

Compact cavity-dressed Hamiltonian framework at arbitrarily strong light-matter coupling

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We present a nonperturbative Hamiltonian mapping method for quantum systems strongly coupled to a quantized field mode (cavity), yielding compact closed-form representations of hybrid light-matter systems, allowing efficient studies of equilibrium and dynamical properties. The mapping builds on an entangling transformation of photonic and atomic degrees of freedom. By truncating the resulting cavity-dressed Hamiltonian (CDH) to successively larger excitation sectors, we construct a series of compact models that converge to the exact limit, outperforming conventional approaches even in the challenging resonant and ultrastrong light-matter regime. We benchmark the CDH framework on the quantum Rabi model, obtaining accurate spectra and thermal observables across weak and strong coupling regimes. We further apply the method to the *open* Rabi model, highlighting its computational advantages for nonunitary dynamics. As a nontrivial application, we analyze the Dicke-Heisenberg lattice model, determine its phase diagram under resonant and strong light-matter coupling, and reveal cavity-mediated spin correlations analytically and numerically, achieving substantial computational savings over brute-force simulations. The closed form and compactness of the CDH provide both physical insight and improved computational efficiency in studies of strongly coupled hybrid light-matter systems.

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Introduction. Under strong light-matter coupling, photons and material excitations hybridize into quasiparticles known as polaritons, with fundamentally transformed properties and behavior than in the bare material [1]. Recent advances in the ultrastrong coupling (USC) regime enabled control over collective phenomena in quantum materials [2], realization of nonlinear optical effects and protection of quantum information [3], modification of chemical reactivity [4–6], and enhancement of transport processes in disordered media [7]. Experiments and supporting calculations have exemplified, e.g., control of chemical reactivity through strong light-matter coupling to vibrational [8–20], electronic excitations [21–30], and engineered qubits [31–36], with the strongest effect showing near resonance between molecular and cavity modes, a regime that remains theoretically challenging [19,37–39]. Beyond molecular systems, cavities have emerged as powerful platforms for materials engineering, where a tailored electromagnetic vacuum induces nonlinear optical properties [40] and stabilizes quantum phases [41–43].

In the ultrastrong coupling regime, light-matter interaction energy becomes comparable to bare matter and light excitation frequencies, while in the *deep-strong coupling* regime, it

exceeds them [1]. Theoretically and computationally, describing the USC regime remains a challenge due to the breakdown of conventional perturbative treatments [44]. Approaches built on, e.g., dressing transformations [38,45–50], variational ansätze [51–53], Markovian embedding [38,54–57], asymptotic decoupling [58], perturbation theory on mean-field reference wavefunctions [59,60], and generalized rotating-wave approximations [45,61,62] have enabled significant progress in addressing the challenges of the USC regime. However, theoretical results are limited to specific models (e.g., the Rabi model [45,63,64]), while computations offer limited insight into the nature of the USC regime, constraining the rational design of cavity-modified properties and dynamics.

In this Letter, we present a general, compact, and analytically tractable framework for deriving cavity-dressed Hamiltonians (CDHs) describing quantum systems interacting with collections of bosonic modes at arbitrary coupling strengths, covering the weak, ultrastrong, and deep-strong regimes. The approach is accurate, systematically convergent, and broadly applicable, encompassing both single-mode cavities and bosonic thermal environments [65], and remains accurate even in the resonant and ultrastrong light-matter coupling regime. The main advantages of our method are (1) its analytical formulation, which provides physical insight and thus pathways to control light-induced matter processes; (2) its exactness in the deep-strong coupling limit for coupled spin-boson systems; and (3) its straightforward numerical implementability.

