an exponential to obtain the relaxation time (Appendix D).

To model our data, we consider energy relaxation from capacitively coupled dielectric loss, $1/f$ flux noise, quasiparticles near the single JJ and within the JJ array, and radiative loss to the control and readout circuitry. Representative curves for both a traditional two-level and the six-level description of each considered loss model are shown in Fig. 2. We find that the model for dielectric loss best describes our data over a majority of the frequency range (Appendix D 2). For this mechanism, we utilize a phenomenological circuit model that can account for loss caused by imperfect materials, interface oxides, and the average behavior of environmental TLS. For this, we consider Johnson-Nyquist voltage noise across a resistance in parallel to the qubit capacitance, giving rise to the energy relaxation rate [39,40]

$$\Gamma_1^C = \frac{16E_C}{\hbar Q_C} |\langle 0|\hat{n}|1\rangle|^2 \coth\left(\frac{\hbar\omega_{01}}{2k_B T}\right), \quad (3)$$

where Q_C is $1/\tan(\delta)$ and $\tan(\delta)$ is the materials loss tangent (Appendix C 1 a). Here, we include a phenomenological

frequency dependence of the quality factor [41],

$$Q_C = Q'_C \left(\frac{6 \times 10^9}{\omega_{ij}/2\pi} \right)^\epsilon. \quad (4)$$

We find that this modification describes the data better than a frequency-independent model (Appendix E 3), similar to other literature [15,18,19]. As we will describe later but preview here for clarity, to compare devices we ultimately use a value Q_C^{eff} , which represents the quality factor for a frequency bin-averaged relaxation rate, accounting for contributions from radiative loss to the control circuitry and $1/f$ flux noise.

In general, the prediction of the six-level description of T_1 processes is similar to the two-level description, with the exception of Purcell loss. We favor the use of the better-motivated, multilevel model in all subsequent analysis. For all models, we assume an effective qubit temperature of 40 mK [42,43] and an effective resonator temperature of 65 mK [44,45].

V. DATA PROCESSING

We observe TLS signatures in the frequency dependence of T_1 and a general trend consistent with the capacitive dielectric loss model. This motivates us to primarily use this model to quantify qubit performance. In service of this, we first account for contributions to T_1 from our models for $1/f$ flux noise and radiative loss to the microwave environment (Appendix C 1 c), using our independent characterizations of each qubit described in Sec. III (Appendix D 2). We exclude data for which the combined prediction from $1/f$ flux noise and radiative losses is greater than 10% of the measured decay rate.

We sample T_1 more finely in frequency near the $\Phi_{\text{ext}}/\Phi_0 = 0$ and $\Phi_{\text{ext}}/\Phi_0 = 0.5$ biases. In these regions, we generally observe fine structure, which we attribute to TLSs [Fig. 3(b)]. To mitigate bias associated with potential oversampling of TLSs, we apply a binned average to the dataset of 8 MHz width corresponding to the approximate average full width at half maximum of the observed TLS features. The frequency spacing between T_1 samples is greater than the bin width for a majority of the frequency tuning range [Fig. 3(c) and Appendix E 1]. With this analysis procedure, we do not distinguish TLS from any background sources of capacitive dielectric loss.

To account for the T_1 dependence on Hamiltonian parameters, we choose to then recast our data into Q_C^{eff} , the quality factor corresponding to the bin-averaged T_1 (as opposed to Q'_C corresponding to an individual measurement of T_1) [Fig. 3(c) and Appendix E 2]. We additionally accommodate for the effect of the resonator Lamb shift due to qubit population in higher levels in our model (an effect not included on the T_1 curves in Figs. 2 or 3), which we detail further in Appendix D 1. Here, the mean, median, and distribution of measured Q_C^{eff} serve as metrics that can be used to compare qubits and fabrication processes inclusive of TLS effects. Though Q_C^{eff} could in principle have different frequency dependence across qubits and fabrication processes, we use the value of ϵ that best describes the combined dataset of eight qubits, $\epsilon = 0.25$, in all analyses for consistency (Appendix E 3).

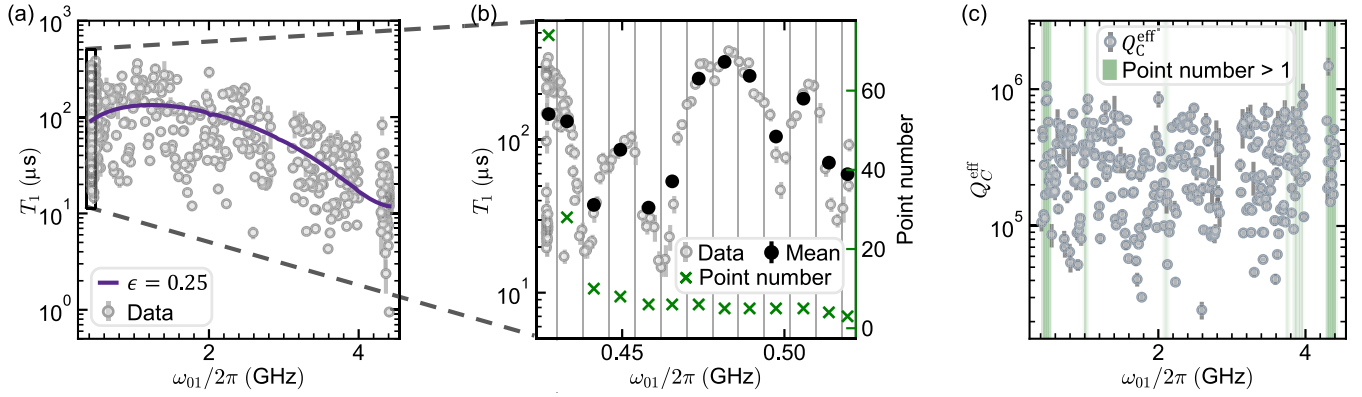


FIG. 3. Transformation of T_1 into Q_C^{eff} for qubit B1. (a) T_1 vs qubit transition frequency, with curve for our multilevel model for capacitive loss, using the quality factor as defined by Eq. (4), with $\epsilon = 0.25$ and $Q_C = 3.11 \times 10^5$, the qubit distribution mean discussed in Sec. VI. (b) Close-up of boxed data in panel (a), where T_1 was more finely sampled, additionally visualizing the binned averaging applied to the dataset. Vertical lines indicate the boundaries of adjacent bins, with the right-side y axis corresponding to the number of points within each bin. (c) Extracted Q_C^{eff} vs qubit transition frequency for the binned T_1 values. Green shading indicates frequencies where the number of points per bin was greater than one.

VI. RESULTS

We compare the distributions of Q_C^{eff} for each qubit, plotted in Fig. 4(a). The distributions contain largely overlapping Q_C^{eff} values and are non-Gaussian distributed. Though the binning protocol reduces point-to-point correlations caused by TLSs, we do still observe some structure in the binned T_1 [Fig. 3(b)]. We attribute the uniqueness of the distributions and approximately 2 orders of magnitude range in the Q_C^{eff} values to the unique spectra and finite number of TLSs that couple to each qubit, which we are not able to fully mitigate in this analysis. We therefore do not expect these distributions to be identical, which limits the applicability of many statistical tests of similarity. We report the mean, median, and standard deviation

of Q_C^{eff} for each qubit in Table II. While the median is practically useful for predicting characteristic qubit performance, we choose to apply an interpretable statistical test, Welch's t -test [46], to compare the means between qubits and fabrication processes. We justify use of this test based on applicability of the central limit theorem to bounded distributions and the assumption that our binning procedure sufficiently reduces point-to-point correlations in the data.

We use Welch's t -test to calculate the 95% self-confidence interval (Appendix G) for each distribution, which bounds the sensitivity of the test: We cannot on average resolve differences in the mean smaller than approximately $\pm 17\%$ of an individual qubit's mean (Table II). To compare qubits, we compute the pairwise confidence interval (see Appendix G and Table III). We find some distributions appear to be similar and some to be statistically different. Comparing designs within the same fabrication process (qubits A1, A2, A3, A4, and A5), a weak trend in the mean as a function of E_J

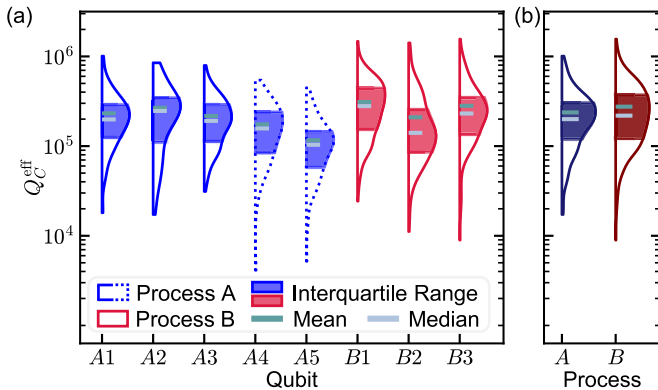


FIG. 4. Extracted Q_C^{eff} distributions. (a) Individual qubit Q_C^{eff} distributions, where the area of each distribution is normalized to show the percent of counts at each Q_C^{eff} . Blue color corresponds to qubits fabricated with process A; red corresponds to process B. Shaded regions within individual distributions represent the interquartile range of the dataset. Overlaid teal and gray lines mark the mean and median of each distribution, respectively. (b) Combined distributions of qubits A1, A2, and A3 labeled as process A (dark blue), and B1, B2, and B3 (process B, dark red) are used to compare like designs across fabrication processes. Dashed outlines in panel (a) indicate exclusion from the combined distributions shown in panel (b).

TABLE II. Summary for individual and combined process Q_C^{eff} distributions. Means, medians, and standard deviations σ of each distribution are reported. We also report the 95% confidence interval (CI) of the mean as returned by Welch's t -test for each distribution as compared to itself, expressed as a percent of the distribution mean.

Qubit	Mean Q_C^{eff}	Median Q_C^{eff}	Std. Q_C^{eff} , σ	95% CI
A1	2.32×10^5	1.98×10^5	1.59×10^5	$\pm 16\%$
A2	2.68×10^5	2.47×10^5	1.96×10^5	$\pm 20\%$
A3	2.17×10^5	1.91×10^5	1.34×10^5	$\pm 16\%$
A4	1.75×10^5	1.57×10^5	1.13×10^5	$\pm 19\%$
A5	1.16×10^5	1.03×10^5	0.81×10^5	$\pm 17\%$
B1	3.11×10^5	2.79×10^5	2.03×10^5	$\pm 10\%$
B2	2.10×10^5	1.41×10^5	2.06×10^5	$\pm 20\%$
B3	2.81×10^5	2.31×10^5	2.14×10^5	$\pm 15\%$
A	2.37×10^5	2.00×10^5	1.65×10^5	$\pm 10\%$
B	2.75×10^5	2.19×10^5	2.11×10^5	$\pm 8\%$

emerges across otherwise highly overlapped distributions. This could possibly be attributed to the increased capacitance from the small junction as its size was increased to vary E_J (Appendix H). Based on observed TLS densities and effective quality factors of typical shadow-evaporated Al-AlOx-Al JJs [21,24], this trend is expected; however, further experiments are needed to definitively quantify the small JJ contribution to dielectric loss in our fluxonium qubits.

To compare fabrication processes, we combine the Q_C^{eff} distributions of A1, A2, and A3 and separately those of the identically designed qubits B1, B2, and B3 (Fig. 4). We exclude A4 and A5 from this comparison, as we did not test qubits from process B targeting the same values of E_J . We find these two composite distributions to be statistically different, with a 95% confidence interval indicating that the mean of distribution of combined B qubits is higher by $13.8\% \pm 8.4\%$ of the mean of the corresponding combined A qubits (Appendix G and Table IV). This indicates that the pad etch treatment of process B did slightly improve the effective materials quality of our fluxoniums compared to process A (presumably from removing residues at the metal-substrate interface). However, the large overlap between the process Q_C^{eff} distributions and the small difference in the distribution means suggests the metal-substrate interface underneath the JJs is not the primary contributor to dissipation in these qubits. In Appendix I, we verify that consideration of a frequency-independent Q_C does not qualitatively change this result. In Appendix J, we show that this result is also preserved when considering the traditional two-level model of qubit energy relaxation.

VII. CONCLUSIONS AND OUTLOOK

We have examined the frequency dependence of T_1 for eight planar, aluminum-on-silicon, fluxonium qubits sampling from two distinct fabrication processes. We employ a multilevel model of the energy relaxation dynamics, and find that the frequency dependence of T_1 is well modeled by a combination of capacitive dielectric loss, radiative loss to the microwave environment, and $1/f$ flux noise. Using these three loss models and accounting for the frequency dependence of the qubit transition matrix elements, we convert T_1 into a capacitive quality factor Q_C^{eff} to compare qubits.

