

Dirac bilayer metasurfaces as an inverse Gires-Tournois etalon

Ki Young Lee^{1,2}, Kwang Wook Yoo,¹ Francesco Monticone,³ and Jae Woong Yoon^{1,2,*}¹Department of Physics, *Hanyang University*, Seoul 04763, South Korea²Research Institute for Natural Sciences, *Hanyang University*, Seoul 04763, South Korea³School of Electrical and Computer Engineering, *Cornell University*, Ithaca, New York 14853, United States

(Received 5 October 2023; accepted 25 September 2025; published 17 October 2025)

Efficient transmissive pure-phase resonances are highly desirable for optical modulation and wave front engineering. Here, we propose a principle to realize a pure-phase resonance in an extremely broad transmission band, as opposed to previous approaches restricted to operating in reflection mode or over a narrow spectral band. We show that a glide-symmetric bilayer metasurface mathematically mimicking a two-dimensional Dirac semimetal induces unidirectional guided-mode excitation and perfect leakage-radiation blazing at the transmission channel. These effects create a peculiar resonant-scattering configuration, similar to the classical reflective Gires-Tournois etalon, but in transmission, providing full 2π phase modulation with constant transmittance near 100%. Most importantly, this effect persists over an extremely wide band, associated with topological effects. Hence, our proposed approach produces a spectrally and parametrically robust pure-phase resonance effect in transmission, which is highly beneficial for practical applications.

DOI: [10.1103/3dw5-nvdq](https://doi.org/10.1103/3dw5-nvdq)

I. INTRODUCTION

Nanophotonic phase-delay modulation enables various optical devices including high-speed phase shifters [1], solid-state beam scanners [2,3], three-dimensional (3D) display pixels [4,5], spatial light modulators [6,7], orbital-angular-momentum generators [8], metasurfaces [9], and many others. In order to obtain remarkably large phase modulation from subwavelength thin-film structures, resonant excitation of localized states is typically required to produce a phase-delay range greatly enhanced from the nonresonant spectral background. Unfortunately, resonant phase delay is usually accompanied by abrupt intensity variations, which is detrimental for most applications.

A pure-phase resonance is a peculiar effect in which a phase delay can be realized over a full 2π range, as a function of certain parameters (e.g., frequency or structural parameters) while keeping the intensity constant, thereby providing an ideal mechanism for nanophotonic optical-phase control devices. A Gires-Tournois (GT) etalon is a well-known classical system that produces pure-phase resonances [10]. An ideal GT etalon consists of two parallel lossless mirrors, one is partially reflecting and the other is totally reflecting. The existence of a totally reflecting mirror makes the reflectivity of the system fixed at 100%, while the reflection phase delay across each interferometric resonance changes over a 2π range. An

identical effect is obtainable in general if any lossless resonance mechanism is included in a totally reflecting optical system. Therefore, nanophotonic pure-phase resonance structures in such configurations have been widely investigated and developed so far [11–14]. However, the effect is produced only in reflection, causing various technical complications in practice, such as the need for additional redirecting optical paths, alignment issues, efficiency degradation, and optical window restriction.

For these reasons, transmissive pure-phase resonances are of great interest and some promising structures have been previously proposed in the literature. Two main principles have been investigated in this context. One is based on the combination of spectrally overlapped electric and magnetic dipole resonances in metasurface structures, i.e., periodic arrays of scatterers, known as Huygens metasurfaces [15–18]. In this scenario, interference between electric and magnetic dipole scattered fields suppresses the backward radiation while enhancing the forward scattering. Combined with high nonresonant transmission through a materially sparse structure, the transmission efficiency can remain remarkably high at a fixed frequency while the transmission phase can span the full 2π range by varying some structural parameter.

Another principle to realize a pure-phase resonance response in transmission is based on the concept of quasibound states in the continuum and the associated polarization vortices in thin-film photonic crystal structures [19–23]. In this approach, certain parametric tuning associated with the vertical asymmetry of the structure slightly perturbs a bound state in the continuum (BIC) so that it starts leaking radiation toward a preferred single-radiation channel, hence causing an asymmetric scattering effect similar to the hybrid electric-magnetic dipole resonances if the preferred radiation channel is on the transmission side.

*Contact author: yoonyjw@hanyang.ac.kr

Although these approaches are capable of producing almost ideal pure-phase resonances in transmission, they are fundamentally narrow-band effects and their parametric sensitivity is inherently high because their intricate coupling configurations must be critically balanced. These features substantially restrict their practical applicability.

In this paper, we propose a principle to realize a pure-phase resonance effect in transmission, which persists across a remarkably wide photonic band and is parametrically robust. Specifically, our approach is based on a glide-symmetric bilayer metasurface with its synthetic band topology being identical to a two-dimensional (2D) Dirac semimetal [24,25]. We find that its characteristic eigensystem structure at a critical topological phase implies anomalous non-Hermitian orthogonality and extreme asymmetry in leakage-radiation distributions. These unique topological properties give rise to unidirectional resonance excitation and perfect leakage-radiation blazing at a single preferred radiation channel over nearly an entire photonic band, resulting in an extremely broad pure-phase resonance band in transmission. We provide a comprehensive theory and full-wave numerical analyses to elucidate this intriguing effect. Finally, we discuss potential applications of the proposed effect. An ultrathin transmissive phase modulator should be an immediate promising application by employing appropriate index-tuning mechanisms such as thermo-optic or electro-optic effects in dielectric media, and electrical carrier effect in semiconductor platforms. In addition to such a rather straightforward application, we present a decent numerical demonstration of a focusing metasurface structure as a rudimentary example of wave front engineering in transmission even in the presence of transversal nonlocality, which is generally inherent in any guided-mode resonance structures including our proposed cases here.

II. GENERAL FEATURES OF PURE-PHASE RESONANCES

In a resonant scattering problem, the outgoing wave complex amplitude at an arbitrary radiation port can be expressed as

$$A(\omega) = A_c + A_d(\omega) = A_c + a_d \frac{i\Gamma}{(\omega - \omega_0) + i\Gamma}, \quad (1)$$

where A_c is the amplitude of the nonresonant pathway through a broad continuum state, $A_d(\omega)$ is the amplitude of the resonant pathway through a narrow discrete state, ω is the frequency of the incident wave, ω_0 is the center frequency of the discrete state excitation, and Γ is the dissipation rate of the discrete state. This expression provides a simplified physical picture (valid for a single narrow resonance) according to which the nonresonant amplitude A_c forms the spectral background and its interference with the resonant amplitude $A_d(\omega)$ produces characteristic Fano line shapes in the intensity and phase of $A(\omega)$. Note that $A_d(\omega)$ is expressed in terms of resonance-excitation strength coefficient a_d and normalized Lorentzian function $i\Gamma(\omega - \omega_0 + i\Gamma)^{-1}$.

An ideal pure-phase resonance appears when the coupling configuration of a system satisfies the condition

$$a_d = -2A_c, \quad (2)$$

so that Eq. (1) reduces to

$$A(\omega) = -A_c e^{2i\alpha(\omega)}, \quad (3)$$

where $\alpha(\omega) = \tan^{-1}[(\omega - \omega_0)/\Gamma]$. Clearly, Eq. (3) implies a frequency-independent, constant intensity, $I = |A|^2 = |A_c|^2$, and an abrupt 2π phase shift following $\arg(A) = \pi + \arg(A_c) + 2\tan^{-1}[(\omega - \omega_0)/\Gamma]$ while ω varies around ω_0 over a sufficiently wide spectral region with respect to the resonance linewidth Γ . In this respect, creating a pure-phase resonance is simply the problem of finding a resonance structure where A_c (nonresonant) and a_d (resonant) are appropriately balanced at a certain desired port to satisfy Eq. (2).

Such a condition can be easily obtained for a nonabsorbing or largely overcoupled resonance state combined with a single port corresponding to a single input channel and a single output radiation channel (for example, a cavity with only a reflection channel, and no transmission, as in a GT etalon). In this case, temporal coupled-mode theory predicts $|\kappa|^2 = 2\Gamma$ and $a_d = -\kappa^2/\Gamma = -2\exp(2i\phi_\kappa)$ with $\phi_\kappa = \arg(\kappa)$, where κ is the coupling constant between the port radiation and the resonance modes. Subsequently, the pure-phase resonance condition in Eq. (2) reduces to $A_c = \exp(2i\phi_\kappa)$. This implies that the ideal pure-phase response for a single-radiation-channel resonance is obtained if $|A_c| = 1$ and $\arg(A_c) = 2i\phi_\kappa$. The strength condition of $|A_c| = 1$ is trivial for any nonabsorbing system with a single output radiation channel due to energy conservation. Favorably, the phase condition of $\arg(A_c) = 2i\phi_\kappa$ is also naturally satisfied in the single port case because the time-reversal symmetry requirement of the scattering process dictates so in general [26,27].

Consider the blue circle in Fig. 1(a) for a phasor diagram of a typical $A(\omega)$ in a single-port resonance case. $A(\omega)$ is a superposition of the frequency-independent A_c and the Lorentzian resonance amplitude $A_d(\omega)$, and it appears as a unit circle over which the phase shifts without any intensity change, as shown in Figs. 1(b) and 1(c). This is exactly the case for the classical Gires-Tournois etalon [10] and for modern reflective dielectric metasurfaces [11–14] in which the zero-order reflection channel is the only output radiation channel for both the resonant and nonresonant pathways.

For a transmissive element supporting a *single* resonance, unfortunately, the single-radiation-channel condition is not generally achievable because of the coexistence of reflection and transmission channels and the constraints imposed by the reciprocity and time-reversal symmetry of linear systems. The coexistence of two radiation channels is not simply a matter of imperfect intensity and phase distributions at some tolerable magnitudes. This intensity minimum is caused by a complete destructive interference between the nonresonant amplitude A_c and the resonant amplitude $A_d(\omega)$. As proved previously [26,27], the time-reversibility restriction for two output-port cases forces the phase difference between $A_d(\omega)$ and A_c to be π at a frequency point where $|A_d(\omega)| = |A_c|$, making the destructive interference between them perfect at the intensity minimum (i.e., zero). A phasor diagram of the typical $A(\omega)$ for a two-port resonance is illustrated as the red circle in Fig. 1(a). It necessarily passes through the origin and the associated spectral profiles involve a zero in the intensity and a π jump in the phase, seriously limiting its utility for phase modulation, as indicated in Figs. 1(b) and 1(c).

