

Robust correlation-induced localization under time-reversal symmetry breakingBikram Pain^{1,2,*}, Sthitadhi Roy^{1,†}, Jens H. Bardarson³, and Ivan M. Khaymovich^{4,‡}¹*International Centre for Theoretical Sciences, Tata Institute of Fundamental Research, Bengaluru 560089, India*²*Nordita, Stockholm University and KTH Royal Institute of Technology, Hannes Alfvéns väg 12, SE-106 91 Stockholm, Sweden*³*Department of Physics, KTH Royal Institute of Technology, 106 91 Stockholm, Sweden*⁴*Université de Toulouse, CNRS, Laboratoire de Physique Théorique, Toulouse, France*

(Received 21 April 2026; accepted 25 June 2026; published 31 August 2026)

We study Anderson localization in a one-dimensional disordered system with long-range correlated hopping decaying as $1/r^a$ with complex hopping amplitudes that break time-reversal symmetry in a tunable fashion by varying their argument. We find analytically a correlation-induced algebraic localization that is robust to a finite strength of the time-reversal symmetry-breaking parameter, beyond which all states delocalize. This establishes a localization-delocalization transition driven by the interplay between long-ranged correlated hopping and time-reversal symmetry breaking. In addition to obtaining the static localization phase diagram, we also investigate the dynamical phase diagram through the lens of wave-packet spreading. We find that the growth in time of the mean-squared displacement of a wave packet, which is subdiffusive for the time-reversal symmetric case, becomes diffusive for any finite value of the time-reversal symmetry-breaking parameter.

DOI: [10.1103/34h6-sn4p](https://doi.org/10.1103/34h6-sn4p)

Introduction. Anderson localization [1,2], a hallmark phenomenon in condensed matter physics, reveals that disorder-induced quantum interference leads to exponential localization of wavefunctions in real space, thereby suppressing diffusion. The canonical model of Anderson localization is a tight-binding Hamiltonian with nearest-neighbor hopping and uncorrelated random on-site potentials. In such systems in one dimension (1D), all eigenstates are localized by any infinitesimal amount of disorder [3].

Despite the apparent robustness of localization in 1D, several notable exceptions exist that have enriched our understanding of localization-delocalization transitions in low-dimensional disordered systems. One prominent example is the Aubry-André (AA) model [4], where quasiperiodic, and thus correlated, on-site potentials lead to a localization-delocalization transition in 1D. Another class of exceptions, allowing for the presence of extended states even in 1D, is the power-law banded random matrix (PLBRM) ensemble [5–8] that features uncorrelated hopping amplitudes between two sites decaying as a power law with the distance between them. The long-range couplings effectively increase the connectivity of the lattice, which facilitates transport.

A related actively debated frontier arises in Anderson localization of light [9,10]. In three-dimensional (3D) disordered media, near-field dipole-dipole interactions between randomly positioned scatterers are believed to hinder a localized phase entirely for vector waves [11,12]. Localization is restored only in some specific cases that depend sensitively on the material characteristics of the scatterers [13,14]. This motivated a model of long-ranged correlated hopping, proposed originally by Logan and Wolynes, and Burin and Maksimov [15–17]. The model combines on-site disorder with translation-invariant hopping correlations mimicking dipole-dipole interactions. While the correlations, translation invariance, and long-range nature of hopping might be expected to promote delocalization, numerical [18] and analytical [19] studies have shown that the above factors can instead favor localization. Unlike the uncorrelated hopping of PLBRM, correlations and translational invariance suppress randomness in hopping pathways, reinforcing localization beyond the convergence of the standard locator expansion [1,5,6]. Later, several works addressed questions regarding the robustness of correlation-induced localization with respect to the nature of the disorder [18,20,21], partial correlations [22,23], dimensionality [24,25], and even interactions [26] and dynamics [27].

In the conventional picture of Anderson localization, an excitation undergoing multiple scattering can follow a self-crossing trajectory, thereby forming loops. A pair of trajectories that are identical except for their traversal of such loops, where they are related by time-reversal symmetry, interfere constructively. These so-called quantum corrections to conductivity enhance the probability of return (coherent backscattering) and thereby inhibit transport. Breaking time-reversal symmetry (TRS) generally alters the relative phases these trajectory pairs accumulate along the loops, weakening this

*Contact author: bikram.pain@icts.res.in†Contact author: sthitadhi.roy@icts.res.in‡Contact author: ivan.khaymovich@gmail.com

interference mechanism. This indeed happens in certain integrable models [28]. This naturally raises the question of how these expectations reconcile with correlation-induced localization. Motivated by this, in this work, we address the robustness of correlation-induced localization [19] in the presence of TRS breaking.