Our results indicate that microwave loss associated with the metal-substrate interface underneath the fluxonium JJs does not play as limiting a role in energy relaxation as it did for transmons using the same materials processing [26]. We hypothesize that the many JJs comprising the linear inductance of the fluxonium play a role in the reduction of relative Q_C^{eff} [approximately 3.48×10^5 for these fluxoniums as opposed to approximately 6.6×10^6 for the transmons in Ref. [26], both applying a frequency-independent capacitive loss model (see Appendix I). Taken together, these results point to the metal-air interface or the JJ barrier itself as the main host of fluxonium-limiting TLSs. Additional studies seeking to understand the microscopic nature of TLSs will be key to future mitigation strategies and improving superconducting qubit performance.

While our characterization technique does not probe the microscopic properties of the TLSs directly, it consolidates

their effects into simple metrics—the Q_C^{eff} distribution means and 95% confidence intervals—with which we compare qubits. Combining data from three qubits per process, we find the 95% confidence interval comparing the composite distributions to themselves is on the order of $\pm 10\%$ or less of their respective means. Measuring more qubits would further reduce our estimation error, at the expense of additional measurement time. This low estimation error indicates that our protocol is sensitive to relatively small changes in performance and can be used in future experiments as an effective probe of new materials, fabrication processes, and qubit designs.

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The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the U.S. Government.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: EXPERIMENTAL SETUP

This experiment was performed in a BlueFors XLD1000 dilution refrigerator used to cool the qubits to a temperature of approximately 12 mK. Devices were wirebonded into gold-plated copper packages and mounted to a cold finger connected to the mixing chamber stage of the cryostat. The samples are shielded with three layers of magnetic shielding at the mixing chamber stage. The innermost shield is made of aluminum, the middle shield of tin-plated copper, and the outer shield is made of Ammuneal A4K.

The dc flux bias is supplied by a coil mounted to the lid of the package, current biased by a Yokogawa GS200 operating in voltage mode, filtered at room temperature by a homemade 10 Hz low-pass RC filter with a 10 k Ω series resistance. Twisted pair lines connect the dc bias to the sample coil. On-chip flux bias lines were present but not used in this experiment. All unused bias lines (charge and flux) were

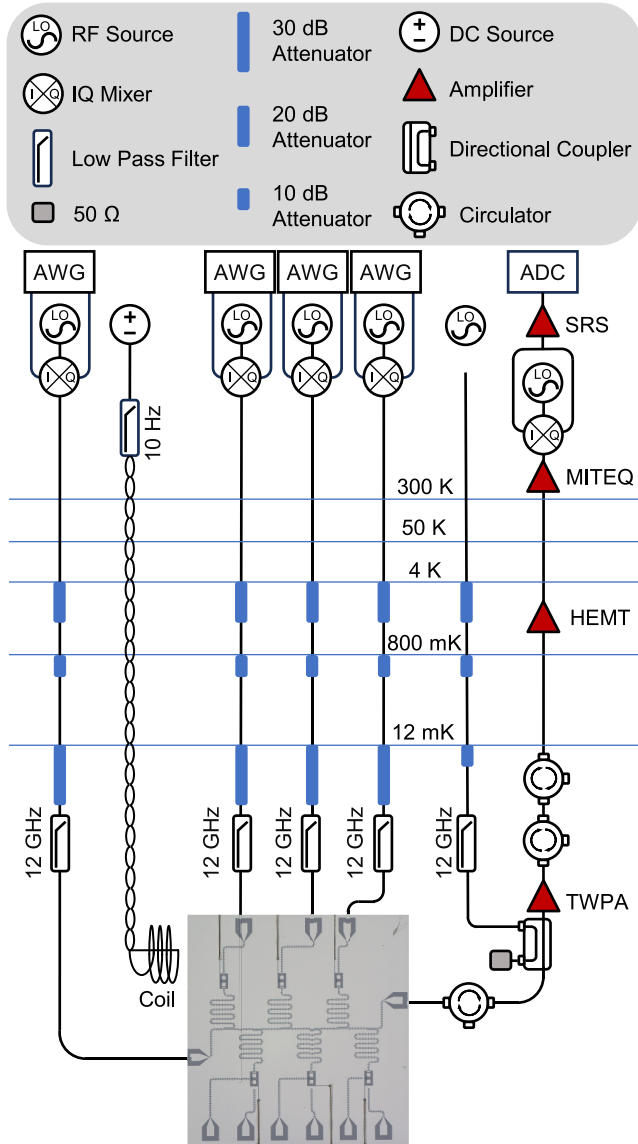


FIG. 5. Wiring diagram of experimental setup. Unused qubit control lines (here, on the lower side of the sample chip) are connected to the 12 mK stage and capped with 50 Ω terminations, not pictured.

capped with 50 Ω terminations away from the qubit package, at the mixing chamber stage.

For microwave qubit control, signals were generated by single-sideband mixing of Keysight PXIe M3202A 1 GSa/s arbitrary waveform generators with a Rohde and Schwarz SGS100A SGMA RF source. Readout signals were generated by single-sideband mixing of PXIe Keysight M3202A 1 GSa/s arbitrary waveform generators with a Keysight MXG microwave signal generator. All ac control and readout signals were attenuated with 60 dB of attenuation distributed throughout the fridge temperature stages, and filtered with a 12 GHz low-pass filter (Fig. 5). All control electronics are synchronized with a 10 MHz Stanford Research Systems (SRS) Rb clock.

Output signals were amplified first by a traveling wave parametric amplifier [47] made by MIT Lincoln Laboratory,

pumped by a Rohde and Schwarz SGS100A SGMA rf source. Signals were further amplified by an Low Noise Factory high electron mobility transistor amplifier at the 4 K stage, and again at room temperature with an MITEQ amplifier, and finally by SRS SR445A amplifier, before being detected by a Keysight M3102A digitizer.

APPENDIX B: DEVICE CHARACTERIZATION

We model the coupled qubit-resonator system with the Hamiltonian $\hat{H} = \hat{H}_{\text{qubit}} + \hat{H}_{\text{res}} + \hat{H}_{\text{int}}$. Here, $\hat{H}_{\text{qubit}}$ is given by Eq. (1), the bare resonator Hamiltonian is

$$\hat{H}_{\text{res}} = \hbar\omega_{\text{res}}(\hat{a}^\dagger\hat{a} + 1/2), \quad (\text{B1})$$

and the interaction Hamiltonian between the qubit and resonator is

$$\hat{H}_{\text{int}} = \hbar g \hat{n}(\hat{a} + \hat{a}^\dagger). \quad (\text{B2})$$

The coupling constant $g = \omega_{\text{res}} \frac{C_c}{C_\Sigma} \sqrt{\pi Z_0 / R_q}$, where the characteristic impedance of the readout resonator $Z_0 = 50 \Omega$, and $R_q = h/(2e)^2$ is the resistance quantum. The frequency shift of the resonator due to the qubit occupying state $|i\rangle$ (otherwise referred to as the dispersive shift or Lamb shift) is then given by [48]

$$\chi_i = \sum_{j \neq i} \frac{2g^2 |\langle i | \hat{n} | j \rangle|^2 \omega_{ij}}{\omega_{ij}^2 - \omega_{\text{res}}^2}, \quad (\text{B3})$$

which is the basis for the dispersive measurement scheme employed in this work.

We show the readout response around the resonator frequency versus applied flux bias for all qubits in Fig. 6, which we fit to Eq. (B3) to extract the bare resonator frequency ω_{res} and qubit-resonator coupling rate $g/2\pi$. We additionally show the measured qubit frequency $\omega_{01}/2\pi$, with the overlay as predicted by each qubit's Hamiltonian parameters in Fig. 7.

1. Heralded measurement

When the qubit transition energy $\hbar\omega_{01}$ becomes small relative to the thermal energy scale $k_B T$, the equilibrium population in $|1\rangle$ reduces readout contrast. In our experiment, at qubit transition frequencies of ≈ 1 GHz and below, a heralded measurement scheme is used for all qubit characterizations [49]. In this scheme, a qubit premeasurement m_1 is performed before the qubit control pulses and is then correlated with a postmeasurement m_2 to determine the change in qubit state as a result of the control pulses. A wait time of 5 μs ($> 5/\kappa$ for all readout resonators) is added between m_1 and any qubit control pulses to allow for measurement photons to dissipate. To determine the results in this heralded scheme, the averaged correlation function of the two measurements $\langle m_1 m_2 \rangle$ is fit to a function specific to each measurement.

2. T_1 measurements

For each T_1 measurement, the qubit and resonator system was characterized at the corresponding flux bias, according to the methods described in Appendix B. By applying a sideband modulated cosine pulse to drive the qubit frequency on the charge bias line, the population in the qubit $|0\rangle$ and $|1\rangle$ states

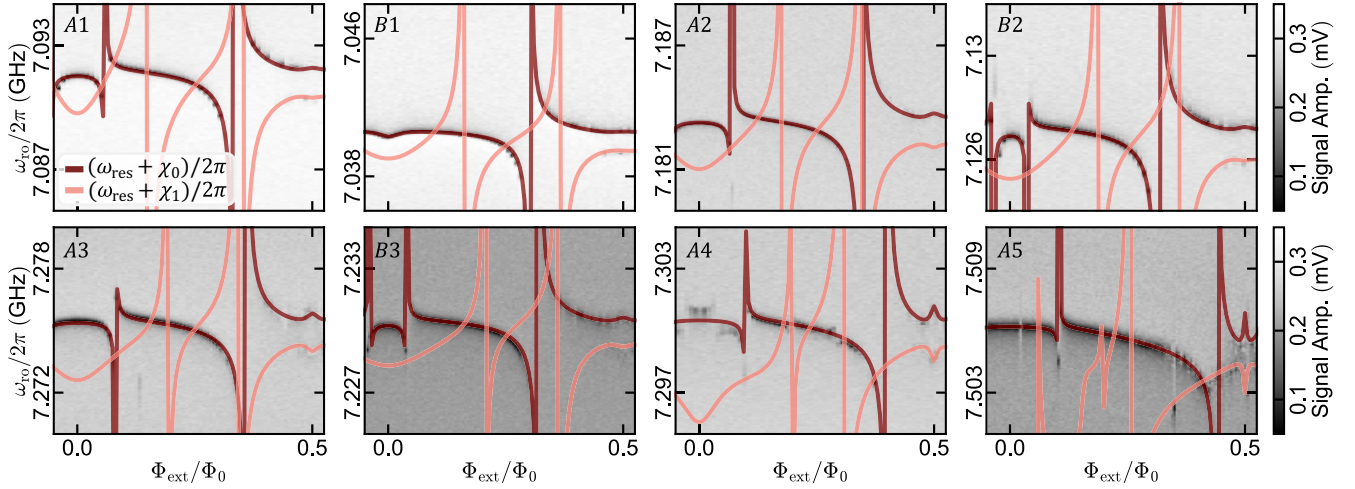


FIG. 6. Readout resonator spectroscopy as a function of applied magnetic flux bias, with theory lines for the sum of bare resonator frequency and dispersive shifts $(\omega_{\text{res}} + \chi_0)/2\pi$ and $(\omega_{\text{res}} + \chi_1)/2\pi$ overlaid. Here, we vary the y axis across plots to accommodate the unique resonator frequencies, while the x axis is shared. The color bar is also shared across all plots.

is inverted, after which the population is allowed to freely decay for a variable amount of time prior to measurement. The delay times were spaced logarithmically for each T_1 trace to capture the wide distribution of decay times. The resultant signal decay curve was fit to an exponential function to extract T_1 , with error bars corresponding to one standard deviation of error as output from our fitting protocol. We exclude from our analysis any T_1 measurements for which this error bar was greater than twice the value of the T_1 time, which indicates a failure of our automated calibration routine. The measured T_1 versus $\omega_{01}/2\pi$ are shown in Fig. 8 for all fluxoniums measured.

3. Dephasing measurements

At $\Phi_{\text{ext}}/\Phi_0 = 0.5$, we characterize the Ramsey and echo dephasing times of each qubit, where each measurement utilized a heralding premeasurement. To characterize the Ramsey dephasing, we apply a resonant $\pi/2$ pulse to create an equal superposition state, wait for a variable delay time, and then apply a second resonant $\pi/2$ pulse (with variable phase to induce oscillations, as opposed to a real frequency

detuning) before dispersively measuring the qubit state. The Echo dephasing time is characterized similarly, adding a π pulse centered between the pair of $\pi/2$ pulses. The measurements are fit to $\langle m_1(\tau)m_2(\tau) \rangle = A \times \exp(-\tau/T_2^{(R,E)}) \times \cos(\omega\tau + \phi)$. The average and standard deviations of repeated measurements of the Ramsey and Echo dephasing times T_2^R and T_2^E are reported in Table I.