We apply the CDH approach onto the quantum Rabi model [64,77] and the Dicke-Heisenberg model [71–75,78–87], archetypal light-matter systems; it can be feasibly applied

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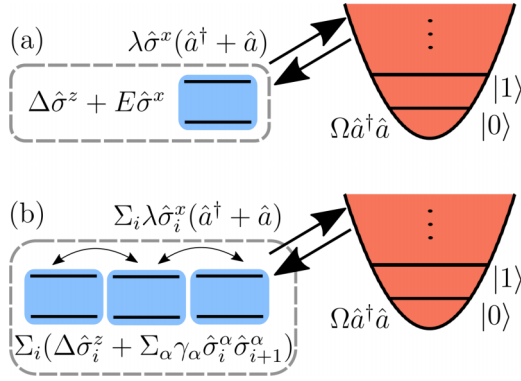


FIG. 1. (a) The quantum Rabi model with a spin coupled to a single boson, and (b) the Dicke-Heisenberg model.

to other cavity-coupled molecular and material Hamiltonians. The Rabi model, which describes a two-level system coupled to a single bosonic mode, has attracted renewed attention with the advent of the ultrastrong and deep-strong coupling regimes in circuit quantum electrodynamics [31] and in emerging quantum simulation platforms [88]. Its integrability [63] makes it an ideal benchmark for testing theoretical methods and resolving basic concepts in quantum optics [89]. The Dicke model extends the Rabi framework to a collection of spins coupled to a common bosonic mode [90]. Moreover, in the Dicke-Heisenberg model, a collection of spins in a chain configuration interact with their neighbors, as well as with a cavity mode, exhibiting cavity-induced collective behavior [71].

Method. We consider a matter system, governed by the Hamiltonian $\hat{H}_S$, coupled to multiple bosonic modes, labeled n , via different system operators $\hat{S}_n$,

$$\hat{H} = \hat{H}_S + \sum_n (\lambda_n \hat{S}_n (\hat{a}_n^\dagger + \hat{a}_n) + \Omega_n \hat{a}_n^\dagger \hat{a}_n). \quad (1)$$

Each bosonic mode (annihilation operator $\hat{a}_n$) is characterized by its frequency Ω_n , and it couples to the system with strength λ_n . Figure 1 depicts examples with a single lossless cavity mode. In the Supplemental Material [65], we generalize this to include dissipation effects from secondary baths.

The CDH mapping consists of a unitary light-matter entangling transformation, $\hat{H} \rightarrow \hat{U}_P \hat{H} \hat{U}_P^\dagger$, with $\hat{U}_P = \exp[\sum_n \frac{\lambda_n}{\Omega_n} \hat{S}_n (\hat{a}_n^\dagger - \hat{a}_n)]$ the familiar polaron transform, generalized to the multimode case [54]. The total Hilbert space is then decomposed into blocks defined by the occupation number of modes, $(\hat{H})_{sr}^{\text{cdh}} = \langle s | \hat{U}_P \hat{H} \hat{U}_P^\dagger | r \rangle$. For example, $|s\rangle = |00\rangle$ corresponds to two bosonic modes in their ground states. Each block is an operator on the system Hilbert space. To evaluate these blocks, we use a momentum representation for each mode and turn the expectation value calculation into an integral over momenta. This approach is particularly powerful when coupling operators $\hat{S}_n$ do not commute [54]. We truncate each mode to its M lowest-energy states in the basis of particle numbers, after the polaron transform. The result is a cavity-dressed Hamiltonian

$$\hat{H}_{(M)}^{\text{cdh}} = (\hat{H}_S)_{(M)}^{\text{cdh}} + \sum_n \left[\Omega_n \hat{N}_{n,(M)} - \frac{\lambda_n^2}{\Omega_n} (\hat{S}_n^2)_{(M)}^{\text{cdh}} \right], \quad (2)$$

where $(\hat{O})_{(M)}^{\text{cdh}} = \sum_{s,r} |s\rangle \langle r| \otimes (\hat{O})_{sr}^{\text{cdh}}$, with (M) indicating the degree of truncation. For details, see the Supplemental Material [65]. $\hat{N}$ is the number operator. To illustrate the applicability of the CDH, in what follows we focus on models with a single-cavity mode.