We show that models with correlated, long-range hopping exhibit robust localization up to a critical strength of the TRS breaking parameter given by the argument of the complex hopping amplitudes. We characterize the resulting localization-delocalization phase diagram by extending the matrix inversion trick of Ref. [19] to TRS-broken systems. Furthermore, we numerically demonstrate that TRS breaking leads to the anomalous dynamics of a wave packet where the core remains localized but the tails diffuse. This is in contrast to the time-reversal symmetric case, where we observe subdiffusive growth of the wave-packet mean-squared displacement.

Model. We consider a one-dimensional disordered model with periodic boundary conditions and fully correlated long-ranged hopping with complex hopping amplitudes. The Hamiltonian is given by

$$H = \sum_n \epsilon_n |n\rangle \langle n| + \sum_{n \neq m} j_{n-m} |n\rangle \langle m|, \quad (1)$$

where $\{|n\rangle\}$ denotes the set of real-space basis states and $\epsilon_n \sim \mathcal{N}(0, W)$ are i.i.d., Gaussian random onsite potentials. As a representative realization of fully correlated hopping, we consider that the translationally invariant hopping matrix elements [29],

$$j_{n-m} = \frac{J_0 e^{i\theta \text{sgn}(n-m)}}{|n-m|^a} \quad (n \neq m), \quad (2)$$

are power-law decaying like in the PLBRM, but fully correlated [30], where J_0 sets the overall energy scale (without loss of generality, we set $J_0 = 1$), $a > 0$ controls the power-law decay of the hopping amplitude between two sites with the distance between them, and the phase θ is a tunable parameter that breaks TRS via the difference in hopping phases to the right and left, given by the sign function. The latter is defined as $\text{sgn}(x) = 1$ for $x > 0$ and $\text{sgn}(x) = -1$ for $x < 0$. We restrict ourselves to $\theta \in [-\pi/2, \pi/2]$, because the phase diagram is periodic beyond this interval.

For $\theta = 0$, the model reduces to a real, symmetric (time-reversal invariant) hopping matrix, first, suggested for 3D case in Refs. [15–17], which has a localized phase even when the locator expansion diverges at $a < 1$ [18,19], unlike the PLBRM model that has random hopping matrix elements and shows delocalized behavior in the above-mentioned long-range regime. $\theta \neq 0$ breaks TRS that induces chiral propagation and suppresses interference between time-reversed paths. In the following, we present how TRS breaking (i.e., at $\theta \neq 0$) affects the correlation-induced localization properties.

Phase diagram. The main result of this work is that the algebraic correlation-induced localization of eigenstates at $a < 1$ remains robust even under TRS breaking up to a critical parameter value $|\theta| = \theta_c = \pi a/2$, above which TRS breaking destabilizes the localized phase (see Fig. 1). Importantly, the phase boundary, $|\theta_c| = \pi a/2$, being a dependent, leads to a

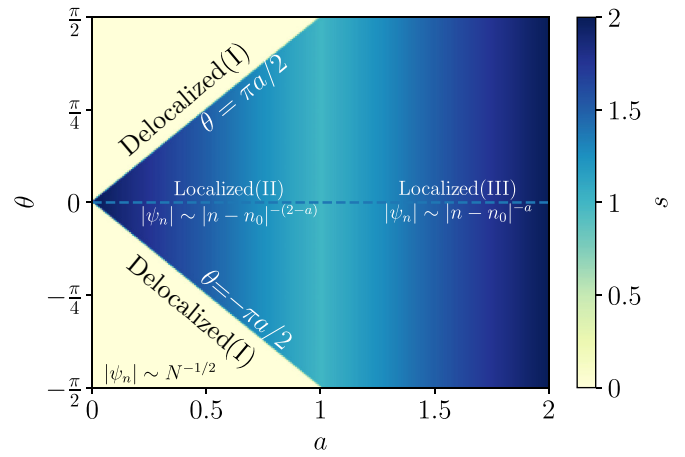


FIG. 1. Phase diagram in the θ - a parameter plane. The system exhibits a localization-delocalization transition with the delocalized phase (I) appearing for $|\theta| \geq \pi a/2$, $a < 1$. Further, the localized phase exhibits two flavors [labeled (II) and (III)] characterized by different power-law decays of the eigenstates wavefunctions. The color map indicates the power-law exponent s from Eq. (3) across the parameter space.

vanishing robustness interval in the integrable limit $a = 0$ corresponding to the crossover between the Richardson's and Russian doll models [28].