4. Characterizing A_ϕ

To characterize the flux noise amplitude of each qubit, repeated interleaved measurements of T_1 and T_ϕ^E were taken over a narrow range around $\Phi_{\text{ext}} = 0.5\Phi_0$. Away from $\Phi_{\text{ext}} = 0.5\Phi_0$, where we expect the dephasing to be limited by $1/f$ flux noise, the data from the spin-echo experiments were fit with a decay envelope $\langle m_1(\tau)m_2(\tau) \rangle = A \times \exp[-\tau/(2T_1) - (\tau/T_\phi^E)^2] \times \cos(\omega\tau + \phi)$. In this fit function, T_1 was constrained to the value obtained from the immediately preceding measurement. As shown in Fig. 9, we analyze the dependence of $\Gamma_\phi^E = 1/T_\phi^E$ on the absolute value of the qubit spectrum slope in flux, where we expect $\Gamma_\phi^E = |\frac{\partial\omega}{\partial\Phi}| \sqrt{A_\phi \ln(2)}$. We fit the

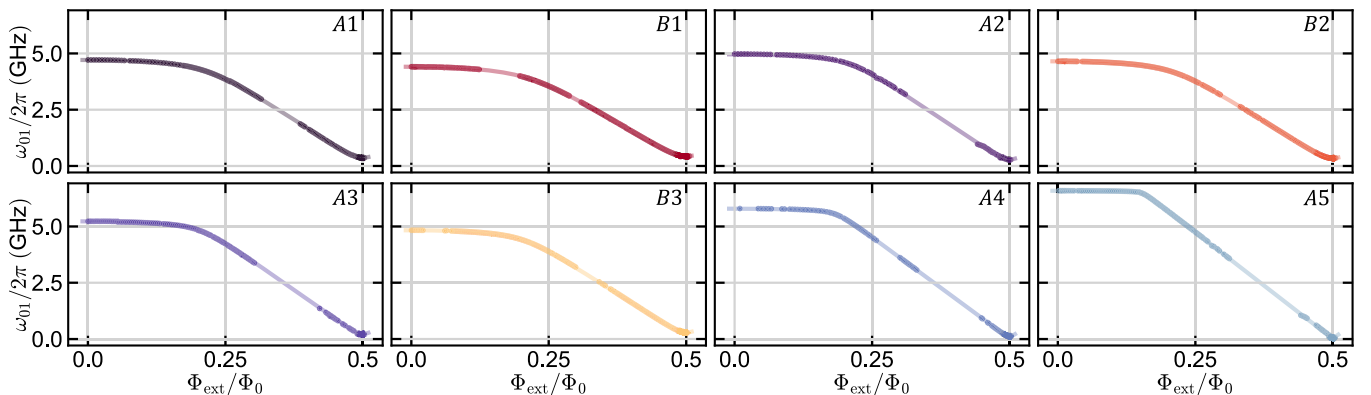


FIG. 7. Qubit transition frequency $\omega_{01}/2\pi$ vs applied magnetic flux Φ_{ext} . Points indicate measured frequencies, and the line is the calculated frequency from the qubit Hamiltonian parameters listed in Table I.

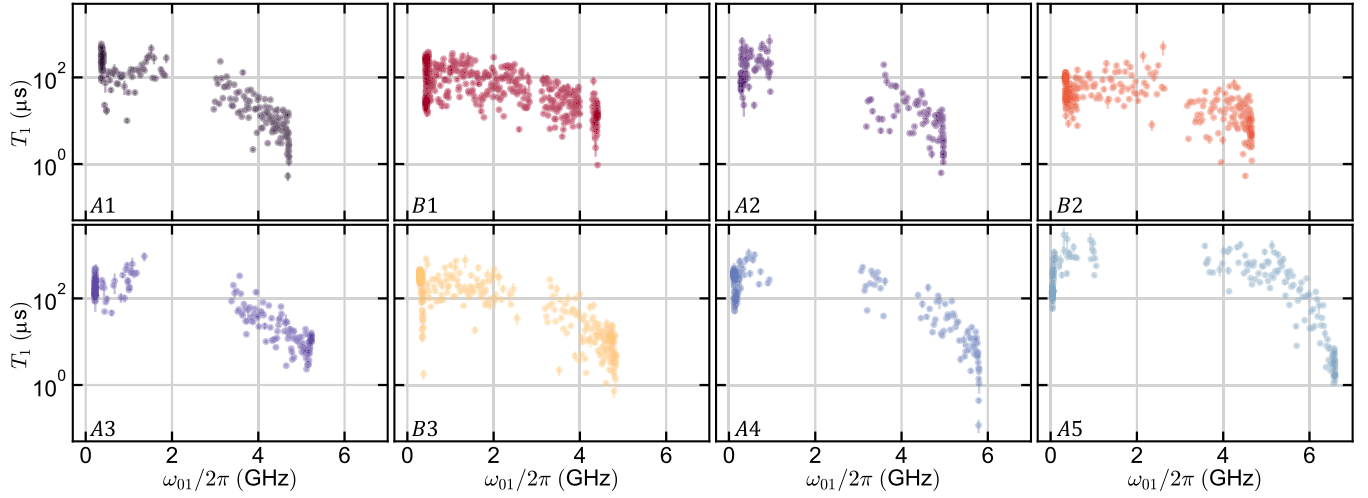


FIG. 8. Measured T_1 for all qubits as a function of transition frequency $\omega_{01}/2\pi$. Gaps in measured data correspond to regions of insufficient readout contrast (see Fig. 6).

data to a linear function to extract $\sqrt{A_\phi}$. Given that dephasing in this model vanishes at $\Phi_{\text{ext}}/\Phi_0 = 0.5$, where $\frac{\partial\omega}{\partial\Phi} = 0$, the fit enforces an intercept with the origin [38]. Measurements at $\Phi_{\text{ext}} = 0.5\Phi_0$, where we do not expect dephasing to be limited by first-order flux noise were not included in the linear fit. We observe a wide range of $\sqrt{A_\phi}$ across the fluxoniums (Fig. 9). Noise that couples to the qubit from the flux bias coil and from defects in the environment contributes to this flux noise. Noise from our instrumentation would appear as correlation between $\sqrt{A_\phi}$ and the effective coupling of the bias line to the qubit (larger coupling corresponds to a lower applied voltage to achieve one Φ_0 of applied bias). Figure 10 shows the flux noise amplitude A_ϕ versus the voltage periodicity for each qubit. We observe a weak correlation between the voltage periodicity and the extracted flux noise amplitude, indicating that global bias noise is not itself sufficient to describe the variation in $\sqrt{A_\phi}$.

APPENDIX C: MODELING T_1

Fermi's golden rule is used to model the transition rate between two-qubit energy eigenstates $|i\rangle$, $|j\rangle$, under the in-

fluence of a perturbatively treated noise source [50], which is given by

$$\Gamma_\lambda = \frac{2}{\hbar^2} |\langle i|\hat{D}_\lambda|j\rangle|^2 S_\lambda(\omega_{ij}). \quad (\text{C1})$$

Here, $S_\lambda(\omega_{ij}) = \frac{1}{2} \int_{-\infty}^{\infty} \langle \lambda(0)\lambda(t) + \lambda(t)\lambda(0) \rangle e^{-i\omega_{ij}t} dt$, which is the symmetrized, two-sided power spectrum of the time-fluctuating noise $\lambda(t)$, and $\hbar\omega_{ij}$ is the transition energy between the eigenstates $|i\rangle$, $|j\rangle$. The coupling of this noise to the qubit system is generically described by a linear interaction term in the Hamiltonian of the form $\hat{D}_\lambda\lambda(t)$. The qubit susceptibility to different noise sources is contained within the transition matrix element $|\langle i|\hat{D}_\lambda|j\rangle|^2$.

1. Circuit models of loss

For circuit-based models of dissipation, we model noise as a fictitious resistor in the qubit circuit. Under this class of models, the power spectrum is obtained by considering either voltage fluctuations across a resistor in parallel to the qubit circuit, or current fluctuations through a resistor in series with a qubit circuit element. Following the Caldeira-Leggett

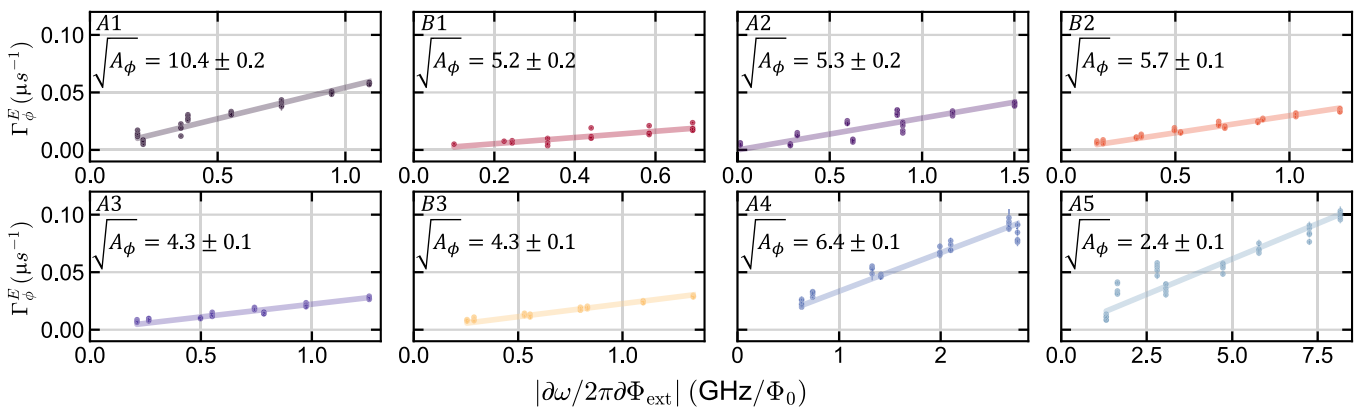


FIG. 9. Measured Γ_ϕ^E vs external flux sensitivity for all qubits with linear fit used to extract $\sqrt{A_\phi}$ for each qubit, which is reported with the standard error on the fit. We note that the y axes are shared across all plots; however, the x axis varies as was necessary to accommodate each qubit's unique transition spectrum and the range over which Γ_ϕ^E was characterized.

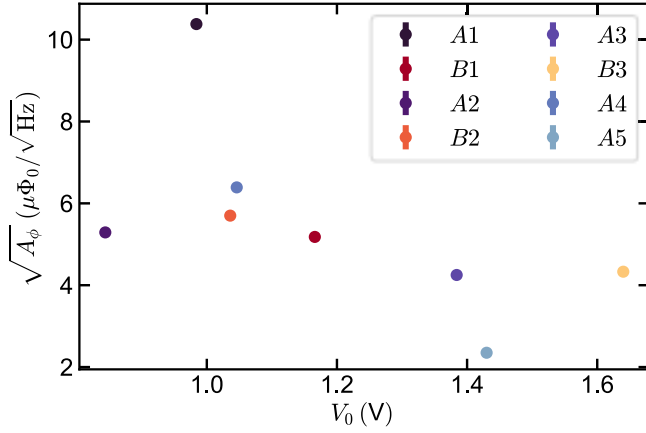


FIG. 10. Extracted $\sqrt{A_\phi}$ vs V_0 , the voltage required to bias the fluxonium to $\Phi_{\text{ext}} = \Phi_0$.

model, the symmetrized noise spectra for Johnson-Nyquist voltage and current fluctuations are, respectively [39,40,51],

$$S_V(\omega_{ij}) = \hbar\omega_{ij}\text{Re}[Z(\omega_{ij})]\coth\left(\frac{\hbar\omega_{ij}}{2k_B T}\right) \quad (\text{C2})$$

and

$$S_I(\omega_{ij}) = \hbar\omega_{ij}\text{Re}[Y(\omega_{ij})]\coth\left(\frac{\hbar\omega_{ij}}{2k_B T}\right), \quad (\text{C3})$$

where $Z(\omega_{ij})$ is the impedance of our effective circuit and $Y(\omega_{ij}) = 1/Z(\omega_{ij})$ is the admittance.

a. Capacitive dielectric loss

Lossy dielectric materials that couple to the electric field of the qubit via the operator $2e\hat{n}$ are modeled in this work as Johnson-Nyquist voltage noise across the qubit capacitor. The relevant impedance for the voltage noise spectrum is calculated from the parallel capacitive and resistive branches, $Z(\omega_{ij}) = (i\omega_{ij}C_\Sigma + 1/R)^{-1}$. All other qubit elements are treated as perfect and lossless in this model. We substitute the quality factor for a parallel resonance circuit, $Q_C = \omega_{ij}RC_\Sigma$ [52], into our impedance expression to obtain $Z(\omega) = \frac{1}{Q_C\omega_{ij}C_\Sigma}$. Together, this gives the energy relaxation rate [53],

$$\Gamma_1^C = \frac{16E_C}{\hbar Q_C} |\langle 0|\hat{n}|1\rangle|^2 \coth\left(\frac{\hbar\omega_{01}}{2k_B T}\right), \quad (\text{C4})$$

where Q_C indicates the quality factor of the lossy capacitor. Though the construction of this model typically assumes quality factor of the circuit to be frequency independent, we lift this assumption [Eq. (4)], which we find better describes our data, similar to other literature [15,17–19,41]. We do not explicitly model the fine frequency structure associated with TLSs, and instead use the frequency-dependent, binned average effective quality factor Q_C^{eff} in our main analysis to characterize and compare qubits.

b. Inductive loss

Dual to the capacitive loss model is a phenomenological model for lossy dielectric materials that couple to the magnetic field of the qubit through the operator $\phi_0\hat{\phi}$, where $\phi_0 = \Phi_0/2\pi$ are modeled in this work as Johnson-Nyquist

current noise through a series resistive element. In this work, we instead consider two microscopically motivated models— $1/f$ flux noise and quasiparticle (QP) tunneling through JJ array—instead of this phenomenological inductive loss model. For more details on this inductive circuit loss model, see Refs. [13,16,19,53] and references therein.

c. Spontaneous emission to control and readout circuits

We utilize circuit-based models for loss through the charge and flux control lines, as well as for resonance-enhanced (Purcell) emission through the readout resonator, described in detail here.