To compute expectation values in the CDH framework, system observables in the laboratory frame are written as [91] $\langle \hat{O} \rangle_\rho = \text{Tr}(\hat{O}\rho) = \text{Tr}(\hat{U}_P \hat{O} \hat{U}_P^\dagger \rho_P)$, with ρ_P the density matrix in the polaron frame. We compute ρ_P assuming a canonical state, taking the CDH as the mean-force Hamiltonian [65,92]. Rotated operators must also be truncated: If $\hat{O}$ does not commute with $\hat{U}_P$, we rotate it including M_P levels per mode and then truncate to dimension M . Setting $M_P = M$ offers the greatest computational efficiency. In certain cases [65], using $M_P > M$ to perform the rotation before truncating back to dimension M was necessary, but never exceeding the dimension N needed for convergence in the conventional-bare approach. Alternatively, for a single-cavity mode, or multiple modes with commuting coupling operators, matrix elements of $\hat{U}_P \hat{O} \hat{U}_P^\dagger$ are found exactly by performing the spectral decomposition of $\hat{S}_n$ [65].

Quantum Rabi model. We illustrate the CDH representation on the quantum Rabi model. After analytically deriving the degree- M CDH, we compute the model's energy spectrum, thermal equilibrium, and dissipative behavior. Comparing these results with exact analytical and numerical calculations, we show the superior convergence of the CDH framework relative to the bare representation, as well as the physical insights provided by its block structure. The model is described by the following Hamiltonian: $\hat{H} = \Delta \hat{\sigma}^z + \Omega \hat{a}^\dagger \hat{a} + \lambda \hat{\sigma}^x (\hat{a}^\dagger + \hat{a})$, with 2Δ the spin splitting, Ω the cavity frequency, and λ the spin-cavity coupling strength. The model is integrable; the analytical solution for its eigenenergies is given as the roots of an infinite sum of recursively defined real functions [63]. When simulations are performed in the bare representation, we denote the dimension of the cavity Hilbert space by N ; for CDH simulations, we use M .

The CDH Hamiltonian for the Rabi model is derived in the Supplemental Material [65]. For $M = 1$, we get $\hat{H}_{(1)}^{\text{cdh}} = \tilde{\Delta} \hat{\sigma}^z - \Omega \epsilon^2 \hat{I}$, which corresponds, in the polaron frame, to a spin coupled to the cavity ground state. Here, $\epsilon = \lambda/\Omega$ and $\tilde{\Delta} = \Delta e^{-2\epsilon^2}$ is the suppressed spin splitting. For $M = 2$, the cavity is truncated to two levels, and the CDH takes the form

$$\hat{H}_{(2)}^{\text{cdh}} = \begin{pmatrix} \tilde{\Delta} \hat{\sigma}^z - \Omega \epsilon^2 \hat{I} & 2\tilde{\Delta} \epsilon i \hat{\sigma}^y \\ -2\tilde{\Delta} \epsilon i \hat{\sigma}^y & \tilde{\Delta} (1 - 4\epsilon^2) \hat{\sigma}^z + \Omega (1 - \epsilon^2) \hat{I} \end{pmatrix}. \quad (3)$$

The eigenenergies of $\hat{H}_{(2)}^{\text{cdh}}$ are

$$E_{\pm, \pm'} = \frac{1}{2} \Omega - \epsilon^2 (\Omega \pm 2\tilde{\Delta}) \pm \sqrt{\tilde{\Delta}^2 (4\epsilon^4 + 1) \pm \tilde{\Delta} \Omega (1 - 2\epsilon^2) + \Omega^2 / 4}. \quad (4)$$

The analytical form of the CDH provides insight into the resulting hybrid states. In the weak coupling limit, $\epsilon \rightarrow 0$, the spectrum reduces to that of a decoupled spin and a truncated free cavity. In the opposite, deep-strong limit and for any truncation degree M , the CDH retains only the terms $\Omega \hat{N}_{(M)} - \Omega \epsilon^2 \hat{I}_{(M)}$ on diagonal blocks, which correspond to the cavity excitation ladder and a global energy shift induced by the spin-cavity coupling. This universal behavior,