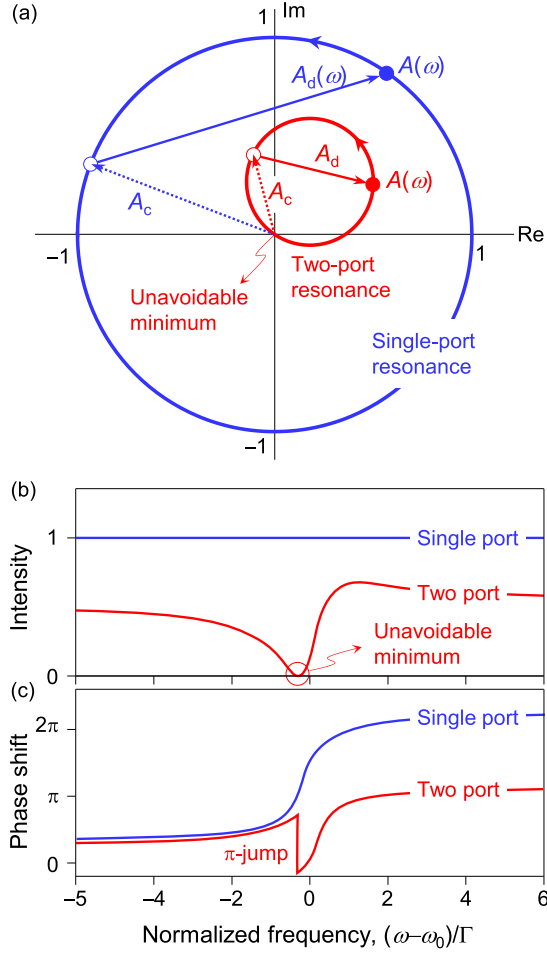


FIG. 1. General feature of pure-phase resonances. (a) Phasor diagrams of the typical $S(\omega)$ for a single-radiation port resonance and two-radiation-port resonance. S_c and S_d are the nonresonant and resonant amplitudes, respectively, and S is their linear superposition. (b), (c) Typical intensity $|S|^2$ and phase $\arg(S)$ spectra for the single-radiation-port and two-radiation-port resonances. The two-port resonance involves an unavoidable intensity zero and an associated π jump in the phase.

Considering these fundamental properties, an ideal transmissive pure-phase resonance is obtained only if the reflection channel is closed in both the nonresonant and resonant pathways. Previously, resonant reflection channel suppression has been obtained by using doubly resonant structures radiating in the same channels, e.g., arrays of nanoparticle meta-atoms where electric and magnetic dipole resonances are simultaneously excited [15–18]. In such structures, the backward scattering channel (reflection) can be suppressed by destructive interference between the electric and magnetic dipole radiation. In this way, one can design meta-atoms that cover the full 2π phase range in transmission, as a function of some structural parameters, while maintaining transmission intensity close to unity. However, this only works within a narrow spectral band in which the induced electric and magnetic dipole moment strengths and phases are coincidentally optimal. In addition, in order to produce an *ideal* transmissive pure-phase resonance, additional structural optimization and

tuning should be performed to suppress also the nonresonant reflection channel without unfavorably affecting the resonant reflection suppression condition. Therefore, the doubly resonant nanoparticle array approach is fundamentally narrow band and demands careful parametric tuning of the unit cell structures and array geometries to obtain high performance and a robust pure-phase resonance response in transmission.

In addition, we note that a transmissive pure-phase resonance is distinguished from induced transparency effects where a narrow transmission peak appears within a broad reflection-resonance band [28–30]. For an induced transparency effect in general, the transparency window includes a characteristic spectral change of phase delay in the transmission. However, this phase-delay change is inevitably correlated with sharp change in the transmission intensity in stark contrast to the transmissive pure-phase resonance where the phase-delay change is obtained without any change in the transmission intensity.

III. EXTREMELY BROAD PURE-PHASE RESONANCE BAND

As an approach to producing a broadband pure-phase resonance response in transmission, we consider guided-mode resonances in a glide-symmetric bilayer grating structure, as shown in Fig. 2(a). Two identical grating layers are shifted from each other by a distance $a\Delta$, where a is the period and Δ is the normalized glide shift. As conventional resonant gratings do, this structure produces narrow scattering resonances along dispersion bands at which any one of the diffraction orders has an in-plane phase distribution matching that of a leaky guided mode [25–27,31–34]. In the zero-order regime below the first-order Rayleigh frequency, a leaky guided mode couples to two radiation ports corresponding to the zero-order reflection and transmission plane-wave channels, and thereby gives rise to a two-port Fano-resonance profile with a typical response as indicated in Figs. 1(b) and 1(c).

Nevertheless, guided-mode resonances in the proposed structure, for certain optimal parameter values, produce an almost ideal pure-phase resonance band similar to what one would expect for single-port resonances, as shown in Figs. 2(b) and 2(c). See figure caption for the optimal parameter set that we considered for these results. In Fig. 2(b), we show the transmittance (T) and transmission phase (ϕ_t) spectra in the (in-plane) wave vector-frequency plane. In the lower vicinity of the first-order Rayleigh frequency $\nu_R(+1)$, we find a band of almost ideal pure-phase resonances, i.e., constant intensity at $T \approx 1$ with full phase range $\Delta\phi_t \approx 2\pi$. Remarkably, this resonance persists over an extremely broad spectral band from $0.19c/a$ (108 THz) to $0.38c/a$ (217 THz). This band is 1382-nm wide in wavelength and covers nearly the entire first-order guided-mode resonance band within the zero-order regime.

Figure 2(c) shows phasor diagrams for the complex transmission coefficient $t(\lambda)$ at selected angles of incidence. For $\lambda_0 = 1.55 \mu\text{m}$, $t(\lambda)$ appears as a unit circle representing an ideal pure-phase resonance. At wavelengths of 1.8 and $2.15 \mu\text{m}$, the resonant phasor circles deviate from the unit circle inward. Therefore, the transmission intensity is not strictly constant any longer but creates an asymmetric

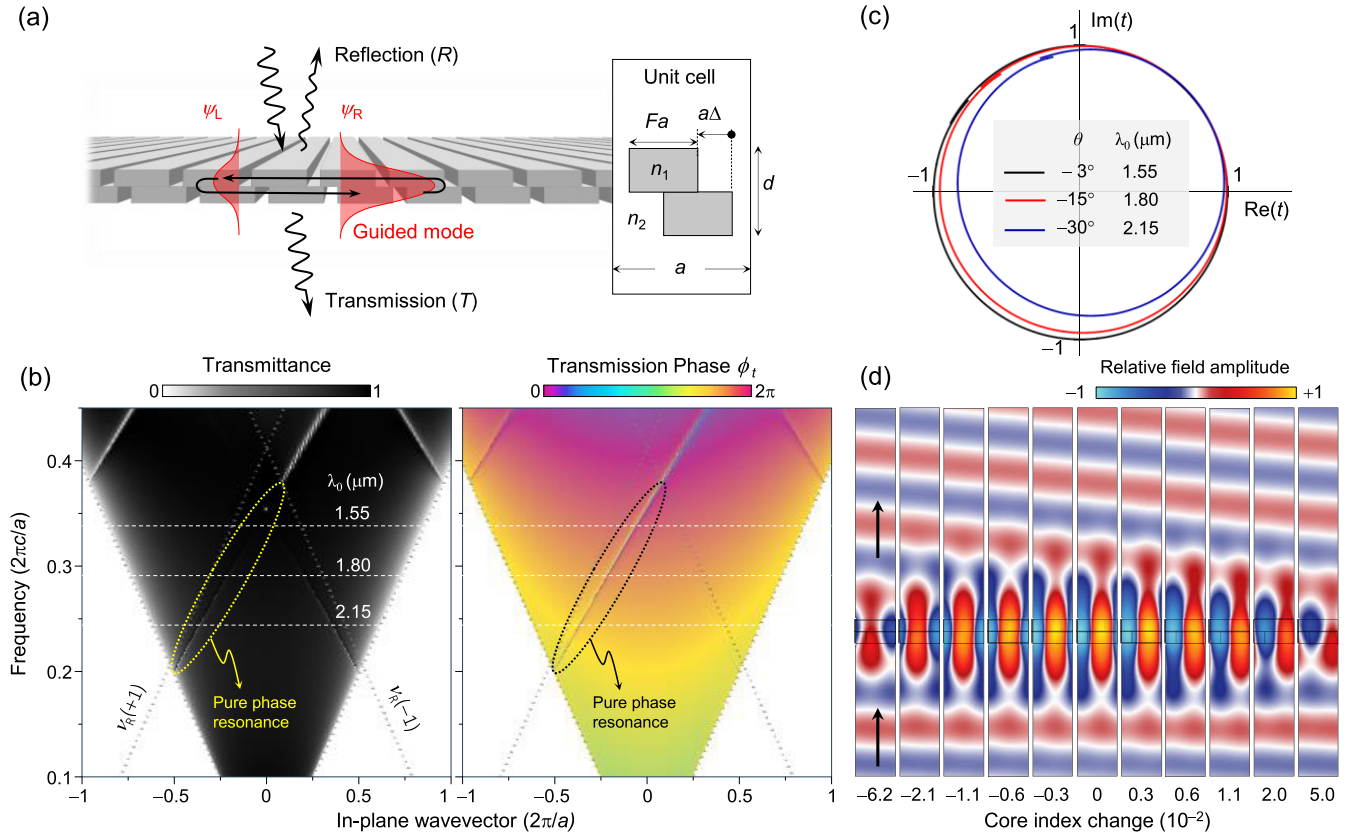


FIG. 2. Extremely broad pure-phase transmission resonance band in a glide-symmetric bilayer waveguide grating. (a) Schematic of the proposed structure. Two identical grating layers with period a , fill factor F , and core and cladding indices of refraction n_1 and n_2 are laterally shifted by a distance $a\Delta$. Guided modes propagating toward the left (ψ_L) and right (ψ_R) induce guided-mode resonances. (b) Transmittance (T) and transmission phase (ϕ_t) spectra for an optimal structure under transverse-electric (TE) polarized light incidence. The parameters are $a = 525$ nm, $F = 0.8$, $d = 200$ nm, $n_1 = 3.48$ (Si), $n_2 = 2.45$ (Si_3N_4), and $\Delta = 1/4$. $\nu_R(\pm 1)$ indicates the first-order Rayleigh frequency. (c) Phasor diagrams of transmission coefficient $t(\lambda)$ at several selected angles of incidence, in which the range of λ is ± 100 nm with respect to the center wavelength λ_0 . These three selected λ_0 's are indicated in panel (b) by the horizontal dotted linecuts. (d) Core-index-dependent transmitted wave front change at the pure-phase resonance for $\lambda = 1.55$ μm and $\theta = -3^\circ$.