Charge control line. We model radiative loss to the charge control line as voltage noise from a $50\ \Omega$ environment coupled to the qubit across the capacitance between the control line and the qubit, C_d . The relaxation rate is written as [11,19,54]

$$\Gamma_1^{\text{cd}} = \frac{16E_C C_d^2}{\hbar^2 e^2} |\langle 0|\hat{n}|1\rangle|^2 \coth\left(\frac{\hbar\omega_{01}}{2k_B T}\right). \quad (\text{C5})$$

From finite element simulations, C_d is estimated to be 20 aF, which is used for all qubits. From the qubit Hamiltonian parameters, the coupling capacitance, and assumed qubit effective temperature, we calculate the contribution of this mechanism to all qubit T_1 times in our dataset. This mechanism is accounted for in the main text comparative analysis, as well as in two-level analysis detailed in Appendix J.

Flux control line. Analogous to the charge control line, we model radiative loss to the flux control line as current noise from a $50\ \Omega$ environment coupled via the mutual inductance between the control line and the qubit, M_d . Combined with the current noise spectrum, this gives the relaxation rate [19],

$$\Gamma_1^{\text{fd}} = \frac{8\pi^2 E_L^2 M_d^2}{\hbar^2 \Phi_0^2} |\langle 0|\hat{\phi}|1\rangle|^2 \coth\left(\frac{\hbar\omega_{01}}{2k_B T}\right). \quad (\text{C6})$$

The flux line design and loop area for all qubits in this dataset are identical to those in Ref. [19], with mutual inductance $M_d = \Phi_0/21.5\ \text{mA}$ used for all qubits in this dataset [19]. From the qubit Hamiltonian parameters, the mutual inductance, and the assumed qubit effective temperature, we calculate the contribution of this mechanism to all qubit T_1 times in our dataset. This mechanism is accounted for in the main text comparative analysis, as well as in the $N = 2$ level analysis detailed in Appendix J.

Readout resonator. We utilize a circuit-based model for Purcell decay through the readout resonator, in contrast to other models in literature that treat the resonator quantum mechanically [13]. This was chosen to more accurately model the frequency dependence of the coupling between all qubit levels and the resonator. We model the readout resonator as a $\lambda/4$ transmission line with characteristic impedance $Z_0 = 50\ \Omega$, capacitively coupled on one end to the qubit and inductively coupled to the feedline on the other. The feedline is modeled as an infinitely long transmission line via termination with a $50\ \Omega$ resistor on either end (Fig. 11). The relaxation rate due to this mechanism is written as [19]

$$\Gamma_1^{\text{P}} = \frac{8e^2\omega_{01}}{\hbar} \left(\frac{C_c}{C_\Sigma}\right)^2 |\langle 0|\hat{n}|1\rangle|^2 \text{Re}[Z_{\text{in}}(\omega_{01})] \coth\left(\frac{\hbar\omega_{01}}{2k_B T}\right), \quad (\text{C7})$$

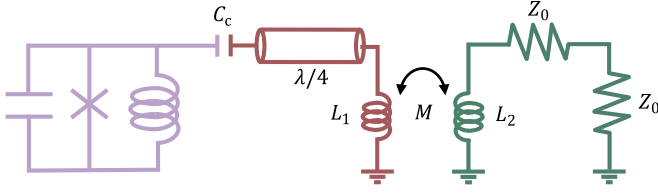


FIG. 11. Effective circuit diagram used for calculation of Purcell loss. Purple elements correspond to the fluxonium qubit circuit, red elements correspond to the readout resonator, and green elements correspond to the microwave feedline.

where the coupling capacitance $C_c = \frac{\hbar g C_\Sigma}{2e\omega_{\text{res}}} \sqrt{\frac{\pi}{2\hbar Z_0}}$ and C_Σ is the total qubit capacitance. $Z_{\text{in}}(\omega_{01})$ is the input impedance of the lossy environment of the feedline filtered by the readout resonator, as seen by the qubit.

For the mode with frequency ω_{res} , Z_{in} is written as

$$Z_{\text{in}}(\omega) = Z_0 \frac{\omega^2 M^2 \cot\left(\frac{\pi\omega}{2\omega_{\text{res}}}\right) + 2jZ_0^2}{2Z_0^2 \cot\left(\frac{\pi\omega}{2\omega_{\text{res}}}\right) + j\omega^2 M^2}, \quad (\text{C8})$$

where M is the mutual inductance between the resonator and feedline [52]. Assuming this coupling is weak such that

$\omega M \ll Z_0$, we rewrite Eq. (C8) as

$$Z_{\text{in}}(\omega) \approx \frac{1}{\frac{\omega_{\text{res}}^2 M^2}{Z_0^2} + \frac{2j\pi\omega}{2\omega_{\text{res}}Z_0}} = \frac{1}{1/R + 2j\omega C}, \quad (\text{C9})$$

where by matching terms with the parallel RLC circuit, we find $R = Z_0^3/\omega_{\text{res}}^2 M^2$ and $C = \pi/2\omega_{\text{res}}Z_0$. The quality factor is defined as $Q = \omega RC$, which for the obtained values of R and C can be rewritten as

$$Q = \frac{\pi Z_0^2}{2\omega_{\text{res}}^2 M^2}. \quad (\text{C10})$$

We do not directly measure the mutual inductance between the readout resonator and the feedline, but we do measure the resonator quality factor, $Q_{\text{res}} = \omega_{\text{res}}/\kappa$, which we use as a substitute for the mutual inductance (assuming that the resonator is overcoupled to the feedline),

$$M = \frac{Z_0}{\omega_{\text{res}}} \sqrt{\frac{\pi}{2Q_{\text{res}}}}. \quad (\text{C11})$$

For qubits with transition frequencies within 1 GHz of their readout resonator frequencies, we find this model for radiative loss through the readout resonator becomes strong enough to limit T_1 in addition to capacitive dielectric loss. Under a six-level model for this mechanism, the limit to T_1 is stronger across the qubit tuning range than in a two-level model, with

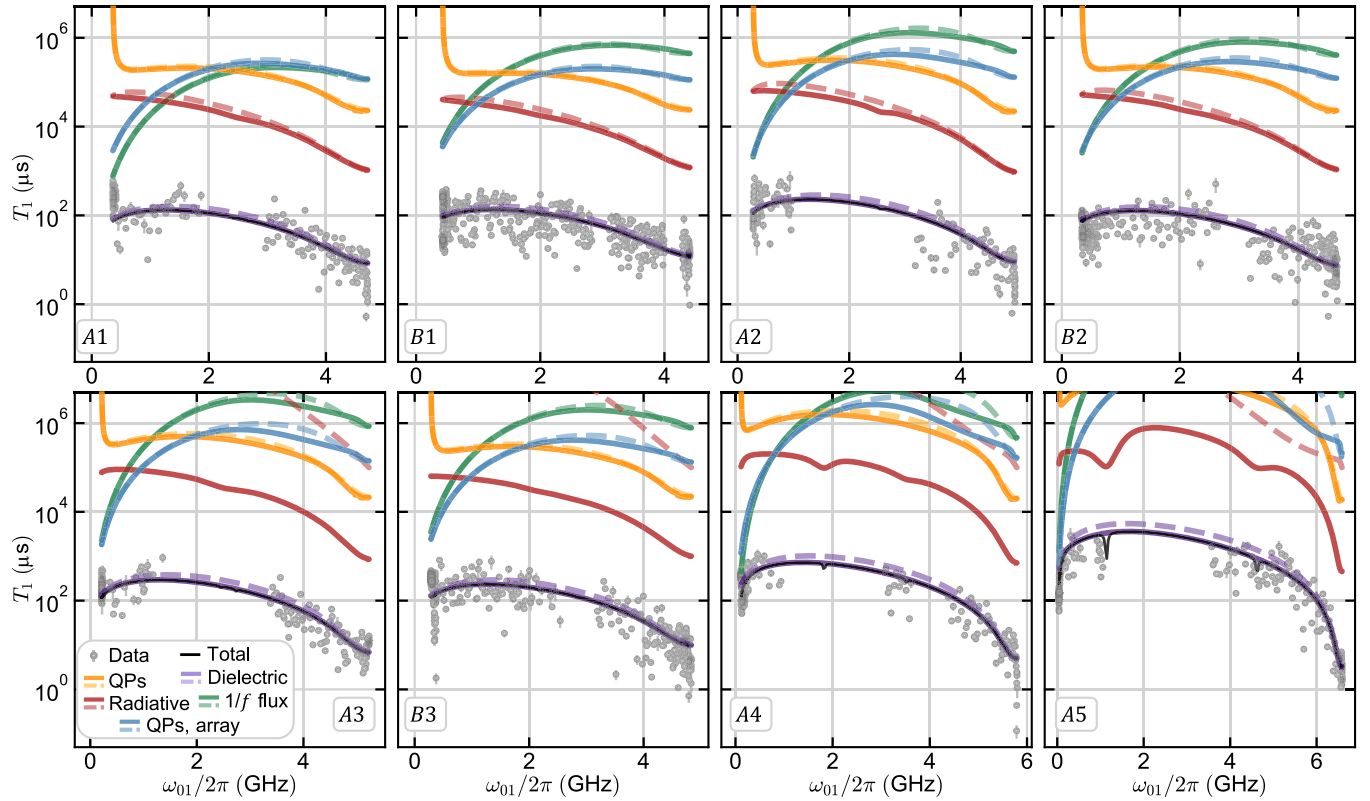


FIG. 12. Measured T_1 vs $\omega_{01}/2\pi$ for each qubit. Note that each subplot's x axis is varied to accommodate each qubit's frequency tuning range. The y axis range is shared for all plots. Overlay lines show predicted T_1 due to different models. Solid lines indicate models for which a six-level treatment is considered; dashed lines indicate a two-level treatment. All models assume an effective qubit temperature of 40 mK and an effective resonator temperature of 65 mK. $1/f$ flux noise, Purcell loss, and dielectric loss curves use qubit-specific parameters summarized in Tables I and II. The phenomenological frequency dependence is used for the dielectric loss model, with $\epsilon = 0.25$. Quasiparticle models assume $x_{\text{QP}} = 1 \times 10^{-9}$.

features corresponding to higher qubit transitions crossing the resonator (see solid red curves in Fig. 12).

2. Quasiparticle-induced loss

Here, we model energy relaxation due to nonequilibrium quasiparticles (QPs) tunneling across the qubit junctions. We first assume the energy hierarchy, $\varepsilon_{\text{QP}}, \hbar\omega_{ij} \ll \Delta$, where ε_{QP} is the characteristic energy of the quasiparticles, $\hbar\omega_{ij}$ is the qubit transition energy, and Δ is the superconducting gap. In this low-energy regime, we additionally assume that the qubit energy is larger than the quasiparticle energy, which allows us to simplify the noise spectral density to [8,55]

$$S_{\text{QP}}(\omega_{ij}) = x_{\text{QP}} \frac{8E_J}{\pi\hbar} \sqrt{\frac{2\Delta}{\hbar\omega_{ij}}}. \quad (\text{C12})$$

The consequence of this assumption is that quasiparticles can only relax the qubit, and not excite it, as they do not have the requisite energy. We reintroduce this possibility into the quasiparticle noise spectral density by adding a contribution reduced by a Boltzmann factor $e^{\hbar\omega_{ij}/k_B T}$, consistent with detailed balance. The loss from quasiparticles tunneling across the junction couples via the $\sin(\hat{\phi}/2)$ operator, which combined with the noise spectrum gives the relaxation rate [8,55]

$$\Gamma_1^{\text{QP}} = \frac{16E_J x_{\text{QP}}}{\pi\hbar} \sqrt{\frac{2\Delta}{\hbar\omega_{01}}} |\langle 0|\sin(\hat{\phi}/2)|1\rangle|^2 (1 + e^{\hbar\omega_{01}/k_B T}), \quad (\text{C13})$$

where x_{QP} is the normalized quasiparticle density. For quasiparticles tunneling in the JJ array inductor, we sum the contribution from every JJ, each with corresponding phase drop $\hat{\phi}/N_{\text{array}}$, where N_{array} is the total number of JJs in series. As N_{array} is typically large (in our case 151), we approximate $\sin \hat{\phi} \approx \hat{\phi}$ to get the rate,

$$\Gamma_1^{\text{QP,A}} = \frac{2E_L x_{\text{QP}}}{\pi\hbar} \sqrt{\frac{2\Delta}{\hbar\omega_{01}}} |\langle 0|\hat{\phi}|1\rangle|^2 (1 + e^{\hbar\omega_{01}/k_B T}). \quad (\text{C14})$$

Assuming a spatially homogeneous, normalized QP density x_{QP} , a value of 1×10^{-6} or higher would be required for tunneling through the single JJ branch to describe the measured T_1 . This quasiparticle density is significantly larger than reported in recent measurements [56,57]. The same x_{QP} predicts a T_1 due to QP tunneling in the inductor JJs that underestimates the data at low transition frequencies. The array tunneling model can align with the data at low frequencies using an $x_{\text{QP}} = 1 \times 10^{-8}$ or less, for which QP tunneling across the single JJ branch would not be limiting. At these frequencies, we continue to see signatures of dielectric loss limitation (TLS-like features), indicating that limitation from quasiparticles is unlikely. Therefore, we assume QP tunneling to be a negligible effect in all analysis.