Due to the long-range nature of the model, a vanishing fraction of eigenstates remain plane-wave-like delocalized for $a < 3/2$ for weak enough disorder [19,31–33] and for any disorder for $a < 1$. However, as in correlation-induced localization [19], the typical eigenstate decay, $|\psi(n)|_{\text{typ}}^2 \equiv \overline{\exp(\ln |\psi(n)|^2)} \sim |n - n_0|^{-2s}$ [34], at sites n away from the localization center n_0 , remains intact and power law under TRS breaking up to θ_c . The decay exponent s is dual with respect to the PLBRM critical point, $a = 1$, i.e., $s(2 - a) = s(a) > 1$, in direct analogy with correlation-induced localization [19]. More specifically, typical eigenstates have the following decay profile:

$$|\psi(n)|_{\text{typ}}^2 = \begin{cases} N^{-1} & \text{for } |\theta| > \frac{\pi a}{2}, \quad a < 1, \\ |n - n_0|^{-2(2-a)} & \text{for } |\theta| < \frac{\pi a}{2}, \quad a < 1, \\ |n - n_0|^{-2a} & \text{for } a > 1. \end{cases} \quad (3)$$

We call these three regimes (I) delocalized phase at strong TRS breaking, $|\theta| > \pi a/2$, and in long-range case, $a < 1$; (II) correlation-induced localized phase in the long-range case, $a < 1$, with weak TRS breaking, $|\theta| < \pi a/2$; and (III) Anderson localized phase in the short-range regime $a > 1$. The eigenstate behavior in regimes (I) and (III) is analogous to the one in the long- and short-range phases of the uncorrelated power-law models, like PLBRM or ultrametric matrices [35].

We present numerical evidence for the decay profile of typical eigenstates in Fig. 2. The data show that typical eigenstates follow power-law localization, Eq. (3), with a power-law exponent that is independent of θ in both localized regimes [Figs. 2(a)–2(c)]. In contrast, for $\theta > \pi a/2$ and $a < 1$, all eigenstates are extended [Fig. 2(d)]. In the following, we derive the whole θ - a phase diagram (Fig. 1) from the hopping spectrum of the above model.

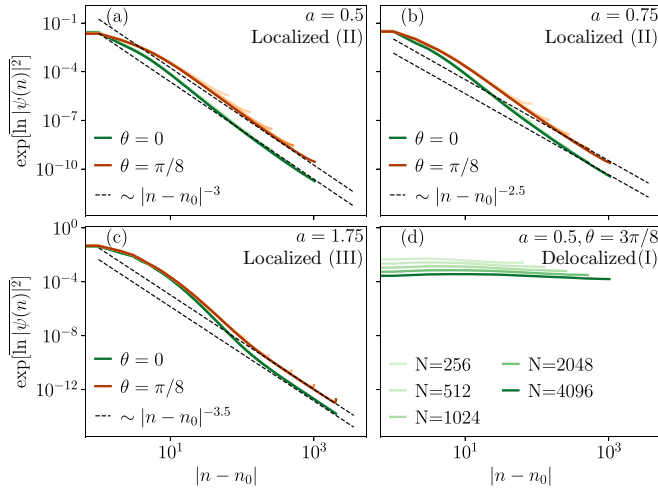


FIG. 2. Spatial decay profile of eigenstates in the bulk of the spectrum. The eigenstates exhibit algebraic localization, as specified in Eq. (3), (a)–(c) which collapses for all system sizes (darker colors represent larger system sizes), and (d) become delocalized for $\theta > \pi a/2$ and $a < 1$. Black dashed lines show the analytical predictions of Eq. (3). The data are averaged over 500 disorder realizations for all system sizes.

Spectrum. Due to the hopping being translation invariant, the momentum basis is another natural basis for the problem [30]. In the Hamiltonian, the hopping and the on-site disorder exchange their roles upon the discrete Fourier transformation,

$$H = \sum_p \tilde{E}_p |p\rangle \langle p| + \sum_{p,q} \tilde{J}_{p-q} |p\rangle \langle q|, \quad (4)$$

where $|p\rangle$ is p -momentum basis state, and $\tilde{E}_p$ and $\tilde{J}_{p-q}$ are defined as

$$\tilde{E}_p = \sum_{n \neq 0} j_n e^{-2\pi i \frac{np}{N}}, \quad \tilde{J}_{p-q} = \frac{1}{N} \sum_{m=0}^{N-1} \epsilon_m e^{-2\pi i \frac{(p-q)m}{N}}, \quad (5)$$

with integer $0 \leq p, q < N$. Here, for simplicity, we have chosen the hopping eigenbasis to be momentum-space one, though any basis with extended eigenstate profile in the real-space basis will not break the reasoning below. The momentum-space hopping terms $\tilde{J}_{p-q}$, which come from the Fourier transforms of the real-space onsite potentials ϵ_n , are i.i.d. Gaussian random variables, $\tilde{J}_{p-q} \sim \mathcal{N}(0, N^{-1/2})$. $\tilde{J}_{p-q}$ are also correlated in the sense that the hopping amplitude between any two momentum modes separated by a fixed momentum $p - q$ is the same.