Fano-resonance profile. This deviation results from imperfect satisfaction of Eq. (4)—the optimal thickness condition with respect to the wavelength. Although the transmission intensity is not constant, the magnitude of the intensity change is insignificant (within 20%).

If we define an operating wavelength range as the range supporting transmission resonances with the intensity minimum exceeding 50% while keeping a full 2π -phase shift, we obtain a 900-nm-wide range from 1.45 to 2.35 μm . Although the phase modulation should be operated around restricted conditions around the guide-mode dispersion band, one can obtain full 2π transmission-phase modulation over almost an 1- μm -wide spectral domain. Importantly, the obtained operation range is an order of magnitude wider than the typical 3%–5% fractional bandwidths obtained with previous approaches based on hybrid electric and magnetic dipole scattering (Huygens metasurfaces) [15,17] or polarization vortices associated with quasibound state in the continuum [21].

This almost ideal pure-phase resonance in transmission is applicable for efficient wave front control devices, e.g., metasurfaces, by suitably tuning its parameters. For example, we calculated transmitted field distributions as a function of core index change on the order of 10^{-2} and the results are shown in

Fig. 2(d). A full one-wavelength shift in the transmitted wave front is obtained without any significant change in the transmitted light intensity. Interestingly, this pure-phase resonance band appears to be accompanied by extreme asymmetry. Under TE illumination, it is produced only at the phase-matching condition for the guided mode (ψ_R) propagating toward the right ($+x$ direction), whereas no resonance feature is observed [see Fig. 2(b)] for the other guided mode (ψ_L) propagating toward the left ($-x$ direction).

The optimal parameter set to observe these effects is not obtained through blind optimization, but by requiring the system to satisfy the following conditions:

$$\text{Optimal film thickness } d \approx \pi j K_c^{-1}, \quad (4)$$

$$\text{Interlayer crossover coupling } \chi_1 = 0, \quad (5)$$

$$\text{Optimal glide shift } \Delta = \pm 1/4. \quad (6)$$

Here, j is a positive integer and $K_c = (\varepsilon_c k_0^2 - k_x^2)^{1/2}$ is the effective out-of-plane wave vector component with ε_c and k_x denoting the effective dielectric constant of the grating layer and the in-plane wave vector of incident light, respectively.

In addition, χ_1 denotes the amplitude of coupling between the left- and right-propagating guided modes through two sequential first-order diffractions, one in the lower grating layer and another in the upper grating layer, thereby we refer to it as the interlayer crossover coupling. See Sec. S1 in the Supplemental Material [35] for a detailed mathematical treatment leading to these conditions. If these conditions are satisfied, both the nonresonant and resonant reflection channels are simultaneously closed and the desired single-radiation channel condition is obtained.

Considering the three requirements in Eqs. (4)–(6) in further detail, Eqs. (4) and (5) in principle depend on frequency and in-plane wave vector k_x , and thereby result in restrictions in operation bandwidth and angular range. In particular, Eq. (5) for interlayer crossover coupling χ_1 reduces to a thickness condition as $d \approx 2\pi j' K_c^{-1}$, where j' is a positive integer. Therefore, Eq. (5) ensures Eq. (4) to be simultaneously satisfied, and thereby both conditions are restricted by identical frequency or k_x dependences. The thickness condition is simply the single-layer antireflection or equivalently weak Fabry-Pérot-resonance condition for the effective homogeneous film, which is much broader than Guided-Mode Resonance (GMRs) or Mie-type resonances in general. Please note that resonance bandwidth inversely scales with resonance quality factor Q , whose order of magnitude is typically 10^4 – 10^2 for conventional GMRs and merely 1 for weak single-layer Fabry-Pérot resonances.

The necessary conditions for the suppression of the resonant reflection channel are Eqs. (5) and (6), and they are associated with unique interaction configurations at a glide-symmetric non-Hermitian topological nodal phase, as we will explain in detail in the next section.

IV. NON-HERMITIAN TOPOLOGICAL PROPERTIES

In the spectral region of interest, guided modes propagating toward right (ψ_R) and left (ψ_L) are internally coupled to each other primarily through second-order diffraction processes and they are externally coupled to the radiation continuum primarily through first-order diffraction processes. Guided-mode eigenstates in the considered structure, at the vanishing interlayer crossover coupling condition in Eq. (5), can be described by a non-Hermitian binary eigenvalue problem

$$[\omega_c + i\gamma + (m + i\gamma)\cos(2\pi\Delta)\sigma_1 + vk_x\sigma_3]|\omega_{\pm}\rangle = \omega_{\pm}|\omega_{\pm}\rangle, \quad (7)$$

where σ_j is the Pauli matrix, ω_c is the center frequency of the second-order stop band, i.e., the frequency of the second-order Bragg condition for the guided modes, γ is the radiation rate of the guided modes in the absence of internal coupling, m is the internal coupling constant, v is the in-plane group speed of the guided modes, k_x is the Bloch wave vector, and $|\omega_{\pm}\rangle = [\psi_{R\pm} \ \psi_{L\pm}]^T$. See Sec. S1 in the Supplemental Material [35] for mathematical details of this description.

It is known from our recent work [24] that Eq. (7) allows us to treat the suggested glide-symmetric grating as a 2D Dirac semimetal, which has its nodal point at $\Delta = \pm 1/4$. The nodal point condition exactly coincides with the conditions in Eqs. (5) and (6), strongly suggesting that the

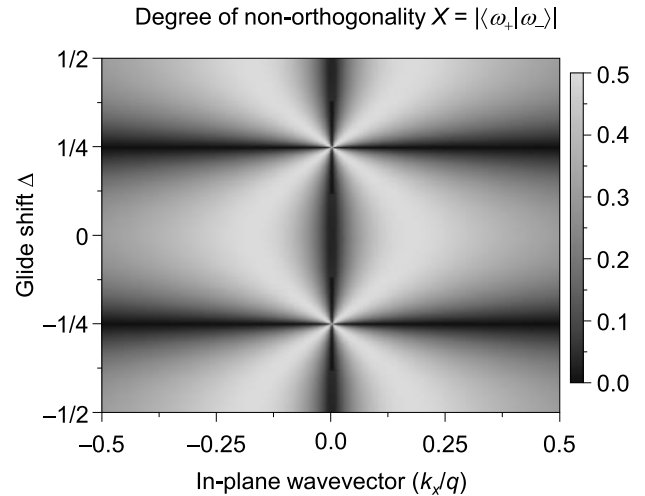


FIG. 3. Anomalous orthogonality of guided-mode eigenstates at the topological nodal phase. Degree of nonorthogonality as a measure of cross-projection magnitude of the two eigenstates is indicated as a function of glide-shift parameter Δ and in-plane wave vector k_x .

transmissive pure-phase resonance effect shares its underlying physics with 2D topological semimetals. Indeed, peculiar eigenstate properties and asymmetric leakage-radiation blazing effects provide a firm physical foundation for the almost ideal pure-phase resonances at the nodal-point condition.

First, the anomalous orthogonality of the eigenstates plays a key role. Eigenstates of non-Hermitian Hamiltonians are inevitably nonorthogonal in general scenarios. In our proposed structure, coupling of guided modes to the radiation continuum is an essential feature and the associated radiation loss results in non-Hermiticity, as parametrized by the leakage-radiation rate γ of the guided mode in Eq. (7). Therefore, at the critical topological phase for $m = 0$ and $\Delta = 0$, the conventional Dirac point splits into two exceptional points and the eigenstates become nonorthogonal in a similar manner as in conventional topological, non-Hermitian, guided-mode resonance effects [36–38].

For $\Delta = \pm 1/4$, however, inter-guided-mode coupling represented by the term with σ_1 in Eq. (7) vanishes. Consequently, the exceptional points merge into a Dirac point, the imaginary eigenvalue splitting vanishes to form a flat imaginary band, and the eigenstates become orthogonal throughout the entire frequency bands even in the presence of non-Hermiticity. We note that these atypical non-Hermitian properties are identical to those of 2D non-Hermitian topological semimetals [24,39].

We numerically confirmed the anomalous orthogonality in the considered case, as shown in Fig. 3(a). We indicate degree of nonorthogonality by taking the cross-projection magnitude $X = |\langle\omega_+|\omega_-\rangle|$ as a function of in-plane wave vector k_x and glide-shift parameter Δ . X vanishes throughout the entire k_x domain at $\Delta = \pm 1/4$. This property is an immediate consequence of the emergence of the Dirac point.

In further detail, Eq. (7) implies an expression for the eigenstates as

$$|\omega_{\pm}\rangle = N(k_x) \begin{bmatrix} \sqrt{\Delta\omega \pm vk_x} \\ \pm\sqrt{\Delta\omega \mp vk_x} \end{bmatrix}, \quad (8)$$

where

$$\Delta\omega = \frac{1}{2}(\omega_+ - \omega_-) = \sqrt{(vk_x)^2 + (m + i\gamma)^2 \cos^2(2\pi\Delta)}, \quad (9)$$

is the complex frequency splitting and $N(k_x) \approx |2\Delta\omega|^{-1/2}$ is a normalization constant. In the weak radiation-loss limit with $|m| \gg \gamma$, the degree of nonorthogonality is then expressed as

$$X = |\langle\omega_+|\omega_-\rangle| \simeq \frac{\gamma|vk_x \cos(2\pi\Delta)|}{(vk_x)^2 + m^2 \cos^2(2\pi\Delta)}. \quad (10)$$

This expression conveniently explains the features observed in Fig. 3(a). The orthogonal eigenstate condition ($X = 0$) is found at $k_x = 0$ (Γ point) and $\Delta = \pm 1/4$. These conditions are evident in Eq. (10) because $X \propto |k_x \cos(2\pi\Delta)|$. The orthogonality at the Γ point is also consistently understood by Eq. (8), which reduces to $|\omega_\pm\rangle = 2^{-1/2} [1 \pm 1]^T$ at $k_x = 0$ regardless of Δ value.