3. $1/f$ flux noise

To model $1/f$ flux noise, we consider fluctuations in flux bias coupling to the fluxonium Hamiltonian [Eq. (1)], which couple to the qubit through the inductor via the operator $\phi_0 \hat{\phi}$.

We write the power spectrum of this noise as

$$S_{\Phi}(\omega_{ij}) = \left(\frac{2\pi A_{\Phi}}{\omega_{ij}} \right)^{\alpha}, \quad (\text{C15})$$

where A_{Φ} is the noise amplitude referenced at 1 Hz [29,58]. The method used to extract A_{Φ} is described in detail in Appendix B 4, assuming $\alpha = 1$. The energy relaxation rate is written as

$$\Gamma_1^{\Phi} = 4\pi \left(\frac{2\pi E_L}{\hbar\Phi_0} \right)^2 |\langle 0|\hat{\phi}|1\rangle|^2 \frac{A_{\Phi}}{\omega_{01}}, \quad (\text{C16})$$

where temperature dependence of this model is included implicitly in the value of A_{Φ} [17,19].

APPENDIX D: N-LEVEL ENERGY RELAXATION MODEL

1. Modeling energy relaxation dynamics

Qubit population transfer through higher energy levels has been shown to be relevant to relaxation of qubits with energy splitting on the order of $k_B T$ [19,59,60]. We consider the population dynamics using the time evolution of the population vector $\vec{p}$, which tracks six lowest energy levels of the circuit (after which we see the results from our modeling converge for all qubits). The vector $\vec{p}$ evolves under the equation $\frac{d}{d\tau} \vec{p}(\tau) = \mathbf{B} \vec{p}(\tau)$ [19]. The matrix $\mathbf{B}$ is defined as

$$\mathbf{B} = \begin{pmatrix} -\sum_{i \neq 1}^N \Gamma_{0 \rightarrow i} & \Gamma_{1 \rightarrow 0} & \cdots & \Gamma_{N \rightarrow 0} \\ \Gamma_{0 \rightarrow 1} & -\sum_{i \neq 1}^N \Gamma_{1 \rightarrow i} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \Gamma_{N \rightarrow N-1} \\ \Gamma_{0 \rightarrow N} & \Gamma_{1 \rightarrow N} & \cdots & -\sum_{i \neq N}^N \Gamma_{N \rightarrow i} \end{pmatrix}, \quad (\text{D1})$$

where elements $\Gamma_{i \rightarrow j}$ are transition rates from state $|i\rangle$ to state $|j\rangle$, where at present we are considering the transitions induced by a *single* loss model. The solution to the differential equation is $\vec{p}(\tau) = e^{\mathbf{B}\tau} \vec{p}(0) = \mathbf{V} e^{\mathbf{S}\tau} \mathbf{V}^{-1} \vec{p}(0)$, where $\mathbf{B}$ is diagonalized by $\mathbf{V} \mathbf{S} \mathbf{V}^{-1}$. Here, $\mathbf{S}$ is a diagonal matrix of real eigenvalues associated with the columns of matrix $\mathbf{V}$, which are composed of the eigenvectors of $\mathbf{B}$.

The population evolves according to each eigenmode of $\mathbf{B}$ as

$$\vec{p}(\tau) = c_0 \vec{v}_0 + c_1 \vec{v}_1 e^{-\gamma_1 \tau} + \cdots c_N \vec{v}_N e^{-\gamma_N \tau}, \quad (\text{D2})$$

with $(c_0 c_1 \cdots c_N)^T = \mathbf{V} \vec{p}(0)$ being the coefficients of the initial state in the eigenbasis of $\mathbf{B}$. We use an equilibrium state as the initial state of the system $\vec{p}(0)$ with occupation probabilities given by the Boltzmann distribution, assuming a temperature of 40 mK. We invert the population between the first two states and solve for the time evolution of $\vec{p}(\tau)$ back to the equilibrium state.

With higher energy levels allowed to participate in population dynamics, T_1 maintains the definition of being the timescale upon which a perturbed qubit population returns to thermal equilibrium; however, these higher levels can introduce interesting effects. The solution to this model is by definition nonexponential; however, our measured decay

envelopes do not noticeably deviate from an exponential shape (this is further discussed and quantified in Fig. 15). We find for the Hamiltonian parameters of the qubits in this dataset, it is important to consider the multirate nature of this model to accurately capture the qubit population dynamics. Furthermore, we find a consequential impact on the decay signal from the higher-state Lamb shifts of the readout resonator, which also requires an accounting of all system rates in order to model. Due to the range of Hamiltonian parameters and observed impact of the readout resonator on the extracted decay timescale, we follow a protocol distinct from that [19], where the eigenvalue of the mode with greatest overlap with the initial state of the system was chosen as the decay rate.

In this work, we solve the rate matrix differential equation for $\vec{p}(\tau)$. Using the extracted qubit Hamiltonian parameters and qubit-resonator coupling, we also calculate the Lamb shifts of the resonator due to qubit population in each considered energy level. We find resonator response for each qubit state $|i\rangle$ according to

$$S_{21}^i(\omega) = 1 - \frac{Q_{\text{Load}}/Q_{\text{Coup}}}{1 + \frac{2iQ_{\text{Load}}\Delta\omega_i}{\omega_{\text{res}} + \chi_i}}, \quad (\text{D3})$$

where Q_{Load} and Q_{Coup} are the loaded and coupling Q s of the resonator, respectively, and $\Delta\omega_i$ is the difference between the probe frequency ω and the dressed frequency of the resonator $\omega_{\text{res}} + \chi_i$. For this modeling, we make the assumption that $Q_{\text{Load}} = Q_{\text{Coup}} = \omega_{\text{res}}/\kappa$, and that we probed the resonator at frequency $(\omega_{\text{res}} + \chi_0)/2\pi$.

The location in in-phase/quadrature (IQ) space of each higher-state resonator frequency can then be calculated via Eq. (D3). To match experiment, we then rotate the simulated response to maximize signal contrast ($S_{21,\text{rot}}^i$). Finally, we calculate an expected T_1 decay signal as

$$s(\tau) = \left| \sum_i^{N=6} p_i(\tau) \text{Re}\{S_{21,\text{rot}}^i\} \right|, \quad (\text{D4})$$

where $p_i(\tau)$ is the population in the i th state as a function of the T_1 measurement delay time and S_{21}^i is the rotated location of the state in the IQ plane. We fit $s(\tau)$ to an exponential to extract the expected decay rate.

To simulate a T_1 time informed by multiple loss channels, the architecture outlined previously is extended by summing the rate matrices $\mathbf{B}_{\text{tot}} = \sum_m \mathbf{B}_m$, where m represents each included loss model. This is analogous to summing the rates due to each loss mechanism in a two-level model, where $\Gamma_1^{\text{tot}} = \sum_m \Gamma_1^m$. For the main text comparisons, we sum loss matrices for capacitive dielectric loss, $1/f$ flux noise, and radiative losses to the microwave environment to describe T_1 (and calculate Q_C^{eff}) of each qubit in the dataset.

2. Identifying limiting T_1 mechanisms

In Fig. 12, we show the measured T_1 for all qubits with each coherence model we consider overlaid. For the plotted six-level models, we do not include the effect of the resonator Lamb shift on the T_1 , and instead fit the decay of $p_1(t)$ to an exponential. We use the device-specific measured parameters for our models of $1/f$ flux noise and Purcell loss, summarized

in Table I. We assume an x_{QP} of 1×10^{-9} for models of quasiparticle tunneling for each device. The dielectric loss model uses the mean Q_C^{eff} reported in Table II and $\epsilon = 0.25$.

We see that the capacitive dielectric loss describes the data over a majority of the qubits' frequency tuning range. For some qubits, limitation from $1/f$ flux noise or Purcell losses are more prevalent in certain regions of the flux range, particularly for qubits A1, A4, and A5.

APPENDIX E: DATA PROCESSING

In this appendix, we describe the data processing done to convert the measured T_1 times to the Q_C^{eff} distributions used in the main text for all comparisons.

1. Binned average

As described in the main text, we apply an 8 MHz binned average to the T_1 data across the frequency tuning range of each device. The binned average reduces the influence of potential oversampling of specific TLSs on the data analysis by averaging over these features. The window size of 8 MHz was chosen as the average observed TLS feature full width at half maximum.

The binning primarily affects data taken near $\Phi_{\text{ext}}/\Phi_0 = 0$ and 0.5. We summarize the number of T_1 samples and the number of points remaining after the 8 MHz binned average is applied for each qubit in Table VI. We see that a majority of the frequency tunable range for each qubit is not impacted by the binning (Fig. 13).

2. Converting to Q_C^{eff}

To extract Q_C^{eff} , we solve the differential equation for population evolution using a rate matrix composed of multiple loss models $\mathbf{B}_{\text{tot}} = \sum_m \mathbf{B}_m$, where indices m represent each loss model. At this stage of the analysis, we only include losses in the matrix $\mathbf{B}_{\text{tot}}$ due to $1/f$ flux noise, radiative emission to the microwave environment, and capacitive dielectric loss. We utilize `scipy.optimize.minimize` [61] with the Nelder-Mead method [62,63] to solve for Q_C^{eff} from our measured T_1 . We define our cost function as squared difference between the predicted T_1 from a trial Q_C^{eff} and the measured T_1 . We exclude data points limited significantly by flux noise and/or radiative losses, defined in this analysis by whether the contributions from those models are greater than 10% of the measured rate. We then assume that the remaining data are representative of the capacitive dielectric loss. The number of points in the final Q_C^{eff} distribution for each qubit is summarized in Table VI. Flux noise at low frequencies as well as radiative losses through the readout resonator are both limiting for qubits A4 and A5 in the dataset, and commensurately they have the fewest number of points remaining after this processing in their Q_C^{eff} distributions (Fig. 13 and Table VI).

3. Frequency dependence of the Q_C^{eff} distributions

To find the ϵ in Eq. (4) that best describes the entire dataset, we calculate Q_C^{eff} for each qubit using varying ϵ from -1 to 1 in steps of 0.05 . For each qubit at each ϵ , we

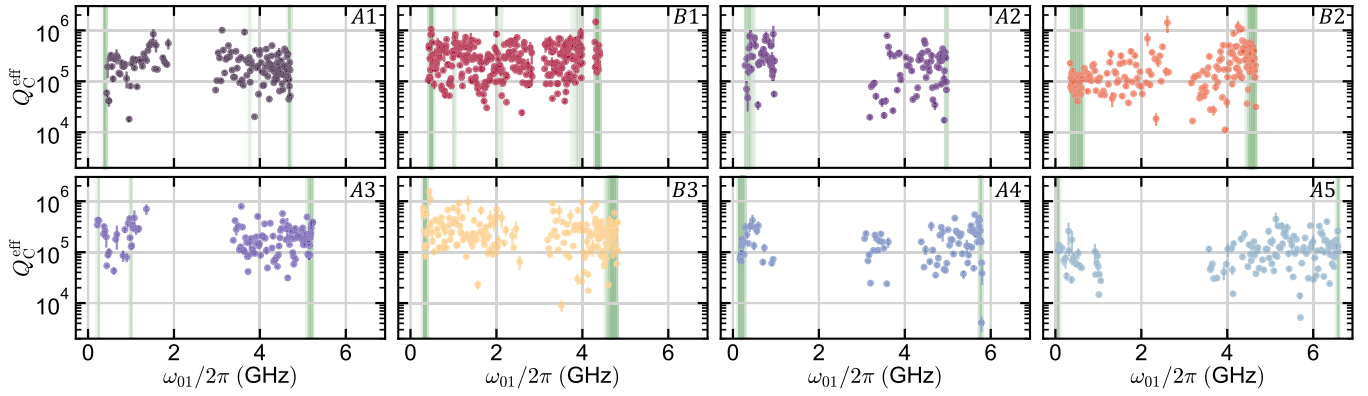


FIG. 13. Extracted Q_C^{eff} data used in the main text, for each qubit, vs qubit transition frequency. The dielectric loss model includes the frequency dependence according to Eq. (4) with $\epsilon = 0.25$. Green vertical shaded regions indicate areas where more than one T_1 data point was within the same bin. In those regions, Q_C^{eff} is calculated from the bin-average T_1 value.

calculate $\log_{10}(Q_C^{\text{eff}}) - \log_{10}(\overline{Q_C^{\text{eff}}})$, where $\overline{Q_C^{\text{eff}}}$ is the mean of the qubit Q_C^{eff} distribution. We minimize the variance of this parameter combining data from all qubits over the range of tested epsilons to find the best fit, $\epsilon = 0.25$. We plot the Q_C^{eff} distributions with this best epsilon versus the qubit transition frequency in Fig. 13.