The diagonal part of the momentum-space Hamiltonian, which is nothing but the spectrum of the real-space hopping Hamiltonian, shows a divergence at small momenta $|p| \ll N$ in the long-ranged regime $a < 1$ (see Fig. 3). For such momenta, $0 < |p| \ll N$, which control the long-range structure of eigenstates, the spectrum reads as

$$\tilde{E}_p = 2\zeta_a + 2\Gamma_{1-a} \sin\left(\frac{\pi a}{2} + \text{sgn}(p)\theta\right) \left(\frac{2\pi|p|}{N}\right)^{a-1}, \quad (6)$$

where ζ_a is the Riemann zeta function at a and Γ_k is the gamma function. Note that the spectrum diverges in both

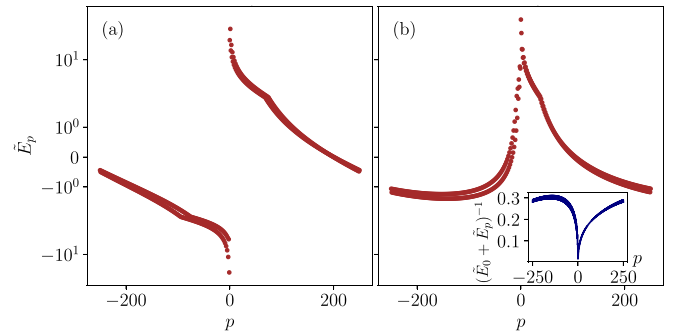


FIG. 3. Spectrum of the momentum-space hopping term $\tilde{E}_p$ for $a = 0.5$. (a) For $\theta > \pi a/2$, the spectrum diverges in both directions, whereas (b) for $\theta < \pi a/2$, it diverges in only one direction. (Inset) Spectrum of the M matrix, which is bounded from both directions.

directions for $|\theta| > \pi a/2$ and diverges only in the positive direction for $|\theta| \leq \pi a/2$ (see Fig. 3). In addition, the spectrum satisfies the relation $\tilde{E}_p(\theta) = -\tilde{E}_p(\theta + \pi)$, which implies that it is sufficient to study the phase diagram in the region $|\theta| \leq \pi/2$.

The high-energy eigenstates, with $|\tilde{E}_p| \gg 1$, remain momentum-space localized (nearly plane waves) under on-site-disorder perturbations because their energy gaps are larger than the momentum-space hopping, i.e., $|\tilde{E}_p - \tilde{E}_q| \gg |\tilde{J}_{p-q}|$ at all $|p|$ or $|q| < p_* \sim N^{1/(4-2a)}$ [32]. In contrast, for typical states, with $p \sim O(N)$, the perturbation series in momentum space diverges: At any order, the level spacing decreases with $|p|$ faster than the hopping amplitude $|\tilde{J}_{p-q}| \sim N^{-1/2}$. At the same time, the real-space perturbation series also diverges, which we discuss next, questioning the origin of a stable algebraic localized phase for $a < 1$. Here, the matrix inversion trick (MxIT), which we explain below, helps to understand how correlations in the hopping cause localization of eigenstates that are in the bulk of the spectrum.

Matrix inversion trick and TRS. The condition for localization is usually understood via Levitov's resonance counting [5,6], which extends Anderson's locator expansion argument to the long-range systems. Localization occurs when the hopping amplitude at distance R , $|j_R| \sim R^{-a}$, is perturbatively small compared to the mean level spacing, $\delta_{\epsilon_R} = \frac{\min_m |\epsilon_n - \epsilon_m|}{|m-n| < R}$,

$$\frac{|j_R|}{\delta_{\epsilon_R}} < R^{-\epsilon}, \quad \epsilon > 0, \quad (7)$$

at all $R > O(1)$. Here, $\delta_{\epsilon_R} \sim W/R^d$ is the typical level spacing in a ball of radius R for a d -dimensional system and W is the disorder strength variance. For $a > d = 1$, this inequality holds and standard perturbative arguments ensure that all eigenstates are localized in all power-law banded models, irrespective of the hopping correlations.

For $a < 1$, however, $|j_R|/\delta_{\epsilon_R} \sim R^{1-a} \rightarrow \infty$, the perturbation series diverges, suggesting delocalization. This reasoning applies to PLBRM models but fails for correlated models: When the series diverges, all perturbative orders contribute, and correlations between matrix elements become essential. The model in Eq. (1) for $\theta = 0$ belongs to this category and

exhibits localization in this regime due to such correlations. Nosov *et al.* [19] resolved this using MxIT, which maps the problem to one with a spectrum bounded on both sides, allowing perturbative expansion to converge even for $a < 1$.

When the spectrum is one-side bounded, in MxIT one uses an offset energy $E_0 \sim O(1)$ below the minimum hopping energy, define an inverted matrix,

$$M \equiv (E_0 + j)^{-1} = \sum_p \frac{1}{E_0 + \tilde{E}_p} |p\rangle \langle p|, \quad (8)$$

and rewrite the effective hopping term in the spectral bulk as $j_{mn}^{\text{eff}} = M_{m-n}(E + E_0 - \epsilon_n)$, keeping the diagonal disorder intact (see, e.g., Eqs. (15) and (16) in Ref. [19]).