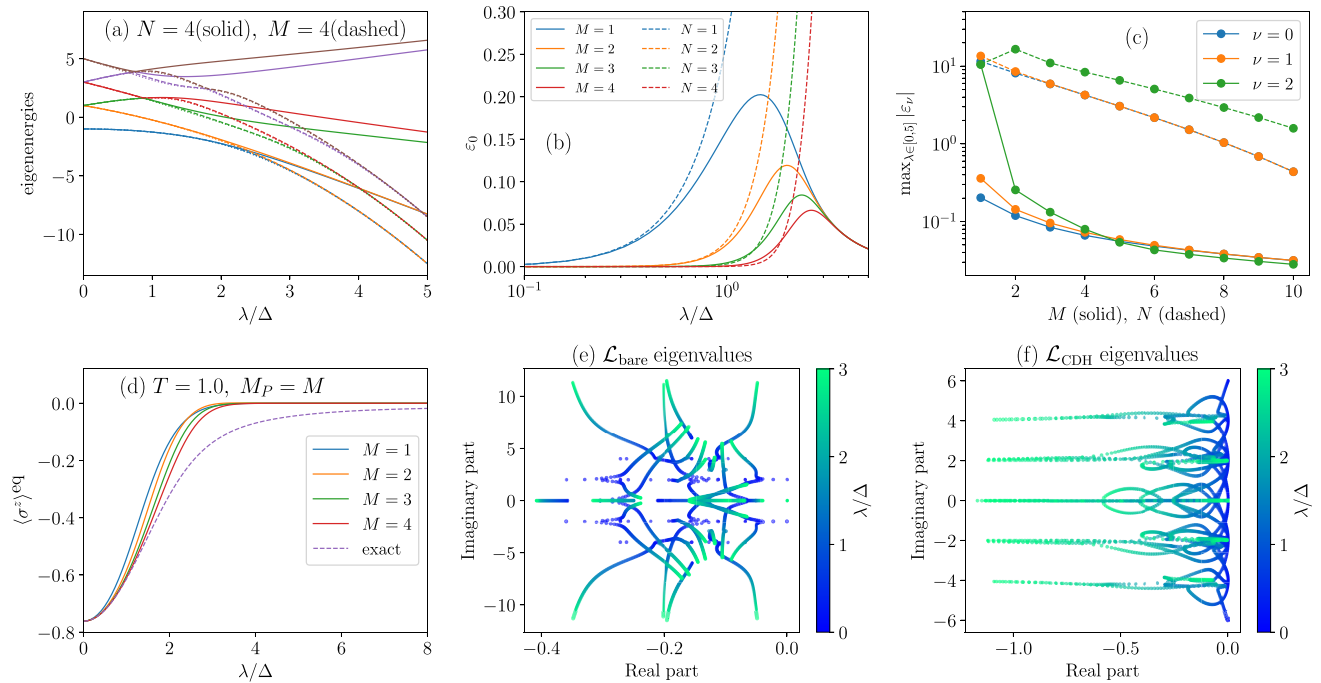


FIG. 2. (a) Eigenenergies of the quantum Rabi model against the coupling strength λ using the bare representation with $N = 4$ (solid) and a CDH of degree $M = 4$ (dashed). Exact eigenvalues [63] (dotted) overlap with CDH results. (b) ε_0 defined as the difference between the zero eigenenergy predicted by the CDH with $M \in [1, 4]$ or the bare model with $N \in [1, 4]$, and the exact result. (c) Maximal error ε_ν over $\lambda \in [0, 5]$ with respect to M and N for the first three eigenenergies, $\nu = 0, 1, 2$. (d) Equilibrium magnetization $\langle \hat{\sigma}^z \rangle_{\text{eq}}$ at $T = 1$ using the CDH with $M \in [1, 4]$, compared to brute-force converging solution (dashed). Parameters are $\Omega = 2$, $\Delta = 1$. (e), (f) Complex eigenvalues of the Redfield generator in the bare ($N = 3$) and the CDH ($M = 3$) representations. For the dissipator, we assume an Ohmic function with $\gamma = 0.01$ as a dimensionless cavity-bath coupling coefficient, high-frequency cutoff, and $T = 1$. Colors correspond to different values of λ . At this truncation, the generator has $(2 \times 3)^2$ eigenvalues for each λ .