At $\Delta = \pm 1/4$, the orthogonality persists over the entire Brillouin zone even in the presence of radiation loss ($\gamma \neq 0$) and, hence, we refer to this effect as “anomalous orthogonality.” This is an immediate consequence of the decoupling between the right and left guided modes (ψ_R and ψ_L) at the topological nodal-phase condition. At this special condition, $\Delta\omega = |vk_x|$ according to Eq. (9) and its application to the eigenstate expression in Eq. (8) results in

$$|\omega_\pm\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (11)$$

The decoupled orthogonal eigenstates at this specific condition have no dependence on k_x and, as discussed in the following, extremely asymmetric responses might be created over the entire resonance band. Here, it appears that the lattice structure at $\Delta = \pm 1/4$ does not provide any net effect on the guided modes because the inter-guided-mode coupling vanishes. However, this specific lattice structure has remarkable effects on the leakage-radiation distribution, leading to an almost ideal pure-phase resonance as explained in the next section.

V. PERFECT BLAZING IN LEAKAGE RADIATION

The decoupled guided-mode states and their interaction with the radiation continuum at $\Delta = \pm 1/4$ induce perfect blazing of leakage radiation toward a single preferred diffraction order and, consequently, lead to the effective realization of a single-port resonance in transmission.

In Figs. 4(a)–4(e), we show Δ -dependent frequency bands and their partial leakage-radiation strengths toward the cover (κ_c) and substrate (κ_s) media for the upper (ω_+) and lower (ω_-) band states. For $\Delta = 0$ or $1/2$, $\omega_\pm$ bands have a typical anticrossing structure and the partial leakage-radiation strengths are symmetric along both vertical and lateral axes, i.e., $|\kappa_c| = |\kappa_s|$ and $\kappa_{c \text{ or } s}(k_x) = \kappa_{c \text{ or } s}(-k_x)$. In the other cases for $0 < D < 1/2$, $|\kappa_c| \neq |\kappa_s|$ and $\kappa_{c \text{ or } s}(k_x) \neq \kappa_{c \text{ or } s}(-k_x)$ in general. This up/down and left/right asymmetry in the leakage-radiation distribution can be intuitively understood as resulting from the broken mirror symmetry of the structure with respect to x - y and y - z planes.

Importantly, we see several very peculiar characteristics in these cases. First, the leakage-radiation distribution is extremely asymmetric at $\Delta = 1/4$. The frequency bands show a typical Dirac point appearing at the topological nodal phase resulting from complete decoupling between the right and left guided modes, i.e., $|\omega_R\rangle = |R\rangle = [0 \ 1]^T$ and $|\omega_L\rangle = |L\rangle = [0 \ 1]^T$ over the entire frequency domain. The leakage-radiation distributions of these completely decoupled guided-mode eigenstates are also completely decoupled in such a way that $\kappa_c = 0$ (no radiation toward the cover) for $|\omega_R\rangle$ while $\kappa_s = 0$ (no radiation toward the substrate) for $|\omega_L\rangle$. We refer to this effect as asymmetric perfect leakage-radiation blazing in a similar sense to the classical blazed gratings that maximize the diffraction efficiency at a selected diffraction order by matching ray-optical and diffractive-optical angles. In our case, however, the blazing happens for the cover and substrate zero-order waves, not different diffraction orders, and its underlying physics is different from the classical blazed gratings.

Although the perfect leakage-radiation blazing appears throughout the entire frequency bands for $\Delta = 1/4$, strong blazing effect is still persistent even for $\Delta \neq 1/4$, as shown in Figs. 4(b) and 4(d). In particular, for these intermediate conditions, almost perfect blazing is found at two opposite k_x points on the ω_- band. Specifically, looking at the radiation rate spectra for the ω_- band in Figs. 4(b) and 4(d), we see $\kappa_s \approx 0$ or $\kappa_c \approx 0$ at $k_x = \pm 0.01 \times 2\pi/a$. The almost perfect blazing for $\Delta \neq 1/4$ at these points is associated with the presence of a bound state in the continuum located at the Γ point ($k_x = 0$) for $\Delta = 0$ and $1/2$, as further elucidated below.

In order to better understand the leakage-radiation blazing conditions, we calculate ratio of the leakage-radiation strengths $|\kappa_c/\kappa_s|$ for ω_+ and ω_- bands on the k_x - Δ plane as shown in Figs. 4(f) and 4(g), respectively. The perfect blazing effect over the entire bands is clearly seen at $\Delta = 1/4$, where the ratio $|\kappa_c/\kappa_s|$ is maximized (radiation toward cover) or minimized (radiation toward substrate). In Fig. 4(h), we show excited eigenstates and their leakage-radiation distributions at $\Delta = 1/4$ at four selected conditions I–IV, indicated in Figs. 4(c), 4(d), and 4(g). We confirm that the perfect blazing is closely related to the eigenstates $|R\rangle$ and $|L\rangle$ such that $|L\rangle$ emits radiation only toward the cover while $|R\rangle$ emits radiation only toward the substrate. For the ω_- band in Fig. 4(g), the map areas for $\Delta \neq 1/4$ reveal additional strong blazing conditions ($|\kappa_c/\kappa_s| > 10^{+2}$ or $|\kappa_c/\kappa_s| < 10^{-2}$) that are connected to the BIC at $\Delta = 0$ and $\pm 1/2$.

To further elucidate this behavior, we can use the coupled-mode theory by Kazarinov and Henry [31] to determine closed-form expressions for the leakage-radiation strength as

$$|\kappa_{c\pm}| = \sqrt{2\gamma} |\psi_{L\pm} \cos[\pi(\Delta - 1/4)] - \psi_{R\pm} \sin[\pi(\Delta - 1/4)]|, \quad (12)$$

$$|\kappa_{s\pm}| = \sqrt{2\gamma} |\psi_{R\pm} \cos[\pi(\Delta - 1/4)] - \psi_{L\pm} \sin[\pi(\Delta - 1/4)]|, \quad (13)$$

and these expressions provide comprehensive understanding of the leakage-radiation blazing properties revealed in the

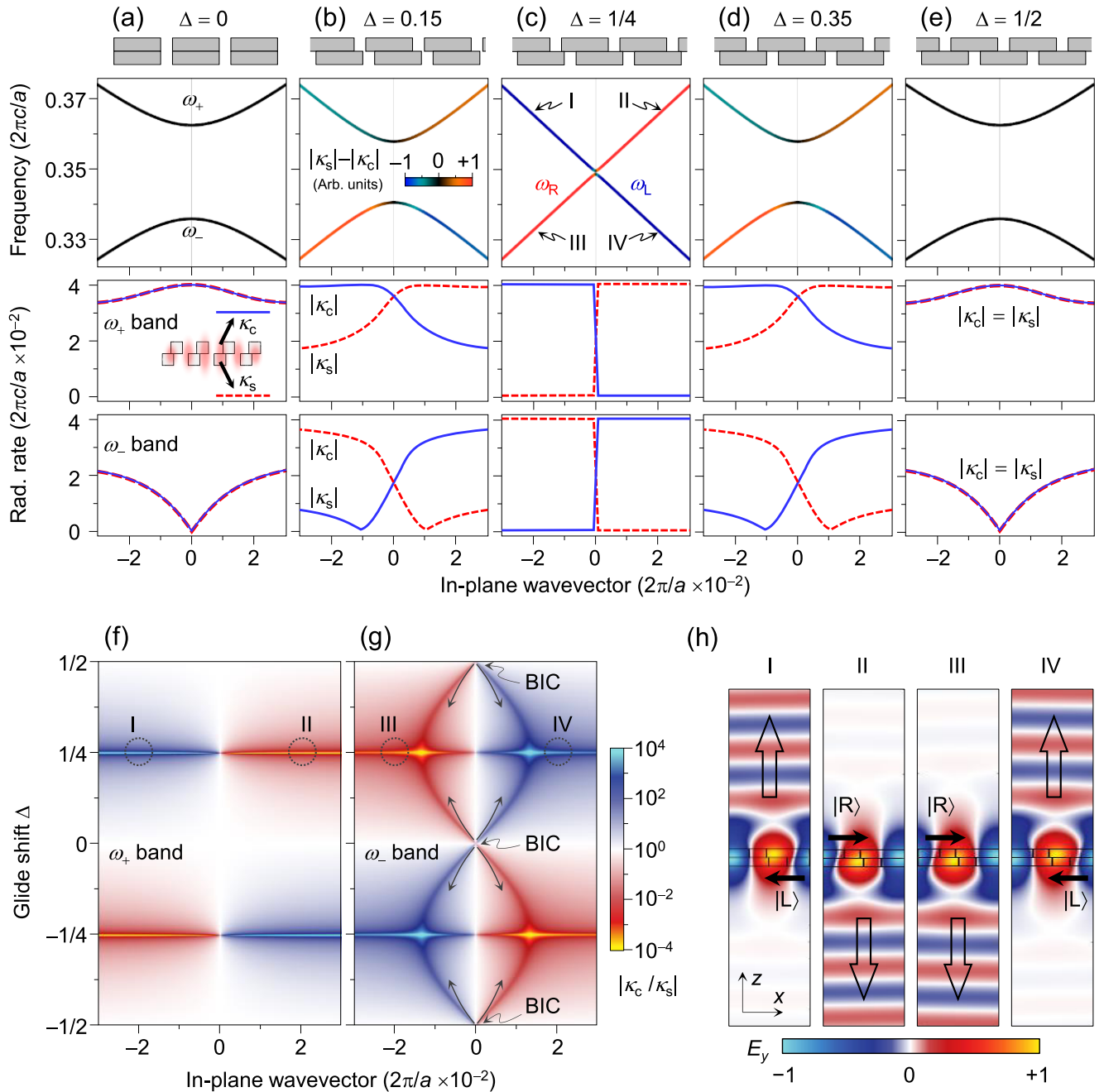


FIG. 4. Perfect blazing of leakage radiation. (a)–(e) Glide-shift-dependent photonic bands (top) and corresponding leakage-radiation rates toward the cover (middle) and substrate (bottom) near the second-order Bragg condition (stop band). $\omega_{\pm}$ denotes the upper (+) and lower (–) band, respectively. κ_c and κ_s represent leakage-radiation rates toward the cover and substrate, respectively. Leakage-radiation asymmetry ratio $|\kappa_c/\kappa_s|$ as a function of glide-shift parameter Δ and in-plane wave vector k_x for ω_+ (f) and ω_- (g) bands. (h) Field distributions for four selected perfect leakage-radiation blazing conditions I, II, III, and IV. Parametric locations of these conditions are indicated in panels (c), (f), and (g).

numerical analyses. Here, $\psi_{R\pm} = \langle R | \omega_{\pm} \rangle$ and $\psi_{L\pm} = \langle L | \omega_{\pm} \rangle$ and the expressions are derived by taking spatial symmetries and interferences between the two grating layers into account. See Sec. S2 in the Supplemental Material [35] for detailed mathematical treatment. Note in Eqs. (12) and (13) that the spatial-symmetry property $\psi_{L\pm}(-k_x) = \psi_{R\pm}(+k_x)$ of the eigenstates results in $|\kappa_{c\pm}(-k_x)| = |\kappa_{s\pm}(+k_x)|$. Therefore, the blazing conditions for the cover and substrate leakage radiations symmetrically appear with respect to the Γ point, as found in Figs. 4(f) and 4(g).