This operation flattens the broad energy spectrum of j_{m-n} , making the spectrum of M bounded, as seen in the inset of Fig. 3(b), and allowing for a perturbative treatment in real space. This process enables us to derive an effective hopping, $j_{mn}^{\text{eff}} \sim |m-n|^{-(2-a)}$, the details of which are discussed in Supplemental Material [36]. In this effective model, Levitov's criterion, Eq. (7), is now satisfied, extending the localized phase of all the spectral bulk eigenstates to $a < 1$ with an effective decay of $2s = 2(2-a)$, as given in Eq. (3).

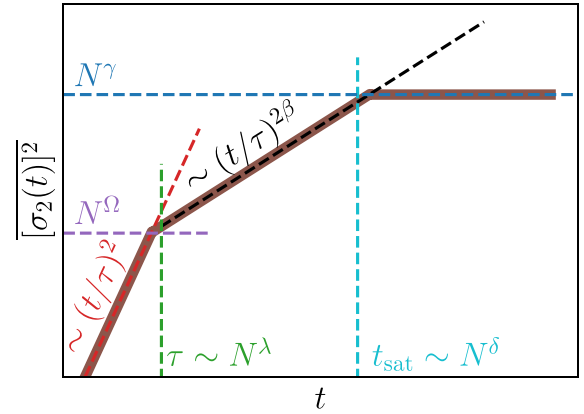
This MxIT construction applies only when the single-particle spectrum is unbounded in one direction; if it is unbounded in both, the method fails and should be generalized as, e.g., in Ref. [28], leading to the ergodically delocalized states. In the latter case, there is no energy $E_0 \sim O(1)$ lying outside the spectrum that can be used to define the inverse matrix M without introducing additional divergences, since in the thermodynamic limit the bulk spectrum becomes continuous and gapless.

In our model, the spectrum $\tilde{E}_p$ is unbounded only from one side for $a < 1$ and $|\theta| < \pi a/2$, whereas it becomes unbounded in both directions for $|\theta| > \pi a/2$, leading to the breakdown of MxIT. This transition can also be understood from the behavior of the group velocities $v_p = d\tilde{E}_p/dp$ (see Fig. 3): For $|\theta| < \pi a/2$, the velocities have opposite signs for positive and negative p , enabling backscattering and promoting localization of the bulk eigenstates. For $|\theta| > \pi a/2$, the velocities share the same sign, leading to the unidirectional motion and, thus, favoring coherent forward scattering and consequently delocalization. Thus, the phase θ induces a transition for $a < 1$ in which the direction(s) of unboundedness directly governs the localization-delocalization physics. In contrast, for $a > 1$, the spectrum is bounded for all θ leading to only a localized phase.

Wave-packet dynamics. We now consider the dynamical phase diagram of the model from the viewpoint of wave-packet dynamics, which shows rich behavior complementing the static phase diagram. To this end, we analyze the dynamics of an initially localized wave packet [27]. Starting from $\langle n|\psi(0)\rangle = \delta_{n,n_0}$, the spreading is quantified by the generalized q th central moment,

$$\sigma_q(t) = \left(\sum_n |n - \langle n \rangle|^q |\psi(n, t)|^2 \right)^{1/q}, \quad (9)$$

where $\psi(n, t) = \langle n|e^{-iHt}|\psi(0)\rangle$ and $\langle n \rangle = \sum_n n |\psi(n, t)|^2$. Owing to the power-law-localized eigenstates, different



	Ω	2β	γ	λ	δ
(I) $ \theta > \frac{\pi a}{2}$	1	1	2	$a - 1$	a
(II) $\theta = 0, a \leq 1$		$\frac{1}{2-a}$	$2 - \frac{1}{4-2a}$		$\frac{1}{2}$
(II) $\theta = 0, a > 1$		1			$a - \frac{1}{4-2a}$
(II) $0 < \theta < \frac{\pi a}{2}$					
(III) $1 < a < \frac{3}{2}$					
(III) $a > \frac{3}{2}$	0	−	0	0	0

FIG. 4. Schematic illustration of the dynamical windows (top) and their associated scaling exponents (bottom). The top panel shows a schematic diagram of the different regimes in the dynamics of the wave-packet width: The red, black, and blue dashed lines represent the ballistic, intermediate (sub)diffusive, and saturation regimes, respectively, while the green and blue vertical dashed lines mark the crossover timescales $\tau \sim N^\lambda$ and $t_{\text{sat}} \sim N^\delta$ between these windows. The table reports the corresponding exponents Ω , 2β , γ , λ , and δ governing the dynamics in different windows.

values of q probe distinct spatial regions of the wave packet: Small q is sensitive to the core, whereas large q emphasizes the algebraic tails. The transition between these regimes is set by a critical index q_* . As shown in Supplemental Material [36],

$$\lim_{t \rightarrow \infty} [\sigma_q(t)]^q \sim N^{q-1/(4-2a)}, \quad (10)$$

which yields $q_* = \frac{1}{4-2a}$. For $q < q_*$, the long-time value of σ_q stays finite in the thermodynamic limit, indicating localization of the core. For $q > q_*$, the moments retain sensitivity to the extended tails and therefore display nontrivial spreading. This analysis is valid for $a < 3/2$ and related to the presence of $p_* \sim N^{1-q_*}$ high-energy delocalized states at $a < 3/2$ and their contribution to the dynamics.