Though we use the same frequency dependence for each qubit in the main text analysis, we note that each qubit has a unique frequency dependence, varying between $\epsilon = -0.15$ and $\epsilon = 0.50$. We show the best ϵ for each qubit for both the six-level analysis and for a two-level analysis in Fig. 14. Most qubits in the dataset show an increase in the average extracted Q_C^{eff} from high to low transition frequency; however, this trend is broken by qubit B2. We note that applying the globally optimized $\epsilon = 0.25$ does not seem to decrease the applicability of this model, given the spread of measured Q_C^{eff} as a function of qubit frequency. Though the frequency trend in Q_C^{eff} is not universal across all devices, by fitting the entire dataset together we are able to compare the distributions on equal footing and mitigate fit bias from qubits with less data at low frequency.

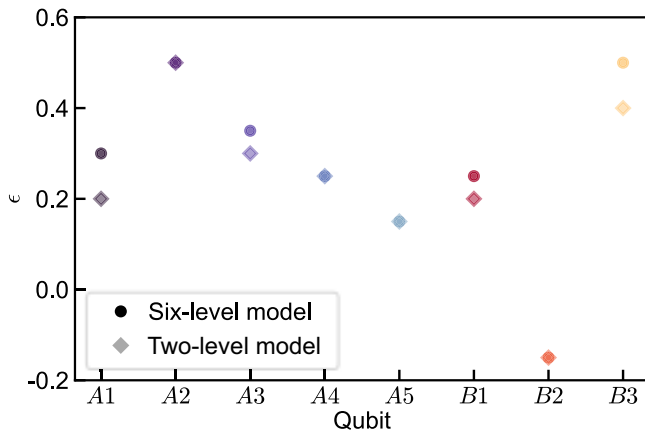


FIG. 14. Extracted frequency dependence of the Q_C^{eff} distributions, ϵ , under the six-level model (circles) and two-level model (diamonds), for each qubit.

APPENDIX F: VALIDATION TESTS FOR SIX-LEVEL RELAXATION MODEL

Here, we justify the application of exponential decay fit functions to our T_1 dataset under the six-level model of population dynamics.

1. Quantifying exponential dynamics

Deviation from exponential decay was not observed in this experiment. It is of note, however, that multiexponential decay dynamics have been observed in other fluxonium coherence studies in literature [8,15]. Following Ref. [19], to quantify our expectations of exponential behavior under a six-level treatment of the population dynamics, we calculate a vector quantity $\vec{\delta}$, defined as

$$\vec{\delta} = \vec{p}(0) - c_0 \vec{v}_0 - c_{k,\max} \vec{v}_{k,\max}. \quad (\text{F1})$$

Here, we subtract from our equilibrium population vector $\vec{p}(0)$ the contributions of the steady state, or “stationary mode” $c_0 \vec{v}_0$, and the most dominant mode $c_{k,\max} \vec{v}_{k,\max}$. We define the eigenvectors to have unit length, such that $||\vec{v}_i|| = 1$. The most dominant mode is defined as the eigenmode of the rate matrix with the greatest overlap with the initial state of the system, $k, \max = \arg\max_{i \neq 0} |c_i|^2$, and should be most characteristic of the decay of $\vec{p}(t)$. Though the main text does not use the decay rate of the most dominant mode as T_1 of the system, we find this method useful to quantify the contribution of all matrix modes to the decay dynamics, which we use to understand the applicability of an exponential fit to the population decay in our system.

The vector $\vec{\delta}$ contains the population in the nondominant, non-steady-state modes of the system. The length of $\vec{\delta}$ is

$$M = \sqrt{\sum_i \delta_i^2}, \quad (\text{F2})$$

which is used as a quantitative metric to benchmark the exponential behavior of our system. For each device, we calculate the decay rates γ_i , the vector overlaps $|c_i|$, and the length of $\vec{\delta}$, M , across the flux tuning range (Fig. 15). We use a rate matrix accommodating for capacitive dielectric loss with the device

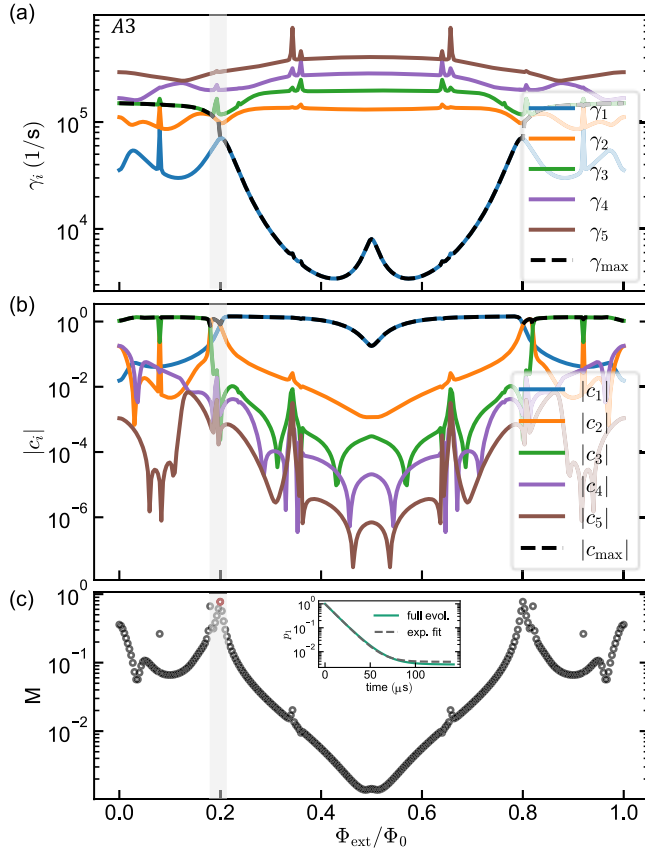


FIG. 15. Quantifying exponential behavior under the six-level model for energy relaxation. Plot uses the parameters of the qubit with least exponential behavior (highest M value), A3. (a) Decay rates γ_i of each rate matrix normal mode vs flux. The rate associated with most dominant mode, γ_{max} is shown with a black dashed line. (b) Overlap with the initial state of each normal mode of the rate matrix. Largest overlap $|c_{\text{max}}|$ is shown with black dashed line. (c) M metric vs flux, where high values correspond to less exponential behavior, and low values correspond to more exponential behavior. The maximum M value is highlighted in red. The vertical shaded region corresponds to the least exponential region (this is symmetric in flux, though we only shade the region within $\Phi_{\text{ext}}/\Phi_0 = 0, 0.5$), as determined by the maximum value of M . Inset shows the decay of p_1 at the flux with the highest M value, with gray dashed overlay corresponding to a single exponential fit of the decay of $p_1(t)$.

average Q_C^{eff} , $\epsilon = 0.25$, $1/f$ flux noise, and radiative losses to the microwave environment. At the greatest values of M for each device, we examine the decay of $p_1(\tau)$ and compare it to a single exponential decay curve.

Across all qubits in this study, our model does not predict a significant deviation from exponential decay even at the least exponential (highest M value) flux bias, where we show an exponential fit to the decay of p_1 at that bias to confirm [Fig. 15(c) inset]. This is consistent with our observations of exponential behavior in experiment.

2. Effect of population leakage on dispersive T_1 measurements

Here, we discuss the importance of the inclusion of the Lamb shifts in the simulation of the energy relaxation dy-

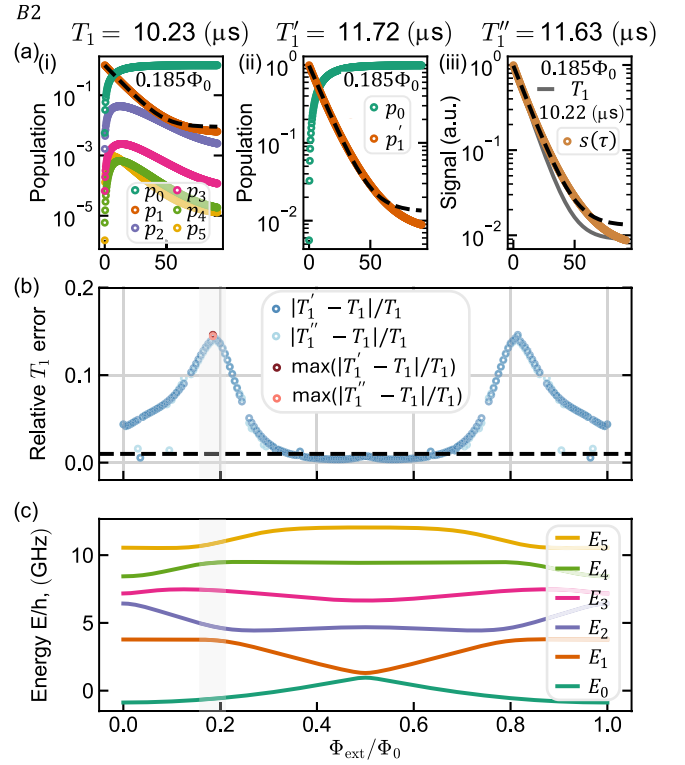


FIG. 16. Simulating the effect of state misassignment of higher-level populations under different measurement techniques. Calculations shown are for the qubit with highest observed relative errors, B2. (a) Comparison of extracted T_1 times under different methods. (i) Full population evolution at $\Phi_{\text{ext}}/\Phi_0 = 0.185$, the flux with highest relative error for heralded measurements. Dashed line is an exponential fit to the decay of p_1 , with the plot title corresponding to the extracted decay timescale. (ii) Decay of p_0, p_1 with all higher-state population assigned to p_1 at $\Phi_{\text{ext}}/\Phi_0 = 0.185$. Plot title corresponding to the extracted decay timescale from an exponential fit to p_1 . (iii) Decay of signal $s(\tau)$ at the bias with highest relative error for dispersive measurements, $\Phi_{\text{ext}}/\Phi_0 = 0.185$. The dark gray curve (behind the orange points) corresponds to T_1 as extracted from fitting the decay of p_1 (where p_1 is not shown), to compare with the dashed black curve exponential fit of $s(\tau)$. (b) Relative error vs flux for the two measurement techniques. Black dashed line indicates 1% relative error. (c) Energy spectrum of qubit B2 for the first $N = 6$ levels. The vertical shaded region corresponds to the biases with the highest relative errors, as observed in panel (b). This is symmetric in flux, though we only shade between $\Phi_{\text{ext}}/\Phi_0 = 0, 0.5$.

namics. We plot the relative error as T_1 extracted from a single exponential fit to p_1 [dark gray curve in Fig. 16(a)(iii)], here labeled as T_1 , minus the decay rate as extracted from a single exponential fit to $s(\tau)$ [our simulated signal influenced by the resonator, corresponding to the black dashed curve in Fig. 16(a)(iii)], here T_1'' , divided by T_1 [Fig. 16(b)]. We see the highest relative error values between bias values of $\Phi_{\text{ext}}/\Phi_0 = 0$ and ≈ 0.2 [Figs. 16(a)(iii) and 16(b)], with a maximum error of about 15% observed for qubit B2. This high relative error indicates that it is indeed necessary to consider the response of the readout resonator for these qubit parameters. Though qubits with different Hamiltonian parameters

may be less sensitive to this effect [19], we choose to apply the same model considerations to all qubits in this dataset.

3. Effect of population leakage on heralded T_1 measurements

In this appendix, we address the potential error in our measured T_1 data due to population leakage outside the first two energy levels of the qubit, assuming the data were measured with the heralded scheme described in Appendix B 1. To model this, we fit an exponential function to the decay $p_1(\tau)$ under two cases: total population misassignment to the $|0\rangle$ state of the qubit and total population misassignment to the $|1\rangle$ state of the qubit.

In the case where all higher-level population is assigned to the $|0\rangle$ state, the decay of p_1 is unaffected and the single exponential fit used to extract T_1 is identical to the case of perfect population assignment. We therefore compare the values of extracted T_1 as obtained from fitting p_1 with and without all higher-level populations assigned to p_1 [Fig. 16(a)(ii)]. We plot the relative error as T_1 extracted from a single exponential fit to p_1 with perfect population assignment, here labeled as T_1 , minus the decay rate as extracted from a single exponential fit to p_1 with all higher-state populations misassigned to it (T_1'), divided by T_1 [Fig. 16(b)]. We see a relative error of up to 13% due to population misassignment in this measurement scheme, with the largest relative errors occurring between $\Phi_{\text{ext}}/\Phi_0 = 0$ and 0.3.