For $\Delta = \pm 1/4$, Eqs. (12) and (13) reduce to

$$|\kappa_{c\pm}| = \sqrt{2\gamma} |\psi_{L\pm}| \& |\kappa_{s\pm}| = \sqrt{2\gamma} |\psi_{R\pm}| \text{ for } \Delta = 1/4$$

$$|\kappa_{c\pm}| = \sqrt{2\gamma} |\psi_{R\pm}| \& |\kappa_{s\pm}| = \sqrt{2\gamma} |\psi_{L\pm}| \text{ for } \Delta = -1/4 \quad (14)$$

Clearly, for $\Delta = 1/4$, the leakage radiation toward the cover is exclusively from $|L\rangle$, while the leakage radiation toward the substrate is solely from $|R\rangle$, as seen in Fig. 4(h). Therefore, perfect blazing at the cover or substrate zero order

occurs for resonance band ω_R or ω_L that takes $|R\rangle$ or $|L\rangle$ as the eigenstate, respectively.

For $\Delta \neq \pm 1/4$ in contrast, Eqs. (12) and (13) imply that perfect blazing ($\kappa_c = 0$ or $\kappa_s = 0$) is obtained for the following conditions:

$$\frac{\psi_{L\pm}}{\psi_{R\pm}} = \pm \sqrt{\frac{\Delta\omega \pm vk_x}{\Delta\omega \mp vk_x}} = \begin{cases} \tan[\pi(\Delta - 1/4)] \text{ for } \kappa_{c\pm} = 0 \\ \cot[\pi(\Delta - 1/4)] \text{ for } \kappa_{s\pm} = 0 \end{cases} \quad (15)$$

Equation (15) quantitatively describes the strong blazing conditions connected to the BICs in Fig. 4(g). In the close vicinity of the BIC that is located at the band edge of the ω_- band for $\Delta \approx 0$, for example, the blazing condition of Eq. (15) reduces to $vk_x/\Delta\omega \approx 2\pi\Delta$ for $\kappa_{c-} = 0$ or $vk_x/\Delta\omega \approx -2\pi\Delta$ for $\kappa_{s-} = 0$. This is exactly consistent with the feature revealed at the bottom of Fig. 4(g) where the blazing conditions for the cover and substrate leakage radiations linearly branch off from the Γ point toward the left and right, respectively, as Δ increases from 0. In addition, one can easily find that there is no sensible solution to Eq. (15) for the ω_+ band, which explains why the blazing condition for the ω_+ band appears only at $\Delta = \pm 1/4$, as shown in Fig. 4(f).

These leakage-radiation properties described by Eqs. (12) and (13) enable a simple and intuitive model that quantitatively describes the transmission intensity and phase spectra. Specifically, we construct a temporal coupled-mode model of scattering amplitudes. For the nonresonant scattering processes in this model, we treat the grating structure as an effective homogeneous thin film. For the resonant scattering processes on the other hand, we take the binary eigenstates in Eq. (7) as resonance states. We assume that coupling strengths between the resonance states and radiation channels are given by Eqs. (12) and (13). Although the model only assumes time-reversal symmetry, reciprocity, and energy conservation for the states given by Eq. (7), with coupling strengths in Eqs. (12) and (13), the calculation results for the angle-dependent intensity and phase spectra in transmission and reflection are quantitatively consistent with rigorous numerical analysis based on the finite-element method (FEM). See Sec. S3 in the Supplemental Material [35] for a detailed mathematical derivation and its comparison with numerical calculation results.

In particular, the model predicts transmission coefficients for cover-side light incidence

$$\tau_{14} = it + \frac{\kappa_1^2}{i(\omega - \omega_0) - \Gamma}, \quad (16)$$

$$\tau_{23} = it + \frac{\kappa_2^2}{i(\omega - \omega_0) - \Gamma}, \quad (17)$$

where τ_{14} and τ_{23} are transmission coefficients for $+k_x$ and $-k_x$, respectively. Here, it represents the nonresonant transmission coefficient, κ_1 and κ_2 are coupling constants of the resonance states with the corresponding transmission channels, $\omega_0 = \text{Re}(\omega_{\pm})$ is the resonance center frequency, and $\Gamma = \text{Im}(\omega_{\pm})$ is the decay rate of the resonance state. For the optimal conditions of $\Delta = 1/4$ and Eq. (5), the parameters take values as $t \approx 1$, $\kappa_1 \approx (2\gamma)^{1/2} \exp(i\pi/4)$, $\kappa_2 \approx 0$, and

$\Gamma \approx \gamma$. See Sec. S3 in the Supplemental Material [35] for the associated closed-form expressions of these parameters.

Then, τ_{14} in Eq. (16) for the transmission toward the perfectly blazed channel reduces to

$$\tau_{14} \approx i + \frac{2\gamma}{\omega - \omega_0 + i\gamma} = i \frac{\omega - \omega_0 - i\gamma}{\omega - \omega_0 + i\gamma}. \quad (18)$$

This is identical to Eq. (1) with nonresonant amplitude $A_c = i$ and $A_d = -2i$, and hence it satisfies the ideal pure-phase resonance condition in Eq. (2), i.e., $A_d = -2A_c$. Note that the right-hand side of Eq. (18) clearly represents frequency-independent constant intensity at $|\tau_{14}| = 1$ and frequency-dependent phase $\arg(\tau_{14}) = \pi/2 - \tan^{-1}[(\omega - \omega_0)/\gamma]$, which is expected for a phasor along the unit circle as indicated by the blue circle in Fig. 1(a) for a single-port resonant-scattering amplitude. Thus, rather strikingly, the considered transmissive element behaves exactly as an open resonator connected to a single-radiation channel over a very broad bandwidth. In contrast, τ_{23} in Eq. (17) for the transmission toward the closed channel reduces to $\tau_{23} \approx i = \text{constant}$ which does not have any frequency-dependent resonance feature in the intensity and phase. Therefore, our topological-state and leakage-radiation theory of the bilayer glide-symmetric grating structure fully captures the underlying physics of the asymmetric pure-phase resonance effect in transmission. We note that Eq. (18) is also in consistency with alternative generic descriptions based on complex poles and zeros for resonance responses as provided in Refs. [40,41].

VI. TRANSMISSION SPECTRA OF THE PURE-PHASE RESONANCE BANDS

In this section, we show how the pure-phase resonance band gradually emerges in consistency with our proposed theory in Secs. III–V. Figure 5 shows the angle-resolved transmittance and phase-delay spectra for Δ increasing from 0 to $1/2$. Note that this corresponds to the numerical calculation results of the transmission spectra for the cases shown in Figs. 4(a)–4(e).

For $\Delta = 0$, the transmittance spectrum shows typical Fano-shape GMR featuring a zero-transmittance minimum, phase-delay change limited within π , and left-right symmetry. A symmetry-protected BIC clearly appears at the lower band edge ω_- ($\theta = 0$) in consistency with Fig. 4(a).

For $\Delta = 0.15$, the BIC for $\Delta = 0$ splits into two narrow-band pure-phase resonances at $\theta = \pm 2^\circ$ and the phase-delay spectrum becomes considerably asymmetric in contrast to the symmetric transmittance (intensity) spectrum. In the phase-delay spectrum, magnitude of the resonant change remarkably decreases the resonant phase-delay change magnitude. The BIC splitting in the ω_- band is also indicated in the leakage-radiation blazing spectrum in Fig. 4(g) and fundamentally identical to the polarization-vortex topological effect that causes narrow-band pure-phase resonances or equivalently unidirectional GMRs, as previously studied in Refs. [19–23].

For $\Delta = 1/4$, the GMR bands in the transmittance spectrum almost entirely vanish, while only a pointlike resonance at $\theta = 0$ is left behind. In contrast, the transmission phase-delay spectrum has a clear resonance band of 2π phase change exclusively along the right-going guided-mode band.

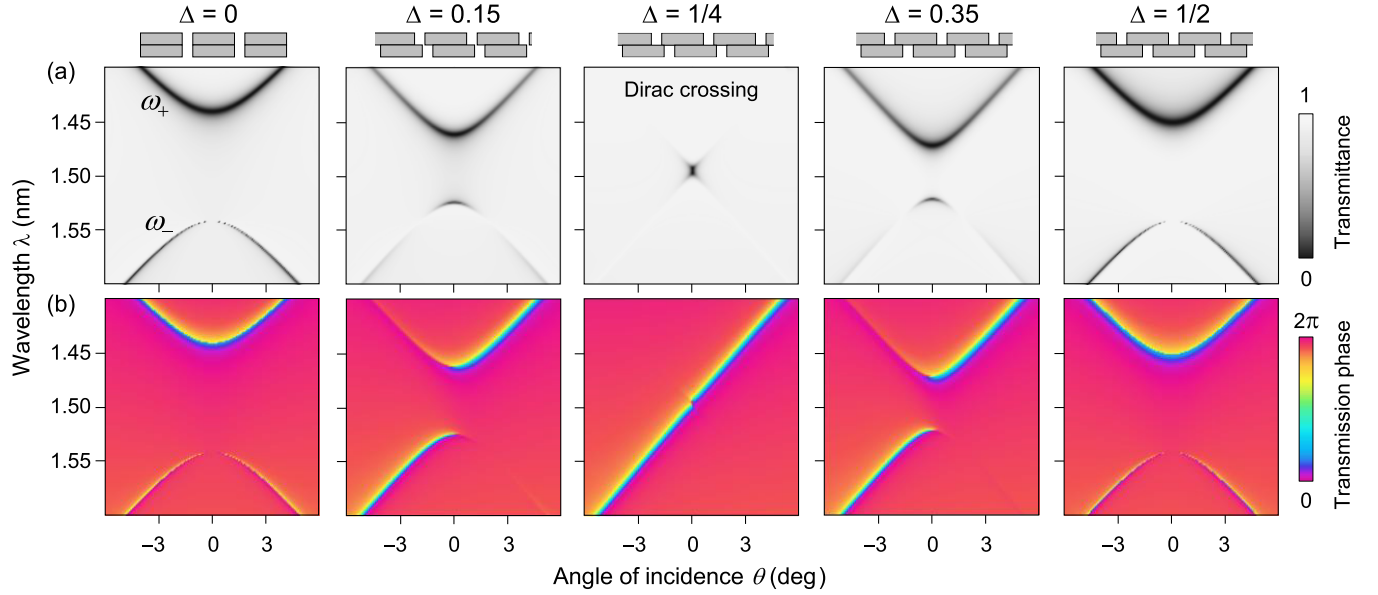


FIG. 5. Glide-shift-dependent resonant transmission features. Numerical calculation results of angle-resolved transmittance (a) and transmission phase (b) spectra of the incident light from the up port according to the grating configurations for a given Δ .