In the following, we focus on the second moment, $\sigma_2(t)$. Since $q_* < 2$ for $a < 3/2$, $\sigma_2(t)$ is dominated by the tails of the wave packet and therefore exhibits dynamics beyond the trivial ballistic front. The disorder-averaged variance, $[\sigma_2(t)]^2$ displays a characteristic three-stage evolution

$$[\sigma_2(t)]^2 \propto \begin{cases} N^\Omega (t/\tau)^2, & 0 < t < \tau \propto N^\lambda, \\ N^\Omega (t/\tau)^{2\beta}, & \tau < t < t_{\text{sat}} \propto N^\delta, \\ N^\gamma, & t > t_{\text{sat}}. \end{cases} \quad (11)$$

We summarize the exponents $(\Omega, 2\beta, \gamma, \lambda, \delta)$ for the different parameter windows in the table of Fig. 4 and present numerical evidence and analytical understanding of the

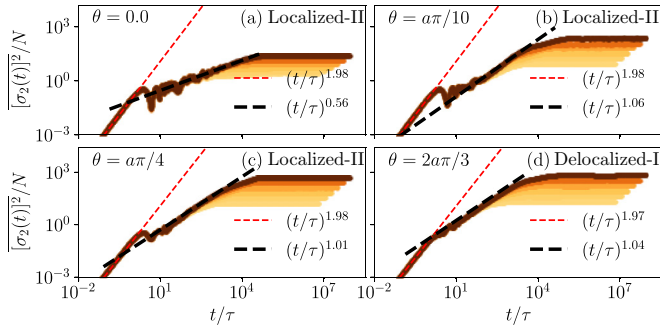


FIG. 5. Dynamics of the width of a wave packet starting from an initially localized state. The panels show the wave-packet dynamics for $a = 0.25$ at different values of θ , with the darkest color corresponding to $N = 8192$: Panel (a) (Regime II with $\theta = 0$) displays an intermediate-time regime with $2\beta = \frac{1}{2-a}|_{a=0.25} \approx 0.58$, panels (b) and (c) (Regime II with $0 < \theta < \pi a/2$) exhibit diffusive dynamics between the ballistic front and saturation, and panel (d) ($\theta = 2\pi a/3$) corresponds to the delocalized phase (I), where the intermediate regime is also diffusive. The data are averaged over 500 disorder realizations for $N \leq 1024$ and 100 otherwise.

ballistic decay in Supplemental Material [36]. We also discuss the origin of the short-time ballistic decay and the scaling of the long-time width, i.e., γ , there.

Remarkably, although the eigenstate properties in the bulk of the spectrum yield three distinct regimes (Fig. 1), the dynamics further splits. For example, in the correlation-induced localized phase (II), the eigenstate spatial decay is insensitive to the choice of θ . In contrast, the dynamics being sensitive to relative phases and correlations between eigenstates displays qualitatively different behavior: In the intermediate time window, second moment is diffusive for $\theta \neq 0$, while it becomes subdiffusive for $\theta = 0$ (see Fig. 5). The (sub)diffusive behavior originates from an extensive yet asymptotically vanishing fraction ($p_*/N \sim N^{-q_*}$) of high-energy delocalized states for $0 < a < 3/2$ [32,37], which dominate the dynamics for lower moments $q < q_*$.

Conclusion and outlook. We summarize the main findings of this work. We investigated localization phenomena in long-range correlated hopping models with broken time-reversal symmetry and established two key results. First, on the static side, we showed that the power-law-localized eigenstates in the bulk of the spectrum remain robust against time-reversal symmetry breaking up to $|\theta| = \pi a/2$, beyond which all states become delocalized. The resulting phase diagram was explained analytically by extending the matrix inversion trick to finite θ , clarifying the stability of the correlation-induced localized phase.

Second, from the dynamical perspective, we analytically determined the saturation properties of the generalized moments $\sigma_q(t)$, demonstrating how different values of q selec-

tively probe the core or the tails of the wave packet. We further showed numerically that the experimentally accessible variance $[\sigma_2(t)]^2$ is strongly sensitive to time-reversal symmetry breaking unlike the static eigenstate spatial decay and exhibits distinct diffusive or subdiffusive behavior depending on θ .