Heralded measurements were used in this experiment when the qubit transition energy $\hbar\omega_{01}$ became small relative to the thermal energy $k_B T$, around $\omega_{01}/2\pi \approx 1$ GHz, or $0.7 \gtrsim \Phi_{\text{ext}}/\Phi_0 \gtrsim 0.3$ on the qubits in this dataset. In that region, the relative error from population misassignment in this scheme is significantly lower, on the order of 1%. Though population misassignment could have occurred in this measurement scheme, we conclude that it is likely a small effect on the measured T_1 values. We therefore choose to neglect the impact of this error in our analysis.

APPENDIX G: STATISTICAL TESTING

We use statistical testing to quantitatively compare the Q_C^{eff} distributions. As our distributions are non-Gaussian distributed, a natural choice would be to use a nonparametric test such as the Kolmogorov-Smirnov (KS) [64,65] test or the Mann-Whitney U (MWU) test [66]. Crucially, however, the presence of TLS features that will be unique to each qubit's local microscopic environment implies that the underlying distributions of Q_C^{eff} in the well-sampled limit will never be identical qubit to qubit. We therefore choose to apply statistical comparison methods that do not rely on the assumption that all moments of the compared distributions are identical. This criterion rules out the KS test, which does have this hypothesis. The MWU test would still be an appropriate choice; however, it loses interpretability for distributions with distinct shapes. Given the variation we observe in our distributions, we favor using Welch's two-sided t -test for its interpretability as a comparison of means.

Welch's two-sided t -test is a statistical test comparing the difference in means of two distributions, accommodating for the unique variance of the distributions [46]. The test calcu-

lates the test statistic t_0 , defined as

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}, \quad (\text{G1})$$

where $\bar{x}_1$ and $\bar{x}_2$ are the means of compared distributions, σ_1^2 and σ_2^2 are their variances, and n_1 and n_2 are the number of their samples.

The test also calculates the degrees of freedom ν , defined in this case as

$$\nu = \frac{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)^2}{\frac{\left(\frac{\sigma_1^2}{n_1}\right)^2}{n_1-1} + \frac{\left(\frac{\sigma_2^2}{n_2}\right)^2}{n_2-1}}. \quad (\text{G2})$$

Using the t -statistic and the degrees of freedom, the p value is calculated from the t -distribution probability density function (PDF), given by

$$f(t, \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{t^2}{\nu}\right)^{-(\nu+1)/2}, \quad (\text{G3})$$

where Γ is the gamma function. The t -distribution PDF is a symmetric function about $t = 0$, greatly resembling a bell curve of mean zero and variance one, but typically with more weight in the tails of the distribution. As ν increases, the t -distribution PDF approaches the normal distribution.

The p value is calculated as the integral of $f(t, \nu)$ from values of $\pm t_0$ to $\pm\infty$. Graphically, it is the area in the tails of the distribution, defined for the regions above t_0 and below $-t_0$. If a distribution is compared to itself under this test, the t -statistic $t_0 = 0$ and the p value would therefore be equal to 1. For two distinct distributions, a small p value indicates that the observed difference between the distributions is unlikely to be random, indicating that according to the test, their underlying distributions are different.

It is additionally insightful to calculate the CI of the t -statistic: The range of values expected to contain the true t_0 of the distribution, to a set degree of confidence. The CI is calculated as

$$\pm t_{\alpha/2, \nu} \times \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}, \quad (\text{G4})$$

where $t_{\alpha/2}$ is the critical t value associated with a set level of confidence $(1 - \alpha) \times 100\%$, for a PDF corresponding to ν degrees of freedom. The critical t value is the value at which the associated PDF has the integrated area $\alpha/2$ under its curve in the regions defined as above the value $t_{\alpha/2}$ and below the values $-t_{\alpha/2}$. A CI that contains the value zero indicates that, to the set level of confidence, the test cannot claim that their means are distinct. A CI containing zero will correspond to a p value above the threshold corresponding to the same set confidence level. A CI that contains only negative or only positive values indicates that the compared sample distribution means are different up to the confidence value of the test. Negative (positive) values indicate the mean of distribution 1 is less (more) than the mean of distribution 2.

We report here the test details for pairwise comparisons of all qubits. We show the p values, t -statistic, and degrees of freedom for this comparison in Fig. 17 and 95% CIs in Table III. CIs for each pairwise comparison are reported as

TABLE III. Confidence interval of 95% significance level for Welch's t -test pairwise comparison of devices in the main text. Intervals are reported as the percentage of the column qubit's Q_C^{eff} distribution mean, where negative (positive) values indicate the row qubit mean is lower (higher) than the column qubit mean.

	$\overline{A1}$	$\overline{A2}$	$\overline{A3}$	$\overline{A4}$	$\overline{A5}$	$\overline{B1}$	$\overline{B2}$	$\overline{B3}$
A1	$0\% \pm 16\%$	$-13\% \pm 17\%$	$7\% \pm 17\%$	$33\% \pm 20\%$	$100\% \pm 26\%$	$-25\% \pm 11\%$	$11\% \pm 19\%$	$-18\% \pm 14\%$
A2	$15\% \pm 20\%$	$0\% \pm 20\%$	$23\% \pm 21\%$	$53\% \pm 26\%$	$131\% \pm 35\%$	$-14\% \pm 14\%$	$28\% \pm 23\%$	$-5\% \pm 17\%$
A3	$-6\% \pm 16\%$	$-19\% \pm 17\%$	$0\% \pm 16\%$	$24\% \pm 19\%$	$87\% \pm 24\%$	$-30\% \pm 11\%$	$4\% \pm 18\%$	$-23\% \pm 13\%$
A4	$-25\% \pm 15\%$	$-35\% \pm 17\%$	$-20\% \pm 16\%$	$0\% \pm 19\%$	$50\% \pm 23\%$	$-44\% \pm 10\%$	$-17\% \pm 18\%$	$-38\% \pm 13\%$
A5	$-50\% \pm 23\%$	$-57\% \pm 15\%$	$-47\% \pm 13\%$	$-34\% \pm 16\%$	$0\% \pm 17\%$	$-63\% \pm 9\%$	$-45\% \pm 16\%$	$-59\% \pm 11\%$
B1	$34\% \pm 15\%$	$16\% \pm 17\%$	$43\% \pm 15\%$	$78\% \pm 19\%$	$168\% \pm 23\%$	$0\% \pm 10\%$	$48\% \pm 18\%$	$10\% \pm 13\%$
B2	$-10\% \pm 17\%$	$-22\% \pm 18\%$	$-3\% \pm 18\%$	$20\% \pm 22\%$	$81\% \pm 28\%$	$-33\% \pm 12\%$	$0\% \pm 20\%$	$-26\% \pm 15\%$
B3	$21\% \pm 17\%$	$5\% \pm 18\%$	$30\% \pm 17\%$	$61\% \pm 21\%$	$142\% \pm 28\%$	$-9\% \pm 12\%$	$34\% \pm 20\%$	$0\% \pm 15\%$

a percent of the mean of the column qubit Q_C^{eff} distribution. We note that we observe CI that are generally wide compared to this mean, indicating that the test cannot easily distinguish between individual qubits. Taken together, the p values and CIs indicate that Welch's t -test identifies differences between distributions as small as approximately 20% of a qubit's mean. From these features, we conclude that our analysis protocol is sensitive enough to resolve differences at this level, and that the distribution of Q_C^{eff} between qubits is quite similar.

We also compare between fabrication processes using the combined distributions of qubits A1, A2, and A3 as one distribution, and the combined distributions of qubits B1, B2, and B3 as the other distribution. We report the 95% confidence intervals for these comparisons in Table IV. We additionally report that for this comparison, the p value was 0.001, the t -statistic $t_0 = -3.22$, and $\nu = 889.6$. This indicates that the pad etch treatment of process B only slightly improves the mean Q_C^{eff} as compared to process A.

APPENDIX H: Q_C^{eff} DEPENDENCE ON P_{JJ}

Here, we investigate whether there is a trend in the mean Q_C^{eff} with the area of the small JJ, expressed as its percent contribution to the total qubit capacitance P_{JJ} . To calculate P_{JJ} , we assume a junction-specific capacitance of $49 \text{ fF}/\mu\text{m}^2$ as extracted by Ref. [37] for qubits utilizing the same fabrication process as our process A. Across designs 1–5, the single junction size of the qubit was varied to give the range in E_J reported in Table I. The varied range in E_J corresponds to approximately an 11%–18% range of the single junction contribution to the total qubit circuit capacitance. We plot the

TABLE IV. Confidence interval of 95% significance level for Welch's t -test pairwise comparison of combined process distributions in the main text. Intervals are reported as the percentage of the column qubit's Q_C^{eff} distribution mean, where negative (positive) values indicate the row qubit mean is lower (higher) than the column qubit mean.

	$\overline{A}$	$\overline{B}$
A	$0\% \pm 10\%$	$-16.0\% \pm 9.7\%$
B	$13.8\% \pm 8.4\%$	$0\% \pm 8\%$

extracted Q_C^{eff} distributions versus their small JJ capacitance participation ratio in Fig. 18. Modeling the total qubit quality factor Q_C^{eff} as

$$1/Q_C^{\text{eff}} = P_{JJ}/Q_{JJ} + (1 - P_{JJ})/Q_{\text{other}}, \quad (\text{H1})$$

where Q_{JJ} is the materials quality factor of the junction and Q_{other} is the materials quality factor of the rest of the circuit, we map between our measured distributions and Q_{JJ} , assuming $Q_{\text{other}} \gg Q_{JJ}$. Lines to guide the eye for different values of Q_{JJ} are included in Fig. 18.

We observe a weak trend in the distribution maximum Q_C^{eff} , as well as the distribution mean, with the junction capacitance participation ratio for process A. This is somewhat expected—there will be a contribution from the metal-substrate interface that was nominally improved by process B in this study that scales with JJ size, and other measurements of JJ barrier quality give similar effective Q_{JJ} [24]. We note that qubits A1, A2, and A3 were fabricated on the same sample, with A4, A5 on a second sample, both from the same wafer. We do not expect a high variation in performance from sample to sample on the same wafer in our fabrication process. We note that this study did not seek to quantify the single-junction branch contribution to the qubit Q_C^{eff} .

APPENDIX I: COMPARISON ASSUMING FREQUENCY INDEPENDENT Q_C^{eff}

In the main text and the majority of analysis, we choose to add a phenomenological frequency dependence to the dielectric quality factor introduced in literature [15,17,18,41], as it more closely matches the trends in our data. Here, we compare the distributions of qubit Q_C^{eff} without this frequency dependence (Fig. 19).

We perform the identical analysis described in the main text and Appendixes E, E 1, and G, substituting $\epsilon = 0$ into Eq. (4). The means, medians, standard deviations, and self-confidence intervals for individual qubit distributions and the combined process distributions are reported in Table V. We see generally higher values of each mean, consistent with the understanding that the value of ϵ used in the main text decreases the Q_C^{eff} values near $\Phi_{\text{ext}}/\Phi_0 = 0.5$. We apply Welch's t -test to these distributions, calculating the 95% CI of the distributions as compared to themselves to obtain an aver-

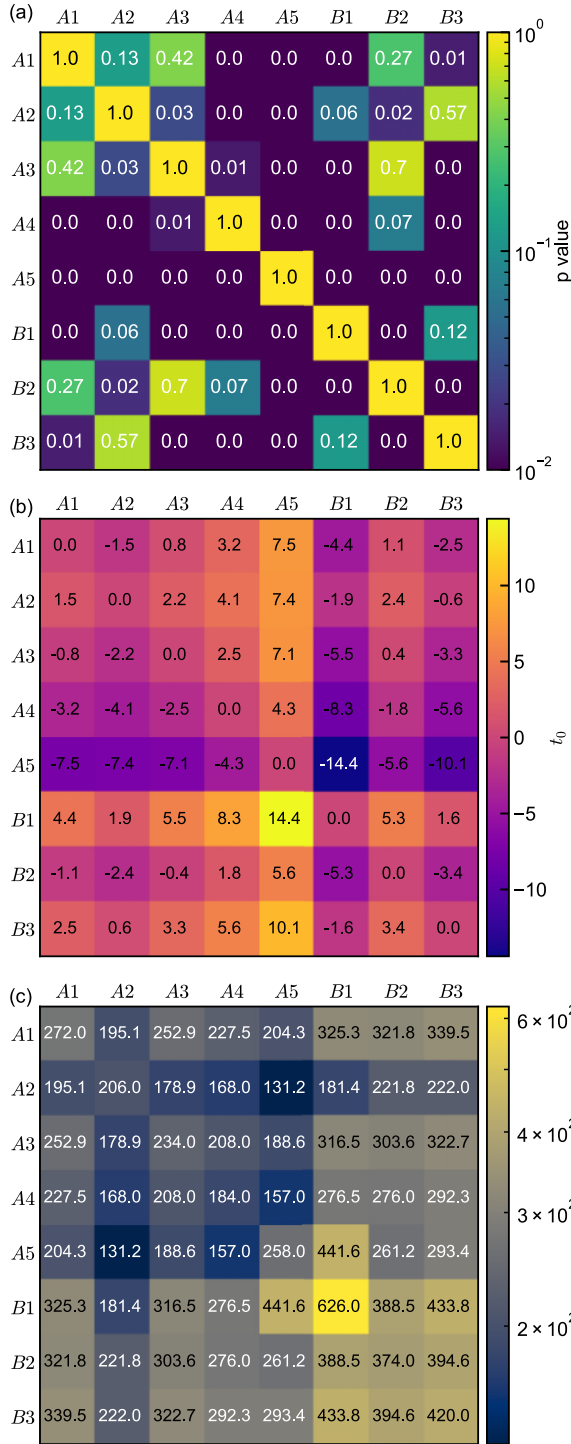


FIG. 17. Supplemental parameters from Welch's t -test for pairwise comparisons of all qubit Q_C^{eff} distributions in the main text (Fig. 4). (a) p values for all pairwise comparisons. For a 95% threshold, a p value of 0.05 or greater indicates similarity in mean according to this test. Values lower than 0.05 indicate that this test finds the distribution means to be distinct. Values less than 0.01 are reported here as 0.0. (b) t -statistic t_0 for all pairwise comparisons. We note that the matrix is symmetric about the diagonal if the t_0 value is multiplied by -1 . (c) Degrees of freedom ν for all pairwise comparisons.