independent of M , explains why even for $M = 1$ the CDH yields accurate predictions for the Rabi model (as well as the Dicke-Heisenberg model below) in the $\lambda \rightarrow \infty$ limit [38]. Between the ultraweak and deep-strong coupling limits, the off-diagonal terms in Eq. (4), as well as in higher-order CDHs [65], reveal that transitions between cavity levels $l \rightarrow l \pm p$ are coupled to system transitions with an effective strength $\frac{2\Delta}{\Omega} \times (\frac{\lambda}{\Omega})^{p-1} \lambda e^{-2\lambda^2/\Omega^2}$. It is thus clear that the resonance condition ($2\Delta = \Omega$) is more favorable for applications than the off-resonant case ($2\Delta < \Omega$), as it effectively enhances light-matter interactions.

In Fig. 2(a), we analyze the first six eigenenergies of the quantum Rabi model as a function of the coupling strength in the resonant case. We present exact results (dotted, overlapped with dashed) [63] and compare them with numerical simulations using the CDH of degree $M = 4$ (dashed) and bare-conventional simulations with $N = 4$ cavity levels (full). In bare simulations with $N = 4$ cavity levels, the ground-state energy diverges beyond $\lambda/\Delta \approx 2$ [see Fig. 2(b)], with excited-state energies exhibiting even larger deviations [Fig. 2(c)]. In contrast, the CDH accurately reconstructs the first six energy levels in all coupling strengths, maintaining a *bounded error* [Fig. 2(b)] in weak, ultrastrong, and deep coupling limits; the residual error in the CDH arises primarily in the coupling regime around $\lambda/\Delta \in [1, 5]$. This represents a substantial reduction in computational cost for numerical simulations. In Fig. 2(c), we study the maximum of the absolute error ε_ν

over $\lambda \in [0, 5]$ for the first three energy levels using the CDH method (M , solid) and the bare truncation (N , dashed). With CDH mapping, we observe a striking improvement in error scaling with Hilbert-space dimension, particularly for small M . Notably, the CDH mapping exhibits comparable error scaling for excited states, whereas the bare-basis calculation becomes progressively less accurate, beginning with the second excited-state energy.

Having established the CDH representation's convergence of eigenenergies across all coupling strengths, we use it to compute thermal equilibrium observables. In Fig. 2(d), we present the equilibrium magnetization as a function of λ , $\langle \hat{\sigma}^z \rangle_{\text{eq}} = \frac{1}{Z} \text{Tr}[\hat{\sigma}^z e^{-\hat{H}_{(M)}^{\text{cdh}}/T}]$; $Z = \text{Tr}[e^{-\hat{H}_{(M)}^{\text{cdh}}/T}]$ is the canonical partition function, $k_B = 1$. CDH simulations are compared to results from the exact solution [63]. It is evident that the series of CDHs yields progressively convergent observables as we increase M , not only in the weak and ultrastrong coupling limits, but also in the intermediate regime. In the Supplemental Material [65], we examine the average magnetization at both zero temperature (ground state) and finite temperature (thermal state). For ground-state observables, $M_P > M$ was required to achieve convergence. We also study there the cavity occupation number, demonstrating the advantage of CDH in convergence over the bare approach, up to the ultrastrong coupling regime.