Combining the wide angle and frequency spectra in Fig. 2(b) with Fig. 5 for $\Delta = 1/4$, we confirm that strong pure-phase resonance effect is persistent over the entire right-going guided-mode resonance band below the first-order Rayleigh frequency $\nu_R(\pm 1)$ beyond which the number of plane wave radiation ports exceeds two. Γ point at $\theta = 0$ is the only exception of the pure-phase resonance within this frequency and angle range below $\nu_R(\pm 1)$. At Γ point, the left-going and right-going guided modes are equally excited and they independently emit leakage-radiation waves toward the reflection and transmission ports, respectively. The excitation of the left-going guided mode opens up the resonant reflection port that was closed for $\theta \neq 0$ and thereby the pure-phase resonance vanishes there.

We note that this drastic broadening of the pure-phase resonance and leakage-radiation blazing band at this quarter-period glide-shift condition is fundamentally independent of the existence of a BIC and associated polarization-vortex topological effect. Instead, it originates from the anomalous non-Hermitian orthogonality of the guided-mode eigenstates and asymmetric interference between diffraction amplitudes as we have explained through Secs. IV and V. The leakage-radiation blazing spectrum in Fig. 4(f) is one direct evidence of the causal independence of our case from the previous polarization-vortex topological effect. In Fig. 4(f), the unidirectional perfect blazing at $\Delta = \pm 1/4$ for the ω_+ band obviously does not have any association with the BIC feature in contrast to Fig. 4(g) for the ω_- band. In further detail, the unidirectional leakage-radiation blazing at $\Delta = \pm 1/4$ is independent of k_x according to Eq. (14) and thereby appears as a broad-band property. For the BIC-connected, polarization-vortex topological feature, it is obtained at a specific k_x value as Eq. (15) describes, implying a narrow-band effect in principle.

If Δ increases further from $1/4$ below $1/2$, i.e., $1/4 < \Delta < 1/2$, the full-band pure-phase resonance quickly fades away while the BIC-connected narrow-band features remain,

as shown in Fig. 5 for $\Delta = 0.35$ in which the transmittance and phase-delay spectra are almost identical to those for $\Delta = 0.15$. We note that this similarity of the transmission intensity and phase spectra with respect to the quarter-period glide-shift condition is fully captured by the leakage-radiation blazing properties according to Eqs. (12) and (13).

For $\Delta = 1/2$, as in the right panels of Figs. 5(a) and 5(b), the transmission intensity and phase spectra become symmetric with respect to k_x because the structure is mirror-symmetric. For $\Delta > 1/2$, the spectra are simply identical to those presented in Fig. 5 with k_x being inverted to $-k_x$.

In summary of our underlying physics description through Secs. II–VI, we identify two fundamental requirements for a transmissive pure-phase resonance in a generic two-radiation-port scattering system and explain how to realize it in a glide-symmetric bilayer GMR structure. The two essential conditions are zero nonresonant transmission and unidirectional emission of the resonant mode to the transmission side. We elaborate a minimal model of guided-mode band structures and leakage-radiation properties in the proposed structure based on two counterpropagating guided modes, three space-harmonic fields, and their diffractive coupling processes.

Importantly, the minimal model reveals fundamental underlying physics along two different lines. One is non-Hermitian band topology. Our model allows a non-Hermitian Hamiltonian description for the resonant states and frequency-dispersive characteristics, which have a direct mathematical correspondence to Dirac semimetals in the presence of leakage losses. The anomalous Dirac point, not conventional exceptional point, at the quarter-period glide-shift condition indicates robust formation of purely unidirectional resonant excitation, which is one necessary condition for pure-phase resonances. In another aspect, our model allows more fundamental, diffraction-interference description of the proposed effect. The anomalous Dirac-point formation is understood by complete destructive interference between the second-order

diffraction amplitudes, i.e., Bragg-reflection amplitudes, in the upper and lower layers. In addition, the leakage-radiation amplitudes toward the upper and lower surrounding media are described in terms of two interfering contributions from the left-going and right-going guided modes whose complex amplitudes are fully determined by the closed-form non-Hermitian Hamiltonian, as indicated in Eqs. (7)–(9), (12), and (13). Intriguingly, these equations fully capture two unidirectional resonance features that independently induce pure-phase resonances in the transmission. One is a broadband feature at the quarter-period glide-shift condition that we mainly propose in this paper. The other one is a narrow-band feature connected to the BIC point, which corresponds to previously proposed, polarization-vortex charge separation. In our model, narrow-band nature of this feature is readily understood with Eq. (15) that it must appear at a specific k_x point where leakage-radiation waves separately from the left-going and right-going guided modes cancel each other. Therefore, our model provides a conceptually simplest physical description of the narrow-band unidirectional photonic-lattice resonance and its abstract polarization-vortex topological properties in terms of elementary wave properties of diffraction and interference.

VII. POTENTIAL APPLICATION

Combined with appropriate index-tuning mechanisms or spatial parameter distributions, i.e., suitable spatial structuring, the pure-phase transmission resonance may have a wide range of application areas as a compact flat optical device, such as phase modulators and wave front control metasurfaces. Considering the wave front engineering capability in particular, precise manipulation of local phase delay is a key feature. Although the proposed pure-phase resonance effect in transmission provides an efficient mean for controlling global phase delay over the cross section of an optical beam in the absence of undesirable intensity variations, it is unclear whether the phase delay in the proposed structure is locally controllable within an acceptable level in practice. This question naturally arises because diffractive properties from guided-mode resonances are inherently collective responses involving multiple unit cells typically on the order of 10–1000. Thereby, precise phase match of the incident light to the dispersion relation and nonlocal properties of the diffracted light are unavoidable [42–44].

In this respect, we perform a set of numerical experiments on metasurface lensing functionality based on the proposed pure-phase resonance effect. We set a 2D FEM model of our proposed bilayer metasurface structure such that its transmission phase-delay distribution $\phi(x)$ follows a quadratic function as

$$\phi(x) = \phi_0 - \frac{\pi x^2}{\lambda f}, \quad (19)$$

where ϕ_0 is a reference phase delay at the center ($x = 0$) and f is the desired focal length. This quadratic phase-delay distribution is wrapped within a $[0, 2\pi)$ range, resulting in piece-wise continuous Fresnel zones. The wrapped phase-delay distribution is coded into a corresponding period distribution $a(x; f)$.

We first tested a trial design based on the structure considered in Fig. 1. The 2D FEM model in this test consists of a 80- μm -wide bilayer metasurface with its period $a(x; f)$ distribution is inferred from its transmission phase spectra combined with Eq. (19) for $f = 40, 60$, and 80 μm at wavelength 1.5 μm . The outer boundaries of the computational domain are surrounded by three perfectly matched layers and one incident port that emits a 40- μm -wide cylindrical Gaussian beam toward the metasurface structure. Unfortunately, we found no clear signature of a focal spot in the FEM simulation results. We attribute the failure in this case to the strong nonlocal response of the structure. Effective propagation length of the guided mode in the first trial cases is $L_{\text{eff}} \sim 230a$, which we estimate from the resonance quality factor $Q \sim 4400$ and group speed $v \sim 0.35c$ using the relation $L_{\text{eff}} \approx (2\pi)^{-1} Q(v/c)a$, which is approximately valid for resonant gratings around the first-order Bragg condition in general. It is therefore not surprising to see no local phase-delay effect in this high- Q and long-propagation-length regime.

In order to significantly reduce the effective propagation length to enable a sufficiently stronger local phase-delay response, we modify the structure. We change the host medium from Si_3N_4 ($n_2 = 2.45$) to SiO_2 ($n_2 = 1.5$) to increase the index contrast. In addition, we modify the thickness of the Si grating layer from 200 to 250 nm and fill factor from 0.8 to 0.6. These modifications are favorable for increasing the radiation decay rate of the guided mode since the confinement factor of the guided mode to the grating layer and the diffraction strength are enhanced with such parametric changes. With these modified design parameters, the resonance quality factor and group speed for this modified structure are $Q = 43$ and $v = 0.37c$, resulting in effective propagation length $L_{\text{eff}} \sim 2.5a$, which seems very promising to expect an improved local phase-delay response in contrast to the original structure with $L_{\text{eff}} \sim 230a$.

In Figs. 6(a) and 6(b), we show transmission intensity T and phase delay $\arg(t)$ spectra. Along the blazed-state resonance condition indicated by the dark-red dashed curve, the T spectrum is persistently high above 0.8 over most of the spectral domain of interest, except for the Γ point, while maintaining a 2π phase shift around it, as expected. For a reference resonance state at 1.5 μm wavelength and 10° angle of incidence, we calculate T and $\arg(t)$ as the grating period a changes from 600 to 800 nm, as shown in Figs. 6(c) and 6(d). These results confirm $T > 0.8$ over the entire period range, while $\arg(t)$ changes over almost a full 2π span. Note that a pure-phase resonance does not occur at normal incidence because both left-going and right-going guided modes are equally excited and consequently the one-port radiation effect is no longer sustainable. Therefore, the focusing effect cannot be produced at the normal-incidence condition unfortunately. Using this period-dependent response, we then create a metasurface with a period distribution that realizes a phase profile according to Eq. (19) for the desired focal length $f = 40, 60$, and 80 μm .