While we have established a clear understanding of both the early-time ballistic regime and the saturation scale of the wave packet, a full analytical description of the (sub)diffusive intermediate-time scaling remains open. This subdiffusive regime also appears in light-localization numerics [38,39]. The above localization of light setting involves dipole-dipole interactions and, thus, also contains long-range correlated hopping, like in the model considered in our paper. Thus, the applications of our methods and investigation of the wave-packet dynamics in such systems offers an exciting opportunity for further exploration. We anticipate that a deeper analytical and experimental investigation of this connection will provide valuable insight into anomalous transport in generic long-range correlated systems.

An independent extension concerns the robustness of correlation-induced localization to non-Hermiticity. Although the present model allows complex hopping phases, the Hamiltonian remains Hermitian; introducing asymmetric non-Hermitian hopping would generalize the Hatano-Nelson model [40] to long-range correlated systems. Whether the algebraically localized phases identified here persist under non-Hermitian spectral flow and associated skin effects remains an open question.

Acknowledgments. I.M.K. acknowledges support by the French National Research Agency (ANR) under the France 2030 program, reference “ANR-25-CFFS-0019” and the TIRIS (Toulouse Initiative for Research’s Impact on Society) Blockbuster Programme, co-financed by ANR under the France 2030 program, the Occitanie Region, and the European Regional Development Fund (ERDF). B.P. acknowledges support from NORDITA through the NORDITA Visiting Ph.D. Fellowship Program, as part of which this work was carried out. B.P. and S.R. acknowledge support from the Department of Atomic Energy, Government of India, under Projects No. RTI4013 and No. RTI4019. S.R. acknowledges support from SERB-DST, Government of India, under Grant No. SRG/2023/000858, and from a Max Planck Partner Group grant between ICTS-TIFR, Bengaluru and MIPPKS, Dresden. J.H.B. received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation program (Grant Agreement No. 101001902), the Knut and Alice Wallenberg Foundation (KAW) via the project Dynamic Quantum Matter (Grant No. 2019.0068), and the Swedish Research Council (VR) through Grant No. 2020-00214.

Data availability. There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] P. W. Anderson, Absence of diffusion in certain random lattices, *Phys. Rev.* **109**, 1492 (1958).
- [2] E. Abrahams, *50 Years of Anderson Localization* (World Scientific, Singapore, 2010).

- [3] E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Scaling theory of localization: Absence of quantum diffusion in two dimensions, *Phys. Rev. Lett.* **42**, 673 (1979).