We now use the CDH formalism to study dissipative open-system dynamics, described by a quantum master equation,

$\dot{\rho} = \mathcal{L}\rho$, where here we choose the dynamics generator $\mathcal{L}$ to be of the Redfield form assuming weak dissipation [68]. In our model, the cavity is coupled to a dissipative thermal bath; the microscopic description is given in the Supplemental Material [65]. We characterize the open-system dynamics and its relaxation toward the steady state through the spectrum of the Liouvillian $\mathcal{L}$. In the doubled Hilbert-space representation, the Liouvillian eigenvalue equation reads $\hat{\mathcal{L}}|\rho_n\rangle\rangle = (-\Gamma_n^R + i\Gamma_n^I)|\rho_n\rangle\rangle$, where the real and imaginary parts of the eigenvalues, Γ_n^R and Γ_n^I , correspond, respectively, to the decay rate and oscillation frequency of the eigenmode $|\rho_n\rangle\rangle$. The index n runs over the full Liouville space, $n \in \{1, \dots, \dim(\hat{\mathcal{H}})^2\}$.

Figures 2(e) and 2(f) illustrate the spectrum of the Redfield generator for the open Rabi model. In the bare representation, the spectrum fails to converge for $N = 3$. In contrast, the CDH spectrum ($M = 3$) is convergent and it exhibits a strikingly organized structure: The eigenvalues separate into distinct branches, vertically spaced by Ω , and shifted horizontally at large λ as the coupling is increased. These branches correspond to different oscillatory modes in the dissipative dynamics. Within each branch, as we increase λ , at least one eigenvalue remains close to the $\Gamma^R = 0$ axis (slow dissipative decay), while another diverges toward $-\infty$ (fast decay). This reveals that a separation of timescales occurs at strong coupling for each frequency component. In the Supplemental Material [65], we illustrate these features in the magnetization dynamics of the open Rabi model. The CDH framework thus allows efficient-accurate calculations of *open* cavity-coupled systems.

Dicke-Heisenberg model. This model includes a Heisenberg spin chain coupled to a single-mode cavity [80,81]. We show that the CDH framework provides analytical insights and efficient simulations into the magnetic phases of the spin system in the Dicke-Heisenberg model. For two sites, the model can be analytically solved by mapping it to the Rabi model [93]. Here, we consider an L -long chain,

$$\hat{H} = \Delta \sum_{i=1}^L \hat{\sigma}_i^z + \Omega \hat{a}^\dagger \hat{a} + \sum_{i=1}^L \left(\frac{\lambda}{\sqrt{L}} \hat{\sigma}_i^x (\hat{a}^\dagger + \hat{a}) - \sum_{\alpha=x,y,z} \gamma_\alpha \hat{\sigma}_i^\alpha \hat{\sigma}_{i+1}^\alpha \right). \quad (5)$$

2Δ is the spin splitting, Ω is the cavity frequency, λ is the interaction strength, and γ_α are the Heisenberg interaction parameters. This model was studied numerically in the context of superradiant phenomena [82–84] in many-body systems [71,85] and for the design of quantum batteries [86]. Here, we focus on uncovering features of the spin system in weak to the deep-strong limit *using a small M* . To build the CDH, we transform this Hamiltonian with the unitary $\hat{U} = \exp[\frac{\lambda}{\Omega\sqrt{L}} \sum_{i=1}^L \hat{\sigma}_i^x (\hat{a}^\dagger - \hat{a})]$ and truncate it to include M excitation sectors [65]:

$$\hat{H}_{(M)}^{\text{cdh}} = \Delta \sum_{i=1}^L (\hat{\sigma}_i^z)_{(M)}^{\text{cdh}} - \frac{\lambda^2}{\Omega L} \sum_{i,j=1}^L (\hat{\sigma}_i^x \hat{\sigma}_j^x)_{(M)}^{\text{cdh}} + \Omega \hat{N}_{(M)} - \sum_{i=1}^L \sum_{\alpha=x,y,z} \gamma_\alpha (\hat{\sigma}_i^\alpha \hat{\sigma}_{i+1}^\alpha)_{(M)}^{\text{cdh}}. \quad (6)$$