The simulation results are provided in Figs. 6(e)–6(g). We show the designed period distributions on the bottom and the corresponding transmitted field intensity distributions on top. The incident Gaussian beam is 40- μm -wide in its full

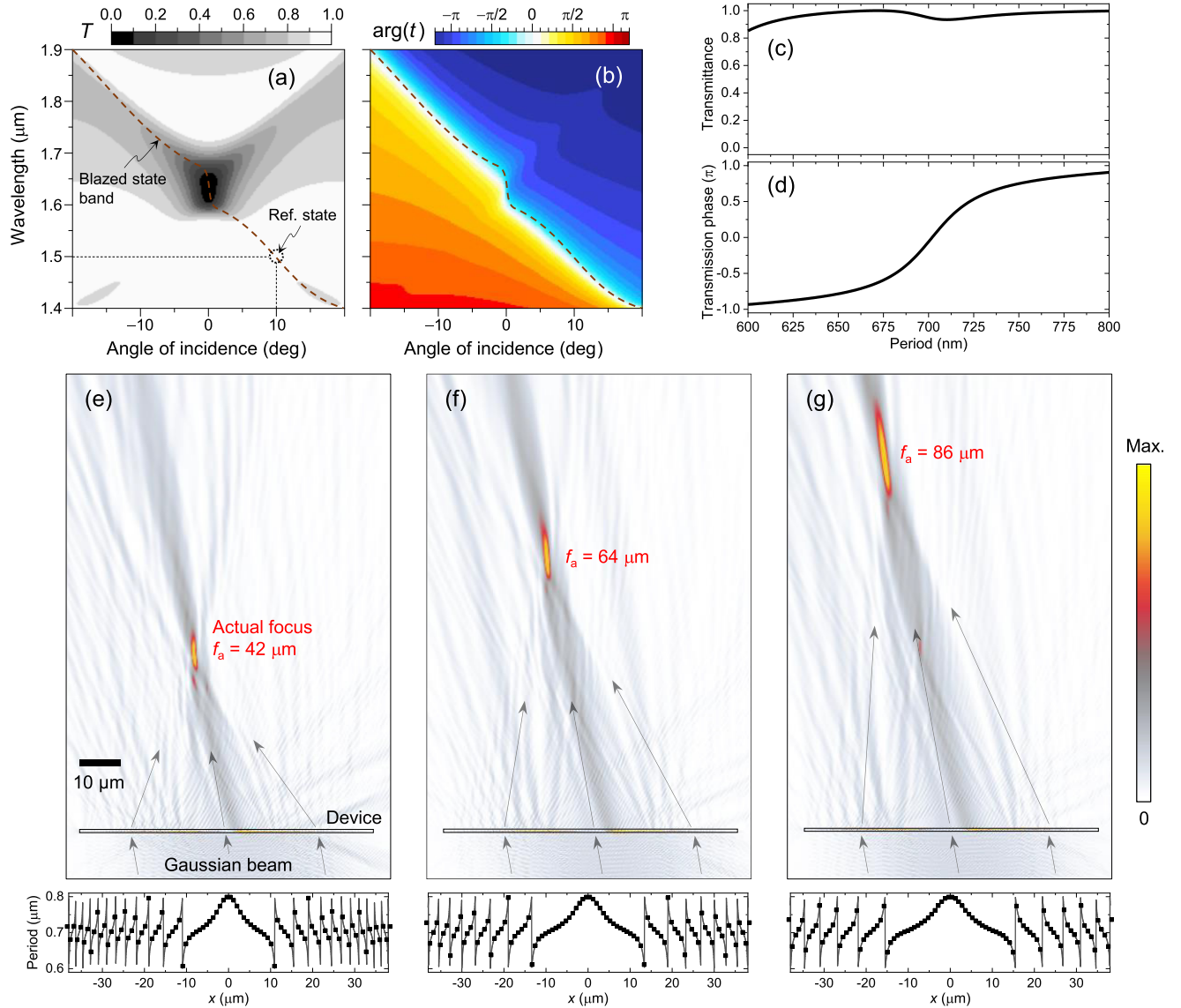


FIG. 6. Numerical demonstration wave front control. (a), (b) Transmittance T and transmission phase $\arg(t)$ spectra for a structure with parameters $a = 700$ nm, $F = 0.6$, $d = 250$ nm, $n_1 = 3.48$ (Si), and $n_2 = 1.5$ (SiO₂). (c), (d) T and $\arg(t)$ as a function of period for a pure-phase resonance condition at wavelength $\lambda = 1.5$ μm and angle of incidence $\theta = 10^\circ$. (e)–(g) Focusing of an incident Gaussian beam for desired focal length $f_d = 40$, 60 , and 80 μm . The period distribution on the bottom of each panel is configured so that the device yields the desired focal length in the ideal local-response regime. f_a denotes focal length obtained through simulation.

width at half maximum for all three cases. As seen clearly, we obtain well-defined focal spots for all cases. The obtained focal efficiencies and numerical aperture are 59% and 0.95 for $f = 42$ μm , 61% and 0.62 for $f = 62$ μm , and 64% and 0.46 for $f = 86$ μm , respectively. The focal efficiency indicates partial optical power within the Gaussian focal spot with respect to the total scattered outgoing power from the metasurface structure. The focal-length errors are around 6% from their desired values. We consider that this error is related to the nonlocal response of the resonant transmission effect, as we discussed earlier in this section. Besides the focal-length error, these simulation results suggest highly promising performance as the reflection of the incident Gaussian beam is lower than 1.7% and the power efficiency of the focal spot, i.e., portion of the optical power contained within the

focal spot area, is higher than 60%. Moreover, different from Huygens metasurfaces based on localized magnetic-electric dipole resonances, our design guarantees similar performance metrics over a very broad wavelength range, albeit for different angles of incidence at different wavelengths, as suggested by the results in Figs. 6(a) and 6(b).

We point out several technical limitations of our exemplary and rudimentary numerical demonstration for its applications to wave front engineering surfaces. Although the pure-phase resonance here persists over a broad spectral window, it should not be confused with a broadband pure-phase resonance in a conventional sense as implying that the pure-phase modulation is available under broadband and wide-angular light incidence. The pure-phase resonance itself in our case is a narrow-band resonance occurring along a specific dispersion

curve as a guided-mode resonance. Therefore, the incident light should be monochromatic and highly directive at an angle prescribed by the dispersion curve, e.g., 10° at wavelength $1.5\ \mu\text{m}$ in Figs. 6(e)–6(g). Change in the operation wavelength requires corresponding shift of the incident angle for securing the desired focusing effect. In addition, the focus is subject to a chromatic aberration as explicitly appearing in Eq. (19).

In spite of these limitations, one may still take some technical advantages of our proposed approach for monochromatic-light applications that are abundant in laser-based optical technologies. For example, Pancharatnam-Berry phase metasurfaces are recognized as an efficient method for achieving broadband transmissive phase modulation through nonresonant effects [45,46]. However, compared to our approach, they have several disadvantages. First, accumulating sufficient polarization-dependent phase difference requires a considerable thickness, similar to traditional diffractive optical elements. Second, attaining near-unity transmission is fundamentally challenging due to inherent limitations of nonresonant refractive optical properties and the unavoidable presence of the zero-order component [47,48]. Third, applying time-varying phase modulation without resonance mechanisms is difficult. Considering these limitations, our proposed method offers advantages such as achieving high transmittance with reduced thickness and enabling temporally variable phase control.

In potential realization in experiments, some promising fabrication techniques have been developed recently. In Ref. [49], sophisticated bilayer metasurfaces were successfully fabricated with a standard overlay lithography with additional conformal deposition-etch steps. In another technique, a glide-symmetric bilayer waveguide grating that is basically identical to our proposed structure was created by utilizing polymer-filling and dry-etch planarizing steps [50]. These techniques do not involve harsh mechanical planarizing processes. Therefore, precise realization of our proposed structure is practically feasible.

VIII. CONCLUSION

In conclusion, we have proposed a Dirac bilayer metasurface in a topological nodal phase as a unique platform to enable pure-phase resonances in transmission with their operating band persisting over an extremely wide spectral window, as opposed to previously proposed structures. We have developed a complete theory to study the topological guided-mode eigenstates, the glide-symmetric leakage-radiation properties, and their contribution in the resonant scattering process. Based on this theoretical framework, we have found that the transmissive pure-phase resonance is enabled by the anomalous non-Hermitian orthogonality of the eigenstates and a perfect leakage-radiation blazing effect, which are, in principle, inherently broadband. We confirm quantitative agreement between our theory and rigorous numerical calculations. Finally, we numerically demonstrate a wave front-controlling metasurface functioning as a transmissive microlens with remarkably high efficiency.

From a fundamental viewpoint, the proposed structure is asymmetric along both the vertical and lateral axes and thereby an asymmetric response may be expected. Nevertheless, our theory suggests that appropriate choices of a structural parameter set, within moderate value ranges, can induce extremely asymmetric responses at certain critical conditions associated with the topological phase of a structured medium. This implies that the proposed effect should be robust against imperfections and parametric drifts generally unavoidable in practice. Therefore, the proposed structure and its underlying physics should be applicable for the realization of various, highly robust, transmissive metasurface elements. Immediate application areas are phase modulators and wave front control devices such as electro-optic phase shifters, solid-state beam steering elements, orbital-angular-momentum generators, and various holographic optical elements, which can take technical advantages of operating efficiently in transmission mode. In addition, the anomalous non-Hermitian orthogonality and leakage-radiation blazing effects may find their own independent applications including unidirectional or antireflecting couplers with virtually maximal efficiencies.

Although we provide simplified numerical analysis of planar focusing by 2D models for the proof-of-concept demonstration, its extension to 3D conical focusing is readily possible in principle by taking a 2D parametric modulation in one-dimensional (1D) periodic or quasiperiodic structures, as previously incorporated in Ref. [51]. Therein, a conical focusing mirror was created by 1D quasiperiodic high-contrast grating structure with its period and fill factor are continuous functions of 2D in-plane position coordinates. The identical approach is directly applicable to our proposed structure.

From another perspective, the extremely asymmetric responses in our system share their underlying physics with the 2D Dirac semimetal phase. This suggests that similar or related effects are potentially obtained in other topological systems such as Weyl-semimetal-like systems as 3D extension of the 2D Dirac semimetal [52,53]. We believe it is of great interest to further investigate theoretically and experimentally how the extreme asymmetry in such a 2D topological system manifests in its 3D extensions and how it may open opportunities for applications.