- [4] S. Aubry and G. André, Analyticity breaking and Anderson localization in incommensurate lattices, *Ann. Israel Phys. Soc.* **3**, 133 (1980).
- [5] L. S. Levitov, Absence of localization of vibrational modes due to dipole-dipole interaction, *Europhys. Lett.* **9**, 83 (1989).
- [6] L. S. Levitov, Delocalization of vibrational modes caused by electric dipole interaction, *Phys. Rev. Lett.* **64**, 547 (1990).
- [7] A. D. Mirlin, Y. V. Fyodorov, F.-M. Dittes, J. Quezada, and T. H. Seligman, Transition from localized to extended eigenstates in the ensemble of power-law random banded matrices, *Phys. Rev. E* **54**, 3221 (1996).
- [8] F. Evers and A. D. Mirlin, Anderson transitions, *Rev. Mod. Phys.* **80**, 1355 (2008).
- [9] P. W. Anderson, The question of classical localization a theory of white paint? *Philos. Mag. B* **52**, 505 (1985).
- [10] M. Segev, Y. Silberberg, and D. N. Christodoulides, Anderson localization of light, *Nat. Photon.* **7**, 197 (2013).
- [11] S. E. Skipetrov and I. M. Sokolov, Absence of Anderson localization of light in a random ensemble of point scatterers, *Phys. Rev. Lett.* **112**, 023905 (2014).
- [12] L. Bellando, A. Gero, E. Akkermans, and R. Kaiser, Cooperative effects and disorder: A scaling analysis of the spectrum of the effective atomic Hamiltonian, *Phys. Rev. A* **90**, 063822 (2014).
- [13] A. Yamilov, S. E. Skipetrov, T. W. Hughes, M. Minkov, Z. Yu, and H. Cao, Anderson localization of electromagnetic waves in three dimensions, *Nat. Phys.* **19**, 1308 (2023).
- [14] A. Yamilov, H. Cao, and S. E. Skipetrov, Anderson transition for light in a three-dimensional random medium, *Phys. Rev. Lett.* **134**, 046302 (2025).
- [15] D. E. Logan and P. G. Wolynes, Anderson localization in topologically disordered systems, *Phys. Rev. B* **31**, 2437 (1985).
- [16] D. E. Logan and P. G. Wolynes, Localizability and dephasing of dipolar excitons in topologically disordered systems, *J. Chem. Phys.* **87**, 7199 (1987).
- [17] A. L. Burin and L. A. Maksimov, Localization and delocalization of particles in disordered lattice with tunneling amplitude with R^{-3} decay, *JETP Lett.* **50**, 338 (1989).
- [18] X. Deng, V. E. Kravtsov, G. V. Shlyapnikov, and L. Santos, Duality in power-law localization in disordered one-dimensional systems, *Phys. Rev. Lett.* **120**, 110602 (2018).
- [19] P. A. Nosov, I. M. Khaymovich, and V. E. Kravtsov, Correlation-induced localization, *Phys. Rev. B* **99**, 104203 (2019).
- [20] X. Deng, S. Ray, S. Sinha, G. V. Shlyapnikov, and L. Santos, One-dimensional quasicrystals with power-law hopping, *Phys. Rev. Lett.* **123**, 025301 (2019).
- [21] A. G. Kutlin and I. M. Khaymovich, Renormalization to localization without a small parameter, *SciPost Phys.* **8**, 049 (2020).
- [22] P. A. Nosov and I. M. Khaymovich, Robustness of delocalization to the inclusion of soft constraints in long-range random models, *Phys. Rev. B* **99**, 224208 (2019).
- [23] A. G. Kutlin and I. M. Khaymovich, Emergent fractal phase in energy stratified random models, *SciPost Phys.* **11**, 101 (2021).
- [24] J. T. Cantin, T. Xu, and R. V. Krems, Effect of the anisotropy of long-range hopping on localization in three-dimensional lattices, *Phys. Rev. B* **98**, 014204 (2018).
- [25] X. Deng, A. L. Burin, and I. M. Khaymovich, Anisotropy-mediated reentrant localization, *SciPost Phys.* **13**, 116 (2022).
- [26] L. F. Santos, F. Borgonovi, and G. L. Celardo, Cooperative shielding in many-body systems with long-range interaction, *Phys. Rev. Lett.* **116**, 250402 (2016).
- [27] X. Deng, I. M. Khaymovich, and A. L. Burin, Superdiffusion in a random two-dimensional system with time-reversal symmetry and long-range hopping, *Phys. Rev. B* **109**, 174208 (2024).
- [28] V. R. Motamarri, A. S. Gorsky, and I. M. Khaymovich, Localization and fractality in disordered Russian doll model, *SciPost Phys.* **13**, 117 (2022).
- [29] Our results do not rely on translational invariance, but rather on the fully correlated nature of the hopping. A fully correlated, nontranslationally invariant example is given by $J_{nm} = J/|r_n - r_m|^a$ for randomly positioned sites $\{r_n\}$, for which the presented numerical and analytical arguments remain valid [18,21].
- [30] Unlike full correlations, the translation invariance of the hopping is not crucial for the consideration. We focus on the translation-invariant case here to avoid any complications with the diagonalization of the hopping matrix.
- [31] A. Rodríguez, V. A. Malyshev, and F. Dominguez-Adame, Quantum diffusion and lack of universal one-parameter scaling in one-dimensional disordered lattices with long-range coupling, *J. Phys. A: Math. Gen.* **33**, L161 (2000).
- [32] A. Rodríguez, V. A. Malyshev, G. Sierra, M. A. Martin-Delgado, J. Rodríguez-Laguna, and F. Dominguez-Adame, Anderson transition in low-dimensional disordered systems driven by long-range nonrandom hopping, *Phys. Rev. Lett.* **90**, 027404 (2003).
- [33] D. B. Balagurov, V. A. Malyshev, and F. Dominguez-Adame, Phase coherence in tight-binding models with nonrandom long-range hopping, *Phys. Rev. B* **69**, 104204 (2004).
- [34] Here and further, the overline denotes an average over disordered Hamiltonians and eigenstates.
- [35] Y. V. Fyodorov, A. Ossipov, and A. Rodríguez, The Anderson localization transition and eigenfunction multifractality in an ensemble of ultrametric random matrices, *J. Stat. Mech.* (2009) L12001.
- [36] See Supplemental Material at <https://link.aps.org/supplemental/10.1103/34h6-sn4p> for a discussion of the matrix inversion trick for broken time-reversal symmetry, an analytical derivation of the scaling of the saturation width and short-time ballistic behavior of the wave packet, and numerical evidence for the different dynamical exponents across several values of θ and a .
- [37] F. A. B. F. de Moura, A. V. Malyshev, M. L. Lyra, V. A. Malyshev, and F. Dominguez-Adame, Localization properties of a one-dimensional tight-binding model with nonrandom long-range intersite interactions, *Phys. Rev. B* **71**, 174203 (2005).
- [38] F. Sgrignuoli and L. Dal Negro, Subdiffusive light transport in three-dimensional subrandom arrays, *Phys. Rev. B* **101**, 214204 (2020).
- [39] F. Sgrignuoli, S. Torquato, and L. Dal Negro, Subdiffusive wave transport and weak localization transition in three-dimensional stealthy hyperuniform disordered systems, *Phys. Rev. B* **105**, 064204 (2022).
- [40] N. Hatano and D. R. Nelson, Localization transitions in non-Hermitian quantum mechanics, *Phys. Rev. Lett.* **77**, 570 (1996).