The mapped one-body terms are identical to those in the Rabi model. The second term in the first line describes a cavity-mediated all-to-all spin coupling in the x orientation. The second line includes Heisenberg two-body terms, modified due to the cavity coupling. In this group, the $\hat{\sigma}_i^x \hat{\sigma}_j^x$ terms remain unchanged under the entangling transformation, but the other two interaction terms transform in a nontrivial way: On diagonal blocks (corresponding to a fixed cavity occupation number M), they undergo mixing,

$$\begin{aligned} \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y &\rightarrow f_M(\epsilon) \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + g_M(\epsilon) \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z, \\ \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z &\rightarrow g_M(\epsilon) \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + f_M(\epsilon) \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z, \end{aligned} \quad (7)$$

where $\epsilon = \lambda/(\Omega\sqrt{L})$ and f_M, g_M are dressing functions of the two-body operators in the M th diagonal block [65]. Intriguingly, in off-diagonal blocks, corresponding to transitions in the cavity, terms proportional to $(\gamma_y - \gamma_z)(\hat{\sigma}_i^y \hat{\sigma}_{i+1}^z + \hat{\sigma}_i^z \hat{\sigma}_{i+1}^y)$ appear. These terms *disappear* for the isotropic Heisenberg interaction. In the Supplemental Material [65], we provide closed-form expressions for the $M = 3$ CDH. We note that in the deep-strong coupling limit, the one-body terms are exponentially suppressed, the Heisenberg interaction energies saturate to a constant, and the all-to-all interaction in the x orientation scales as $\frac{\lambda^2}{\Omega}$. To understand the interplay between the different terms in Eq. (6), we examine the phase diagram of the Dicke-Heisenberg model using two order parameters: ground-state average magnetization, $M_z = \frac{1}{L} \sum_i \langle \hat{\sigma}_i^z \rangle_{\text{GS}}$ and ground-state entanglement entropy $S = -\text{Tr}(\rho_A \ln \rho_A)$, where the subsystem A consists of the first $L/2$ spins (assuming an even number of spins).

In Figs. 3(a) and 3(b), we present the ground-state phase diagram of the Dicke-XX Heisenberg model based on the magnetization; a similar crossover is captured with the entanglement entropy [65]. In the bare representation, at weak cavity coupling, we observe an abrupt transition from a ferromagnetic to a paramagnetic phase as we reduce Δ and reach $\Delta/\Omega \approx 0.25$, corresponding to $\Delta \approx \gamma_x + \gamma_y$. This transition is already accurately captured by the CDH of order $M = 1$ [65]. Physically, when $\Delta > \gamma_x + \gamma_y$, the single-spin term dominates, resulting in a fully polarized ground state with $M_z = -1$ and vanishing entanglement entropy. Conversely, when $\Delta < \gamma_x + \gamma_y$, the Heisenberg interaction dominates, entangling spins into singletlike pairs and driving M_z to 0 with $S > 0$. In contrast, the phase diagram of the isotropic Dicke-XXX Heisenberg model does not show such a transition at weak coupling [65], as the $\text{SU}(2)$ symmetry of the spin interaction prevents the spins from aligning on the x - y plane.

Turning on the cavity coupling, we observe a continuous suppression of magnetization as the spin splitting Δ is reduced or the cavity coupling λ increases. This behavior can be understood in terms of a cavity-mediated all-to-all interaction between spins along the x direction. At strong coupling, this interaction dominates, generating spin entanglement and reducing magnetization along the z axis. In Fig. 3(c), we plot the structure factors $S_\alpha = \sum_{i,j} \langle \hat{\sigma}_i^\alpha \hat{\sigma}_j^\alpha \rangle_{\text{GS}} / L^2$, where $\alpha \in \{x, y, z\}$, against the renormalized coupling λ/Ω . We observe a monotonic increase of S_x with λ , driven by cavity-mediated all-to-all interactions, suppression of single-spin splitting terms, and renormalization of the XX Heisenberg interaction. Correspondingly, S_z exhibits the opposite trend.