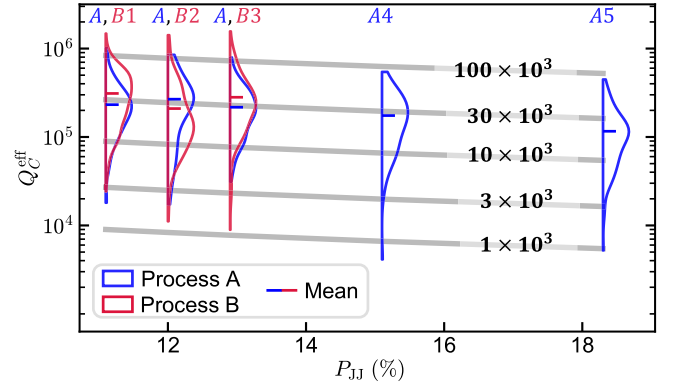


FIG. 18. Qubit Q_C^{eff} distributions vs small JJ participation ratio. The area of each distribution is normalized to show the percent of counts at each Q_C^{eff} . Horizontal lines for different values of Q_JJ are meant to guide the eye.

age test sensitivity of $\pm 18\%$ of an individual qubit's mean (Table V).

Combining the distributions of A1, A2, and A3 and B1, B2, and B3, we compare the two fabrication processes used in this study. Welch's t -test reports these distributions to be different, with a 95% confidence interval indicating that mean of distribution of B qubits is higher by $16.2\% \pm 11.0\%$ of the mean of the corresponding A qubits. This is consistent with the main text result that process B did reduce the loss from the metal-substrate interface, but that this interface is not the dominant limitation to the performance of these fluxoniums.

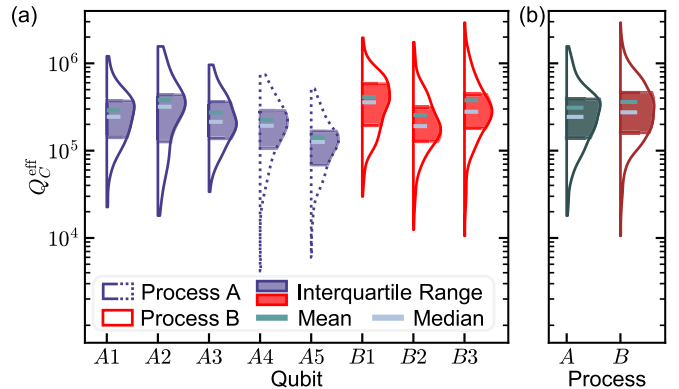


FIG. 19. Frequency-independent Q_C^{eff} analysis: qubit Q_C^{eff} distributions, where under Eq. (4), $\epsilon = 0$. The area of each distribution is normalized to show the percent of counts at each Q_C^{eff} . (a) Individual qubit Q_C^{eff} distributions. Blue color corresponds to qubits fabricated with process A; red corresponds to process B. Shaded regions within individual distributions represent the interquartile range of the dataset. Overlaid teal and gray lines mark the mean and median of each distribution, respectively. (b) Combined distributions of qubits A1, A2, and A3 marked as process A (dark teal), and B1, B2, and B3 (process B, dark red) are used to compare like designs across fabrication processes. Dashed outlines in panel (a) indicate exclusion from the combined distributions shown in panel (b).

TABLE V. Frequency independent Q_C^{eff} analysis: Q_C^{eff} distribution summary for each qubit and for the combined process distributions, with $\epsilon = 0$. Means, medians, and standard deviations σ of each qubit distribution are reported. 95% CI of the mean as returned by Welch's t -test for each Q_C^{eff} distribution as compared to itself, expressed as a percent of the distribution mean. This interval demonstrates the sensitivity with which this method can be used to resolve differences between distributions.

Qubit	Mean Q_C^{eff}	Median Q_C^{eff}	σ	95% CI
A1	2.90×10^5	2.44×10^5	2.05×10^5	$\pm 17\%$
A2	3.80×10^5	3.19×10^5	3.33×10^5	$\pm 24\%$
A3	2.73×10^5	2.14×10^5	1.95×10^5	$\pm 18\%$
A4	2.26×10^5	1.93×10^5	1.52×10^5	$\pm 20\%$
A5	1.41×10^5	1.26×10^5	0.99×10^5	$\pm 17\%$
B1	4.12×10^5	3.58×10^5	2.84×10^5	$\pm 11\%$
B2	2.56×10^5	1.91×10^5	2.36×10^5	$\pm 18\%$
B3	3.78×10^5	2.79×10^5	3.49×10^5	$\pm 18\%$
A	3.11×10^5	2.44×10^5	2.50×10^5	$\pm 12\%$
B	3.61×10^5	2.75×10^5	2.99×10^5	$\pm 9\%$

APPENDIX J: COMPARISON USING A TWO-LEVEL MODEL

The analysis presented in the main text utilizes a six-level model to describe the energy relaxation dynamics in the fluxonium qubits. For comparison with literature, here we perform the identical analysis assuming the population dynamics remain solely within the qubit states $|0\rangle$ and $|1\rangle$.

We again apply the 8 MHz binned average to the dataset, and transform the downselected dataset into a two-level Q_C^{eff} . As done in the main text, we combine three models to describe

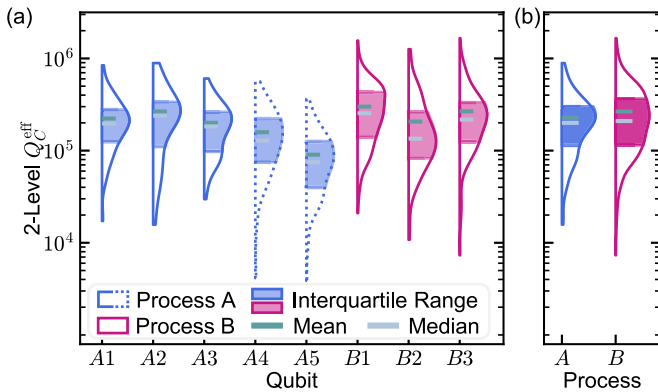


FIG. 20. Two-level analysis: Comparison of the extracted Q_C^{eff} distributions. The area of each distribution is normalized to show the percent of counts at each Q_C^{eff} . (a) All individual qubit Q_C^{eff} distributions. Blue distributions correspond to qubits fabricated with process A; pink distributions correspond to process B. Shaded regions within device distributions represent the interquartile range of the dataset, with overlaid teal and gray lines marking the mean and median of each distribution, respectively. (b) Combined distributions of qubits A1, A2, and A3, marked as process A (dark blue), and B1, B2, and B3 (process B, dark pink) are used to compare like designs across fabrication processes. Dashed outlines in panel (a) indicate exclusion from the combined distributions shown in panel (b).

TABLE VI. Total number of T_1 measurements taken across the flux range of each qubit, number of T_1 points after 8 MHz binned average, number of points in each extracted multilevel Q_C^{eff} distribution, and in the two-level Q_C^{eff} distributions. Fewer points in the Q_C^{eff} column than the “Postbinning” column indicate that some points were limited by the models used for $1/f$ flux noise or spontaneous emission to the control lines. Differences in the six-level Q_C^{eff} and two-level Q_C^{eff} indicate that the model description is important for classifying whether points are limited by the dielectric loss model, according to the imposed rate requirements detailed in the text.

Qubit	No. T_1	No. Postbinning	No. Six-level Q_C^{eff}	No. Two-level Q_C^{eff}
A1	209	144	137	137
B1	530	314	314	314
A2	155	107	104	104
B2	398	188	188	188
A3	257	120	118	119
B3	427	213	211	211
A4	221	102	93	93
A5	270	135	130	130

the measured T_1 versus device transition frequency: capacitive dielectric loss, $1/f$ flux noise, and radiative loss to the microwave environment. We invert the following addition of rates:

$$\Gamma_1^{\text{meas}} = \Gamma_1^C + \Gamma_1^\Phi + \Gamma_1^P + \Gamma_1^{\text{cd}} + \Gamma_1^{\text{fd}}, \quad (\text{J1})$$

where Γ_1^{meas} is the measured relaxation rate, to solve for Q_C^{eff} (contained within Γ_1^C). We find under this model that a phenomenological frequency dependence of the quality factor as described by Eq. (4) describes the data best.

The number of points in the distributions before and after the binning is applied is the same as in the case of the N -level analysis. We summarize the number of points at each analysis step in Table VI. A main difference in this analysis from that

TABLE VII. Two-level analysis: Q_C^{eff} distribution summary for each qubit and for the combined process distributions. Means, medians, and standard deviations σ of each qubit distribution are reported. 95% CI of the mean as returned by Welch's t -test for each Q_C^{eff} distribution as compared to itself, expressed as a percent of the distribution mean. This interval demonstrates the sensitivity with which this method can be used to resolve differences between distributions.

Qubit	Mean Q_C^{eff}	Median Q_C^{eff}	σ	95% CI
A1	2.23×10^5	1.97×10^5	1.43×10^5	$\pm 15\%$
A2	2.66×10^5	2.39×10^5	1.98×10^5	$\pm 20\%$
A3	2.00×10^5	1.83×10^5	1.22×10^5	$\pm 16\%$
A4	1.58×10^5	1.28×10^5	1.11×10^5	$\pm 20\%$
A5	0.90×10^5	0.75×10^5	0.65×10^5	$\pm 18\%$
B1	3.00×10^5	2.54×10^5	2.05×10^5	$\pm 11\%$
B2	2.07×10^5	1.35×10^5	2.05×10^5	$\pm 20\%$
B3	2.67×10^5	2.17×10^5	2.10×10^5	$\pm 15\%$
A	2.28×10^5	2.00×10^5	1.57×10^5	$\pm 10\%$
B	2.66×10^5	2.09×10^5	2.10×10^5	$\pm 10\%$

in the main text is the shape and magnitude of the limitation due to Purcell loss through the readout resonator, though we see no difference in the number of points included between our two model descriptions (Fig. 12).

Points for which the predicted rate from the combination of the two-level models for flux noise and radiative losses is greater than 10% of the measured rate were removed from this analysis. All remaining data are assumed to be limited by capacitive dielectric loss. We find that $\epsilon = 0.2$ best describes the extracted Q_C^{eff} for all qubits, which is slightly different than $\epsilon = 0.25$ that fits the dataset under the six-level analysis. The distributions are then compared to each other using Welch's t -test (Fig. 20). We observe similar results to the main text under this two-level analysis. The extracted Q_C^{eff} distributions are again largely overlapped and non-Gaussian distributed, with a measured range of similar width. We calculate the 95% self-CI of the distributions and obtain an average test

sensitivity of $\pm 17\%$ of an individual qubit's mean (Table VII). Comparing designs within process A, we observe a similar, weak trend in the Q_C^{eff} mean with E_J as shown for the six-level model in Appendix H.

We combine the distributions of A1, A2, and A3 and those of B1, B2, and B3 to compare the like designs across fabrication processes. Welch's t -test finds these two composite distributions to be distinct, with a 95% confidence interval that the mean of distribution of B qubits is higher by $16.6\% \pm 9.8\%$ of the mean of the corresponding A qubits. Under this analysis, we again see that the Process B did reduce the loss from the metal-substrate interface as compared to Process A. This small improvement indicates that the metal-substrate interface is not dominating the dissipation in these fluxonium. Though this analysis utilizes a less well-motivated description of the decay dynamics than in the main text, we obtain results consistent with the main text conclusions.

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