ACKNOWLEDGEMENTS

This research was supported in part by the Leader Researcher Program (NRF-2019R1A3B2068083), and the research fund of Hanyang University (HY-202000000000513). K.Y.L. acknowledges the NRF Sejong Science fellowship (NRF-2022R1C1C2006290) funded by the MSIT of the Korean government. F.M. acknowledges support from the U.S. Air Force Office of Scientific Research (FA9550-22-1-0204).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

- [1] A. Melikyan *et al.*, High-speed plasmonic phase modulators, *Nat. Photonics* **8**, 229 (2014).
- [2] A. L. Holsteen, A. F. Cihan, and M. L. Brongersma, Temporal color mixing and dynamic beam shaping with silicon metasurfaces, *Science* **365**, 257 (2019).
- [3] J. Park *et al.*, All-solid-state spatial light modulator with independent phase and amplitude control for three-dimensional LiDAR applications, *Nat. Nanotechnol.* **16**, 69 (2021).
- [4] H. Sasaki *et al.*, Large size three-dimensional video by electronic holography using multiple spatial light modulators, *Sci. Rep.* **4**, 6177 (2014).
- [5] B. Schwarz, Mapping the world in 3D, *Nat. Photonics* **4**, 429 (2010).
- [6] Y. Horie, A. Arbabi, E. Arbabi, S. M. Kamali, and A. Faraon, High-speed, phase-dominant spatial light modulation with silicon-based active resonant antennas, *ACS Photonics* **5**, 1711 (2017).
- [7] A. M. Shaltout, V. M. Shalae, and M. L. Brongersma, Spatiotemporal light control with active metasurfaces, *Science* **364**, eaat3100 (2019).
- [8] H. Sroor *et al.*, High-purity orbital angular momentum states from a visible metasurface laser, *Nat. Photonics* **14**, 498 (2020).
- [9] M. Khorasaninejad and F. Metalenses Capasso, Versatile multifunctional photonic components, *Science* **358**, eaam8100 (2017).
- [10] F. Gires and P. Tournois, An interferometer useful for pulse compression of a frequency modulated light pulse, *CR Acad. Sci.* **258**, 6112 (1964).
- [11] Y. W. Huang *et al.*, Gate-tunable conducting oxide metasurfaces, *Nano Lett.* **16**, 5319 (2016).
- [12] J. Y. Kim *et al.*, Full 2π tunable phase modulation using avoided crossing of resonances, *Nat. Comm.* **13**, 2103 (2022).
- [13] S. Colburn, A. Zhan, and A. Majumdar, Tunable metasurfaces via subwavelength phase shifters with uniform amplitude, *Sci. Rep.* **7**, 40174 (2017).
- [14] Q. Song, M. Odeh, J. Zúñiga-Pérez, B. Kanté, and P. Genevet, Plasmonic topological metasurface by encircling an exceptional point, *Science* **373**, 1133 (2021).
- [15] M. Decker *et al.*, High-efficiency dielectric Huygens' surfaces, *Adv. Opt. Mater.* **3**, 813 (2015).
- [16] C. Gigli *et al.*, Fundamental limitations of Huygens' metasurfaces for optical beam shaping, *Laser Photonics Rev.* **15**, 2000448 (2021).
- [17] S. Q. Li *et al.*, Phase-only transmissive spatial light modulator based on tunable dielectric metasurface, *Science* **364**, 1087 (2019).
- [18] F. Monticone, N. M. Estakhri, and A. Alu, Full control of nanoscale optical transmission with a composite metascreen, *Phys. Rev. Lett.* **110**, 203903 (2013).
- [19] H. M. Doeleman, F. Monticone, W. den Hollander, A. Alù, and A. F. Koenderink, Experimental observation of a polarization vortex at an optical bound state in the continuum, *Nat. Photonics* **12**, 397 (2018).
- [20] M. M. Salary and H. Mosallaei, Tunable all-dielectric metasurfaces for phase-only modulation of transmitted light based on quasi-bound states in the continuum, *ACS Photonics* **7**, 1813 (2020).
- [21] X. Yin, J. Jin, M. Soljačić, C. Peng, and B. Zhen, Observation of topologically enabled unidirectional guided resonances, *Nature (London)* **580**, 467 (2020).
- [22] H. Zhou *et al.*, Perfect single-sided radiation and absorption without mirrors, *Optica* **3**, 1079 (2016).
- [23] Z. Zhang *et al.*, Observation of intensity flattened phase shifting enabled by unidirectional guided resonance, *Nanophotonics* **10**, 4467 (2021).
- [24] K. Y. Lee *et al.*, Synthetic topological nodal phase in bilayer resonant gratings, *Phys. Rev. Lett.* **128**, 053002 (2022).
- [25] M. F. Lapa, G. Y. Cho, and T. L. Hughes, Bosonic analog of a topological Dirac semimetal: Effective theory, neighboring phases, and wire construction, *Phys. Rev. B* **94**, 245110 (2016).
- [26] S. Fan, W. Suh, and J. D. Joannopoulos, Temporal coupled-mode theory for the Fano resonance in optical resonators, *J. Opt. Soc. Am. A* **20**, 569 (2003).
- [27] J. Yoon, K. H. Seol, S. H. Song, and R. Magnusson, Critical coupling in dissipative surface-plasmon resonators with multiple ports, *Opt. Express* **18**, 25702 (2010).
- [28] D. R. Abujetas, Á. Barreda, F. Moreno, A. Litman, J. M. Geffrin, and J. A. Sánchez-Gil, High-Q transparency band in all-dielectric metasurfaces induced by a quasi bound state in the continuum, *Laser Photonics Rev.* **15**, 2000263 (2021).
- [29] L. Michaeli, H. Suchowski, and T. Ellenbogen, Near-infrared tunable surface lattice induced transparency in a plasmonic metasurface, *Laser Photonics Rev.* **14**, 1900204 (2020).
- [30] X. Zhao, R. Huang, X. Du, Z. Zhang, and G. Li, Ultrahigh-Q metasurface transparency band induced by collective-collective coupling, *Nano Lett.* **24**, 1238 (2024).
- [31] R. Kazarinov and C. Henry, Second-order distributed feedback lasers with mode selection provided by first-order radiation losses, *IEEE J. Quantum Electron.* **21**, 144 (1985).
- [32] D. Rosenblatt, A. Sharon, and A. A. Friesem, Resonant grating waveguide structures, *IEEE J. Quantum Electron.* **33**, 2038 (1997).
- [33] Y. Ding and R. Magnusson, Band gaps and leaky-wave effects in resonant photonic-crystal waveguides, *Opt. Express* **15**, 680 (2007).
- [34] F. Monticone and A. Alu, Leaky-wave theory, techniques, and applications: From microwaves to visible frequencies, *Proc. IEEE* **103**, 793 (2015).
- [35] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/3dw5-nvdq> for mathematical treatments of the guided-mode resonance eigenvalue problem, the temporal coupled-mode theory description, and additional simulation data confirming the validity of the theoretical descriptions.
- [36] K. Y. Lee *et al.*, Topological guided-mode resonances at non-Hermitian nanophotonic interfaces, *Nanophotonics* **10**, 1853 (2021).
- [37] K. Y. Lee, S. Yoon, S. H. Song, and J. W. Yoon, Topological beaming of light, *Sci. Adv.* **8**, eadd8349 (2022).
- [38] Y. S. Choi, K. Y. Lee, and J. W. Yoon, Topological surface plasmon resonance in deep subwavelength structure, *Curr. Appl. Phys.* **45**, 72 (2023).
- [39] H. Xue *et al.*, Non-Hermitian dirac cones, *Phys. Rev. Lett.* **124**, 236403 (2020).
- [40] E. Mikheeva *et al.*, Asymmetric phase modulation of light with parity-symmetry broken metasurfaces, *Optica* **10**, 1287 (2023).

- [41] R. Colom *et al.*, Crossing of the branch cut: The topological origin of a universal 2π -phase retardation in non-Hermitian metasurfaces, *Laser Photonics Rev.* **17**, 2200976 (2023).
- [42] K. Shastri and F. Monticone, Nonlocal flat optics, *Nat. Photonics* **17**, 36 (2023).
- [43] A. Overvig and A. Alù, Diffractive nonlocal metasurfaces, *Laser Photonics Rev.* **16**, 2100633 (2022).
- [44] A. C. Overvig *et al.*, Selection rules for quasibound states in the continuum, *Phys. Rev. B* **102**, 035434 (2020).
- [45] D. Lin, P. Fan, E. Hasman, and M. L. Brongersma, Dielectric gradient metasurface optical elements, *Science* **345**, 298 (2014).
- [46] N. Yu and F. Capasso, Flat optics with designer metasurfaces, *Nat. Mat.* **13**, 139 (2014).
- [47] C. Liu, Y. Bai, Q. Zhao, Y. Yang, H. Chen, J. Zhou, and L. Qiao, Fully controllable Pancharatnam-Berry metasurface array with high conversion efficiency and broad bandwidth, *Sci. Rep.* **6**, 34819 (2016).
- [48] C. McDonnell, J. Deng, S. Sideris, T. Ellenbogen, and G. Li, Functional THz emitters based on Pancharatnam-Berry phase nonlinear metasurfaces, *Nat. Commun.* **12**, 30 (2021).
- [49] A. H. Dorrah *et al.*, Free-standing bilayer metasurfaces in the visible, *Nat. Commun.* **16**, 3126 (2025).
- [50] Z.-P. Zhuang, X. Zhou, H.-L. Zeng, M.-Y. Li, Z.-M. Chen, X.-T. He, X.-D. Chen, L. Zhou, and J.-W. Dong, Overcoming intrinsic dispersion locking for achieving spatio-spectral selectivity with misaligned bilayer metagratings, *eLight* **5**, 13 (2025).
- [51] D. Fattal, J. Li, Z. Peng, M. Fiorentino, and R. G. Beausoleil, Flat dielectric grating reflectors with focusing abilities, *Nat. Photonics* **4**, 466 (2010).
- [52] N. P. Ong and S. Liang, Experimental signatures of the chiral anomaly in Dirac–Weyl semimetals, *Nat. Rev. Phys.* **3**, 394 (2021).
- [53] A. A. Burkov, M. D. Hook, and L. Balents, Topological nodal semimetals, *Phys. Rev. B* **84**, 235126 (2011).