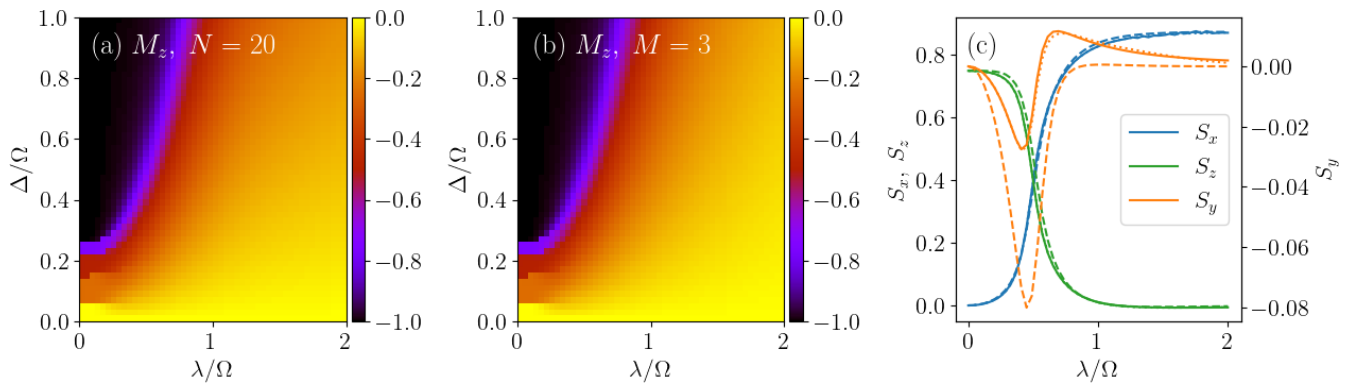


FIG. 3. Ground-state phase diagrams of the Dicke-XX Heisenberg model with periodic boundary conditions (a) in the bare representation, with $N = 20$, and (b) using the $M = 3$ CDH. The order parameter is the average magnetization, M_z . (c) Structure factors S_α for a resonance situation, $2\Delta = \Omega$. We compare numerically converged simulations at $N = 20$ (solid; this choice ensured that the maximal absolute error in magnetization remained below 0.01) to the CDH with $M = 1$ (dashed) and $M = 3$ (dotted), with $M_p = M$. Fixed parameters are $L = 8$, $\gamma_x = \gamma_y = \Omega/8$, $\gamma_z = 0$, $\Omega = 2$, and we vary λ and Δ .

The behavior of S_y , on the other hand, is nonmonotonic, reflecting the mixing of spin terms, described in Eq. (7). We note that the structure factor readily converges to the numerically converged result (full) as we increase M . In fact, for $S_{x,z}$, results are quantitatively correct for $M = 1$, while for S_y , a CDH of degree $M = 3$ is quantitatively correct.

Discussion. We introduced a nonperturbative and computationally efficient mapped Hamiltonian representation for simulating the structure, equilibrium behavior, and relaxation dynamics of cavity-coupled quantum systems. The approach favorably converges in both resonant and off-resonant regimes, and at all coupling strengths. The technique combines a polaronlike transformation with a truncation of the cavity to M levels. The resulting *closed-form* cavity-dressed Hamiltonians provide insights into light-matter hybrid effects. We tested the framework on the quantum Rabi impurity model and the Dicke-Heisenberg lattice model. The CDH captures essential features of the phase diagram of the Dicke-Heisenberg model

with a low-degree truncation, offering a substantial computational advantage over conventional tools. The method can be applied to multimode cavities, including cases where modes couple to the system through distinct, noncommuting system operators [54,55,65]. The CDH can be integrated with other open-system formalisms to efficiently describe relaxation dynamics of strongly coupled hybrid light-matter systems, from the weak to the ultrastrong dissipative regimes.

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Data availability. All data presented in this Letter are available in the referenced GitHub repository [94].

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